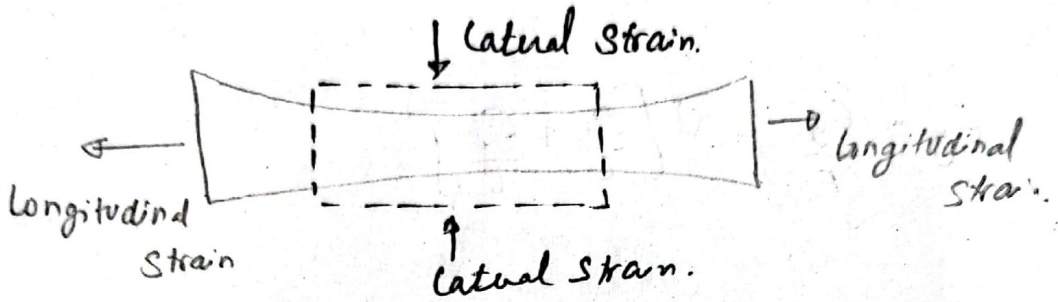
PED (Process Equipment Design

* Poisson's Ratio. (ν)

$$\nu = \frac{\text{Lateral Strain or Transverse Strain}}{\text{Longitudinal strain.}}$$



$$\boxed{\nu = \frac{\epsilon_c \text{ (lateral)}}{\epsilon_e \text{ (axial)}}}$$

* Biaxial stress

- contraction in x-dir^n.

$$\epsilon_c = \nu \epsilon_e \qquad \epsilon_c = \nu \epsilon_2 = \nu \frac{\sigma_y}{E}$$

Net elongation, ϵ_x

- $\epsilon_x = \epsilon_1 - \epsilon_c = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$ — (I)
- $\epsilon_y = \epsilon_2 - \epsilon_c$

$$= \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} \quad \text{— (II).}$$

$$\boxed{\sigma_x = \frac{(\epsilon_x + \nu \epsilon_y) E}{1 - \nu^2}}, \quad \boxed{\sigma_y = \frac{(\epsilon_y + \nu \epsilon_x) E}{1 - \nu^2}}$$

* For triaxial stress:

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$= \frac{\sigma_x}{E} - \nu\left[\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right] = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{\sigma_y}{E} - \nu\left[\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right]$$

$$\epsilon_z = \frac{\sigma_z}{E} - \nu\left[\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right]$$

* Stress concentration

* Thermal stress

coeff. of thermal expansion.

$$\frac{\Delta L}{L} = \alpha(T_2 - T_1)$$

$$\text{strain} = \alpha(T_2 - T_1)$$

$$\sigma = -E\alpha(T_2 - T_1)$$

$$\sigma_x = \sigma_y = -\frac{E\alpha(T_2 - T_1)}{1-\nu}$$

$$\sigma_x = \sigma_y = \sigma_z = -\frac{E\alpha(T_2 - T_1)}{1-2\nu}$$

4).

$E = 200 GPa$

$\nu = 0.3$

$\epsilon_x = 400 \times 10^{-6}$

$\epsilon_y = 300 \times 10^{-6}$

$\epsilon_z = 230 \times 10^{-6}$

(a)

$$400 \times 10^{-6} = \frac{1}{200 \times 10^{9}} \left[\sigma_x - 0.3(\sigma_y + \sigma_z) \right]$$

$$8 \times 10^{7} = \sigma_x - 0.3(\sigma_y + \sigma_z) \quad \text{— (I)}$$

$$6 \times 10^{7} = \sigma_y - 0.3(\sigma_x + \sigma_z) \quad \text{— (II)}$$

$$4.6 \times 10^{7} = \sigma_z - 0.3(\sigma_x + \sigma_y). \quad \text{— (III)}$$

(I) − (III).

