CS215 Assignment-2

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1 Multivariate Gaussian

1.1 Random Number Generation

We know that $Y = AW + \mu$ and A is found using the Covariance matrix

We know that $Y = AW + \mu$ and $C = AA^T$

→ Explanation to get A from C :

$[Q , D] = \text{eig}(c)$

eig(c) returns orthogonal matrix Q which has columns as eigenvector of C and D is a diagonal matrix with diagonal elements as eigenvalues corresponding to eigenvector present in Q.

$\therefore\ CQ = QD$

as $|Q| \neq 1$

$Q^{-1}CQ = D$

Q is orthogonal

$\therefore\ Q^{-1} = Q^T$

$Q^T C Q = D$

$Q^T A A^T Q = D$

$$(Q^T A)(Q^T A)^T = \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ & & & \sqrt{\lambda_n} \end{bmatrix} \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ & & & \sqrt{\lambda_n} \end{bmatrix}$$

$Q^T A = \text{sqrt}(D)$

$\underline{\underline{A = Q * \text{sqrt}(D)}}$

W can be generated using randn

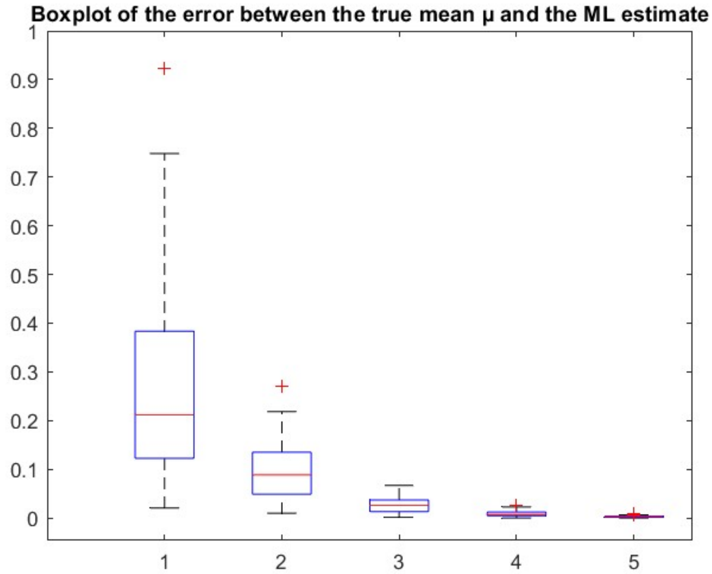
Instructions to run the code:

Run multivariate_gaussuian.m , it generates random numbers based on given PDF of multivariate gaussian.

1.2 Boxplot of the error between the true mean μ and the ML estimate μ_N

ML estimate μ_N:

Value of N	Value of ML estimate μ_N
10	[-0.3284 , 3.8600]’
100	[0.9162 , 2.2217]’
1000	[1.0109 , 1.9373]’
10000	[0.9985 , 2.0079]’
100000	[0.9998 , 2.0037]’

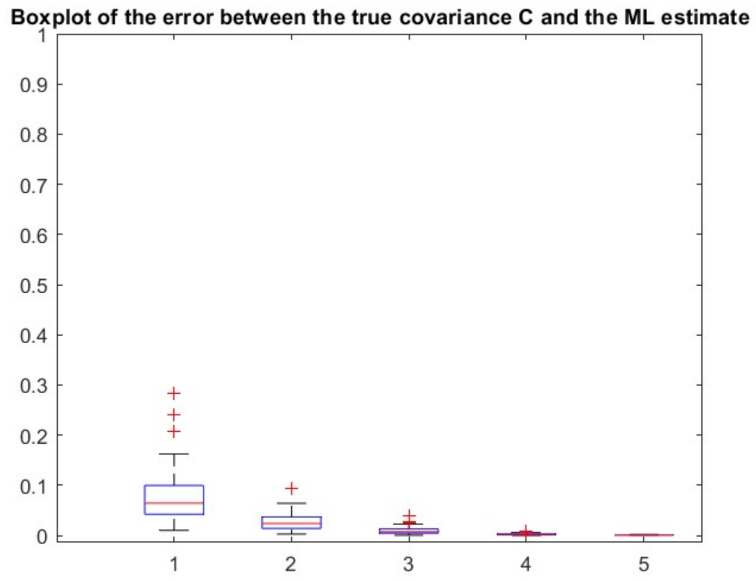


Instructions to run the code:
Run multivariate_mean.m from code folder of Q2 for ML estimate of the mean, mean_Boxplot.m for Boxplot of errors in empirical mean.

