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Cartesian Contact Force Estimation for Robotic Manipulators using Kalman Filters and the Generalized Momentum

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***Abstract*— Estimating contact forces and torques in Cartesian space enables force-controlled robotic applications as well as collision detection without costly additional sensing. A new approach towards online estimation of contact forces and torques at the tool center point from motor torques as well as joint angles and speeds is presented, which is based on analyzing the generalized momentum of the manipulator. Existing generalized momentum based methods are extended by designing a Kalman filter to estimate the generalized momentum as well as contact forces and torques simultaneously. This approach introduces additional degrees of freedom in the design of the estimator that are exploited to increase robustness with respect to disturbances such as uncertainty in joint friction. The method is verified by simulation results obtained from a dynamic model of an ABB YuMi, a dual-arm collaborative robot with 7DOF each arm.**

I. Introduction

While industrial manipulators frequently operate position-controlled, robotic force control allows to extend the area of application to numerous tasks such as grinding, deburring, and robust assembly [1]. Other force-controlled applications include active lead-through schemes for simplified robot teaching and collision detection and impact mitigation [2], [3]. With the advent of collaborative manipulators that are to operate in partially unstructured environments (such as ABB YuMi, Fig. 1), unexpected contact with the environment is much more likely compared to traditional scenarios with fenced manipulators. All applications named above either require knowledge on contact forces and torques with the environment or may gain in robustness based on the additional information. While this knowledge can be obtained from dedicated force/torque sensors, the additional equipment requires mechanical integration and is usually expensive.

Therefore, estimating external wrench (i.e. contact forces and torques) has constantly been receiving attention over past decades. Early approaches using observers for force estimation are reported in [4], [5], [6] and are extended towards force estimation for robotic applications in [7]. Disturbance observer approaches to contact force estimation are reported in [8], [9] and an extended Kalman filter is proposed in [10]. In [11], [12], a simultaneous state and input estimation scheme is proposed, which relies on joint torque measurements. Approaches relying solely on motor signals (i.e. motor torques, joint angles and speeds) are given in [13], [14]. In [15], contact forces are estimated by de-tuning joint controllers and inferring wrench from resulting position errors. In [16], an actuator fault detection and isolation scheme for robotic manipulators is introduced based on the generalized momentum. The approach is extended towards contact force estimation in [2], [3], [17], [18] and compared to a least-squares filtering approach in [19].

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This paper extends the generalized momentum observer approach to Cartesian Contact Force Estimation (CCFE) at the Tool Center Point (TCP) by combining it with principles of disturbance observers. The main contribution is based on augmenting the generalized momentum description of manipulator dynamics with a dynamic model of external wrench. For the augmented system, a disturbance observer is realized as a standard Kalman filter. The benefit of this approach is twofold. First, uncertainties in the manipulator model, e.g. inaccurate friction estimates, can systematically be taken into account. Second, the resulting observer matrix has more degrees of freedom compared to existing generalized momentum observer approaches. These additional degrees of freedom allow to improve robustness, especially for redundant manipulators.

The paper is structured as follows. After giving a detailed problem description in Section II, the generalized momentum observer approach to contact force estimation is recapitulated in Section III. Section IV presents the main result of this paper, i.e. the Kalman filter design based on the generalized momentum. Simulation results obtained from a model of an ABB YuMi robot are given in Section V, before a conclusion is drawn in Section VI.

Fig. 1. ABB YuMi (www.abb.com/yumi), a dual-arm collaborative manipulator with 7DOF each arm – A dynamic model of the manipulator is employed to evaluate the proposed estimation scheme for contact forces and torques at the TCP in Cartesian space.

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II. Preliminaries

A. Notation

Throughout the paper, bold lower-case letters represent column vectors. An $n \times m$-dimensional matrix of zeros is written as $\mathbf{0}_{n\times m}$, while $\boldsymbol{I}_n$ abbreviates an identity matrix of dimension n. A normally distributed random variable $\boldsymbol{v}$ with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$ is denoted by $\boldsymbol{v} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. A priori (i.e. predicted) estimates are written as $\check{\boldsymbol{x}}$, while $\hat{\boldsymbol{x}}$ denotes a posteriori estimates (i.e. corrected by measurement). For a matrix $\boldsymbol{A} \in \mathbb{R}^{n\times m}$ with $n \geq m$, the Moore-Penrose-Inverse (also called pseudo-inverse) of $\boldsymbol{A}$ is written as $\boldsymbol{A}^+ \in \mathbb{R}^{m\times n}$ and fulfills $\boldsymbol{A}^+ \cdot \boldsymbol{A} = \boldsymbol{I}_m$. The notation $\boldsymbol{B} = \mathrm{diag}\,(b_1, \ldots, b_n)$ abbreviates (block-)diagonal matrices and $\exp(\cdot)$ represents the matrix exponential function. In discrete time systems, the k-th sample is denoted with the superscript $k \in \mathbb{Z}^+$ (e.g. $\boldsymbol{A}^k$, $\hat{\boldsymbol{x}}^k$). In accordance with existing literature, the term CCFE involves estimation of both contact forces and torques.

