Important Questions (MAT 203)

MODULE 1

1. Show that $p \wedge (\neg p \wedge q)$ is a contradiction. Use Truth table
2. Show that the implication $\neg(p \to q) \to \neg q$ is a Tautology
3. Define the terms: Converse, Inverse and Contra-positive.
4. Show the following implication without constructing a truth table:
   $(P \wedge Q) \implies P \to Q$
5. Write the negation of the following statement. " If I drive, then I will not walk"
6. Prove the validity of the following argument: If Rochelle gets the supervisor's position and works hard, then she will get a pay raise. If she gets the pay raise, then she will buy a new car. She has not purchased a new car. Therefore either Rochelle did not get the Supervisor's position or she did not work hard.
7. Negate and simplify the statement: $\forall x[p(x) \to q(x)]$
8. Establish the validity of the following argument by the method of proof by contradiction:
   $\neg P \leftrightarrow q, \quad q \to r, \quad \neg r$ Therefore p.
9. Let p(x), q(x) and r(x) be the following open statements. $p(x) : x^2 - 7x + 10 = 0$; $q(x) : x^2 - 2x–3 = 0$; $r(x) : x < 0$
   Determine the truth or falsity of the following statements, where the universe is all integers.
   (i) $\forall x[p(x) \to \neg r(x)]$
   (ii) $\forall x[q(x) \to r(x)]$
   (iii)$\exists x[q(x) \to r(x)]$
   (iv) $\forall x[p(x) tor(x)]$
10. Check whether the propositions $p \wedge (\neg q \vee r)$ and $p \vee (q \wedge \neg r)$ are logically equivalent or not
11. Show that $((p \to q) \wedge (q \to t)) \to (p \to r)$ is a tautology

12. Let p, q,r be the statements given as p:Arjun studies.q:He plays cricket.r:He passes Data Structures.
Let P_1, P_2, P_3 denote the Premises
p_1: If Arjun studies, then he will pass Data Structures'
p_2: If he doesn't play cricket, then he will study'
P_3: He failed Data Structures Determine whether the argument
$(P_1 \wedge p_2 \wedge p_3) \rightarrow q$ is valid

MODULE 2

1. Explain binomial theorem. Determine the coefficient of x^9y^3 in the expansion of $(x+y)^{12}, \quad (x+2y)^{12}, \quad and \quad (2x-3y)^{12}$ using binomial theorem.

2. How many 5 digit numbers can be formed from the digits 1,2,3,4,5 using the digits without repetition.
(i) How many of them are even?
(ii) How many are even and greater than 30,000?

3. How many positive integers not exceeding 100 are divisible by 4 or 6.

4. There are 8 guests in a party. Each guest brings a gift and receives another gift in return. No one is allowed to receive the gift they bought. How many ways are there to distribute the gifts?

5. Six papers are set in an examination of which two are mathematical. Only one examination will be conducted in a day. In how many different orders ,can the papers be arranged so that
(i) Two mathematical papers are consecutive?
(ii) Two mathematical papers are not consecutive?

6. What is pigeon hole principle? Explain. If you select any five numbers from 1 to 8 then prove that at least two of them will add up to 9.

7. In how many ways can the letters of the word ALLAHABAD be arranged ?

8. Find the number of permutations of 1,2,3,4,5,6,7 that are not derangements?

9. In how many ways can we distribute eight identical white balls into 4 distinct containers so that
(i) no container is left empty
(ii) the 4th container has an odd no. of balls in it ?

10. An auditorium has a seating capacity of 800. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials? (You may use pigeonhole principle)

11. A woman has 11 friends among which 2 are married to each other (i) In how many ways can she invite 5 of them to dinner? (ii) In how many ways can she invite 5 of them to dinner if the married couple attends the function together(iii) In how many ways can she invite 5 of them to dinner if the married couple will not attend the function.

12. Prove (i) ${}^nC_0 +^n C_1 +^n C_2 + ...^nC_n = 2^n$.
(ii) ${}^nC_0 -^n C_1 +^n C_2 - ... + (-1)^n.^nC_n = 0$
where n is a positive integer

MODULE 3

1. Let R be a relation on z by xRy iff 4 divides (x-y). Then find all equivalence classes.

2. Show that the divisibility relation ' / ' is a partial ordering on the set Z^+

3. Let $A = \{a, b, c, d, e, f, g, h\}$, $B\{1, 2, 3, 4, 5\}$. How many elements are there in $P(A \times B)$, the power set of $A \times B$?

4. Does the formula $f(x) = \frac{1}{x^2-2}$ define (i)a function $f : R \to R$? (ii)a function $f : Z \to R$?

5. The relation R on Z + is defined by aRb if 'a divides b'. Check whether R is (i) reflexive (ii) symmetric (iii) transitive. Is R an equivalence relation

6. Let $A = \{1, 2, 3\}$. Consider the relation R on A defined as $R = \{(1, 2), (2, 1), (2, 3)\}$. Is R symmetric?, anti-symmetric?

