Stress and Strain

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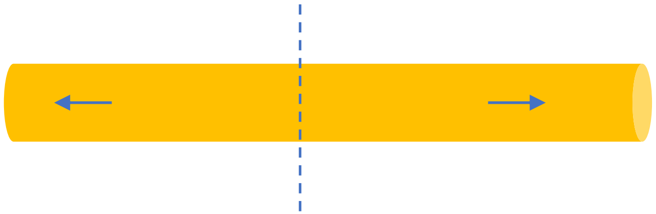
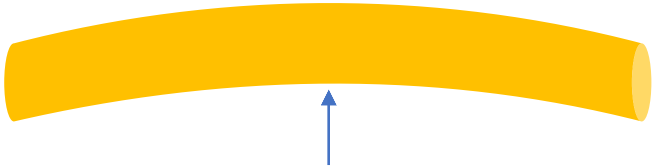
Stress

- The term Stress is used to expressed the loading in terms of force applied to a certain cross-section area of an object.
- The force of resistance per unit cross-section area offered by a body against deformation.

$$\sigma = \frac{F}{A}$$

- Uniform or non-uniform





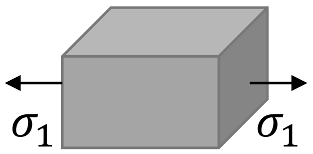
Types of Stress

- Tensile stress
  - Material subjected to pulling or stretching.
  - Dimension increases.
- Compressive stress
  - Force acts inwards of the material body.
  - Dimension decreases.
- Shear stress
  - Force acts parallel to surface.
  - Shape of the object changes; deformation.

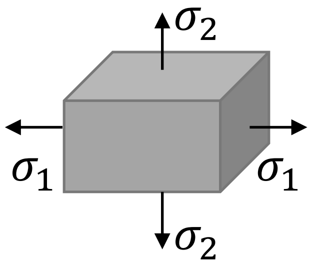
Normal Stress

Normal Stress

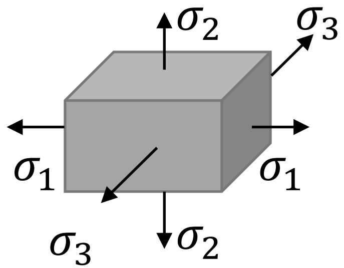
Uniaxial normal stress



Biaxial normal stress



Triaxial normal stress



Strain

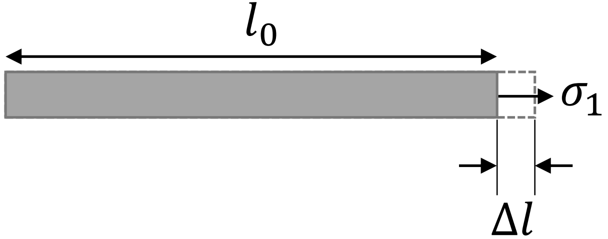
- Defined as the ratio of the change in dimensions of a material to the original dimensions, as a result of an applied stress.

$$\varepsilon = \frac{\Delta l}{l}$$

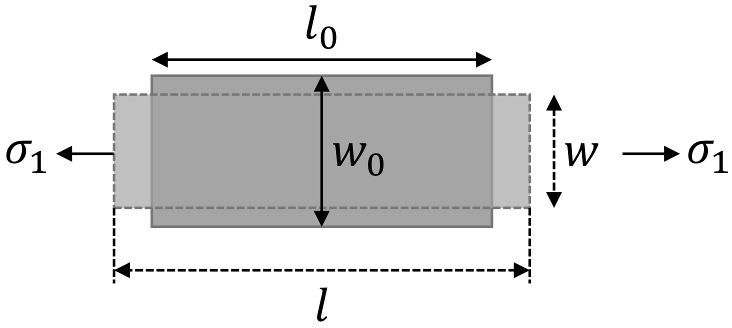
- Increase in dimension is considered a positive deviation; decrease is negative deviation.
  - Tensile → positive strain.
  - Compression → negative strain.