B. Problem Description

In this paper, robotic manipulators described by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\dot{\boldsymbol{q}}, \boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{\tau}_{\text{fric}}(\dot{\boldsymbol{q}}, \boldsymbol{q}) + \boldsymbol{\tau}_{\text{ext}} = \boldsymbol{\tau}_{\text{mot}} \quad (1)$$

are considered. For a manipulator with N degrees of freedom, $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{N\times N}$ is the inertia matrix, which is positive definite according to [20, Section 2.3]. Coriolis and centripetal effects are captured by $\boldsymbol{C}(\dot{\boldsymbol{q}}, \boldsymbol{q}) \in \mathbb{R}^{N\times N}$. Joint torques resulting from gravity are summarized in $\boldsymbol{G}(\boldsymbol{q}) \in \mathbb{R}^N$, $\boldsymbol{\tau}_{\text{fric}} \in \mathbb{R}^N$ resembles friction torques in the joints, and $\boldsymbol{\tau}_{\text{mot}} \in \mathbb{R}^N$ are the motor torques driving the manipulator.

The reaction torques $\boldsymbol{\tau}_{\text{ext}} \in \mathbb{R}^N$ in the robotic joints result from contact forces and torques. External wrench $\boldsymbol{f} \in \mathbb{R}^{n_{\text{ext}}}$ occurring at the TCP is linked to reaction torques $\boldsymbol{\tau}_{\text{ext}}$ by the manipulator Jacobian $\boldsymbol{J}(\boldsymbol{q}) \in \mathbb{R}^{n_{\text{ext}}\times N}$, i.e.

$$\boldsymbol{\tau}_{\text{ext}} = \boldsymbol{J}^T(\boldsymbol{q}) \cdot \boldsymbol{f}, \quad (2)$$

according to [20, Section 1.10]. In the general case, the dimension of wrench is $n_{\text{ext}} = 6$ and its elements in Cartesian base-frame coordinates are defined as

$$\boldsymbol{f} = \begin{bmatrix} f_x & f_y & f_z & \tau_x & \tau_y & \tau_z \end{bmatrix}^T \in \mathbb{R}^{n_{\text{ext}}}, \quad (3)$$

where f_x, f_y, and f_z are contact forces and τ_x, τ_y, and τ_z are contact torques. It is to be noticed that $n_{\text{ext}} < 6$ can be considered if it is a priori known that contact forces and torques cannot occur in certain directions. This is e.g. the case for planar manipulators.

Based on the model (1), the objective of the paper is to estimate external wrench $\boldsymbol{f}$ at the TCP expressed in Cartesian base-frame coordinates as depicted in Fig. 2. We emphasize that only motor torques $\boldsymbol{\tau}_{\text{mot}}$, joint angles $\boldsymbol{q}$ and joint speeds $\dot{\boldsymbol{q}}$ are assumed to be available for the estimation scheme.

C. Manipulator Dynamics Expressed in Terms of the Generalized Momentum

In this section, an alternative way of expressing the manipulator dynamics (1) already presented e.g. in [16] is briefly summarized. To simplify notation, the arguments of the matrices and vectors are omitted in this section and the remainder of the paper.

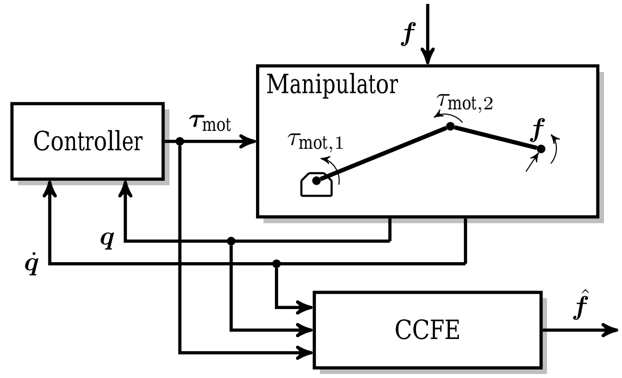

Fig. 2. Overview of Cartesian contact force estimation (CCFE) based on motor signals – A robot controller generates motor torques $\boldsymbol{\tau}_{\text{mot}}$ that drive a manipulator. The manipulator can get into contact with the environment, resulting in external wrench at the tool center point (TCP). The objective of the proposed Cartesian contact force estimation scheme is to obtain an estimate $\hat{\boldsymbol{f}}$ of this external wrench based on motor signals only (i.e. motor torques $\boldsymbol{\tau}_{\text{mot}}$, joint angles $\boldsymbol{q}$, and joint speeds $\dot{\boldsymbol{q}}$) without additional sensing.