7. Consider the set $A = \{a, b, c\}$. Show that $Q(A)$, the set of all proper subsets of A is a partially ordered set under the relation $\subseteq$, the set inclusion. Draw the Hasse diagram for the poset $(Q(A), \subseteq)$. Is it a lattice?

8. Define a Distributive lattice. Give an example with justification.

9. Let $A = \{1, 2, 3, 4, ...11, 12\}$ and let R be the equivalence relation on $A \times A$ defined by (a,b)R(c,d) $\iff$ a+d = b+c. Prove that R is an equivalence relation and find the equivalence class of (2 ,5).

10. Let R and S be two relations on a set A . If R and S are symmetric, Prove that $(R \cap S)$ is also symmetric.

11. Suppose $f(x) = x + 2$, $g(x) = x - 2$, and $h(x) = 3x$ for $x \in R\}$, where R is the set of real numbers. Find $(g \circ f)$, $(f \circ g)$, $(f \circ f)$ and $(g \circ g)$.

MODULE 4

1. solve $2a_n - 3a_{n-1} = 0, \quad n \geq 0, \qquad a_4 = 81$

2. What is meant by exponential generating function? Explain.

3. Find the generating function for the sequence 1,1,1. . . ,1.0,0,0. . . ..where the first n+1 terms are 1.

4. Find the coefficient of x^7 in the expansion of $(1 + x + x^2 + x^3 + ...)^15$.

5. Solve the recurrence relation $a_{n+1} = 2a_n + 1, \quad n \geq 0, \quad a_0 = 0$

6. Solve the recurrence relation $a_{n+2} = a_{n+1} + a_n, \quad n \geq 0, a_0 = 0, a_1 = 1$.

7. Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200, \quad n \geq 0, \quad a_0 = 3000, \quad a_1 = 3300$

8. Determine the sequence generated by the exponential generating function $f(x) = \frac{1}{1-x} + 2x^3$

9. Determine the coefficient of x^{20} in $f(x) = (x^2 + x^3 + x^4 + ...)^5$

10. Solve the recurrence relation $ar - 7a_{r-1} + 10a_{r-2} = 0$ for $r \geq 2$; Given $a_0 = 0$; $a_1 = 41$. using generating functions.

11. Solve the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = (r + 1)^2$ using generating function.

MODULE 5

1. Let $N = \{0, 1, 2, 3, ...\}$. Define $f : N \to N$ as $f(m) = 3^m$. Show that f is a monoid homomorphism from $(N, +) \to (N, .)$ where (N, +), (N, .) are monoids under usual addition and multiplication respectively.

2. Z_n is the group of integers mod n under modular addition. List the generators of
(i)Z_{12} $\qquad$ $(ii)Z_p$, where p is prime.

3. Let (G, o) and (H,*) be groups with respective identities e_G and e_H. If $f : G \to H$ is a homomorphism, prove that, for all $a \in G$ and $n \in Z$
(i) $f(e_G) = e_H$ (ii)$f(a^{-1}) = [f(a)]^{-1}$ (iii) $f(a^n) = [f(a)]^n$

4. Show that the set of idempotent elements of any commutative monoid forms a submonoid.

5. State and prove Lagrange's theorem.

6. If $A = \{1, 2, 3\}$.List all permutation on A. prove that it is a group

7. show that every subgroup of a cyclic group is cyclic.

8. Show that the direct product of two group is a group.

9. If H and K are subgroups of a group G, prove that $H \cap K$ is also a subgroup of G.

10. Let $G = (Z, +)$ be the group of integers under addition. Let $H = \{\ldots, -8, -4, 0, 4, 8, \ldots\}$. Show that H is a subgroup of G. Write all left cosets of H in G.

11. If $f : (R^+, .) \to (R, +)$ as $f(x) = ln(x)$ where R^+ is the set of positive real numbers. show that f is a monoid isomorphism from R^+ onto R.

12. Prove that the set 'Q' of rational numbers other than 1 forms an abelian group with respect to the operation ' * ' defined by $a * b = a + b - ab$.

13. Let $(A, *)$ be a group. Show that $(A, *)$ is an abelian group if and only if $a^2 * b^2 = (a * b)^2$ for all 'a' and 'b' in A.

14. Define (i) semigroup (ii) monoid (iii) group. Give one example each, different from one another. Is R, the set of real numbers, a group under multiplication? Justify

15. Prove that a group is abelian iff $(a * b)^{-1} = a^{-1} * b^{-1}$.

16. Define binary operation * on Z defined by $x * y = x + y + 1$ verify that (Z , *) is an abelian group.

17. Show that every group of prime order is cyclic.