CS215 Assignment-1

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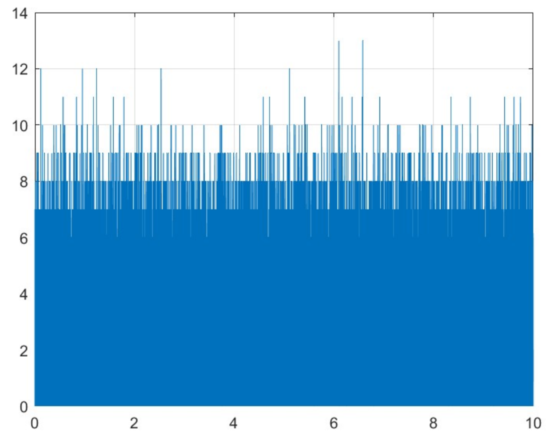
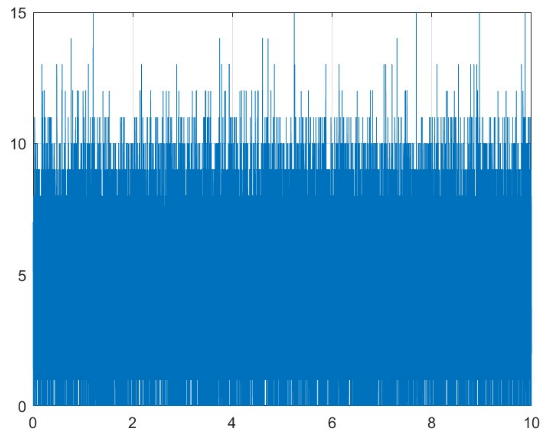
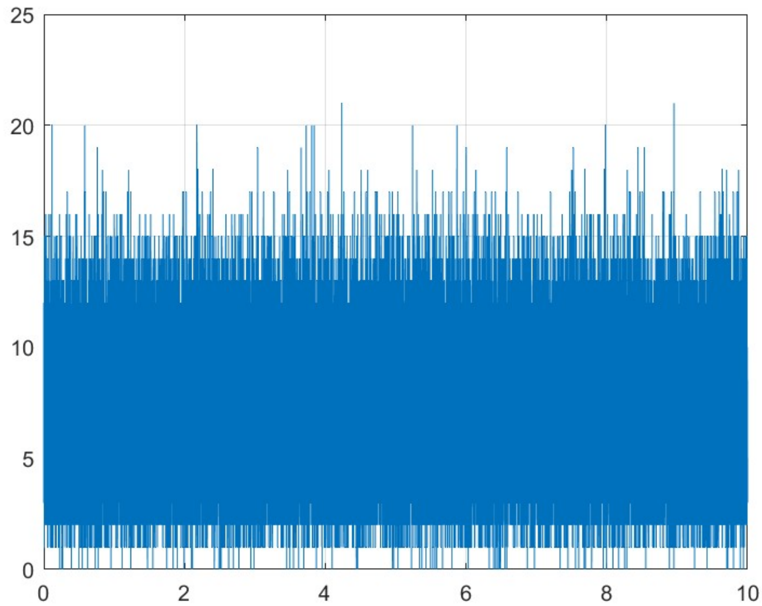
1 Part 1

Given two independent random variables X and Y with $\lambda x = 3$ and $\lambda y = 4$. For a random poisson distribution graphs and data are collected from the running the program POISSONHIST.m , they are :-







Figures in first row are plot for the random variable X, Y respectively and Figure in 2nd row is plot for random variable Z = X + Y.