$$3.4 \times 10^{7} = \sigma_x - 0.3\sigma_z - \sigma_z + 0.3\sigma_x$$

$$= 1.3\sigma_x - 1.3\sigma_z$$

$$\frac{3.4 \times 10^{7}}{1.3} = \sigma_x - \sigma_z = 2.61 \times 10^{7}$$

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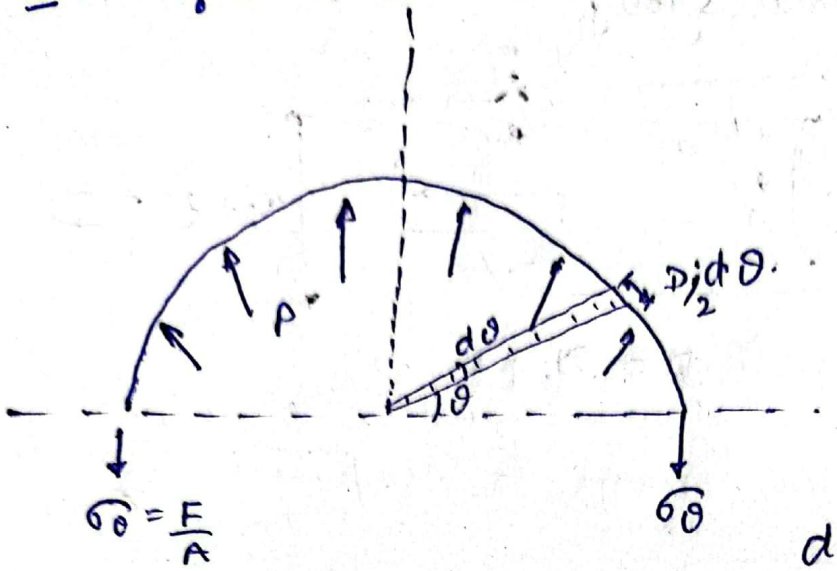
* **Hoop stress** $\{\sigma_\theta = \sigma_c = \sigma_y = \sigma_H\}$

{Hoop stress or Circumferential stress or Tangential stress}



(cylinder).

$$dA = \left[\frac{D_i}{2} d\theta \times L\right]$$

$$F_y = \left[\frac{D_i}{2} d\theta \times L\right] P \sin\theta.$$

$$2\int_0^{\pi/2} F_y = 2\int_0^{\pi/2} \frac{D_i L P}{2} \sin\theta \, d\theta.$$

$$F = D_i L P$$

$$\boxed{2\sigma_\theta = \frac{D_i L P}{t L}}$$

$$\sigma_\theta = \frac{D_i P}{2t}$$

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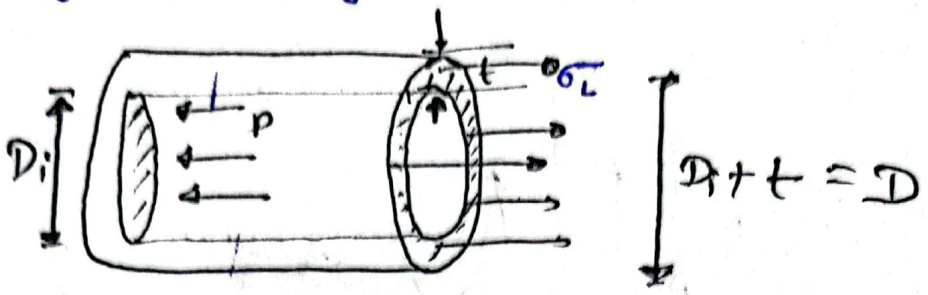
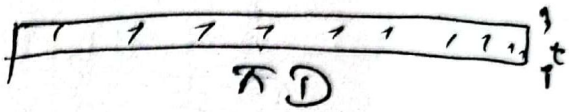
* Longitudinal stress $(\sigma_1 = \sigma_n = \sigma_h) = \sigma_l$

{ Axial stress }



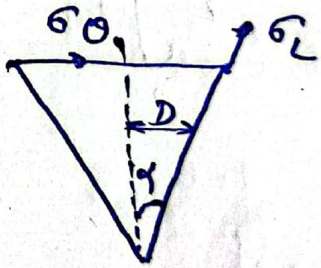
$$D = D_i + t$$



$$P \frac{\pi D_i^2}{4} = \sigma_L (\pi D t)$$

$$\boxed{\sigma_l = \frac{P D_i^2}{4 D t}} \simeq \frac{P D_i}{4 t}$$

↳ For thin-walled conical vessel.