Types of Strain

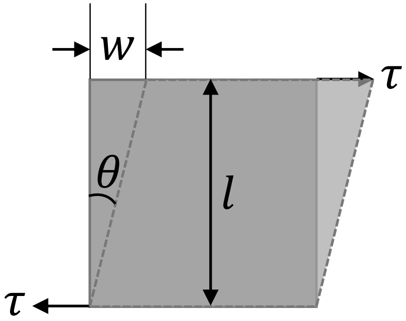
- Longitudinal strain



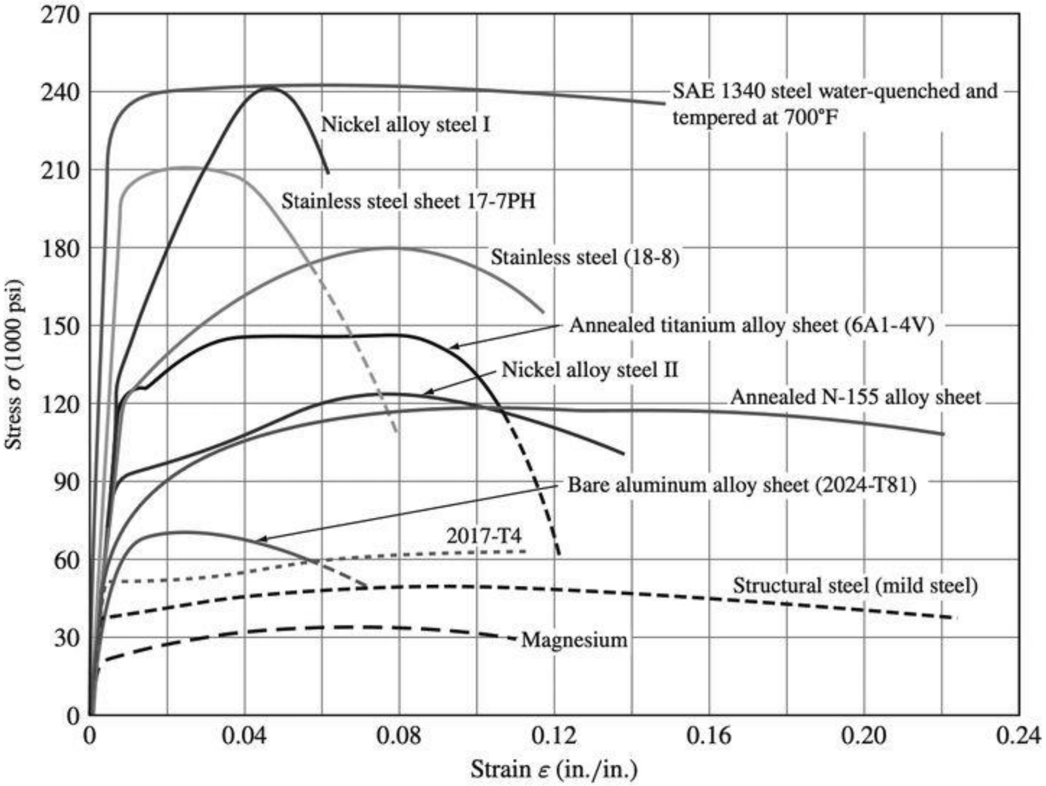
- Lateral or Transverse strain

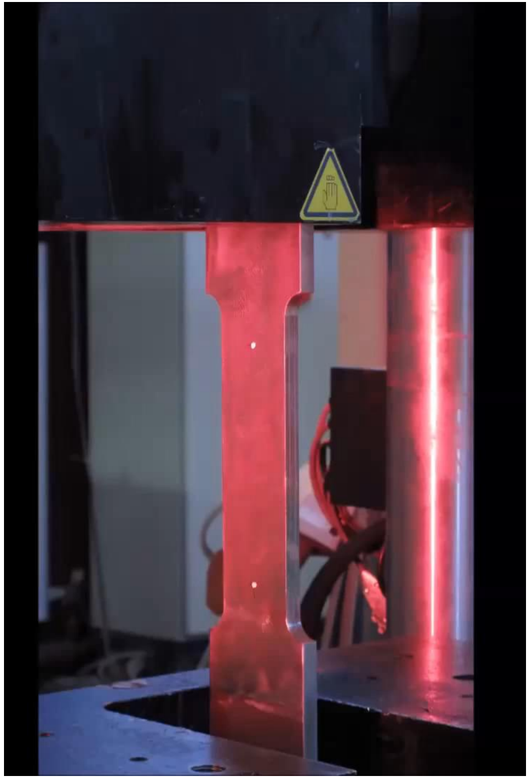


- Volumetric strain
- Shear strain



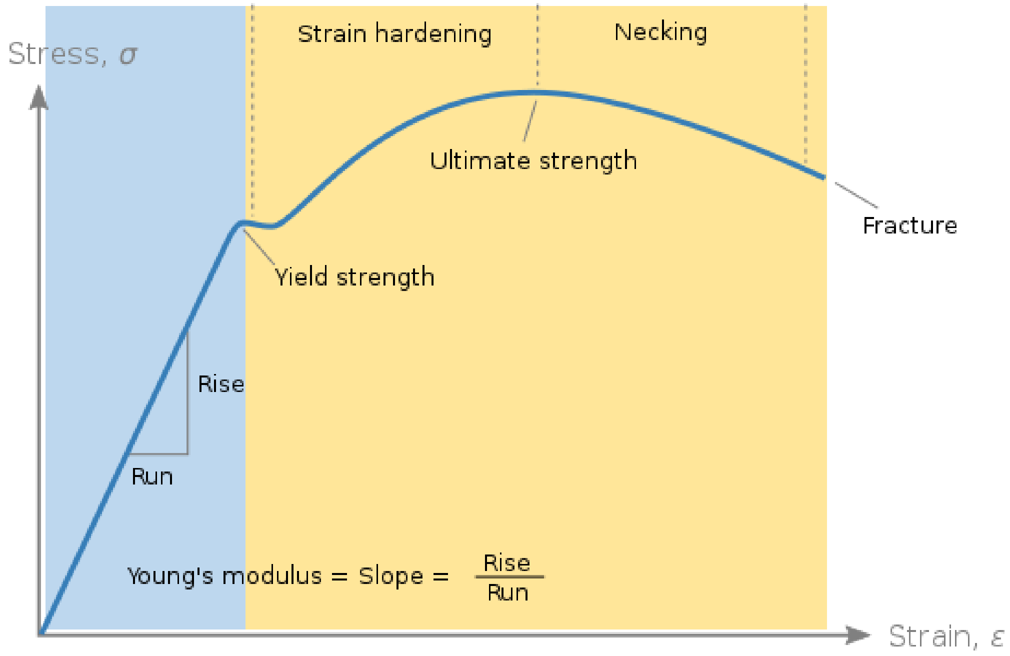
Stress-Strain Relationship





(https://extrudesign.com/stress-strain-curve/; https://en.wikipedia.org/wiki/File:Tensile_test_-_steel_sample.ogv)

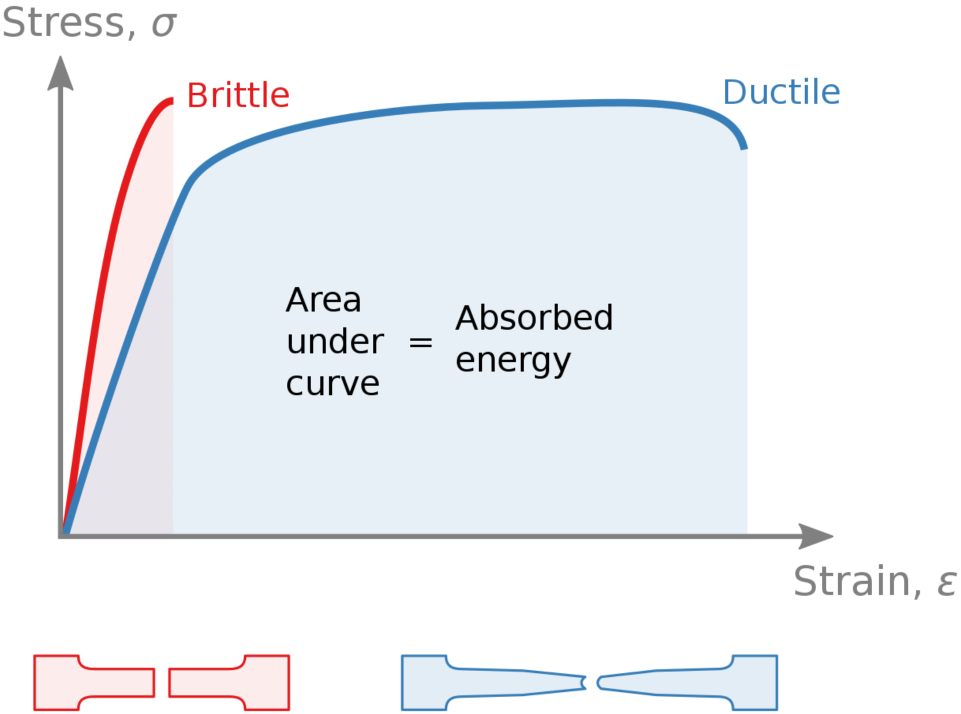
Stress-Strain Relationship



(https://en.wikipedia.org/wiki/File:Stress_strain_ductile.svg)

- Stages
  - Linear elastic region
    - Hooke's law – Young's modulus
  - Strain hardening
    - Plastic deformation (yield strength – initiation)
    - Ultimate strength
  - Necking
    - Heterogeneous
  - Fracture

Type of material



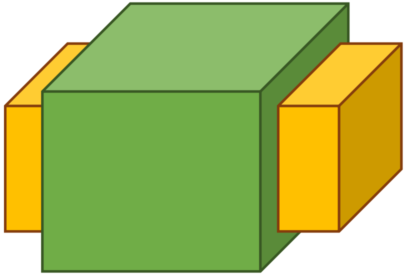
(https://en.wikipedia.org/wiki/File:Stress_strain_comparison_brittle_ductile.svg)

Poisson’s Ratio

- Poisson effect – materials expand in directions perpendicular to the direction of compression.

