HW 4a

Problem 1 (2 = 1 + 2 points) Show that if k_1 and k_2 are mercer kernels then

$$k(\boldsymbol{x}, \boldsymbol{x}^{'}) = k_1(\boldsymbol{x}, \boldsymbol{x}^{'}) + k_2(\boldsymbol{x}, \boldsymbol{x}^{'})$$

is also mercer a kernel where $\boldsymbol{x}, \boldsymbol{x}^{'} \in R^d$.
Prove k is symmetric and and associated gram matrix is positive definite.

Problem 2.(2 points.) Let say $\mathcal{X}$ is a d dimensional input space. Given any $x, x^{'} \in \mathcal{X}$, RBF kernel is $k(x, x^{'}) = \exp(-\frac{\|x-x^{'}\|}{2\sigma^2})$. Prove that RBF kernel maps the d dimensional input space $\mathcal{X}$ into the surface of an unit hypersphere(Infact it is infinite dimentional hypersurface but no need to show infinite part). Note for any point z on unit hyper sphere $\|z\|_2^2 = 1$.

Problem 3.(2 points.)
Let say we have a dataset $\mathcal{D} = \{x_i, y_i\}_i^N$ where $x \in R^d$ and $y_i \in R$. In linear regression $y_i = w^T x_i + \epsilon_i$ estimate of w is $\hat{w} = (X^T X)^{-1} X^T Y)$. Hence residual vector $\mathcal{E}$ against fitted line is $\mathcal{E} = Y - X\hat{w}$. Show that residual vector is orthogonal to columns of X. Note X is a matrix containing observations along rows and Y is the response vector containing all the response y_i along columns.

Problem 4.(1 points.) From data mining book Chapter 5: Q3