Q). (1) $D_i = 3m$

$L = 12m$ $\quad P = 3 \times 10^6 \, N/m^2$ (gauge).

$\sigma_L = 200 \times 10^6 \, N/m^2$

$2 \cancel{00 \times 10^6} = \cancel{3 \times 10^6} \times 3$

$t = \cancel{29}$

800

$$\sigma_{max} = \sigma_\theta = \frac{PD_i}{2t}$$

$$200\times10^{6} = \frac{(3\times10^{6}\times1.05)\times3}{2t}$$

$$\boxed{t = 0.0236}$$

$$= 0.0236\ m$$

$$= 2.36\ cm$$

$\frac{t}{D} << (1-10\%)$ $t \to$ is neglible

Find σ_L / σ_1 & $\sigma_\theta / \sigma_2 / \sigma_\theta$

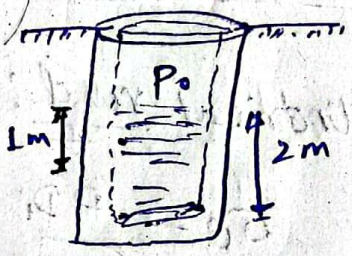
Q). (2)

$R = 1m$

$t = 1mm$

$\rho_w = 1000\ kg/m^3$

$g = 10\ m/s^2$



$P = \rho g h$.

$= 1000\times10\times7$

$= 70\times10^{3}\ N/m^2$

$P = \rho g h + P_0$

{Gauge $P^{ressure}$ $P_0 = 0$} axial $\to (\sigma_L)$

circumferential stress (σ_θ) at mid pt.

$\Rightarrow$ $[(1.05)P_0$ or $(P)]_{max}$ (P_0 is not given).

$P = \rho g h$ $\{P_0 = 0\}$.

$= 70\ kN/m^3$

$$\boxed{\sigma_L = \frac{PD_i}{4t}}$$

$$\sigma_L = \frac{70\times10^{3}\times4}{4\times1\times10^{-3}} = 5\ MN/m^3$$

$$\sigma_\theta = \frac{PD_i}{2t}$$

$$= \frac{70 \times 10^3 \times 2}{2 \times 1 \times 10^{-3}}$$

$$= 70\ MN/m^3$$

* Dilation of pressure vessel

- Radial growth or change in the shell thickness.

$$\boxed{\delta = \epsilon r}$$

ϵ → strain

r → inner radius.

$$\boxed{\epsilon = \frac{\sigma_\theta}{E} - \frac{\nu \sigma_l}{E}}$$

- For cylindrical vessel:

$$\sigma_\theta = \frac{PD_i}{2t}$$

$$\sigma_l = \frac{PD_i}{4t}$$

$$\frac{\delta}{r} = \frac{PD_i}{2tE}\left(1 - \frac{\nu}{2}\right)$$

$$\boxed{\delta = \frac{Pr^2}{tE}\left(\frac{2-\nu}{2}\right)} = \frac{Pr^2}{2tE}(2-\nu)$$

