Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Examination December 2021 (2019 scheme)

Course Code: MAT203

Course Name: Discrete Mathematical Structures

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Show that $p\wedge(\neg p \wedge q)$ is a contradiction. Use Truth table. (3)

2 Show the following implication without constructing a truth table: (3)

$(P \wedge Q) \Rightarrow P \rightarrow Q$

3 Find the number of permutations of 1,2,3,4,5,6,7 that are not derangements? (3)

4 Prove (i) $nC_0 + nC_1 + nC_2 + \ldots\ldots nC_n = 2^n$ (3)

(ii) $nC_0 - nC_1 + nC_2 - \ldots\ldots + (-1)^n nC_n = 0$, where n is a positive integer.

5 Let A= {a, b, c, d, e, f, g, h}, B= {1, 2, 3, 4, 5}. How many elements are there in P(AxB), the power set of AxB ? (3)

6 Does the formula $f(x) = \frac{1}{x^2 - 2}$ define (i)a function f: R →R ? (ii)a function f: Z →R ? (3)

7 Find the generating function for the sequence 1,1,1...,1.0,0,0.....where the first n+1 terms are 1. (3)

8 Find the coefficient of x^7 in the expansion of $(1+x+x^2+x^3+\ldots)^{15}$ (3)

9 Let N={0, 1, 2, 3, ...}. Define f: N→N as $f(m) = 3^m$. Show that f is a monoid homomorphism from (N, +) → (N, .) where (N, +), (N, .) are monoids under usual addition and multiplication respectively. (3)

10 Z_n is the group of integers mod n under modular addition. List the generators of (i) Z_{12} (ii) Z_p, where p is prime (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a) Prove the validity of the following argument: (8)

If Rochelle gets the supervisor's position and works hard, then she will get a pay raise. If she gets the pay raise, then she will buy a new car. She has not purchased a new car. Therefore either Rochelle did not get the Supervisor's position or she did not work hard.

(b) Negate and simplify the statement: $\forall x[p(x) \rightarrow q(x)]$ (6)

12(a) Establish the validity of the following argument by the method of proof by contradiction: (8)

$\neg P \leftrightarrow q,\ q \rightarrow r,\ \neg r$ Therefore p.

(b) Let p(x), q(x) and r(x) be the following open statements. (6)

$p(x):\ x^2 - 7x+10 = 0$; $q(x):\ x^2 - 2x - 3 = 0$; $r(x):\ x < 0$

Determine the truth or falsity of the following statements, where the universe is all integers.

(i) $\forall x[p(x) \rightarrow \neg r(x)]$ (ii) $\forall x[q(x) \rightarrow r(x)]$

(iii) $\exists x[q(x) \rightarrow r(x)]$ (iv) $\forall x[p(x) \rightarrow r(x)]$

Module 2

13(a) In how many ways can we distribute eight identical white balls into 4 distinct containers so that (i) no container is left empty (ii) the 4th container has an odd no. of balls in it ? (8)

(b) An auditorium has a seating capacity of 800. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials? (You may use pigeonhole principle) (6)

14(a) A woman has 11 friends among which 2 are married to each other (i) In how many ways can she invite 5 of them to dinner? (ii) In how many ways can she invite 5 of them to dinner if the married couple attends the function together(iii) In how many ways can she invite 5 of them to dinner if the married couple will not attend the function. (8)

(b) Determine the coefficient of x^9y^3 in the expansion of $(2x - 3y)^{12}$ (6)

Module 3

15(a) The relation R on Z^+ is defined by aRb if 'a divides b'. Check whether R is (i) reflexive (ii) symmetric (iii) transitive. Is R an equivalence relation ? (8)

(b) Let A= { 1, 2, 3 }. Consider the relation R on A defined as R= {(1,2), (2,1), (2,3)}. Is R symmetric?, antisymmetric? (6)

16(a) Consider the set A= {a, b, c}. Show that Q(A), the set of all proper subsets of A is a partially ordered set under the relation $\subseteq$, the set inclusion. Draw the Hasse diagram for the poset (Q(A), $\subseteq$). Is it a lattice? (8)

(b) Define a Distributive lattice. Give an example with justification. (6)

Module 4

17(a) Solve the recurrence relation $a_{n+2} + a_n = 0,\ \ n \geq 0,\ a_0 = 0,\ a_1 = 3$. (8)

(b) Determine the sequence generated by the exponential generating function $f(x) = \dfrac{1}{1-x}$ (6)

18(a) Solve the recurrence relation $a_{n+2} - 10a_{n+1} + 21a_n = 7(11)^n$. (8)

(b) Find the unique solution of the recurrence relation $2a_n - 3a_{n-1} = 0\ ;\ n \geq 1, a_4 = 81$ (6)

Module 5

19(a) Define (i) semigroup (ii) monoid (iii) group. Give one example each, different from one another. Is R, the set of real numbers, a group under multiplication? Justify. (8)

(b) If H and K are subgroups of a group G, prove that H$\cap$K is also a subgroup of G. (6)

20(a) Let (G, o) and (H, *) be groups with respective identities $e_{G,}\, e_H$. If $f: G \to H$ is a homomorphism, prove that, for all a$\in$G and n$\in$Z (8)

(i) $f(e_G) = e_H$ (ii) $f(a^{-1}) = [f(a)]^{-1}$ (iii) $f(a^n) = [f(a)]^n$

(b) Let G = (Z, +) be the group of integers under addition. Let H = {….,-8,-4,0,4,8,…}. Show that H is a subgroup of G. Write all left cosets of H in G. (6)

*****