- The ratio of the lateral (transverse) contraction to the axial (or longitudinal) elongation is known as Poisson’s ratio.

$$\nu = \frac{\varepsilon_c}{\varepsilon_e} = \frac{lateral\ strain}{longitudinal\ strain}$$

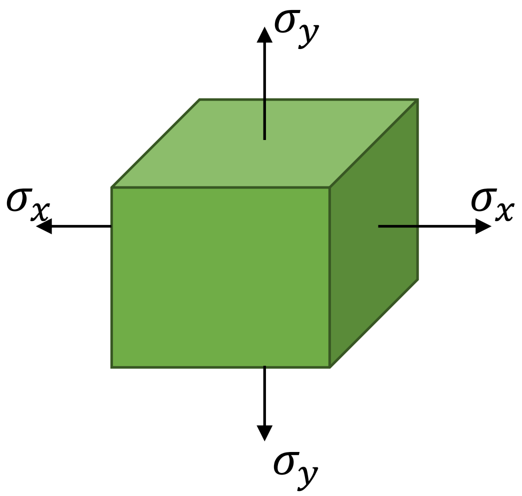


- Constant within the elastic limit.

Poisson's Ratio (2)

When stresses are acting on X direction, compressive strain in the Y direction is,

If stresses are acting on both X & Y directions, strain along Y axis is:



Note: *Tensile stresses are considered positive and compressive stresses are considered negative.*

Poisson's Ratio (3)

Solving for the stress values:

For triaxial stress:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

Numerical 2.1

Stresses are to be determined at the inside corner of an opening in a cylindrical shell by applying strain gauges at the location. The cylindrical shell is of carbon steel with $E = 200\ GPa$ and $\nu = 0.3$. The strain readings from the three gauges are:
$\varepsilon_x = +400 \times 10^{-6}, \varepsilon_y = +300 \times 10^{-6},\ \varepsilon_y = +230 \times 10^{-6}$.

(a) Find the stresses in the three directions at the opening.

(b) Find the stresses in the two direction of the cylindrical shell if $\sigma_z = 0$.

Mechanical Properties of Materials (1)

- Strength
  - Capacity of material to withstand external forces.
  - Types: tensile, compressive, shear, impact (based on force).
  - Induces internal stress and deformations.

- Stiffness/Rigidity
  - Ability of material to resist deformation.
  - Modulus of rigidity or shear modulus.

- Elasticity
  - Ability of material to regain original shape when external load is removed.

Mechanical Properties of Materials (2)

- Ductility
  - Measure of deformability of the material to sustain plastic deformation under tensile stress, before failure.
  - Percentage elongation or reduction of area.

- Toughness
  - Ability of material to absorb energy in deformation in the plastic range.

- Hardness
  - Surface characteristics; resistance to scratching.

- Creep
  - Tendency of a solid material to undergo slow deformation while subject to persistent mechanical stresses.

Stress Concentration

- At specific points, intensity of stress may increases significantly at any location in comparison to its surrounding.
  - Stress raiser, stress riser, notch sensitivity.
  - Brittle materials generally fail at these points.
  - Fatigue cracks propagate from these points.

- Cracks, sharp corners, holes, etc. may act as stress risers.

Thermal Stresses

- Expansion and contraction associated with material
  - Temperature fluctuation.
  - Temperature difference between different parts.
  - Materials with different thermal expansion coefficients.

Unit change in dimension due to thermal expansion (strain) = $\alpha(T_2 - T_1)$

Uniaxial stress, $\sigma = -E\alpha(T_2 - T_1)$

If the component is constrained along 2 axis, the net stress along each axis would be,

$$\sigma_x = \sigma_y = -\frac{E\alpha(T_2 - T_1)}{1 - \nu}$$

If the component constrained along 3 axis, $\sigma_x = \sigma_y = \sigma_z = -\frac{E\alpha(T_2 - T_1)}{1 - 2\nu}$

Note: if component restricted from contraction, tensile stress will be produced.

Criteria of Failure

- Maximum Stress Theory
- Maximum Strain Theory
- Maximum Shear Theory
- Theory of Constant Energy Distortion