Life Expectancy Prediction

Vipul Gharde, Chaitanya Bachhav

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Abstract

We have implemented a Linear Regression model to predict the life expectancy of the human population with our best model having an adjusted R-squared value of 0.8239, RMSE of 3.71, and MAE of 2.85, from the processed dataset having ~1650 observations of ~20 variables related to life expectancy and health factors for 193 countries provided by the Global Health Observatory (GHO) data repository under the World Health Organization (WHO). Several model building techniques including Forward Selection, Backward Elimination, and Stepwise Regression were used to obtain the candidate models, which were then evaluated with K-Fold Cross Validation to yield the model with the lowest RMSE value. Our best model passes the normality assumption and has no issues with the multicollinearity of the variables.

Introduction

Life expectancy is an estimate of the expected average number of years of life (or a person's age at death) for individuals who were born into a particular population. It is one of the most used summary indicators for the overall health of a population. Its levels and trends direct health policies, and researchers try to identify the determining risk factors to assess and forecast future developments. (Luy et al. 2020)

The goal of this project is to build a Linear Regression model that can predict the life expectancy of the human population based on several factors such as the average Body Mass Index (BMI), the Gross Domestic Product (GDP) of a country, the amount of alcohol consumption in a country, immunization of various vaccines among 1-year-olds such as Hepatitis B, Polio, and Diphtheria vaccines, and more, and also derive insights into what factors are significant in determining a higher or lower life expectancy of the human population.

Materials and Methods

Software and Packages Used

We have used R with the RStudio Integrated Development Environment for our analysis and for building the Linear Regression models. We have also used the R packages corrplot, ggplot, car, olsrr, and caret to aid in our analysis and model building.

Dataset

The data related to life expectancy and health factors for 193 countries is taken from the Global Health Observatory (GHO) data repository under the World Health Organization (WHO). Its corresponding economic data was collected from the United Nations website for a period of 16 years (2000-2015).

The dataset is available at https://www.kaggle.com/datasets/kumarajarshi/life-expectancy-who.

Life Expectancy Data.csv contains the following fields:

- Country Country Observed.
- Year Year Observed.
- Status Developed or Developing status.
- Life.expectancy Life Expectancy in age.
- Adult.Mortality Adult Mortality Rates on both sexes (probability of dying between 15-60 years/1000 population).
- infant.deaths Number of Infant Deaths per 1000 population.
- Alcohol Alcohol recorded per capita (15+) consumption (in litres of pure alcohol).
- percentage.expenditure Expenditure on health as a percentage of Gross Domestic Product per capita (%).
- Hepatitis.B Hepatitis B (HepB) immunization coverage among 1-year-olds (%).
- Measles Number of reported Measles cases per 1000 population.
- BMI Average Body Mass Index of entire population.
- under.five.deaths Number of under-five deaths per 1000 population.
- Polio Polio (Pol3) immunization coverage among 1-year-olds (%).
- Total.expenditure General government expenditure on health as a percentage of total government expenditure (%).
- Diphtheria Diphtheria tetanus toxoid and pertussis (DTP3) immunization coverage among 1-year-olds (%).
- HIV.AIDS Deaths per 1000 live births due to HIV/AIDS (0-4 years).
- GDP Gross Domestic Product per capita (in USD).
- Population Population of the country.
- thinness..1.19.years Prevalence of thinness among children and adolescents for Age 10 to 19 (%).
- thinness. 5.9. years Prevalence of thinness among children for Age 5 to 9 (%).
- Income.composition.of.resources Human Development Index in terms of income composition of resources (index ranging from 0 to 1).
- Schooling Number of years of Schooling (years).

In total, there are 2938 observations of 22 variables with 20 of them being numerical and 2 categorical (Country and Status).

We are using Life.expectancy to predict the life expectancy of the human population with the given independent variables in the dataset.

For data cleaning, we have dropped any observation that does not contain any value in any of its columns. This shrinks our dataset to 1649 observations.

We have plotted a boxplot and a histogram for all the numerical variables in the dataset. For categorical variables, we have plotted a barplot indicating the counts of each category of the variable. This can be viewed in Appendix A of the report.

Feature Selection

We have removed some of the variables for building the model due to the reasons mentioned below:

Country - Contains too many levels with no additional information to predict Life.expectancy. Year - Contains time series data with no additional information to predict Life.expectancy.

We have also mutated Hepatitis.B, Polio, and Diphtheria for building the model since the range between their minimum values and their 1st Quartiles are too wide. We have mutated their values into 2 categorical values: '<90% Covered' and '>=90% Covered'.

This leaves us with 1649 observations of 20 variables with 16 of them being numerical and 4 categorical (Status, Hepatitis.B, Polio, and Diphtheria).