The alternative system description is based on the generalized momentum of the manipulator, which is defined as

$$\boldsymbol{p} = \boldsymbol{M}\dot{\boldsymbol{q}}. \quad (4)$$

Differentiating the generalized momentum with respect to time results in

$$\dot{\boldsymbol{p}} = \dot{\boldsymbol{M}}\dot{\boldsymbol{q}} + \boldsymbol{M}\ddot{\boldsymbol{q}}. \quad (5)$$

Solving the manipulator dynamics (1) for $\boldsymbol{M}\ddot{\boldsymbol{q}}$ and substituting into the time derivative of the generalized momentum (5), we obtain

$$\dot{\boldsymbol{p}} = \dot{\boldsymbol{M}}\dot{\boldsymbol{q}} + \boldsymbol{\tau}_{\text{mot}} - \boldsymbol{C}\dot{\boldsymbol{q}} - \boldsymbol{G} - \boldsymbol{\tau}_{\text{fric}} - \boldsymbol{\tau}_{\text{ext}}. \quad (6)$$

While $\boldsymbol{C}$ is not uniquely defined, it is assumed for the remainder of the paper that the Coriolis matrix is expressed using Christoffel symbols [20, Section 2.3]. As a consequence, $\dot{\boldsymbol{M}} - 2\boldsymbol{C}$ is a skew-symmetric matrix, and with symmetry of $\boldsymbol{M}$ this implies

$$\dot{\boldsymbol{M}} = \boldsymbol{C} + \boldsymbol{C}^T. \quad (7)$$

Therewith, equation (6) can further be simplified to

$$\dot{\boldsymbol{p}} = \boldsymbol{\tau}_{\text{mot}} + \boldsymbol{C}^T\dot{\boldsymbol{q}} - \boldsymbol{G} - \boldsymbol{\tau}_{\text{fric}} - \boldsymbol{\tau}_{\text{ext}}. \quad (8)$$

Equation (8) can be regarded as an alternative way of describing the manipulator dynamics (1).

III. Generalized Momentum Observer Approach

Existing CCFE schemes relying directly on the robot dynamics (1) typically involve computation of joint accelerations $\ddot{\boldsymbol{q}}$ [8] or inversion of the inertia matrix $\boldsymbol{M}$ [9]. The former approach requires numerical differentiation of joint speeds going along with the amplification of measurement noise. The latter approach may be computationally costly and prone to numerical issues for manipulators with many degrees of freedom. As a solution, a different approach is presented in [16]. It relies on the alternative description of manipulator dynamics (8) and has been interpreted from a controls perspective in [14], [19]. The key advantage of describing the robot dynamics based on the generalized momentum (8) compared to the original description in (1) is that no joint angle accelerations $\ddot{\boldsymbol{q}}$ are involved.

Assuming an ideal estimate of friction torques $\hat{\boldsymbol{\tau}}_{\text{fric}} = \boldsymbol{\tau}_{\text{fric}}$ and introducing the abbreviation

$$\overline{\boldsymbol{\tau}} = \boldsymbol{\tau}_{\text{mot}} + \boldsymbol{C}^T \dot{\boldsymbol{q}} - \boldsymbol{G} - \boldsymbol{\tau}_{\text{fric}}, \tag{9}$$

an observer for the generalized momentum can be written as

$$\dot{\hat{\boldsymbol{p}}} = \overline{\boldsymbol{\tau}} + \boldsymbol{L}\left(\boldsymbol{p} - \hat{\boldsymbol{p}}\right), \tag{10}$$

where $\boldsymbol{p} = \boldsymbol{M}\dot{\boldsymbol{q}}$ can be calculated from measurements of $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$. An estimate $\hat{\boldsymbol{\tau}}_{\text{ext}}$ of the joint torques due to external wrench is obtained from

$$\hat{\boldsymbol{\tau}}_{\text{ext}} = -\boldsymbol{L}\left(\boldsymbol{p} - \hat{\boldsymbol{p}}\right). \tag{11}$$

Transforming equations (8) and (10) into Laplacian domain and introducing the error variable $\boldsymbol{e} = \boldsymbol{p} - \hat{\boldsymbol{p}}$, we obtain

$$\left(s\boldsymbol{I}_N + \boldsymbol{L}\right)\boldsymbol{e} = -\boldsymbol{\tau}_{\text{ext}}. \tag{12}$$

From (11) and (12), $\boldsymbol{\tau}_{\text{ext}}$ and $\hat{\boldsymbol{\tau}}_{\text{ext}}$ are related by

$$\hat{\boldsymbol{\tau}}_{\text{ext}} = \boldsymbol{L}\left(s\boldsymbol{I}_N + \boldsymbol{L}\right)^{-1}\boldsymbol{\tau}_{\text{ext}}. \tag{13}$$

Assuming a diagonal observer matrix $\boldsymbol{L} = \text{diag}\left(l_1, \ldots, l_N\right)$ allows to evaluate (13) element-wise by

$$\hat{\tau}_{\text{ext},i} = \frac{l_i}{s + l_i} \cdot \tau_{\text{ext},i}, \;\; i = 1, \ldots, N. \tag{14}$$