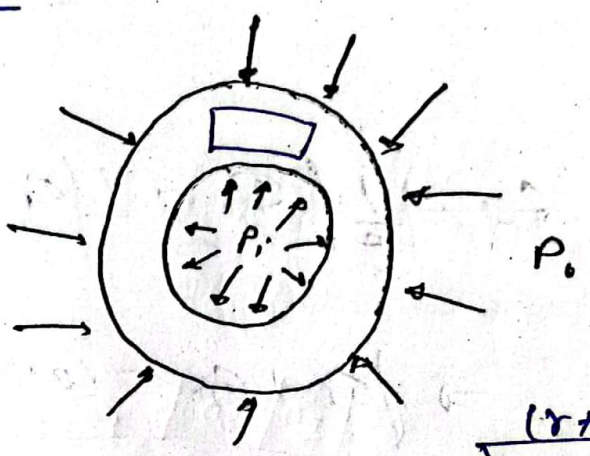
• For spherical vessel

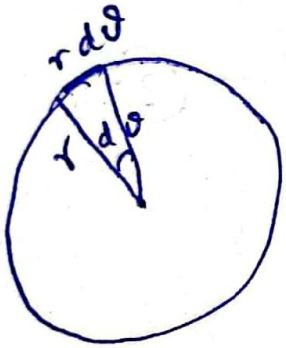
$\delta = [$

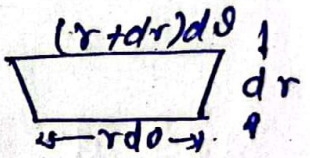
* Thin-walled pressure vessel.

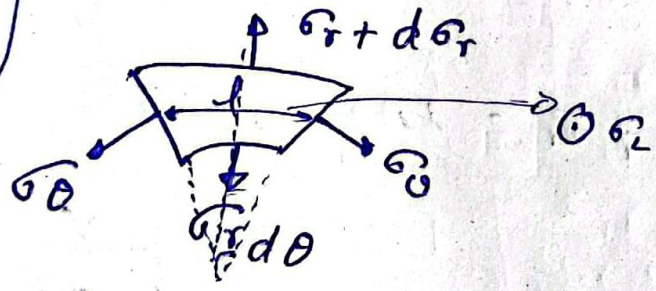
• High pressure & high Temp. Rx^n:
→ Haber process, hydrogenation of petrochemicals & coal, urea synthesis, etc.

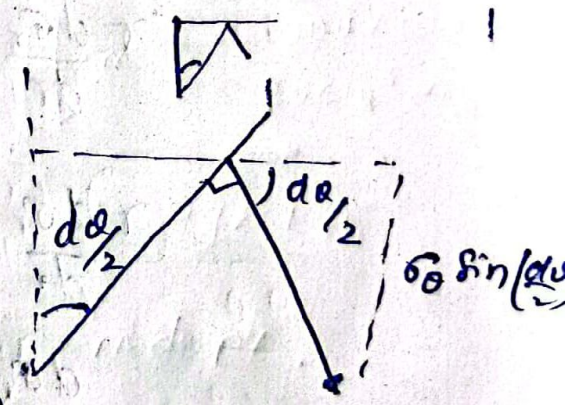
⊛ Lame's stress Analysis











$\Sigma F_y = 0$

$$\underbrace{[(r+dr)d\theta \times l]}_{\text{Area}}(\sigma_r + d\sigma_r) = (\sigma_r) \times [r\,d\theta \times l] + \sigma_\theta \sin\left(\frac{d\theta}{2}\right) \times 2\,[dr \times l]$$

$$\left\{\sin\left(\frac{d\theta}{2}\right) \simeq \frac{d\theta}{2}\right\}.$$

$$[(r+dr)d\theta \times l](\sigma_r + d\sigma_r) = (\sigma_r)[r\,d\theta \times l] + \sigma_\theta\, d\theta\,(dr \times l)$$

$\cancel{r\sigma_\theta d\theta = \sigma_r r d\theta}$

$$\cancel{r\sigma_r + r d\sigma_r + \sigma_r dr = \sigma_r r + \sigma_\theta d\theta}$$

$$r d\sigma_r + \sigma_r dr = \sigma_\theta dr$$

$$r\frac{d\sigma_r}{dr} + \sigma_r = \sigma_\theta \quad \text{— (1)}$$

Triaxial

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_r + \sigma_\theta)\right]$$

$$\Rightarrow \boxed{\sigma_r + \sigma_\theta = \text{constant} = 2A} \quad \text{— (2)}$$

$$r\frac{d\sigma_r}{dr} + \sigma_r = 2A - \sigma_r$$

$$r\frac{d\sigma_r}{dr} + 2\sigma_r = 2A$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$I.F = e^{\int P(x)dx}$$

