CS215 Assignment-3

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1 Maximum Likelihood Estimate

Likelihood function:

$$
\begin{aligned}
Likelihood\ function &= \prod_{i=1}^{N} G(x_i, \mu, \sigma^2) \\
&= \prod_{i=1}^{N} G(\mu, x_i, \sigma^2)
\end{aligned}
$$

A negative exponent can be written as

$$
\frac{\left[N\mu^2 - 2\left(\sum_{i=1}^{N} x_i\right)\mu\right]}{2\sigma^2} + terms\ independent\ of\ \mu
$$
$$
\frac{\left[\mu^2 - 2\left(\sum_{i=1}^{N} x_i/N\right)\mu\right]}{2(\sigma^2/N)} + terms\ independent\ of\ \mu
$$

Thus, likelihood is proportional to $G(\mu; mean = \sum_{i=1}^{N} x_i/N, variance = \sigma^2/N)$

2 MAP estimate for mean

2.1 When prior is gaussian

We know that product of two Gaussians: $G(z, \mu_1, \sigma_1^2) \times G(z, \mu_2, \sigma_2^2)$ is also a gaussian $G(z, \mu_3, \sigma_3^2)$ with

$$
\begin{aligned}
\mu_3 &= \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \\
\sigma_3^2 &= \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2}
\end{aligned}
$$

Given prior is $P(M = \mu) = G(\mu; mean = \mu_0, variance = \sigma_0^2)$
posterior $\propto$ prior $\times$ likelihood
MAP estimate for the mean $\hat{\mu}^{MAP1}$ is :

$$
\hat{\mu}^{MAP1} = \frac{(\sum_{i=1}^{N} x_i/N)\sigma_0^2 + \mu_0(\sigma^2/N)}{\sigma_0^2 + (\sigma^2/N)}
$$

2.2 When prior is uniform distribution

Given prior is uniform distribution in [9.5,11.5],
If $\sum_{i=1}^{N} x_i/N$ of given data lies in the given range [9.5,11.5] then

$$
\hat{\mu}^{MAP2} = \sum_{i=1}^{N} x_i/N
$$

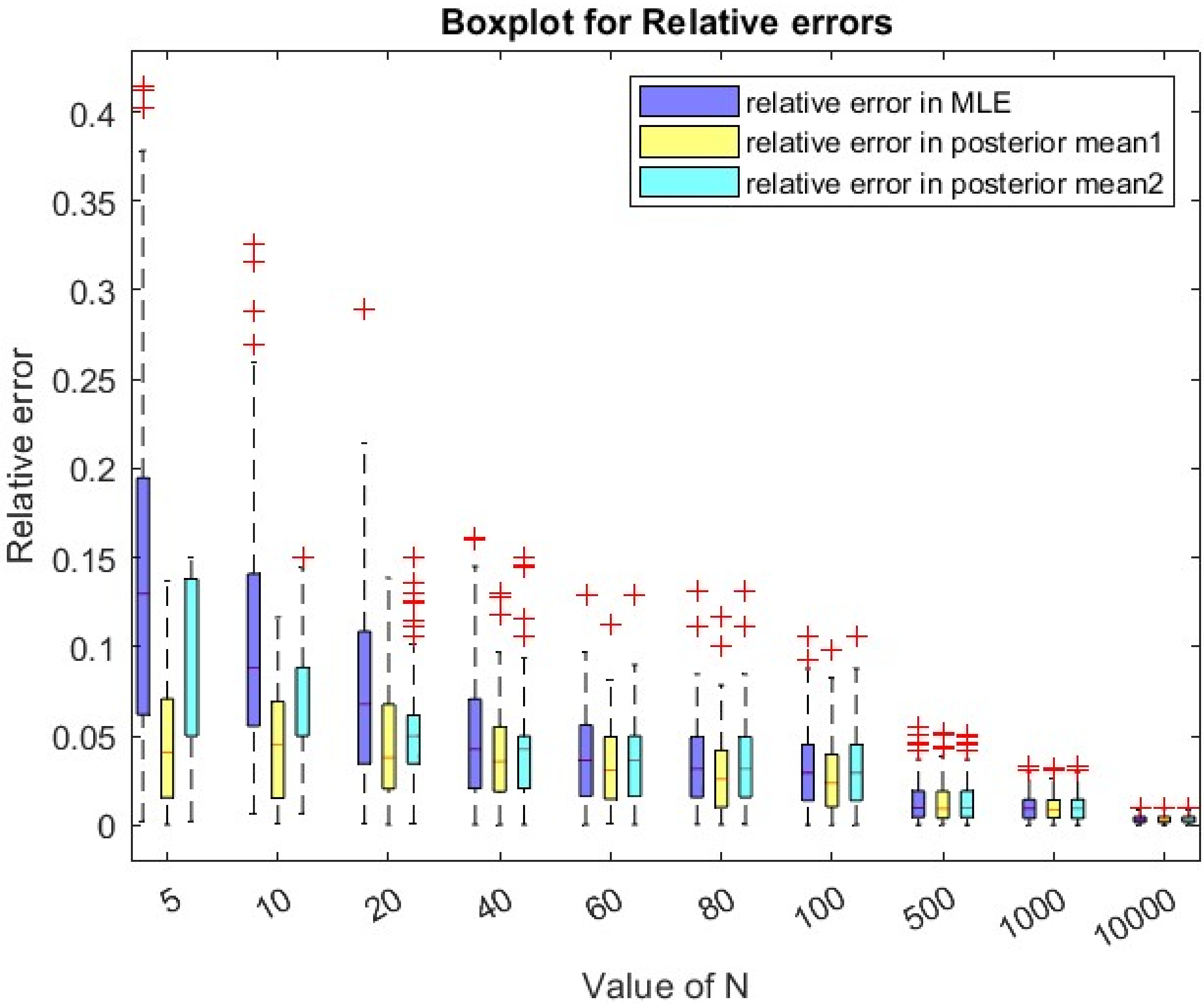
If $\sum_{i=1}^{N} x_i/N$ is less than lower bound of given range i.e 9.5 then

$$
\hat{\mu}^{MAP2} = 9.5
$$

If $\sum_{i=1}^{N} x_i/N$ is greater than upper bound of given range i.e 11.5 then

$$
\hat{\mu}^{MAP2} = 11.5
$$

3 BOXPLOT



4 Interpretation

1. The error range decreases as the value of N increases as the data taken increases the sample mean tends to be the true mean
2. The gaussian prior gives the better estimate as it gives the boxplot with the least range of errors, as the gaussian prior is the conjugate prior of the Gaussian distribution, which gives the best estimate of the mean.

5 Instructions to run the code

Run the error_boxplot.m in the codes section of the Q1 folder to get the boxplot