Consequently, the estimates $\hat{\tau}_{\text{ext},i}$ are low-pass filtered versions of the actual torques $\tau_{\text{ext},i}$. From the estimate $\hat{\boldsymbol{\tau}}_{\text{ext}}$, the wrench estimate $\hat{\boldsymbol{f}}$ is obtained using (2) as

$$\hat{\boldsymbol{f}} = \left(\boldsymbol{J}^T\right)^+ \hat{\boldsymbol{\tau}}_{\text{ext}}. \tag{15}$$

Summarizing (10), (11), and (15), the generalized momentum observer approach to CCFE is described by

$$\dot{\hat{\boldsymbol{p}}} = \overline{\boldsymbol{\tau}} + \boldsymbol{L}\left(\boldsymbol{p} - \hat{\boldsymbol{p}}\right), \tag{16a}$$

$$\hat{\boldsymbol{f}} = -\left(\boldsymbol{J}^T\right)^+ \cdot \boldsymbol{L}\left(\boldsymbol{p} - \hat{\boldsymbol{p}}\right), \tag{16b}$$

with a positive definite diagonal matrix $\boldsymbol{L}$.

Remark 1: In the original publication [16], the generalized momentum observer (16) is written in an integral form as

$$\hat{\boldsymbol{\tau}}_{\text{ext}}(t) = \boldsymbol{L}\left[\boldsymbol{p}(t) - \boldsymbol{p}(0) - \int_0^t \left(\overline{\boldsymbol{\tau}}(\xi) + \hat{\boldsymbol{\tau}}_{\text{ext}}(\xi)\right) d\xi\right]. \tag{17}$$

We rely on the differential formulation (16) to ease the comparison of the standard generalized momentum observer approach to the extension proposed in this paper.

Remark 2: Notice that (15) minimizes $\left\|\hat{\boldsymbol{\tau}}_{\text{ext}} - \boldsymbol{J}^T\hat{\boldsymbol{f}}\right\|_2$ if no prior information is available. As described e.g. in [14], [15], prior knowledge on the contact forces and torques can be taken into account to obtain an improved estimate $\hat{\boldsymbol{f}}$ from $\hat{\boldsymbol{\tau}}_{\text{ext}}$.

IV. Kalman Filter Approach Based on the Generalized Momentum

The approach summarized in Section III assumes ideal friction estimation and is limited to diagonal observer matrices $\boldsymbol{L}$. In this section, we introduce a new approach that is capable of taking model uncertainties and disturbances into account. Furthermore, it provides more degrees of freedom in the observer design by dropping the constraint of a diagonal observer matrix.

A. General Idea

The key idea is to combine the description of the manipulator dynamics based on the generalized momentum (8) with well known disturbance observer approaches.

To this end, contact forces and torques are modeled as

$$\dot{\boldsymbol{f}} = \boldsymbol{A}_f \boldsymbol{f} + \boldsymbol{w}_f, \tag{18}$$

with $\boldsymbol{A}_f \in \mathbb{R}^{n_{\text{ext}} \times n_{\text{ext}}}$, $\boldsymbol{w}_f \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{Q}_{c,f})$, and $\boldsymbol{Q}_{c,f} \in \mathbb{R}^{n_{\text{ext}} \times n_{\text{ext}}}$. The subscript c indicates that $\boldsymbol{Q}_{c,f}$ is related to the dynamics of a continuous time system. The matrix $\boldsymbol{A}_f$ determines the dynamics assumed for the external wrench. The natural choice in a disturbance observer approach is $\boldsymbol{A}_f = \boldsymbol{0}_{n_{\text{ext}} \times n_{\text{ext}}}$. However, a negative definite diagonal matrix $\boldsymbol{A}_f$ allows to mitigate constant offsets in the wrench estimates due to disturbances. This aspect is further discussed in Section IV-C.

While advanced friction estimation and compensation techniques are available [21], [22], inaccuracies in friction estimation are inevitable. Details of friction estimation are beyond the scope of this paper. Focusing on estimating Cartesian contact forces and torques, a joint friction estimate $\hat{\boldsymbol{\tau}}_{\text{fric}}$ is assumed to be available, where uncertainties in friction estimates are modeled as random variables, i.e.

$$\boldsymbol{w}_p = \hat{\boldsymbol{\tau}}_{\text{fric}} - \boldsymbol{\tau}_{\text{fric}} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{Q}_{c,p}), \tag{19}$$

with $\boldsymbol{Q}_{c,p} \in \mathbb{R}^{N \times N}$. Conceptually, the term $\boldsymbol{w}_p$ can also describe other model uncertainties in the generalized momentum dynamics. However, based on an accurate model of the manipulator dynamics, inaccuracies in friction estimation are expected to have highest significance. Taking $\boldsymbol{w}_p$ into account and employing $\overline{\boldsymbol{\tau}}$ as introduced in (9), the generalized momentum dynamics (8) can be expressed as

$$\dot{\boldsymbol{p}} = \overline{\boldsymbol{\tau}} - \boldsymbol{J}^T \boldsymbol{f} + \boldsymbol{w}_p. \tag{20}$$