$$\cancel{\frac{d\sigma_r}{dr} = \frac{2(A - \sigma_r)}{r}}$$

$$\cancel{d\sigma_r = 2(A - \sigma_r)\int \frac{dr}{r}}$$

$$\cancel{\sigma_r = 2(A - \sigma_r)\ln r}$$

$$2\sigma_r r + r\frac{d\sigma_r}{dr} - 2Ar = 0$$

$$\frac{1}{dr}\left[\sigma_r r^2 - Ar^2\right] = 0$$

$$\sigma_r r^2 - Ar^2 = -B$$

$$\sigma_r = \frac{Ar^2 - B}{r^2}$$

$$\boxed{\sigma_r = \left(A - \frac{B}{r^2}\right)}$$

from ② eq

$$\sigma_r + \sigma_\theta = 2A$$

$$\sigma_\theta = 2A - \sigma_r$$

$$\boxed{\sigma_\theta = A + \frac{B}{r^2}}$$

Lame's Eq^n.

Lame's Eq^n.

$$\sigma_\theta = A + \frac{B}{r^2}$$

$$\sigma_r = A - \frac{B}{r^2}$$

$r = r_i$, $\sigma_r = -P_i$

$r = r_o$, $\sigma_r = -P_o$

$$-P_i = A - \frac{B}{r_i^2} \quad \text{--- (I)}$$

$$-P_o = A - \frac{B}{r_o^2} \Rightarrow A = -P_o + \frac{B}{r_o^2} \quad \text{--- (II)}$$

$$-P_i = -P_o + \frac{B}{r_o^2} - \frac{B}{r_i^2}$$

$$(P_o - P_i) = B\left(\frac{1}{r_o^2} - \frac{1}{r_i^2}\right)$$

$B = __$ if we can easily find A from (I) or (II) eq.

$$B = (P_o - P_i)\left(\frac{r_i^2 \cdot r_o^2}{r_i^2 - r_o^2}\right)$$

$$A = -P_o + \frac{B}{r_o^2}$$

$$= \frac{(P_o r_o^2 - P_i r_i^2)}{r_i^2 - r_o^2}$$

$$A = \frac{P_o r_o^2 - P_i r_i^2}{r_i^2 - r_o^2}$$

$$B = \frac{(P_o - P_i) r_i^2 r_o^2}{r_i^2 - r_o^2}$$

* For internal pressure vessel, $(P_o = 0)$.

$$\sigma_r = \frac{-P_i r_i^2}{r_i^2 - r_o^2} + \frac{P_i r_i^2 r_o^2}{(r_i^2 - r_o^2)} \cdot \frac{1}{r^2}$$

$$= \frac{-P_i r_i^2}{(r_i^2 - r_o^2)}\left[1 - \frac{r_o^2}{r^2}\right]$$

$$\sigma_\theta = \frac{-P_i r_i^2}{r_i^2 - r_o^2} - \frac{P_i r_i^2 r_o^2}{(r_i^2 - r_o^2)} \cdot \frac{1}{r^2}$$

$$= \frac{-P_i r_i^2}{(r_i^2 - r_o^2)}\left[1 + \frac{r_o^2}{r^2}\right]$$

$$(\sigma)_{max} = (\sigma_\theta)_{r=r_i} = \frac{P_o r_i^2}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r_i^2}\right]$$

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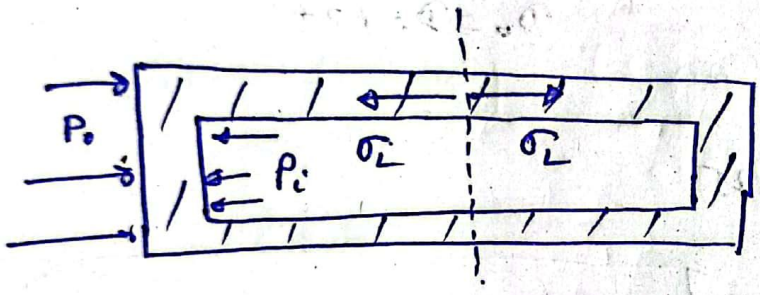
weld-efficiency factor,

$$\underbrace{fJ}_{\text{Max. allowable stress.}} = \sigma_{max}$$

f → (design stress) allowable stress of a material.