1.3 Boxplot of the error between the true covariance C and the ML estimate $\mathbf{C}_N$

ML estimate C_N:

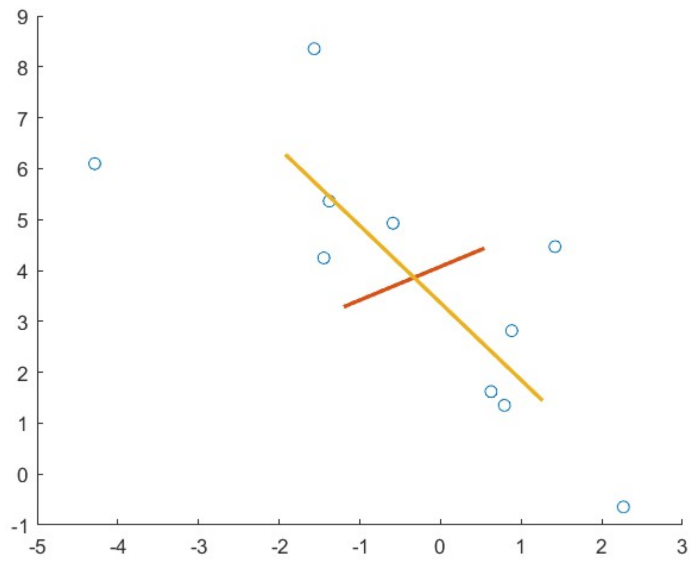
Value of N	Value of ML estimate C_N
10	$\begin{bmatrix} 5.0702 & -5.8263 \\ -5.8263 & 9.6411 \end{bmatrix}$
100	$\begin{bmatrix} 1.8754 & -2.2874 \\ -2.2874 & 4.6412 \end{bmatrix}$
1000	$\begin{bmatrix} 1.6770 & -1.9291 \\ -1.9291 & 3.6975 \end{bmatrix}$
10000	$\begin{bmatrix} 1.6275 & -1.9109 \\ -1.9109 & 3.7212 \end{bmatrix}$
100000	$\begin{bmatrix} 1.6179 & -1.9393 \\ -1.9393 & 3.8656 \end{bmatrix}$



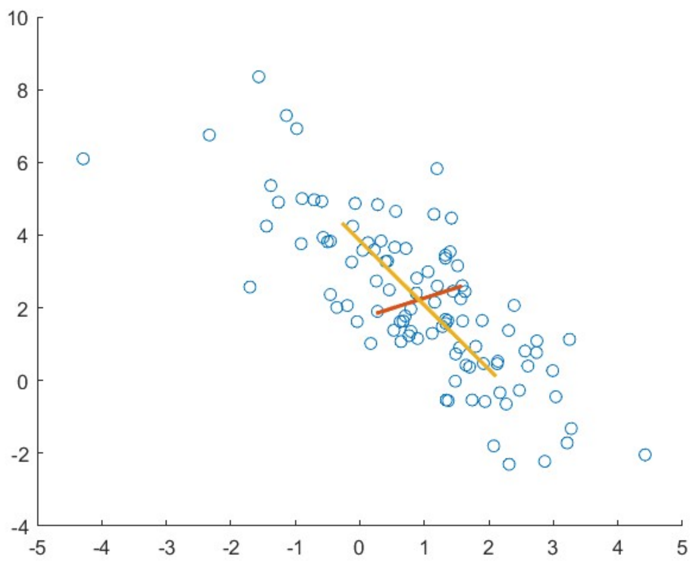
Instructions to run the code:
Run multivariate_covariance.m from code folder of Q2 for ML estimate of covariance, covariance_Boxplot.m for Boxplot of errors in empirical covariance