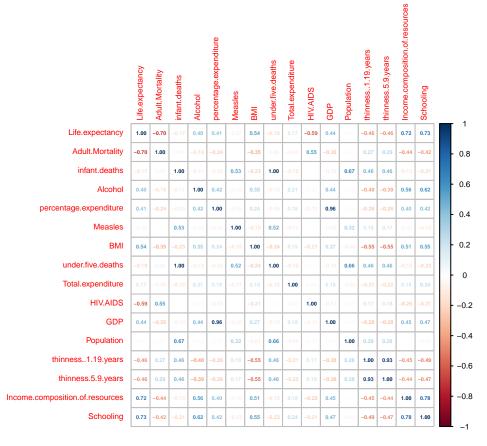
summary(life)

```
##
                       Life.expectancy Adult.Mortality infant.deaths
       Status
##
   Length: 1649
                              :44.0
                                       Min. : 1.0
                                                        Min.
                                                                   0.00
                       Min.
                       1st Qu.:64.4
                                        1st Qu.: 77.0
   Class : character
                                                        1st Qu.:
                                                                   1.00
##
   Mode :character
                       Median:71.7
                                       Median :148.0
                                                        Median :
                                                                   3.00
##
                       Mean
                              :69.3
                                       Mean
                                               :168.2
                                                        Mean
                                                                  32.55
##
                       3rd Qu.:75.0
                                       3rd Qu.:227.0
                                                        3rd Qu.:
                                                                  22.00
##
                       Max.
                              :89.0
                                       Max.
                                              :723.0
                                                        Max.
                                                               :1600.00
                     percentage.expenditure Hepatitis.B
##
       Alcohol
                                                                   Measles
          : 0.010
##
   Min.
                     Min.
                            •
                                 0.00
                                             Length: 1649
                                                                Min.
                                                                             0
    1st Qu.: 0.810
                     1st Qu.:
                                37.44
                                             Class : character
                                                                1st Qu.:
   Median : 3.790
                     Median: 145.10
                                            Mode :character
                                                                Median :
##
                                                                             15
   Mean : 4.533
                                                                          2224
##
                     Mean : 698.97
                                                                Mean
##
   3rd Qu.: 7.340
                     3rd Qu.: 509.39
                                                                3rd Qu.:
                                                                           373
##
   Max.
          :17.870
                     Max.
                            :18961.35
                                                                Max.
                                                                       :131441
##
         BMI
                    under.five.deaths
                                         Polio
                                                          Total.expenditure
##
           : 2.00
                               0.00
                                      Length: 1649
                                                                 : 0.740
   Min.
                    Min.
                                                          Min.
##
   1st Qu.:19.50
                               1.00
                                                          1st Qu.: 4.410
                    1st Qu.:
                                      Class : character
   Median :43.70
                               4.00
                                      Mode :character
                                                          Median : 5.840
                    Median:
   Mean
         :38.13
                    Mean : 44.22
                                                                : 5.956
##
                                                          Mean
##
   3rd Qu.:55.80
                    3rd Qu.: 29.00
                                                          3rd Qu.: 7.470
##
   Max.
          :77.10
                    Max.
                          :2100.00
                                                          Max.
                                                                 :14.390
    Diphtheria
                          HIV.AIDS
##
                                              GDP
                                                               Population
   Length: 1649
                              : 0.100
                                                                    :3.400e+01
##
                       Min.
                                        Min.
                                               :
                                                      1.68
                                                             Min.
                       1st Qu.: 0.100
   Class : character
                                         1st Qu.:
                                                    462.15
                                                             1st Qu.:1.919e+05
##
   Mode :character
                       Median : 0.100
                                        Median :
                                                   1592.57
                                                             Median :1.420e+06
##
                       Mean
                             : 1.984
                                        Mean
                                              : 5566.03
                                                             Mean
                                                                    :1.465e+07
                       3rd Qu.: 0.700
##
                                         3rd Qu.:
                                                  4718.51
                                                             3rd Qu.:7.659e+06
##
                              :50.600
                                               :119172.74
                                                                    :1.294e+09
                       Max.
                                         Max.
                                                             Max.
##
   thinness..1.19.years thinness.5.9.years Income.composition.of.resources
   Min. : 0.100
                         Min. : 0.100
                                                    :0.0000
##
                                             Min.
##
   1st Qu.: 1.600
                         1st Qu.: 1.700
                                             1st Qu.:0.5090
   Median : 3.000
                         Median : 3.200
                                             Median :0.6730
##
   Mean
          : 4.851
                         Mean
                               : 4.908
                                             Mean
                                                    :0.6316
   3rd Qu.: 7.100
                         3rd Qu.: 7.100
##
                                             3rd Qu.:0.7510
   Max.
           :27.200
                                :28.200
                                                    :0.9360
##
                         Max.
                                            Max.
##
      Schooling
          : 4.20
   Min.
   1st Qu.:10.30
##
   Median :12.30
##
  Mean
          :12.12
   3rd Qu.:14.00
##
## Max.
           :20.70
```

Correlations

We would want to look at the correlation matrix to see what variables are correlated with the target variable, and also to check if any independent variable is also correlated with another independent variable. Correlated independent variables in a model can negatively impact its performance, and so if any such pair of independent correlated variables is found, we would keep only one of them in the model.

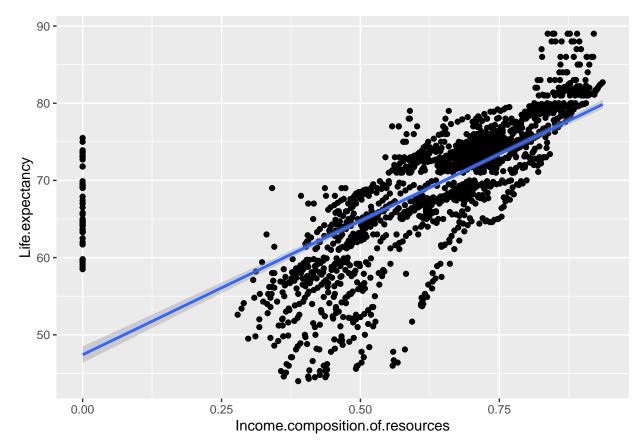
Since the number of variables is moderately large, we have plotted the correlation plot of the dataset rather than printing the correlation matrix by itself. The colors and their shades easily guide us to show what 2 variables are correlated.

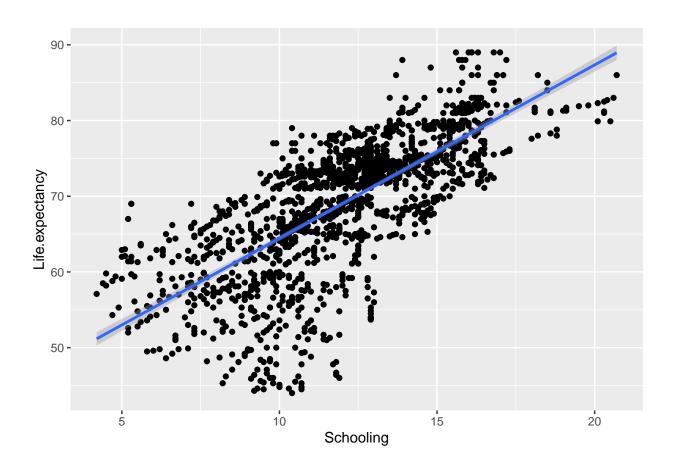


There are a few takeaways from this correlation plot:

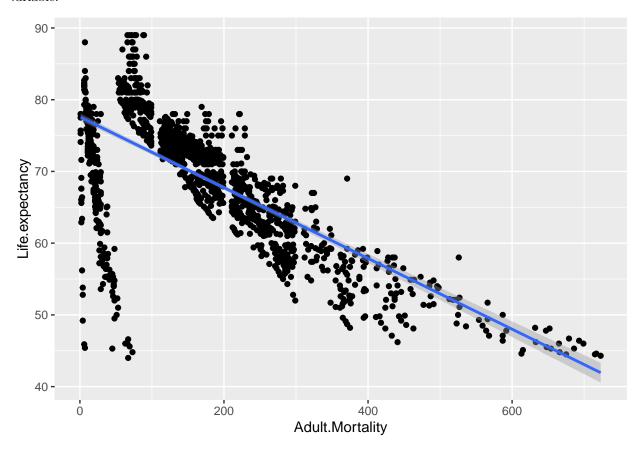
- Life.expectancy has a strong positive correlation with Income.composition.of.resources and Schooling.
- Life.expectancy has a negative correlation with Adult.Mortality, which makes sense since if the adult mortality rate is high, then obviously the life expectancy will be low.
- Life.expectancy has a very weak correlation with Measles and Population.
- There is a very strong correlation between infant.deaths and under.five.deaths, percentage.expenditure and GDP, and thinness.1.19.years and thinness.5.9.years, indicating multicollinearity between them. Therefore, we have removed under.five.deaths, percentage.expenditure, and thinness.5.9.years for building the model.

It is evident from the scatterplot of Life.expectancy against Income.composition.of.resources and Life.expectancy against Schooling that there is a positive trend in the life expectancy of the human population with the increase in the values of these independent variables.

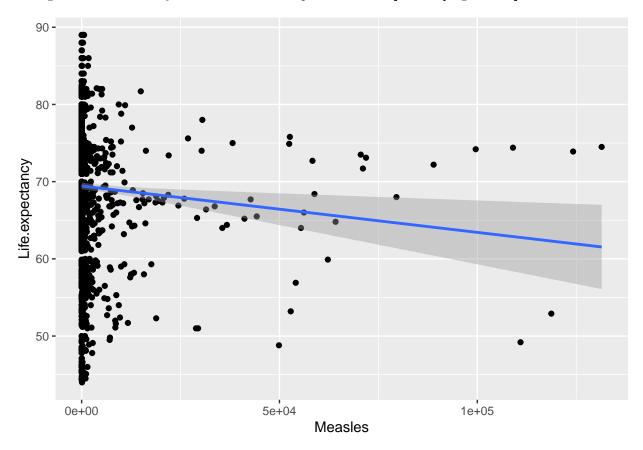


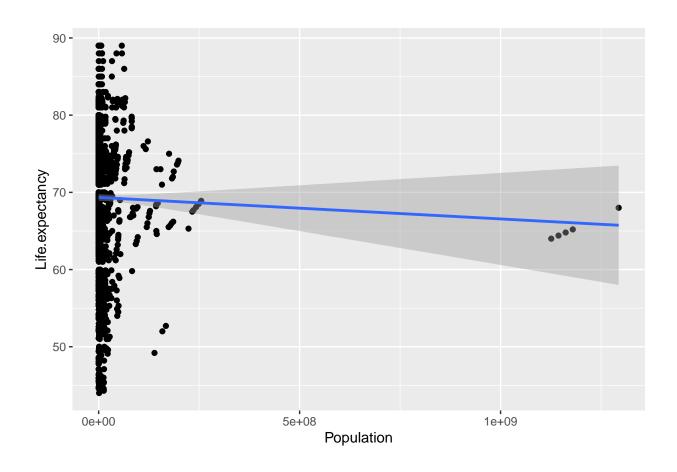


It is evident from the scatterplot of Life.expectancy against Adult.Mortality that there is a negative trend in the life expectancy of the human population with the increase in the value of this independent variable.



It seems in the scatterplot of Life.expectancy against Measles and Life.expectancy against Population that there is a negative trend in the life expectancy of the human population with the increase in the values of these independent variables, but since the bulk of the data falls on the lower range of their values, there exist some high leverage and high influence points that appear to drive the regression line downward toward the negative. This is clearly evident in the scatterplot of Life.expectancy against Population.





Model Building

We now build a Linear Regression Model using all the remaining variables to predict the life expectancy of the human population. We will set the level of α to be 0.05 throughout the analysis.

```
lmod_all = lm(Life.expectancy ~ ., data = life)
summary(lmod_all)
```

```
##
## Call:
## lm(formula = Life.expectancy ~ ., data = life)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                    30
                                            Max
##
   -17.0274
            -2.1069
                       0.0579
                                2.3922
                                        11.5489
##
## Coefficients:
##
                                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                    5.506e+01 8.102e-01 67.965 < 2e-16 ***
## StatusDeveloping
                                   -9.930e-01 3.464e-01 -2.867
                                                                   0.0042 **
## Adult.Mortality
                                   -1.785e-02 9.663e-04 -18.477
                                                                  < 2e-16 ***
## infant.deaths
                                   -3.057e-03 1.260e-03
                                                          -2.425
                                                                   0.0154 *
## Alcohol
                                   -1.539e-01 3.381e-02
                                                          -4.552 5.72e-06 ***
## Hepatitis.B>=90% Covered
                                   -6.990e-01 3.174e-01
                                                          -2.202
                                                                   0.0278 *
## Measles
                                    1.668e-05 1.078e-05
                                                           1.547
                                                                   0.1220
## BMI
                                                           5.884 4.85e-09 ***
                                    3.591e-02
                                               6.103e-03
## Polio>=90% Covered
                                    5.337e-01
                                               4.437e-01
                                                           1.203
                                                                   0.2293
## Total.expenditure
                                    7.441e-02 4.170e-02
                                                           1.784
                                                                   0.0745
## Diphtheria>=90% Covered
                                    9.665e-01 4.888e-01
                                                           1.977
                                                                   0.0482 *
## HIV.AIDS
                                   -4.278e-01
                                               1.850e-02 -23.124
                                                                  < 2e-16 ***
## GDP
                                    6.096e-05 9.637e-06
                                                           6.326 3.24e-10 ***
## Population
                                    2.558e-09
                                               1.766e-09
                                                           1.449
                                                                   0.1476
                                                          -1.720
## thinness..1.19.years
                                   -4.799e-02
                                               2.791e-02
                                                                   0.0857 .
## Income.composition.of.resources 1.041e+01
                                               8.503e-01
                                                          12.244
                                                                  < 2e-16 ***
                                    8.790e-01 6.158e-02 14.274 < 2e-16 ***
## Schooling
## ---
## Signif. codes:
                 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 3.688 on 1632 degrees of freedom
## Multiple R-squared: 0.826, Adjusted R-squared: 0.8243
## F-statistic: 484.1 on 16 and 1632 DF, p-value: < 2.2e-16
```

