True mean and variance :

Consider a particular random walker. Let him take k steps forward, which makes $n-k$ backward steps.

Probability for taking k steps forward

$$= {}^nC_k \times (1/2)^k (1/2)^{n-k}$$

$$= {}^nC_k (1/2)^n$$

Displacement of this particular random walker

$$x = dk - d(n-k)$$

$$x = d(2k-n)$$

Therefore probability of having the final location x

$$P(x=x) = {}^nC_{\left(\frac{n}{2}+\frac{x}{2d}\right)} (1/2)^n$$

$n \to \infty$

for calculation simplicity $k = n/2 + \frac{x}{2d}$

$$P(x=x) = {}^nC_k (1/2)^n$$

We have the approximation $n! = n^n \times e^{-n} \times \sqrt{2\pi n}$

$$P(x=x) = \frac{n!}{k!\,(n-k)!} \left(\frac{1}{2}\right)^n$$

$$P(x) = \left(\frac{n}{2k}\right)^k \times \left(\frac{n}{2(n-k)}\right)^{n-k} \times \sqrt{\frac{n}{2\pi k(n-k)}}$$

$$= e^{-\frac{\delta^2}{2npq}} \quad \text{where } \delta = k - n/2$$

$$P(x) = e^{-\frac{\left(k-\frac{n}{2}\right)^2}{2npq}} \times \frac{1}{n} \sqrt{\frac{1}{2\pi \frac{k(n-k)}{n}}}$$

$$= \frac{1}{\sqrt{2\pi pqn}} \times e^{-\frac{\left(k-\frac{n}{2}\right)^2}{2npq}}$$

AS $k = \frac{x}{2d} + \frac{n}{2}$

True mean and variance:

Consider a particular random walker. Let him/her take k Super

$$P(x) = \frac{1}{\sqrt{2\pi pqn}} \times e^{\frac{-\left(\frac{x}{2d}\right)^2}{2npq}}$$

where $p = q = 1/2$,

n = no. of steps

d = distance travelled in each step

$$P(x) = \sqrt{\frac{2}{\pi n}} \; e^{\frac{-x^2}{2nd^2}} \longrightarrow \frac{(x-\mu)^2}{2\sigma^2}$$

so from standard formula $\mu(\text{mean}) = 0$ &

$$\sigma^2 = nd^2$$

$$\sigma^2 = 10^3 \times (10^{-3})^2$$

$$= 0.001$$

True mean and variance is 0.00 and 0.001