Max Shearing Stress

$$\tau = \frac{\sigma_\theta - \sigma_r}{2}$$



$$P_o(\pi r_o^2) - P_i(\pi r_i^2) = \sigma_L[\pi(r_o^2 - r_i^2)]$$

$$\sigma_L = \frac{P_o r_o^2 - P_i r_i^2}{r_o^2 - r_i^2}$$

$$fJ = \sigma_{max} = \frac{P(r_i^2 + r_o^2)}{r_o^2 - r_i^2}$$

$$P = \frac{fJ(r_o^2 - r_i^2)}{(r_i^2 + r_o^2)}$$

$r_o = r_i + t$

$$P = \frac{fJt}{\left[r_i + \frac{t(r_i+t)}{2r_i+t}\right]} = \frac{fJt}{\left[r_i + t\left(\frac{1+t/r_i}{2+t/r_i}\right)\right]}$$

$$= \frac{2fJt}{\left[D_i + t\underbrace{\left(\frac{1+2t/D_i}{1+t/D_i}\right)}_{\approx 1.2}\right]} \qquad \frac{D_o}{D_i} \le 1.5$$

$$D_o = D_i + 2t$$

When, $\frac{D_o}{D_i} = 1.5$

$$\left(\frac{t}{D_i}\right)_{max} = 0.25$$

$$\boxed{P = \frac{2fJt}{D_i + t} = \frac{2fJt}{D_o - t}}$$

$$PD_o - Pt = 2fJt$$

$$\boxed{t = \frac{PD_o}{2fJ+P}}$$

$$\boxed{t = \frac{PD_o}{2fJ+P}}$$

$$\boxed{t = \frac{PD_i}{2fJ-P}}$$

Min thickness of shell plate.

~~1.50~~ 6

~~1.5~~ 5 $\boxed{1.2}$

$P_{abs} = P_{gauge} + P_{atm}$

Gauge pressure.

9). D_o = Nominal diameter = 1.2 m

$P_{abs} = 800\,kN/m^2$ → Design pressure

$f = 118\,MN/m^2$ ✓

$P = 1.05 \times \underbrace{[800 \times 1000 - 101325]}_{\text{Gauge pressure}}\,N/m^2$

$\eta = 0.85$ ✓

$$t = \frac{800 \times 1.2}{2 \times 118 \times 0.85 + 800}$$

$= 733.608\,kPa.$

$$t = \frac{733.608 \times 10^3 \times 1.2}{2 \times 118 \times 10^6 \times 0.85 + 733.608 \times 10^3}$$

$= 0.00437$

$= 4.37$ mm (excluding corrosion allowance).

$\Rightarrow$ 6.37

$t_{new} = 7\,mm$

$\frac{D_o}{D_i} = 1.0011 \leq 1.5$

$D_o = D_i + 2t$

$D_i = D_o - 2t$

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Ex-3.4 :

$D_i = 350\ mm$ $\quad P_i = 160\ MPa$

$D_o = 450\ mm$ $\quad P_o = 80\ MPa$

$2t = 100\ mm$ $\quad r_i = 175\ mm$

$t = 50\ mm$ $\quad r_o = 225\ mm.$

$$\sigma_r = \left(A - \frac{B}{r^2}\right) \qquad B = (P_o - P_i)\left(\frac{r_o^2 \cdot r_i^2}{r_i^2 - r_o^2}\right)$$

$$\sigma_\theta = \left(A + \frac{B}{r^2}\right) \qquad A = \left(\frac{P_o r_o^2 - P_i r_i^2}{r_i^2 - r_o^2}\right)$$

$$\sigma_r = \frac{(P_o r_o^2 - P_i r_i^2)}{r_i^2 - r_o^2} - \frac{(P_o - P_i) r_i^2 r_o^2}{r_i^2 - r_o^2} \frac{1}{r^2}$$