There are a few takeaways from this full model:

- The p-value of the model is 2.2e-16 < 0.05, indicating that it is significant.
- The adjusted R-squared value of the model is 0.8243, indicating that about 82.43% of the variability observed in the target variable can be explained by the independent variables in the model, which is quite a good result and can possibly be improved even further with model selection.
- Adult.Mortality, Alcohol, BMI, HIV.AIDS, GDP, Income.composition.of.resources and Schooling are the most significant variables with p-value < 0.05.
- From the model we can interpret that Income.composition.of.resources has a strong positive effect on life expectancy.
- From the model we can interpret that StatusDeveloping, Adult.Mortality, infant.deaths, Alcohol, HIV.AIDS, and thinness..1.19.years may have a negative effect on life expectancy.
- A peculiar result we can interpret from the model is that Hepatitis.B>=90% Covered may also have a negative effect on life expectancy.

We now generate models by using different techniques like Forward Selection method, Backward Elimination method and Stepwise Regression method.

Forward Selection Method

The Forward Selection method involves building a model starting with no variables in the model, testing the addition of each variable using a chosen model fit criterion, adding the variable (if any) whose inclusion gives the most statistically significant improvement of the fit, and repeating this process until none improves the model to a statistically significant extent.

For building the model using Forward Selection method, we have used the default p-to-enter value of 0.3. ols step forward p(lmod all)

## ##	Selection Summary							
## ##			Adj.					
	Step	Entered	R-Square	R-Square	C(p)	AIC	RMSE	
## ##	1	Schooling	0.5294	0.5292	2767.6453	10612.7157	6.0362	
##	2	HIV.AIDS	0.7304	0.7301	885.2760	9696.3271	4.5704	
##	3	Adult.Mortality	0.7871	0.7867	355.5223	9308.9473	4.0627	
##	4	Income.composition.of.resources	0.8092	0.8087	150.4656	9130.3986	3.8474	
##	5	BMI	0.8144	0.8138	103.6254	9086.7666	3.7957	
##	6	GDP	0.8197	0.8190	55.8305	9040.9050	3.7422	
##	7	Diphtheria	0.8214	0.8206	42.0679	9027.4602	3.7258	
##	8	Alcohol	0.8226	0.8217	32.8339	9018.3638	3.7144	
##	9	thinness1.19.years	0.8235	0.8225	26.3197	9011.9039	3.7061	
##	10	Status	0.8244	0.8233	19.8462	9005.4410	3.6977	
##	11	Hepatitis.B	0.8248	0.8236	17.7935	9003.3780	3.6943	
##	12	Total.expenditure	0.8251	0.8239	16.7491	9002.3193	3.6920	
##	13	infant.deaths	0.8254	0.8240	16.6117	9002.1684	3.6907	
##	14	Measles	0.8256	0.8241	16.5671	9002.1083	3.6895	
##	15	Population	0.8258	0.8242	16.4465	9001.9689	3.6882	
##	16	Polio	0.8260	0.8243	17.0000	9002.5080	3.6877	

The Forward Selection method included all the variables in the model.

Backward Elimination Method

The Backward Elimination method involves building a model starting with all candidate variables, testing the deletion of each variable using a chosen model fit criterion, deleting the variable (if any) whose loss gives the most statistically insignificant deterioration of the model fit, and repeating this process until no further variables can be deleted without a statistically significant loss of fit.

For building the model using Backward Elimination method, we have used the default p-to-remove value of 0.3.

ols_step_backward_p(lmod_all)

[1] "No variables have been removed from the model."

The Backward Elimination method did not eliminate any variables from the model.

Stepwise Regression Method

The Stepwise Regression method is a combination of the above two methods, starting with no variables in the model and testing at each step for variables to be included or excluded.

For building the model using Stepwise Regression method, we have used the default p-to-enter value of 0.1 and the default p-to-remove value of 0.3.

ols_step_both_p(lmod_all)

## ## ##	Stepwise Selection Summary							
##	Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	
##		Schooling	addition	0.529	0.529	2767.6450	10612.71	
##	2	HIV.AIDS	addition	0.730	0.730	885.2760	9696.32	
##	3	Adult.Mortality	addition	0.787	0.787	355.5220	9308.94	
##	4	<pre>Income.composition.of.resources</pre>	addition	0.809	0.809	150.4660	9130.39	
##	5	BMI	addition	0.814	0.814	103.6250	9086.76	
##	6	GDP	addition	0.820	0.819	55.8310	9040.90	
##	7	Diphtheria	addition	0.821	0.821	42.0680	9027.460	
##	8	Alcohol	addition	0.823	0.822	32.8340	9018.36	
##	9	thinness1.19.years	addition	0.823	0.823	26.3200	9011.90	
##	10	Status	addition	0.824	0.823	19.8460	9005.44	
##	11	Hepatitis.B	addition	0.825	0.824	17.7930	9003.378	
##		Total.expenditure	addition	0.825	0.824	16.7490	9002.31	
##								

A total of 12 variables were included in the model built using Stepwise Regression method.