The average value of Z over 10^6 trials is 7.003274. which is close to the theoretical value $\lambda Z = 7$ ($\lambda Z = \lambda X + \lambda Y$)

(i) Frequencies of P(Z= k) for k = 0,1,2,......,25 are given below:

FREQUENCY OF 0 :901 PMF calculated is :0.000901

FREQUENCY OF 1 :6388 PMF calculated is :0.006388

FREQUENCY OF 2 :22126 PMF calculated is :0.022126

FREQUENCY OF 3 :52098	PMF calculated is :0.052098
FREQUENCY OF 4 :91363	PMF calculated is :0.091363
FREQUENCY OF 5 :127482	PMF calculated is :0.127482
FREQUENCY OF 6 :149081	PMF calculated is :0.149081
FREQUENCY OF 7 :149544	PMF calculated is :0.149544
FREQUENCY OF 8 :130130	PMF calculated is :0.130130
FREQUENCY OF 9 :101393	PMF calculated is :0.101393
FREQUENCY OF 10 :70890	PMF calculated is :0.070890
FREQUENCY OF 11 :45102	PMF calculated is :0.045102
FREQUENCY OF 12 :26772	PMF calculated is :0.026772
FREQUENCY OF 13 :14079	PMF calculated is :0.014079
FREQUENCY OF 14 :7024	PMF calculated is :0.007024
FREQUENCY OF 15 :3229	PMF calculated is :0.003229
FREQUENCY OF 16 :1449	PMF calculated is :0.001449
FREQUENCY OF 17 :594	PMF calculated is :0.000594
FREQUENCY OF 18 :228	PMF calculated is :0.000228
FREQUENCY OF 19 :76	PMF calculated is :0.000076
FREQUENCY OF 20 :37	PMF calculated is :0.000037
FREQUENCY OF 21 :13	PMF calculated is :0.000013
FREQUENCY OF 22 :1	PMF calculated is :0.000001
FREQUENCY OF 23 :0	PMF calculated is :0.000000
FREQUENCY OF 24 :0	PMF calculated is :0.000000
FREQUENCY OF 25 :0	PMF calculated is :0.000000

(ii)

Average theoretical value is 7. Theoretical PMF is given by -

$$\mathbf{P(Z = k) = \frac{e^{-7}7^k}{k!}}$$

However from the average obtained from 10^6 instances, PMF looks like,

$$\mathbf{P(Z = k) = \frac{e^{-7.000616}(7.000616)^k}{k!}}$$

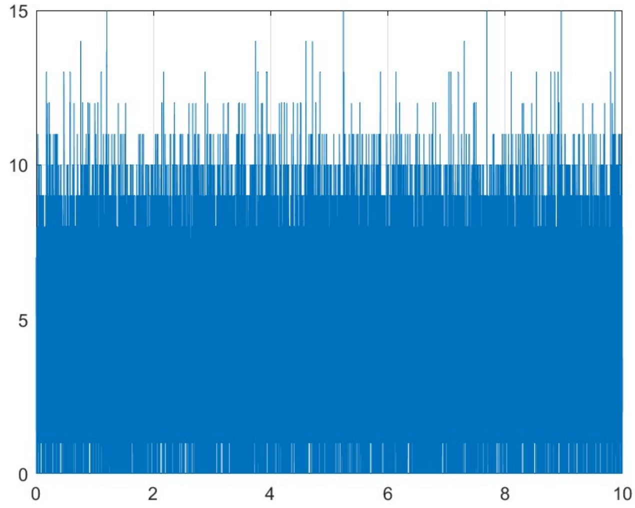
(iii)Comparison

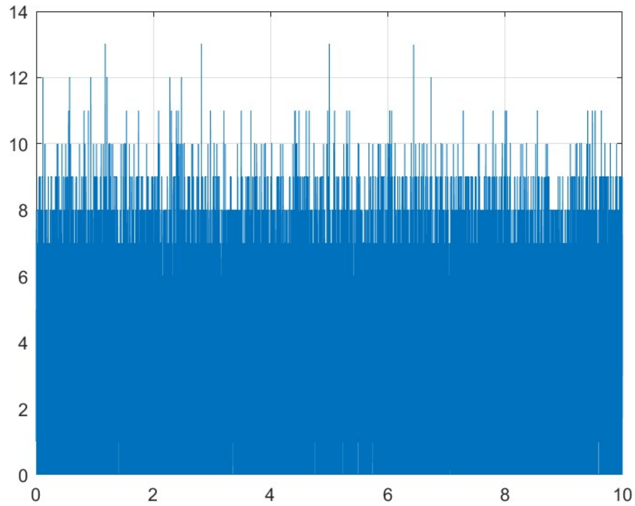
Given are the $\hat{P}$(Z=K) and P(Z=K) values for k = 0,1,2,...,25