$(r = r_o)$:

$$(\sigma_r)_{max.} = \frac{(P_o r_o^2 - P_i r_i^2)}{r_i^2 - r_o^2} - \frac{(P_o - P_i) r_i^2}{r_i^2 - r_o^2}$$

$$= \frac{[80 \times (225)^2 - 160 \times (175)^2] \times 10^6}{(175)^2 - (225)^2}$$

$$- \frac{(-80) \times 10^6 \times (175)^2}{(175)^2 - (225)^2}$$

122.5×10^6

$$= 42.5 \times 10^6 - 122.5 \times 10^6$$

$$= -80\ MN/m^2 = -80\ MPa = -P_o$$

$$\sigma_r = A - \frac{B}{r^2}$$

$$\left.\begin{aligned} (\sigma_r)_{min} &\rightarrow r = r_i \\ (\sigma_r)_{max} &\rightarrow r = r_o \end{aligned}\right\}$$

$\boxed{r = r_i}$

$$(\sigma_r)_{min} = 42.5 \times 10^6 - [202.5] \times 10^6$$

$$= -160 \text{ MPa}$$

$$= -P_i$$

$$\sigma_\theta = A + \frac{B}{r^2}$$

$$(\sigma_\theta)_{max} \rightarrow r = r_i$$

$$\sigma_\theta = \frac{(P_o r_o^2 - P_i r_i^2)}{r_i^2 - r_o^2} + \frac{(P_o - P_i) r_o^2}{r_i^2 - r_o^2}$$

$$= 42.5 \times 10^6 + 202.5 \times 10^6$$

$$= 245 \text{ MPa}$$

$\boxed{r = r_o}$

$$(\sigma_\theta)_{min} = 42.5 \times 10^6 + 122.5 \times 10^6$$

$$= 165 \text{ MPa}$$

$$\begin{array}{r} 122 \\ 42 \\ \hline 64 \end{array}$$

$$\tau = \frac{\sigma_\theta - \sigma_r}{2} \qquad 2\left[\frac{(P_o - P_i)\, r_i^2 r_o^2}{r_i^2 - r_o^2} \cdot \frac{1}{r^2}\right].$$

$$= 2\left[\frac{(P_o - P_i)\, r_i^2 r_o^2}{r_i^2 - r_o^2} \cdot \frac{1}{r^2}\right].$$

$$= \cancel{\frac{122.5 \times 2 \times 10^6}{r^2}}$$

$$= \cancel{122.5}$$

at $(\tau)_{max}$ $\boxed{r = r_i}$

$$= 2\left[\frac{(P_o - P_i)\, r_o^2}{r_i^2 - r_o^2}\right].$$

$(\tau)_{max} = 202.5$ MPa,,
at $r = r_i$

$(\tau)_{min} = 122.5$ MPa,,
$r = r_o$

---

9). $D_i = 100$ mm

$P_i = 50$ MPa.

$P_o = 0$

$t = ?$

Max. Allowable Stress $= 250$ MPa.

a).

$$t = \frac{PD_i}{2fJ - P}$$

Design stress $= \frac{\text{Yield s}}{\text{FOS}}$

(f)

$f = 148$

$$fJ = \sigma_{yield} = \frac{250\ MPa}{2}$$

$$= 125\ MPa.$$

$$P = 1.05 \times 50\ MPa.$$

$$= 52.5\ Pa.$$

$$t = \frac{52.5 \times 0.1}{250 - 52.5}$$

$$= 0.026\ m$$

$$D_o = D_i + 2t$$

$$= 0.1 + 2 \times 0.$$

$$= 0.152$$

$$\frac{D_o}{D_i} = \frac{0.152}{0.1} = 1.5$$

b).