In summary, the variables chosen by the methods are indicated in the following table (x denotes the variable was chosen by the corresponding method):

Model Selection Method	Status	Adult.Mortality	infant.deaths	Alcohol
Forward Selection	X	X	X	X
Backward Elimination	X	X	X	X
Stepwise Regression	X	X		X

Model Selection Method	Hepatitis.B	Measles	BMI	Polio	Total.expenditure
Forward Selection	X	X	X	X	X
Backward Elimination	X	X	X	X	x
Stepwise Regression	X		X		x

Model Selection Method	Diphtheria	HIV.AIDS	GDP	Population
Forward Selection	X	X	X	X
Backward Elimination	X	X	X	X
Stepwise Regression	X	X	X	

Model Selection Method	thinness1.19.years	<pre>Income.compostition.of.resources</pre>	Schooling
Forward Selection	X	x	x
Backward Elimination	X	X	X
Stepwise Regression	X	X	X

Both the Forward Selection method and Backward Elimination method have chosen the same set of variables.

K-Fold Cross Validation

We are left with 2 models - the full model and the model built using Stepwise Regression method - as our candidate models. To find out which model is the better one to pick as our final model, we ran K-Fold Cross Validation on both models and subsequently picked the model with the lowest mean RMSE value as our final model. We chose the value of K to be 5.

In K-Fold Cross-Validation, the original sample of the dataset is randomly partitioned into K equal sized subsamples. Of the K subsamples, a single subsample is retained as the validation data for testing the model, and the remaining K-1 subsamples are used as training data. The cross-validation process is then repeated K times, with each of the K subsamples used exactly once as the validation data. The K results are then averaged to produce a single estimation, which in our case is the mean RMSE value.

```
# Define training control
set.seed(13245)
train.control = trainControl(method = 'cv', number = 5)
```

Cross-Validation for the full model:

```
# Train the model
CV_all = train(
  Life.expectancy ~ .,
  data = life,
  method = 'lm',
  trControl = train.control
)
```

```
# Summarize the results
CV_all
## Linear Regression
##
## 1649 samples
##
     16 predictor
##
## No pre-processing
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 1318, 1319, 1319, 1320, 1320
## Resampling results:
##
##
     {\tt RMSE}
               Rsquared
                          MAE
##
     3.719367 0.8214392 2.853294
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
Cross-Validation for model chosen by Stepwise Regression method:
# Train the model
CV_stepwise = train(
 Life.expectancy ~ Schooling + HIV.AIDS + Adult.Mortality + Income.composition.of.resources + BMI + GD
 data = life,
 method = 'lm',
  trControl = train.control
)
# Summarize the results
CV_stepwise
## Linear Regression
##
## 1649 samples
##
     12 predictor
## No pre-processing
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 1319, 1320, 1318, 1320, 1319
## Resampling results:
##
##
    RMSE
               Rsquared
                          MAE
##
     3.718134 0.8211816 2.853578
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
Results of 5-Fold Cross-Validation on the 2 models:
rbind(CV_all$results, CV_stepwise$results)
##
     intercept
                   RMSE Rsquared
                                        MAE
                                               RMSESD RsquaredSD
                                                                      MAESD
## 1
          TRUE 3.719367 0.8214392 2.853294 0.1466741 0.01735040 0.1035478
          TRUE 3.718134 0.8211816 2.853578 0.2043897 0.01790453 0.1159896
## 2
```

Since the model chosen by Stepwise Regression method <code>lmod_stepwise</code> has a lower RMSE value, we have selected this model to be our final model.

Results

Out of the 2 candidate models, we have picked the model chosen by the Stepwise Regression method to be our final model. This decision was based on the fact that the Stepwise Regression model had the lowest mean RMSE value when evaluated with K-Fold Cross Validation (K = 5).

```
lmod_final = lm(
  Life.expectancy ~ Schooling + HIV.AIDS + Adult.Mortality + Income.composition.of.resources + BMI + GD
  data = life
)
summary(lmod final)
##
## Call:
## lm(formula = Life.expectancy ~ Schooling + HIV.AIDS + Adult.Mortality +
       Income.composition.of.resources + BMI + GDP + Diphtheria +
##
##
       Alcohol + thinness..1.19.years + Status + Hepatitis.B + Total.expenditure,
##
       data = life)
##
## Residuals:
##
       Min
                  1Q
                      Median
                                   3Q
                                           Max
  -16.9828 -2.1039
                      0.0534
                               2.3728
                                       11.4923
##
## Coefficients:
##
                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                   5.505e+01 8.059e-01 68.312 < 2e-16 ***
## Schooling
                                   8.931e-01 6.129e-02 14.572 < 2e-16 ***
## HIV.AIDS
                                  -4.269e-01 1.846e-02 -23.131 < 2e-16 ***
## Adult.Mortality
                                  -1.781e-02 9.641e-04 -18.471 < 2e-16 ***
## Income.composition.of.resources 1.034e+01 8.471e-01 12.204 < 2e-16 ***
## BMI
                                   3.577e-02 6.082e-03
                                                         5.881 4.94e-09 ***
## GDP
                                   6.079e-05 9.646e-06
                                                          6.302 3.77e-10 ***
## Diphtheria>=90% Covered
                                                          4.215 2.64e-05 ***
                                   1.452e+00 3.444e-01
## Alcohol
                                   -1.607e-01 3.355e-02
                                                         -4.789 1.83e-06 ***
## thinness..1.19.years
                                                         -2.726 0.00648 **
                                   -6.820e-02
                                              2.502e-02
## StatusDeveloping
                                  -9.968e-01 3.465e-01
                                                         -2.877 0.00407 **
## Hepatitis.B>=90% Covered
                                  -6.338e-01 3.140e-01
                                                         -2.018 0.04371 *
## Total.expenditure
                                   7.257e-02 4.164e-02
                                                         1.743 0.08155 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 3.692 on 1636 degrees of freedom
## Multiple R-squared: 0.8251, Adjusted R-squared: 0.8239
## F-statistic: 643.3 on 12 and 1636 DF, p-value: < 2.2e-16
```

The final model contains 12 variables: Schooling, HIV.AIDS, Adult.Mortality, Income.composition.of.resources, BMI, GDP, Diphtheria>=90% Covered, Alcohol, thinness..1.19.years, Status, Hepatitis.B>=90% Covered, and Total.expenditure.