PMF calculated is :0.000901 PMF theoretical: 0.000910548857645124
PMF calculated is :0.006388 PMF theoretical: 0.006375174136494603
PMF calculated is :0.022126 PMF theoretical: 0.022317772917611955
PMF calculated is :0.052098 PMF theoretical: 0.05208568710835406
PMF calculated is :0.091363 PMF theoretical: 0.09116900277967949
PMF calculated is :0.127482 PMF theoretical: 0.12766327994176463
PMF calculated is :0.149081 PMF theoretical: 0.14897162182848453
PMF calculated is :0.149544 PMF theoretical: 0.1490027568974467
PMF calculated is :0.130130 PMF theoretical: 0.13040466116443347
PMF calculated is :0.101393 PMF theoretical: 0.10144704557447977
PMF calculated is :0.070890 PMF theoretical: 0.0710277736049034
PMF calculated is :0.045102 PMF theoretical: 0.04520893898791888
PMF calculated is :0.026772 PMF theoretical: 0.02637739279943096
PMF calculated is :0.014079 PMF theoretical: 0.014206179978590945
PMF calculated is :0.007024 PMF theoretical: 0.007104574535103236
PMF calculated is :0.003229 PMF theoretical: 0.003316161049217834
PMF calculated is :0.001449 PMF theoretical: 0.0014511236805087402
PMF calculated is :0.000594 PMF theoretical: 0.000597646397500339
PMF calculated is :0.000228 PMF theoretical: 0.00023246661884343984
PMF calculated is :0.000076 PMF theoretical: 8.56634963456551e-05
PMF calculated is :0.000037 PMF theoretical: 2.9988490005736973e-05
PMF calculated is :0.000013 PMF theoretical: 9.998252533382723e-06
PMF calculated is :0.000001 PMF theoretical: 3.1819270535061545e-06
PMF calculated is :0.000000 PMF theoretical: 9.68614979731407e-07
PMF calculated is :0.000000 PMF theoretical: 2.8257174757646654e-07
PMF calculated is :0.000000 PMF theoretical: 7.913662542007881e-08

2 Part 2

Given Y = 4, we need to thin the Y with probability factor of 0.8, The function poissrnd(.) creates 10^5 instances of poisson distribution and for each instance we pass the instance through binornd(.) this gives the random instance of binomial distribution for each instance of poisson distribution. Given are the graphs of random variables Y and Z in the next page.
Theoretically λZ = λY $\times$ P, i.e theoretical λZ = 0.8 $\cdot$ 4 which is 3.2, the λZ obtained from the random poisson generator is 3.209300, which are close to our theoretica 3.2





(i) Frequencies of P(Z= k) for k = 0,1,2,......,25 are

FREQUENCY OF 0: 4056	PMF calculated is :0.004056
FREQUENCY OF 1: 13156	PMF calculated is :0.013156
FREQUENCY OF 2: 20677	PMF calculated is :0.020677
FREQUENCY OF 3: 22184	PMF calculated is :0.022184
FREQUENCY OF 4: 17698	PMF calculated is :0.017698
FREQUENCY OF 5: 11432	PMF calculated is :0.011432
FREQUENCY OF 6: 6242	PMF calculated is :0.006242
FREQUENCY OF 7: 2798	PMF calculated is :0.002798
FREQUENCY OF 8: 1169	PMF calculated is :0.001169