Defining the state vector $\boldsymbol{x} = [\boldsymbol{p}^T \;\; \boldsymbol{f}^T]^T \in \mathbb{R}^{N+n_{\text{ext}}}$ and combining process noise as $\boldsymbol{w} = [\boldsymbol{w}_p^T \;\; \boldsymbol{w}_f^T]^T \in \mathbb{R}^{N+n_{\text{ext}}}$ with the covariance matrix $\boldsymbol{Q}_c = \text{diag}\left(\boldsymbol{Q}_{c,p}, \boldsymbol{Q}_{c,f}\right) \in \mathbb{R}^{(N+n_{\text{ext}}) \times (N+n_{\text{ext}})}$, equations (18) and (20) can be combined into the augmented system dynamics

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{p}} \\ \dot{\boldsymbol{f}} \end{bmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} \boldsymbol{0}_{N \times N} & -\boldsymbol{J}^T \\ \boldsymbol{0}_{n_{\text{ext}} \times N} & \boldsymbol{A}_f \end{bmatrix}}_{\boldsymbol{A}_c} \underbrace{\begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{f} \end{bmatrix}}_{\boldsymbol{x}} + \underbrace{\begin{bmatrix} \boldsymbol{I}_N \\ \boldsymbol{0}_{n_{\text{ext}} \times N} \end{bmatrix}}_{\boldsymbol{B}_c} \underbrace{\overline{\boldsymbol{\tau}}}_{\boldsymbol{u}} + \boldsymbol{w}. \tag{21}$$

It is to be noticed that the augmented system matrix $\boldsymbol{A}_c$ contains the manipulator Jacobian $\boldsymbol{J}(\boldsymbol{q})$. Hence, the system (21) is time-varying.

Since $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ are assumed to be available from measurements, $\boldsymbol{p} = \boldsymbol{M}\dot{\boldsymbol{q}}$ can be regarded as a measurement. Assuming measurement noise $\boldsymbol{v} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{R}_c)$, the output equation of the augmented system dynamics can be written as

$$\underbrace{\boldsymbol{p}}_{\boldsymbol{y}} = \underbrace{\begin{bmatrix} \boldsymbol{I}_N & \boldsymbol{0}_{N\times n_{\text{ext}}} \end{bmatrix}}_{\boldsymbol{C}_c} \underbrace{\begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{f} \end{bmatrix}}_{\boldsymbol{x}} + \boldsymbol{v}. \tag{22}$$

Analyzing $\boldsymbol{A}_c$ and $\boldsymbol{C}_c$ reveals that the augmented system (21), (22) is structurally observable except for the manipulator residing in a singularity, i.e. observability is only lost if $\boldsymbol{J}(\boldsymbol{q})$ has a rank deficit. Furthermore, it is to be noticed that $\boldsymbol{y} = \boldsymbol{p} \in \mathbb{R}^N$. Thus, the number of available measurements increases with the number of links N, while the dimension of wrench is constant ($n_{\text{ext}} = 6$ in the general case). Consequently, redundant manipulators with $N > 6$ facilitate external wrench estimation, since more information is available compared to $N = 6$.

The approach proposed in this paper is to design a Kalman filter for the augmented system (21), (22). While one option is to design a continous-time Kalman-Bucy filter, an alternative way is to discretize the system in a first step and then designing a standard discrete time Kalman filter. Following [23, p. 215], the discretized matrices $\boldsymbol{A}^k$ and $\boldsymbol{B}^k$ can be obtained from

$$\begin{bmatrix} \boldsymbol{A}^k & \boldsymbol{B}^k \\ \boldsymbol{0}_{N\times(N+n_{\text{ext}})} & \boldsymbol{I}_N \end{bmatrix} = \exp\left(\begin{bmatrix} \boldsymbol{A}_c & \boldsymbol{B}_c \\ \boldsymbol{0}_{N\times(N+n_{\text{ext}})} & \boldsymbol{0}_{N\times N} \end{bmatrix} T_s\right) \tag{23}$$

for a given sampling time T_s. The discretized output matrix and measurement noise covariance matrix are given by

$$\boldsymbol{C} = \boldsymbol{C}_c, \quad \boldsymbol{R} = \frac{1}{T_s}\boldsymbol{R}_c. \tag{24}$$

From the results presented in [24], the discretized process noise covariance matrix can be inferred as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{A}_c & \boldsymbol{Q}_c \\ \boldsymbol{0}_{(N+n_{\text{ext}})\times(N+n_{\text{ext}})} & -\boldsymbol{A}_c^T \end{bmatrix}, \tag{25a}$$

$$\begin{bmatrix} \boldsymbol{M}_{11}^k & \boldsymbol{M}_{12}^k \\ \boldsymbol{0}_{(N+n_{\text{ext}})\times(N+n_{\text{ext}})} & \boldsymbol{M}_{22}^k \end{bmatrix} = \exp\left(\boldsymbol{H}\cdot T_s\right), \tag{25b}$$

$$\boldsymbol{Q}^k = \boldsymbol{M}_{12}^k \left(\boldsymbol{M}_{11}^k\right)^T. \tag{25c}$$

For an in-depth treatment of system discretization for Kalman filters, we refer to [25].

