

## CS215 Assignment-1

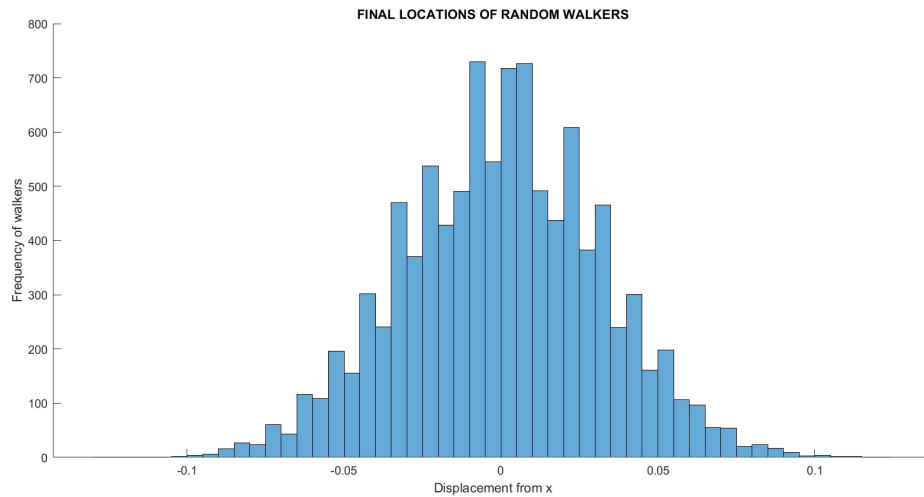
210050115 Patil Vipul Sudhir

210050119 Hari Prakash Reddy

# 1 Part - I

## 1.1 Histogram Plot

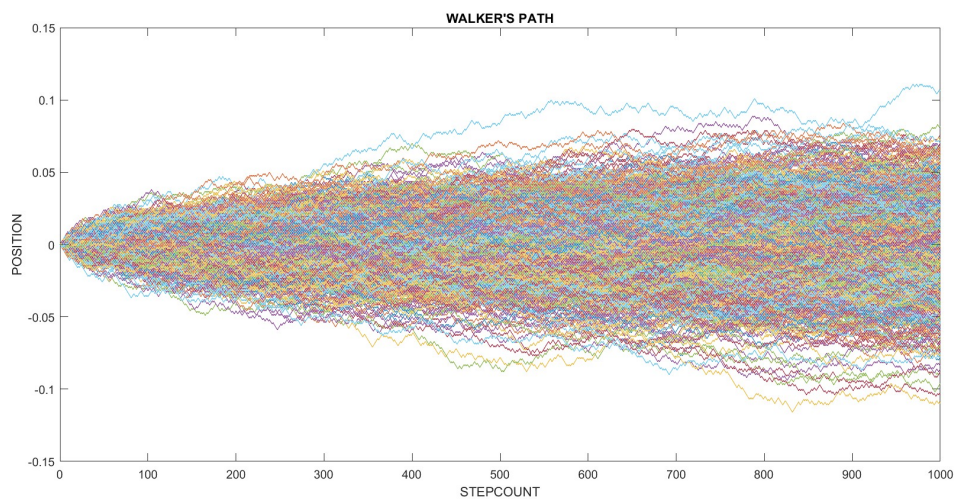
We are required to plot the histogram of all the final locations. In the MATLAB WALKERHIST.m, the function WALKERHIST is used to plot the histogram of their final locations. As walkers can have 2 options - either go towards right or towards left so We used binornd(.) function to decide increasing or decreasing at each step.



This plot is generated on running the MATLAB code final WALKERHIST.m and graph is saved to the results directory.

## 1.2 Space-Time curve

Plot for the first  $10^3$  walkers is given below with different colors. We have randomly generated  $10^4$  walkers and plotted the displacement -time graph of each each walker as shown below



### 1.3 Sub part -3

We have Random variable  $X$  and the dataset  $(X_1, \dots, X_N)$  from the distribution  $X$

#### 1.3.1 Mean

**Law of large numbers(LLN)** : result obtained over large number of trails should be close to the expected value and will tend to be more closer as number of trails increases. As  $X_1, \dots, X_N$  are from the distribution  $X$  mean of them over  $N \rightarrow \infty$  should be the mean of the distribution  $X$  from the above LLN theorem. therefore

$$\hat{M} = \frac{(X_1 + \dots + X_N)}{N}$$

#### 1.3.2 Variance

General variance relation is:

$$\begin{aligned} \hat{V} &= \sum_{i=0}^N \frac{(X - \hat{M})^2}{N} = \sum_{i=0}^N \frac{X^2 - 2X\hat{M} + \hat{M}^2}{N} \\ &= \sum_{i=0}^N \frac{X^2}{N} - \hat{M}^2 \end{aligned} \quad (1)$$

The above is the another form of variance-mean relation, now by applying law of large numbers (LLN) on the first summation term  $\sum_{i=0}^N \frac{X^2}{N}$ , we consider new Random variable  $Y = X^2$  and apply LLN on the first summation term we replace the average of squares of  $X$  by the expected square of  $X$  we get expected square of  $X$  from the true  $\text{Var}(X)$  for  $N \rightarrow \infty$  case, as we have already proved that mean is true mean of  $X$  in  $N \rightarrow \infty$  case.

$$\text{Var}(X) = (\text{expected square of } X) - \hat{M}^2(\text{expected square of } X) = \text{Var}(X) + \hat{M}^2 \quad (2)$$

Therefore substituting the 1 in 2, we get

$$\hat{V} = (\text{Var}(X) + \hat{M}^2) - \hat{M}^2 \hat{V} = \text{Var}(X)$$

Hence that is the required result for us

### 1.4 Mean and Variance of final locations

Empirical mean variance their values in this case are

$$\hat{M} = -0.002000$$

$$\hat{V} = 0.000000$$

### 1.5 True mean and Variance

#### 1.5.1 True mean

Consider a path which has final displacement  $+x$ , now interchange the forward with backward and viceversa (i.e if a man makes a particular step forward interchange with backward) for this path we get the new final location  $-x$ , therefore for every path  $+$  and  $-$  comes in pairs which makes the mean of all final locations to be 0, however detailed proof is given below

#### 1.5.2 True variance

**Attached a Handwritten PDF - PROOF**

## 1.6 Error

$$\hat{M} = -0.002000 - 0$$

$$\hat{M} = -0.002000$$

$$\hat{V} = 0.0000000 - 0.001000$$

$$\hat{V} = -0.001000$$

### Instructions to run the code

Run the WALKERHIST.m code in the codes section on MATLAB, this gives the histogram of final locations of Walkers.

Run the WALKERTRAVERSAL.m code in the codes section on MATLAB, this gives the space-time curve of each walker.

Run the WALKERSTATISTICS.m code in the codes section on MATLAB, this gives the mean and median of the WalkTraversal graph.