1.4 Principal modes of variation of the data

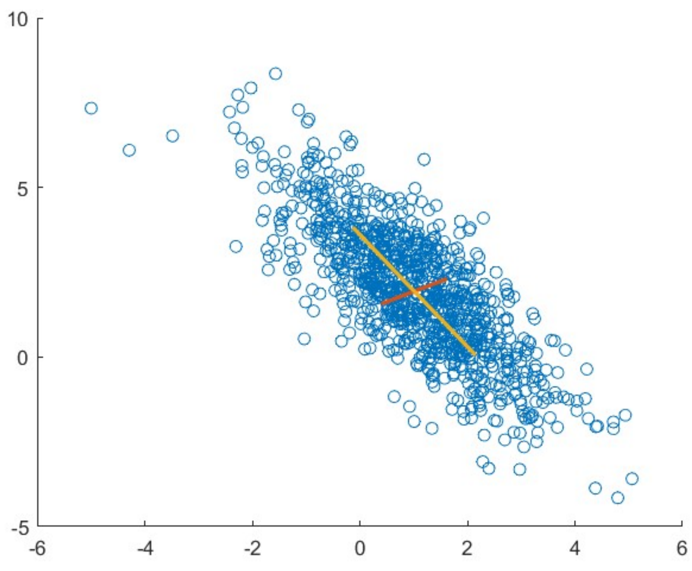
N=10



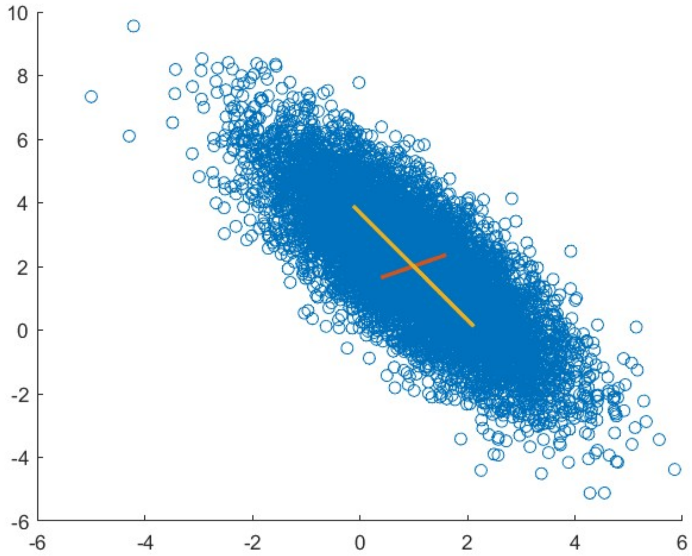
N=100



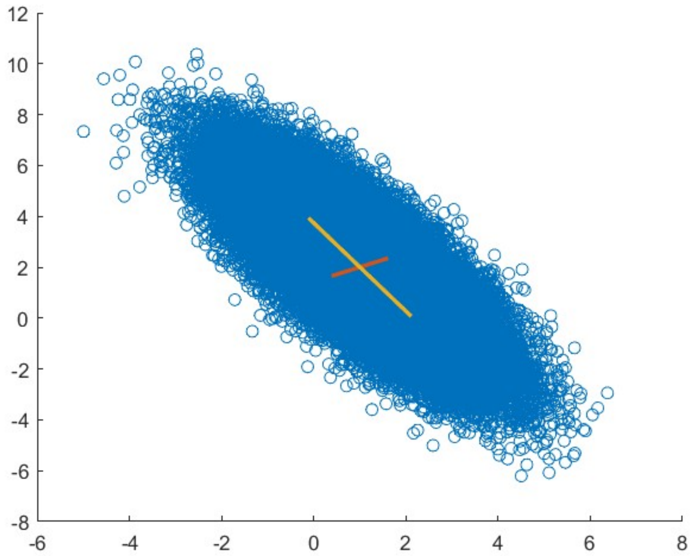
N=1000



N=10000



N=100000



Instructions to run the code:

Run q2d.m with argument N where N belongs to {10,100,1000,10000,100000} , It gives scatterplot