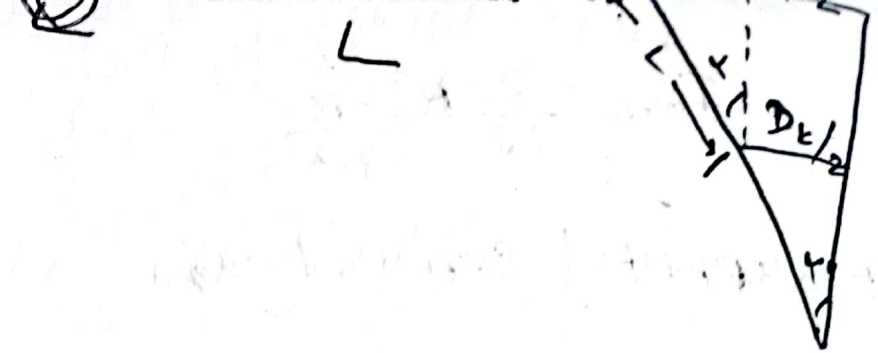
Max. Shear stress $(\tau) = \frac{\text{Yield stress}}{2 \times \text{FOS}}$

$$= \frac{250}{2 \times 2}$$

$$\frac{D_o}{D_i} = 1.5$$

$$D_o = 1.5 \times 0.1$$

$$= 1.5 \times 0.1$$



Q).

$P_{guage} = 500\ KN/m^2$

$D_o = 1.2\ m$

$f = 118\ MN/m^2$

$J = 0.85$

$P = 500 \times 1.05\ KN/m^2$

$t = C D_{eff} \sqrt{\frac{P}{f}}$

$C = 0.45$

$D_{eff} = D_i$

$P = 500 \times 1.05\ KN/m^2$

$D_i = D_o - 2t_s$ shell.

$$t_{shell} = \frac{PD_o}{2fJ + P}$$

$J = 0.85$

$f = 118\ MN/m^2$

$$\boxed{t_s = t_{shell} + 2\ mm}$$

~~$t_{shell} = 1.199\ m$~~

$t_{shell} = 0.0031\ m$

$= 3.13\ mm$

$t_s = 3.13 + 2$

$= 5.13\ mm \simeq 6$

$$t = C D_i \sqrt{\frac{P}{f}}$$

$$= 0.45\left(1.2 - 2 \times 6 \times 10^{-3}\right)\sqrt{\frac{500 \times 1.05 \times 10^{-3}}{118 \times 10^{6}}}$$

$= 0.5346 \times 0.0357$

$= 35.7\ mm$

$\simeq 36\ mm$

$t = 35.7 + 2$

$= 37.7 \simeq 38\ mm$

(2). $\alpha = 30^\circ$

$\cos\alpha = \frac{r}{R} = \frac{r_i}{R}$

for way..

$$t = \frac{P D_k}{2fJ - P} \times \frac{1}{\cos(\alpha)}$$

D_i

$\boxed{t_s = 6\,mm}$

~~$D_k = D_o - 2t_{shell}$~~

~~$t_s = 6\,mm$~~

mean knuckle.

$$t = \frac{P D_o C}{2fJ} = \frac{500 \times 1.05 \times 10^3 \times 1.2 \times 1.35}{2 \times 118 \times 10^6 \times 0.85}$$

$= 4.2\,mm$

$D_k = D_o - 2t_s - 2L\sin\alpha$

$t = 7\,mm$

$L = 0.5\sqrt{\frac{D_o t}{\cos(\alpha)}}$ $\{D_e = D_o\}$

$\boxed{C = 1.35}$

$$L = 0.5\sqrt{\frac{1.2 \times 7 \times 10^{-3}}{\cos(30^\circ)}}$$

$= 0.0492$

$= 49.2\,mm$

$D_k = 1.2 - 2 \times 6 \times 10^{-3} - 2 \times 49.2 \times 10^{-3} \sin 30^\circ$

$= 1.138$

$= 1.14\ m$

$$t = \frac{P D_k}{2 f J - P} \times \frac{1}{\cos \phi}$$

$= 3.63$

$t_{fin} = 3.63 + 2$

$= 5.63$

$\approx 6\ mm$

$6.5\ mm \times \boxed{1.06}$