There are a few takeaways from this final model:

- The p-value of the model is 2.2e-16 < 0.05, indicating that it is significant.
- The adjusted R-squared value of the model is 0.8239, indicating that about 82.39% of the variability observed in the target variable can be explained by the independent variables in the model.
- Pretty much all the variables in the model are the most significant variables with p-value < 0.05.
- From the model we can interpret that Income.composition.of.resources has a strong positive effect
 on life expectancy.
- From the model we can interpret that HIV.AIDS, Adult.Mortality, Alcohol, thinness..1.19.years, and StatusDeveloping, may have a negative effect on life expectancy.
- Hepatitis.B>=90% Covered may also have a negative effect on life expectancy, the same observation we had seen previously on the full model.

Model Error Estimation

We have primarily used the R-squared, the adjusted R-squared, the root-mean-square error (RMSE), and the mean absolute error (MAE) as the metric for evaluating our models.

The estimates for the final model are derived from the results of K-Fold Cross Validation (K = 5), and are summarized in the following table:

Estimate
0.8219568
3.711009
2.846318

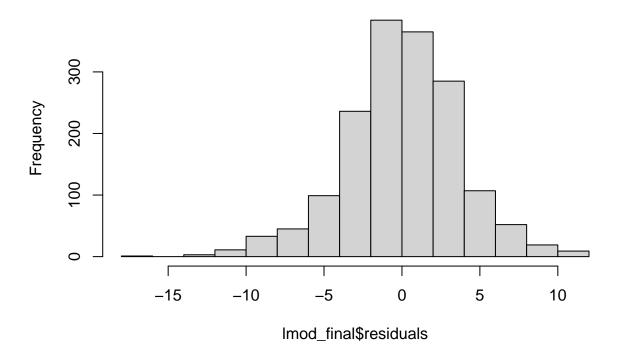
Model Adequacy Checking

To make sure our final model behaves as expected in terms of inference and prediction, we need to test that the assumptions made in building the Linear Regression model are not broken. These assumptions are:

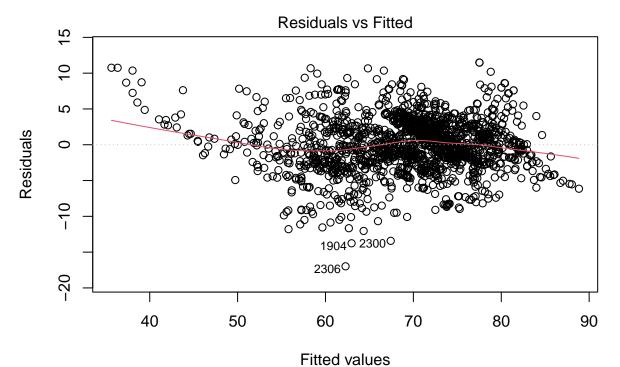
- 1. The relationship between the response y and the regressors is linear, at least approximately.
- 2. The error term ϵ has zero mean.
- 3. The error term ϵ has constant variance σ^2 .
- 4. The errors are uncorrelated.
- 5. The errors are normally distributed. (Montgomery, Peck, and Vining 2021)

We will be testing these assumptions by checking whether the residuals of the model are normally distributed, whether the plot of the residuals against the fitted values show any pattern, and whether there are any anomalies in the Q-Q plot.

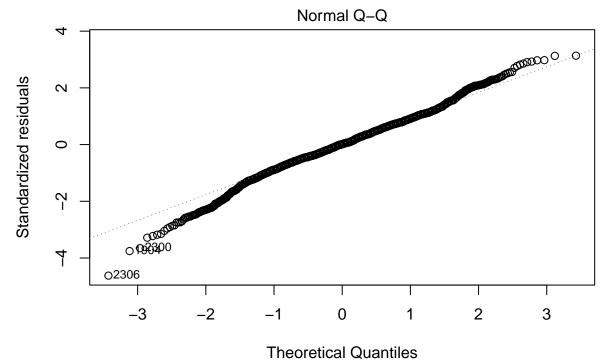
Histogram of Imod_final\$residuals



Most of the residuals seem to be distributed in the center, indicating that they are distributed normally.



Im(Life.expectancy ~ Schooling + HIV.AIDS + Adult.Mortality + Income.compos ...



Im(Life.expectancy ~ Schooling + HIV.AIDS + Adult.Mortality + Income.compos ...

There is no obvious observable pattern in the above plots, and so we conclude that our final model is appropriate.

We also run a multicollinearity test to see if there is any multicollinearity between the variables in the model. We have used the Variance Inflation Factor (VIF) to determine the multicollinearity between the variables.

vif(lmod_final)

##	Schooling	HIV.AIDS
##	3.548828	1.498703
##	Adult.Mortality	<pre>Income.composition.of.resources</pre>
##	1.764485	2.908091
##	BMI	GDP
##	1.745170	1.481404
##	Diphtheria	Alcohol
##	3.511705	2.209220
##	thinness1.19.years	Status
##	1.600520	1.818933
##	Hepatitis.B	Total.expenditure
##	2.981721	1.108283

A VIF > 10 implies serious problems with multicollinearity.

Since the VIF for all of the predictors is less than 10, there seems to be no issue with multicollinearity.

Discussion

We have implemented a Linear Regression model that predicts the life expectancy of the human population using 12 variables related to life expectancy and health factors. We have shown that several of these independent variables have some correlations with life expectancy, suggesting that fitting a Linear Regression model to this dataset would be appropriate. Our model also passes the normality assumption and has no issues with the multicollinearity of the variables.