In summary, the discretized dynamics of the augmented system at time step k is given by

$$\boldsymbol{x}^{k+1} = \boldsymbol{A}^k\boldsymbol{x}^k + \boldsymbol{B}^k\boldsymbol{u}^k + \boldsymbol{w}^k, \tag{26a}$$

$$\boldsymbol{y}^k = \boldsymbol{C}\boldsymbol{x}^k + \boldsymbol{v}^k. \tag{26b}$$

For the time-varying discrete time linear system (26), a standard Kalman filter is designed based on the discretized covariance matrices $\boldsymbol{Q}^k$ and $\boldsymbol{R}$. Details of the implementation are given in the following section.

B. Implementation of the Approach

To implement the approach, the covariance matrix $\hat{\boldsymbol{P}}^0$ and initial estimate of the augmented system $\hat{\boldsymbol{x}}^0$ have to be initialized. Assuming the manipulator to be at standstill without external wrench, $\hat{\boldsymbol{P}}^0 = \boldsymbol{I}_{N+n_{\text{ext}}}$ and $\hat{\boldsymbol{x}}^0 = \boldsymbol{0}$ can be chosen. After parameterizing the matrices $\boldsymbol{A}_f$, $\boldsymbol{Q}_c$, and $\boldsymbol{R}_c$, the matrix $\boldsymbol{R}$ is calculated from (24). With this initialization, the following steps are to be executed in each time step.

1) Measure $\boldsymbol{q}^k$, $\dot{\boldsymbol{q}}^k$, $\boldsymbol{\tau}_{\text{mot}}^k$ and calculate $\boldsymbol{p}^k = \boldsymbol{M}(\boldsymbol{q}^k)\dot{\boldsymbol{q}}^k$
2) Compute friction estimate $\hat{\boldsymbol{\tau}}_{\text{fric}}$ and $\boldsymbol{u}^k = \overline{\boldsymbol{\tau}}^k = \boldsymbol{\tau}_{\text{mot}}^k + \boldsymbol{C}^T(\boldsymbol{q}^k, \dot{\boldsymbol{q}}^k)\dot{\boldsymbol{q}}^k - \boldsymbol{G}(\boldsymbol{q}^k) - \hat{\boldsymbol{\tau}}_{\text{fric}}^k$
3) Discretize the system using equations (23) and (25), resulting in $\boldsymbol{A}^k$, $\boldsymbol{B}^k$, and $\boldsymbol{Q}^k$
4) Predict state and covariance estimates

$$\check{\boldsymbol{x}}^k = \boldsymbol{A}^k\hat{\boldsymbol{x}}^{k-1} + \boldsymbol{B}^k\boldsymbol{u}^k, \tag{27a}$$

$$\check{\boldsymbol{P}}^k = \boldsymbol{A}^k\hat{\boldsymbol{P}}^{k-1}{\boldsymbol{A}^k}^T + \boldsymbol{Q}^k \tag{27b}$$

5) Calculate Kalman gain

$$\boldsymbol{K}^k = \check{\boldsymbol{P}}^k\boldsymbol{C}^T\left(\boldsymbol{C}\check{\boldsymbol{P}}^k\boldsymbol{C}^T + \boldsymbol{R}\right)^{-1} \tag{28}$$

6) Correct state and covariance estimates with measurement update

$$\hat{\boldsymbol{x}}^k = \check{\boldsymbol{x}}^k + \boldsymbol{K}^k\left(\boldsymbol{y}^k - \boldsymbol{C}\check{\boldsymbol{x}}^k\right), \tag{29a}$$

$$\hat{\boldsymbol{P}}^k = \left(\boldsymbol{I}_{N+n_{\text{ext}}} - \boldsymbol{K}^k\boldsymbol{C}\right)\check{\boldsymbol{P}}^k\left(\boldsymbol{I}_{N+n_{\text{ext}}} - \boldsymbol{K}^k\boldsymbol{C}\right)^T + \ldots$$
$$\ldots + \boldsymbol{K}^k\boldsymbol{R}{\boldsymbol{K}^k}^T \tag{29b}$$

7) Extract CCFE estimate $\hat{\boldsymbol{f}}^k$ from corrected state estimate $\hat{\boldsymbol{x}}^k$

$$\hat{\boldsymbol{f}}^k = \begin{bmatrix} \boldsymbol{0}_{n_{\text{ext}}\times N} & \boldsymbol{I}_{n_{\text{ext}}} \end{bmatrix}\hat{\boldsymbol{x}}^k \tag{30}$$

