CS215 Assignment-3

210050115 Patil Vipul Sudhir

210050119 Hari Prakash Reddy

1 PDF of random variable Y

PDF of X is given by:

$$p(X = x) = 1 \ \ for\ 0 \leq x \leq 1$$

If Y=g(X) and pdf of X is p(x) then pdf of Y,

$$q(Y = y) = p(g^{-1}(y)) \times |\frac{d}{dy}(g^{-1}(y)|$$

Given $Y = -\frac{1}{\lambda}log(X)$ and it's inverse function $x = e^{-\lambda y}$

PDF of Y, q(Y=y) is

$$\begin{aligned} q(y) &= p(e^{-\lambda y}) \times |\frac{d}{dy}e^{-\lambda y}| \\ &= \lambda e^{-\lambda y} \end{aligned}$$

2 Maximum Likelihood Estimation (for sample size N)

$$\begin{aligned} Likelihood\ function &= \prod_{i=1}^{N} \lambda e^{-\lambda y_i} \\ &= \lambda^N e^{-\lambda \times \sum_{i=1}^{N} y_i} \end{aligned}$$

MLE for λ is the derivative of the Likelihood function with respect to λ and equating it to zero

$$\frac{d}{d\lambda}(\lambda^N e^{-\lambda \times \sum_{i=1}^{N} y_i}) = 0$$

$$\lambda_{MLE} = \frac{N}{\sum_{i=1}^{N} y_i}$$

3 Posterior Distribution

Given prior is Gamma distribution

$$prior = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

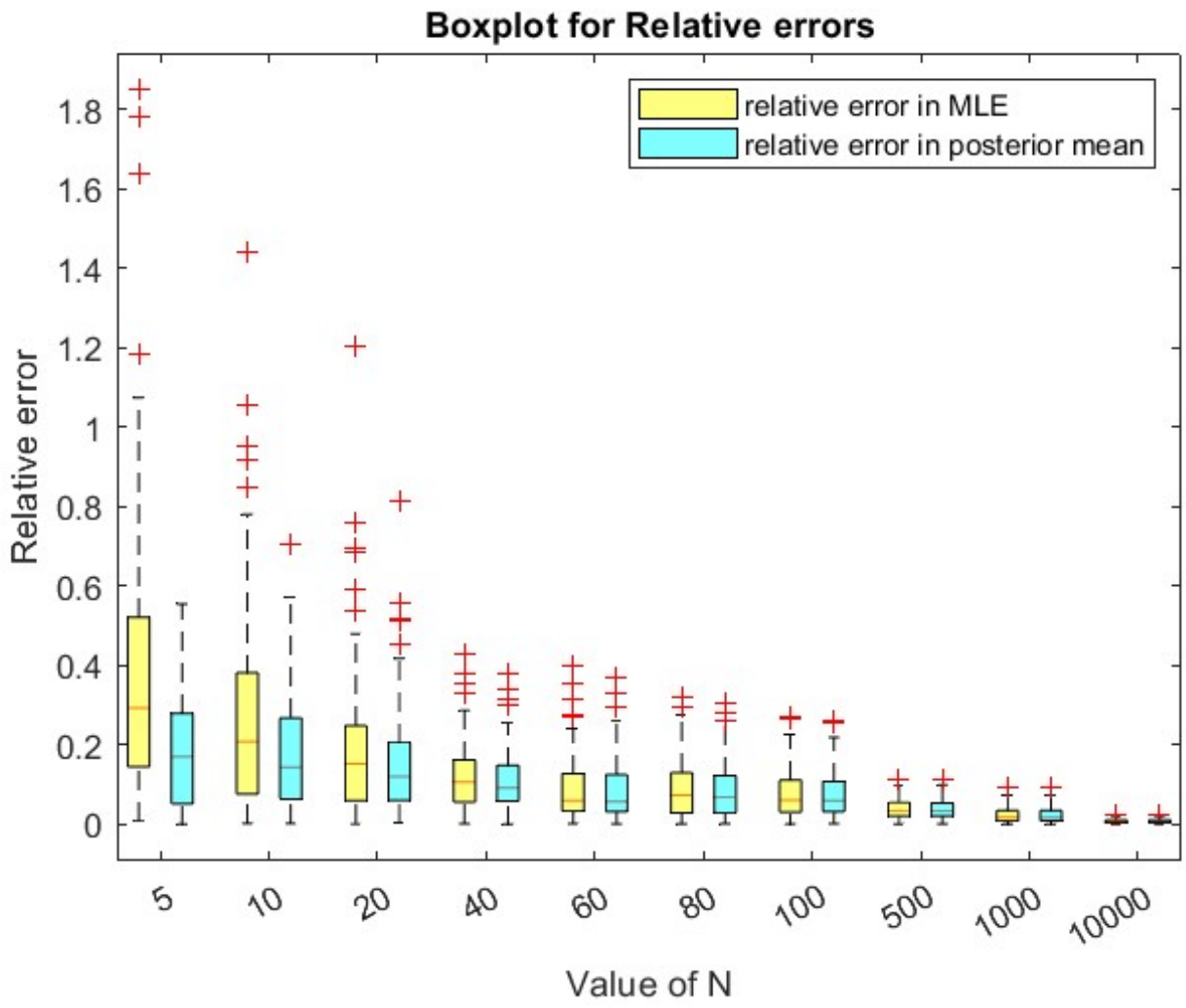
Posterior $\propto$ Prior$\times$ Likelihood function

$$\begin{aligned} Posterior &\propto \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \times \lambda^N e^{-\lambda \times \sum_{i=1}^{N} y_i} \\ &\propto \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha+N-1} e^{-(\beta + \sum_{i=1}^{N} y_i)\lambda} \end{aligned}$$

So posterior will be Gamma distribution with parameters α+N and β+$\sum_{i=1}^{N} y_i$

$$Posterior\ mean = \frac{\alpha + N}{\beta + \sum_{i=1}^{N} y_i}$$

4 BOXPLOT



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5 Interpretation

1. The error range decreases as the value of N increases as the data taken increases the sample mean tends to be the true mean
2. The gamma prior gives the better estimate as it gives the boxplot with the least range of errors, as the gamma prior estimates the mean of the given data with less error by 4 times as compared to the ML estimate.

6 Instructions to run the code

Run the Q2boxplot.m in the codes section of the Q2 folder to get the boxplot