While all the independent variables in our model are significant, the inferences made from this model should be taken with a grain of salt - we do not believe that an increase in Hepatitis B (HepB) immunization coverage among 1-year-olds should be associated with a declining life expectancy in a country. This calls for further investigation.

We had planned to include the All Possible Regressions method and the Best Subsets Regression method for building our candidate models, but since the computational time for running these methods was too high on our machines, we have omitted these methods in our analysis.

Due to the limited amount of time for this analysis, we have not explored working with Polynomial Regression and including interaction terms in our models. We believe that in particular, a squared term for BMI in the model would be appropriate since it is well-known that too low or too high of a BMI poses health risks for individuals. It is also possible to convert this numerical variable into categorical values, for example, 'Underweight' for BMI < 18.5, 'Healthy' for $18.5 \le BMI < 25$, 'Overweight' $25 \le BMI < 30$, and 'Obese' for BMI ≥ 30 .

We are curious to know how our model compares to other regression techniques such as KNN Regression, Support Vector Regression, Decision Tree Regression, and also Random Forest Regression. We have left this for future work.

Though we believe that the variables we have in our dataset are relevant in predicting the life expectancy of the human population, we think that having more relevant variables like sex, exercise, smoking, and environment pollution would be even more helpful for inference and prediction. It is also always beneficial to collect more observations of data related to life expectancy and health factors for building our model.

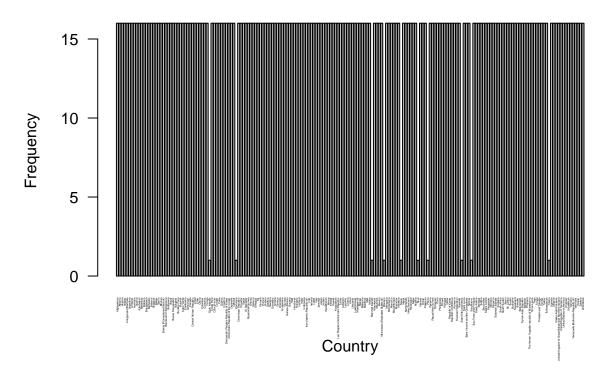
Literature Cited

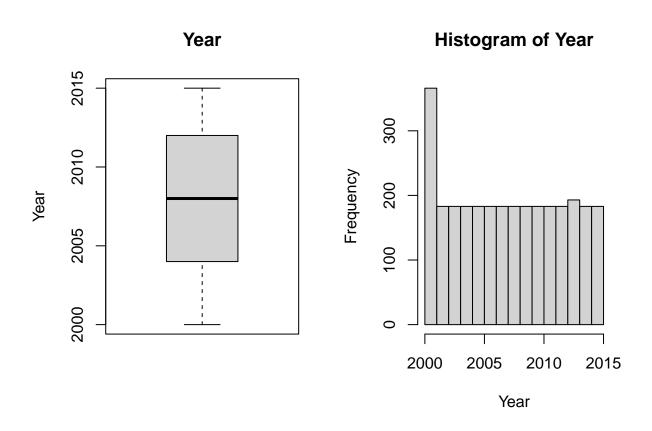
Luy, Marc, Paola Di Giulio, Vanessa Di Lego, Patrick Lazarevič, and Markus Sauerberg. 2020. "Life Expectancy: Frequently Used, but Hardly Understood." *Gerontology* 66 (1): 95–104.

Montgomery, Douglas C, Elizabeth A Peck, and G Geoffrey Vining. 2021. *Introduction to Linear Regression Analysis*. John Wiley & Sons.

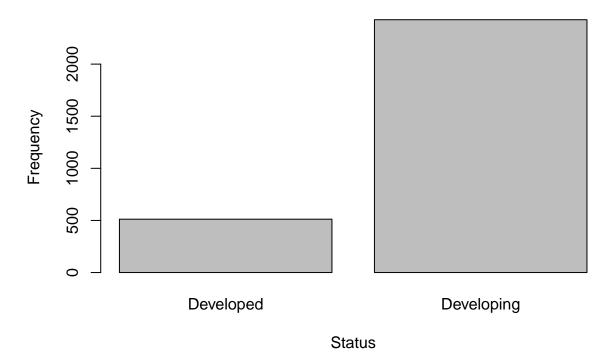
Appendix A - Variable Distribution Plots

