Neural Networks Homework2

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1 Forward Pass

$$z_1^{(1)} = \sum_{i=1}^{n} W_{z_1,i} \cdot X_{z_1,i} = 1 * 2 + 0.5 * 4 - 0.5 * 4 = 2$$

$$a_1^{(1)} = sigmoid(z_1^{(1)}) = \frac{1}{1 + e^{-z_1^{(1)}}} = 0.881$$

$$z_2^{(1)} = \sum_{i=1}^{n} W_{z_2,i} \cdot X_{z_2,i} = 1 * (-1) + 0.5 * 1 - 0.5 * (-2) = 0.5$$

$$a_2^{(1)} = sigmoid(z_2^{(1)}) = \frac{1}{1 + e^{-z_1^{(2)}}} = 0.622$$

$$p(x) = e^{z_1^{(2)}} = e^{W_{z_1^{(2)},1} \cdot a_1^{(1)} + W_{z_1^{(2)},2} \cdot a_2^{(1)} + 1*2} = e^{-3*0.881 - 1*0.622 + 1*2} = 0.282$$

2 Backward Pass

First, let's tackle $\frac{\partial J}{\partial W_{11}^{(2)}}$:

$$\frac{\partial J}{\partial W_{11}^{(2)}} = \frac{\partial J}{\partial p} \cdot \frac{\partial p}{\partial z_1^{(2)}} \cdot \frac{\partial z_1^{(2)}}{\partial W_{11}^{(2)}}$$

We need the derivative of p with respect to $z_1^{(2)}$, which is simply:

$$\frac{\partial p}{\partial z_1^{(2)}} = e^{z_1^{(2)}}$$

We also need the derivative of J with respect to p, which is:

$$\frac{\partial J}{\partial p} = p - y$$

And to end it, we need the derivative of $z_1^{(2)}$ with respect to $W_{11}^{(2)}$:

$$\frac{\partial z_1^{(2)}}{\partial W_{11}^{(2)}} = a_1^{(1)}$$

Now, substitute this into the expression for $\frac{\partial J}{\partial W_{11}^{(2)}}$:

$$\frac{\partial J}{\partial W_{11}^{(2)}} = (p - y) \cdot e^{z_1^{(2)}} \cdot a_1^{(1)}$$

Plotting the values from the previous exercises, we have that

$$\frac{\partial J}{\partial W_{11}^{(2)}} = (0.282 - y) \cdot 0.282 \cdot 0.881$$

Now, for the $\frac{\partial J}{\partial W_{11}^{(1)}}$:

$$\frac{\partial J}{\partial W_{11}^{(2)}} = \frac{\partial J}{\partial p} \cdot \frac{\partial p}{\partial z_1^{(2)}} \cdot \frac{\partial z_1^{(2)}}{\partial a_1^{(1)}} \cdot \frac{\partial a_1^{(1)}}{\partial z_1^{(1)}} \cdot \frac{\partial z_1^{(1)}}{\partial w_{11}^{(1)}}$$

We already know the values of the derivative of J with respect to p, and the derivative of p with respect to $z_1^{(2)}$, so let us calculate the others:

$$\frac{\partial z_1^{(2)}}{\partial a_1^{(1)}} = W_{11}^{(2)}$$

$$\frac{\partial a_1^{(1)}}{\partial z_1^{(1)}} = \sigma(z_1^{(1)}) \cdot (1 - \sigma(z_1^{(1)}))$$

$$\frac{\partial z_1^{(1)}}{\partial w_{11}^{(1)}} = X_{11}$$

Now, putting everything in our equation, we have:

$$\frac{\partial J}{\partial W_{11}^{(1)}} = (0.282 - y) \cdot 0.282 \cdot -3 \cdot 0.881 \cdot (1 - 0.881) \cdot 0.5$$