C. Tuning the Estimator

As described in Section IV-A, the matrices $\boldsymbol{Q}_c$ and $\boldsymbol{R}_c$ describe the process and measurement noise covariances. They are not known, but their values are to be tuned for the implementation to obtain good contact force estimates. Taking symmetry into account, the matrices $\boldsymbol{Q}_c$ and $\boldsymbol{R}_c$ contain $(N+n_{\text{ext}})(N+n_{\text{ext}}+1)/2$ and $N(N+1)/2$ variables, respectively. However, the tuning procedure can be simplified to a large extent. To this end, a diagonal matrix $\boldsymbol{R}_c$ with common diagonal elements can be assumed if the noise level in all joint angle/speed measurements is the same. For the process noise covariance matrix, a diagonal matrix $\boldsymbol{Q}_c$ also allows to obtain good estimates. The larger the weights of $\boldsymbol{Q}_{c,f}$ are chosen, the faster the estimator will respond to changes in $\boldsymbol{f}$. To obtain the same level of responsiveness in all n_{ext} components of wrench, common weights can be chosen. The key factor to exploit knowledge on friction estimates are the weights of $\boldsymbol{Q}_{c,p}$. The more uncertainty is expected in the friction estimate of a joint, the larger the corresponding weight should be chosen. The intuitive interpretation is that therewith, the Kalman filter will rely less on the dynamic model for the corresponding joint.

For parameterization of the contact force model (18), $\boldsymbol{A}_f = \boldsymbol{0}_{n_{\text{ext}}\times n_{\text{ext}}}$ can be regarded as a default choice. Choosing the fading matrix as $\boldsymbol{A}_f = -\text{diag}\left(a_{f,1}, \ldots, a_{f,n_{\text{ext}}}\right)$

with $a_{f,i} > 0$ can serve to mitigate the effects of model uncertainties. In practice, the values of $a_{f,i}$ can gradually be increased during the tuning process.

V. Results

A. YuMi Simulation

To verify applicability of the proposed method, it is employed to estimate contact forces acting onto the tool center point (TCP) of an ABB YuMi manipulator (formerly known as ABB Dual-Arm Concept Robot, DACR [26]). To do so, the manipulator is simulated, where the model incorporates rigid body dynamics, motor inertia, joint friction, and measurement noise. In the simulation, the manipulator is operated in position control mode. The TCP follows a predefined trajectory and during the movement of the manipulator, an abrupt contact force in y-direction with magnitude $10N$ is simulated as well as an incipient contact force in x-direction. As depicted in Fig. 2, joint angles $\boldsymbol{q}$, joint speeds $\dot{\boldsymbol{q}}$ and motor torques $\boldsymbol{\tau}_{\text{mot}}$ are the inputs to the CCFE scheme.

In the estimation scheme described in Section IV-B, the diagonal elements of $\boldsymbol{Q}_{c,p}$ are chosen to reflect the different levels of uncertainty in the corresponding joint friction torques, where the values have been obtained from friction estimation experiments. To enable fast tracking of abrupt contact forces and torques, common weights are chosen for the diagonal elements of $\boldsymbol{Q}_{c,f}$. These elements describing the covariance of external wrench are selected to have higher values (four orders of magnitude) compared to the covariance values for $\boldsymbol{Q}_{c,p}$. The measurement covariance matrix $\boldsymbol{R}_c$ contains common diagonal elements, where the elements are designed to limit noise amplification in the contact force estimates.

For comparison, a generalized momentum observer is implemented using (16). The diagonal elements of the observer matrix are chosen such that the response time of the observer to contact forces is similar to the one achieved by the Kalman filter. Fig. 3 visualizes the simulated and estimated contact forces. It can be observed that the methods do not differ much with respect to response time and noise level in the estimates. However, the generalized momentum observer suffers from inaccuracies in joint friction estimation and shows considerable offsets in the estimates, while the proposed new approach results in improved accuracy.

B. Discussion

As elaborated in Section III, the key feature of the generalized momentum observer approach introduced in [2], [16] is that no joint accelerations are involved and the inertia matrix does not have to be inverted. Furthermore, the approach comes with an inherent decoupling property, since the estimated joint torque $\hat{\tau}_{\text{ext},i}$ only depends on $\tau_{\text{ext},i}$ as shown in equation (14). This enables to apply the approach for collision localization along the manipulator links [3],

