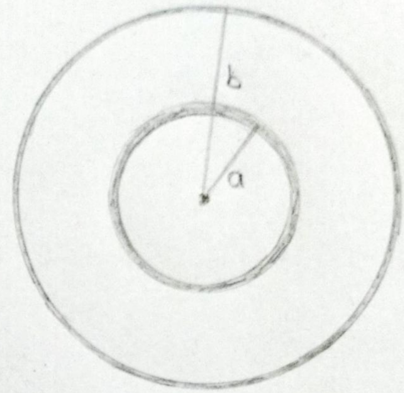
Q.1 a) Explanation of square root on uniform distribution
($d = \sqrt{rand}$)

$$\frac{P(b<X<b+dx)}{P(a<X<a+dx)} = \frac{2\pi b \times dx}{2\pi a \times dx} = \frac{b}{a}$$

$$\Rightarrow \frac{P(x=b)}{P(x=a)} = \frac{b}{a}$$



$\Rightarrow P(x=r) = kr$, k = positive constant

$$\int_0^R P(x=r)\,dr = \int_0^R (kr)\,dr = 1$$

$$\frac{kR^2}{2} = 1$$

$$\boxed{k = \frac{2}{R^2}} \quad \therefore P(x=r) = \frac{2r}{R^2}$$

$(x,y) = (a\cos\theta, b\sin\theta)$

$\left(\frac{x}{a}, \frac{y}{b}\right) = (\cos\theta, \sin\theta)$ [circle of radius 1, stretched radii : semi-major axis, semi-minor axis]

for $R=1$

$P(X=r) = 2r$

$CDF(X=r) = r^2$

$U = [0,1]$, Let $X = T(U)$; U = uniform distribution

$$P(X \le x) = x^2 = P(T(U) \le x)$$
$$= P(U \le T^{-1}(x))$$
$$= T^{-1}(x)$$

$\therefore T(x) = \sqrt{x}$

$\therefore X = \sqrt{U}$