FREQUENCY OF 9: 408	PMF calculated is :0.000408
FREQUENCY OF 10: 139	PMF calculated is :0.000139
FREQUENCY OF 11: 30	PMF calculated is :0.000030
FREQUENCY OF 12: 9	PMF calculated is :0.000009
FREQUENCY OF 13: 2	PMF calculated is :0.000002
FREQUENCY OF 14: 0	PMF calculated is :0.000000
FREQUENCY OF 15: 0	PMF calculated is :0.000000
FREQUENCY OF 16: 0	PMF calculated is :0.000000
FREQUENCY OF 17: 0	PMF calculated is :0.000000
FREQUENCY OF 18: 0	PMF calculated is :0.000000
FREQUENCY OF 19: 0	PMF calculated is :0.000000
FREQUENCY OF 20: 0	PMF calculated is :0.000000
FREQUENCY OF 21: 0	PMF calculated is :0.000000
FREQUENCY OF 22: 0	PMF calculated is :0.000000
FREQUENCY OF 23: 0	PMF calculated is :0.000000
FREQUENCY OF 24: 0	PMF calculated is :0.000000
FREQUENCY OF 25: 0	PMF calculated is :0.000000

(ii)

Average theoretical value is 3.2. Theoretical PMF is given by -

$$\mathbf{P(Z = k) = \frac{e^{-3.2} 3.2^k}{k!}}$$

However from the average obtained from 10^6 instances, PMF looks like,

$$\mathbf{P(Z = k) = \frac{e^{-3.209300} (3.209300)^k}{k!}}$$

(iii) Comparison

Given are the $\hat{P}$(Z=K) and P(Z=K) values for k = 0,1,2,...,25

PMF calculated is :0.04056	PMF theoretical: 0.04062
PMF calculated is :0.13156	PMF theoretical: 0.13013
PMF calculated is :0.20677	PMF theoretical: 0.20844
PMF calculated is :0.22184	PMF theoretical: 0.22256
PMF calculated is :0.17698	PMF theoretical: 0.17824
PMF calculated is :0.11432	PMF theoretical: 0.11419
PMF calculated is :0.06242	PMF theoretical: 0.06096
PMF calculated is :0.02798	PMF theoretical: 0.06096

PMF calculated is :0.01169	PMF theoretical: 0.01117
PMF calculated is :0.00408	PMF theoretical: 0.003976
PMF calculated is :0.00139	PMF theoretical: 0.0012737
PMF calculated is :0.00030	PMF theoretical: 0.00037
PMF calculated is :0.00009	PMF theoretical: 9.90189e-05
PMF calculated is :0.00002	PMF theoretical: 2.43994e-05
PMF calculated is :0.000000	PMF theoretical: 1.1915e-06
PMF calculated is :0.000000	MF theoretical: 2.387005e-07
PMF calculated is :0.000000	PMF theoretical: 4.49788e-08
PMF calculated is :0.000000	PMF theoretical: 8.004622e-09
PMF calculated is :0.000000	PMF theoretical: 1.34955e-09
PMF calculated is :0.000000	PMF theoretical: 2.1615e-10
PMF calculated is :0.000000	PMF theoretical: 3.2977e-11
PMF calculated is :0.000000	PMF theoretical: 4.80e-12
PMF calculated is :0.000000	PMF theoretical: 6.6865e-13
PMF calculated is :0.000000	PMF theoretical: 8.9248e-14
PMF calculated is :0.000000	PMF theoretical: 1.14358e-14

Instructions to run code

- Run POISSONHIST.m in codes section on MATLAB to get the graphs of λ=3,4,7 on first page.
- Run POISSON.m in codes section on MATLAB to get the Experimental frequency,probability and mean of Poisson distribution in part a.
- Run THINNING.m in codes section on MATLAB to get Experimental frequency, probability and mean of Poissonthinning distribution in part b.