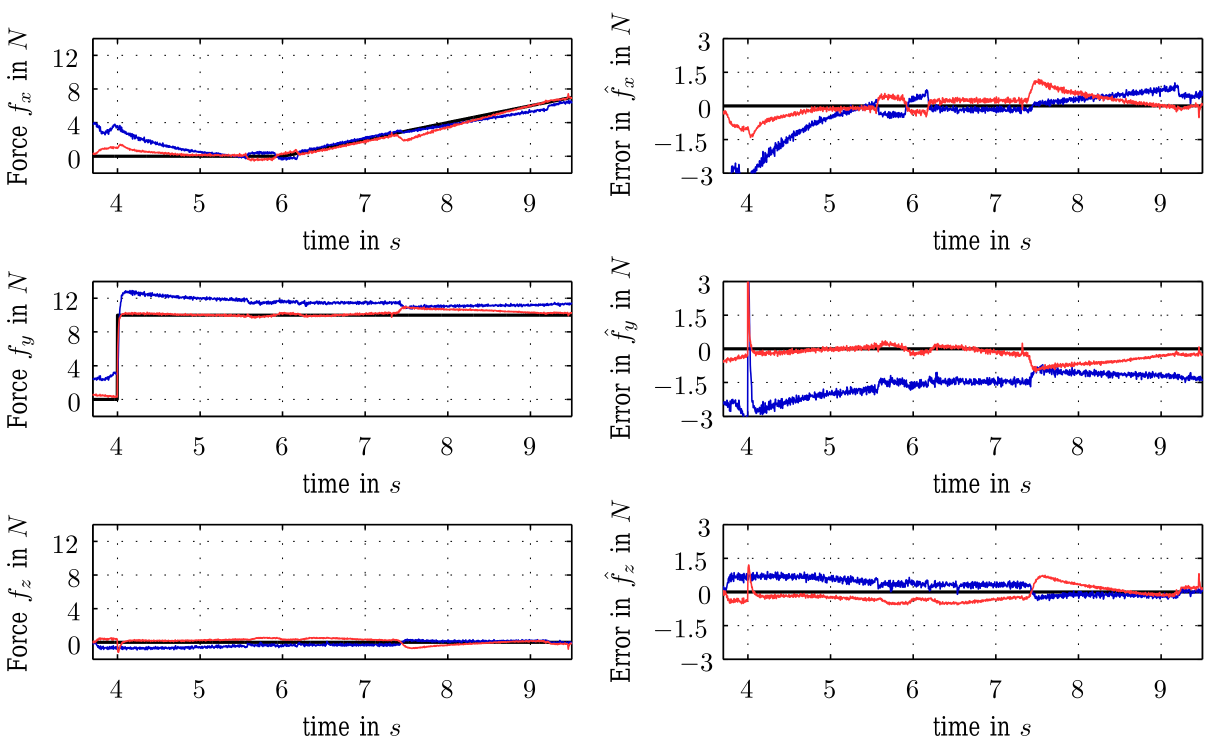

Fig. 3. Comparison of contact force estimates for the standard generalized momentum observer approach (dark blue lines) and the proposed Kalman filter approach (light red lines) – The left column shows simulated contact forces (black lines) as well as corresponding estimates for f_x, f_y, and f_z. At $t = 4s$, an abrupt force of $10N$ acts onto the tool center point (TCP) of the manipulator in y-direction. From $t = 6s$, an incipient contact force in x-direction affects the TCP, while the contact force in z-direction is constantly 0. The right column displays the corresponding errors in the contact force estimates. While both approaches are comparable in terms of estimation speed and noise attenuation, the proposed new solution achieves improved estimation accuracy, especially for f_y in this case.

[17]. While the generalized momentum observer is comparatively easy to tune due to the limited number of parameters (diagonal elements of $\boldsymbol{L}$), an ideal friction estimation is assumed and disturbances are not handled systematically.

The Kalman filter for the augmented system involving the generalized momentum of the manipulator and a model of contact forces shares the benefit of avoiding inversion of $\boldsymbol{M}$ and computation of $\ddot{\boldsymbol{q}}$. In the proposed form, it does not come with the decoupling property of the generalized momentum observer, which is not required for estimating wrench at the TCP. In fact, the decoupling property constrains the structure of the estimator as elaborated in [27]. In the proposed new approach, the constraint is dropped and the additional degrees of freedom are exploited to systematically take disturbances into account. For redundant manipulators, the additional degrees of freedom are therewith implicitly exploited to increase robustness. Furthermore, introducing fading in the model of contact forces as in equation (18) has the potential to further reduce estimation errors.

VI. Conclusion and Future Work

We presented a new approach to Cartesian contact force estimation using motor torques as well as joint angles and speeds only. It relies on extending the well established generalized momentum observer based approach with a disturbance observer. The design allows to take inevitable inaccuracies in the model (e.g. friction estimates) into account and is capable of exploiting the additional degrees of freedom in redundant manipulators to obtain improved wrench estimates.

While the method has proven beneficial in simulations, the natural next step is to verify the approach with experimental data. Furthermore, simplifying the parameterization of the proposed approach will be subject to future research. In the paper, Gaussian distributions were assumed for both process and measurement noise. The results might be further improved by considering other distributions (see e.g. [13] for an analysis of inaccuracies in friction estimation) and adapting the estimator for the augmented system accordingly. Another direction of research could be to extend the results from external wrench estimation at the TCP to estimating contact forces and torques at arbitrary locations on the manipulator.

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