Barplot of Country

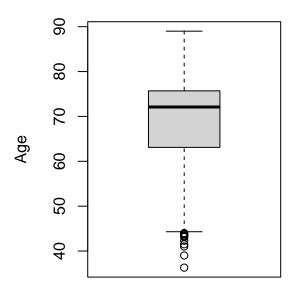




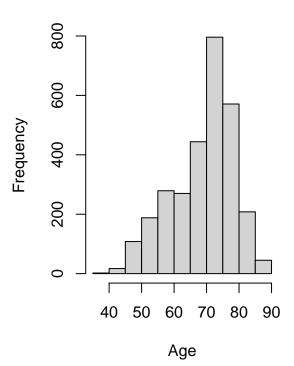
Barplot of Status



Life Expectancy

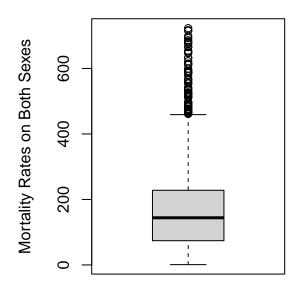


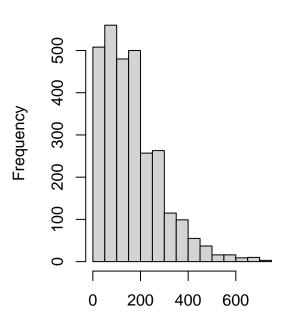
Histogram of Life Expectancy



Adult Mortality Rate

Histogram of Adult Mortality Rate

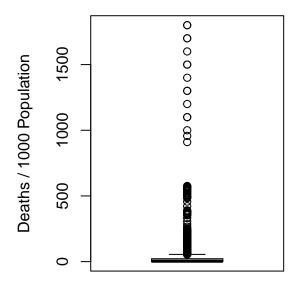


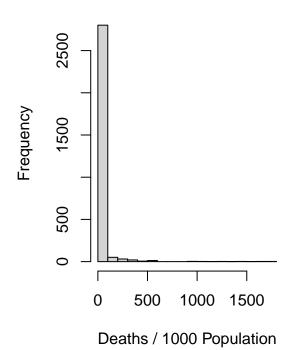


Mortality Rates on Both Sexes

Infant Deaths

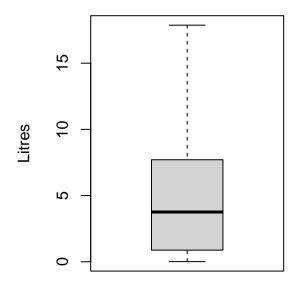
Histogram of Infant Deaths

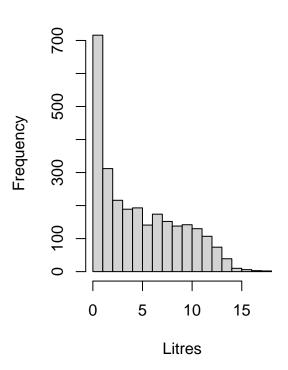


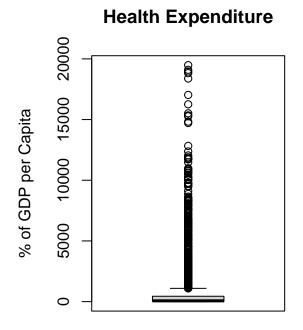


Alcohol Consumption

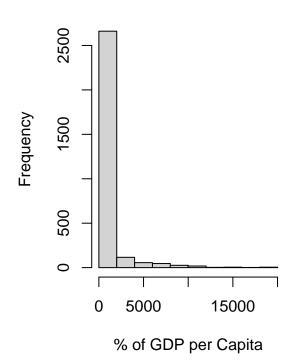
Histogram of Alcohol Consumption





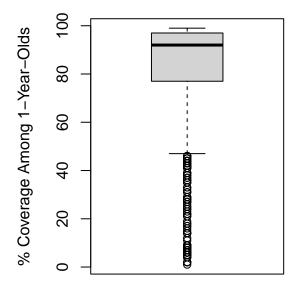


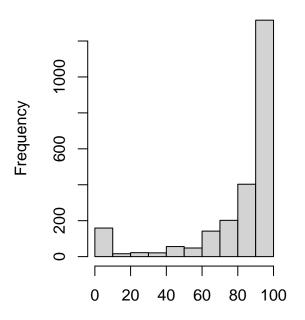
Histogram of Health Expenditure



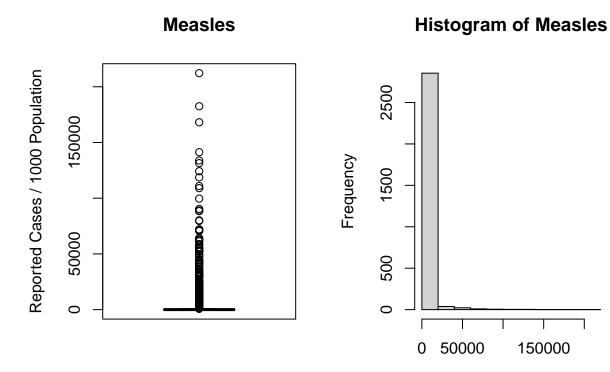
Hepatitis B (HepB) Immunization

Histogram of HepB Immunization



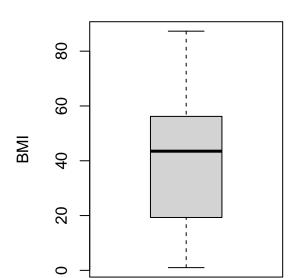


% Coverage Among 1-Year-Olds

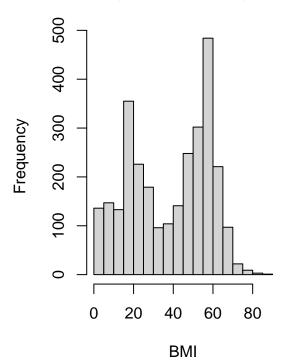


Reported Cases / 1000 Population

Average BMI

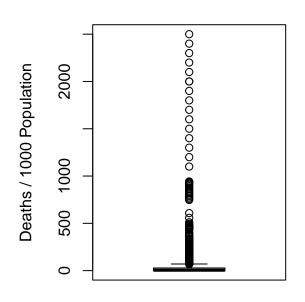


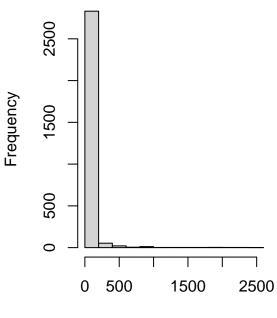
Histogram of Average BMI





Histogram of Under-Five Deaths

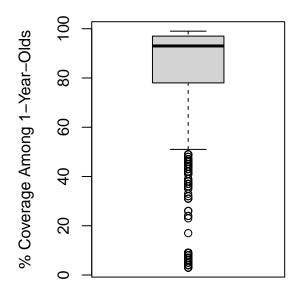


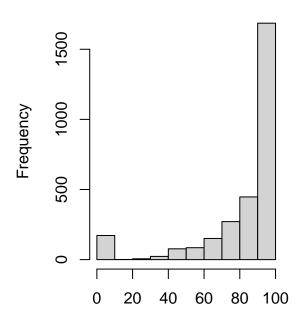


Deaths / 1000 Population

Polio (Pol3) Immunization

Histogram of Pol3 Immunization

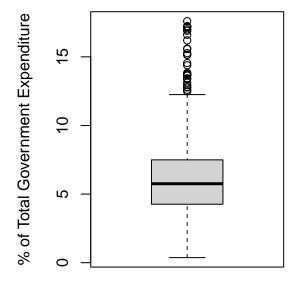


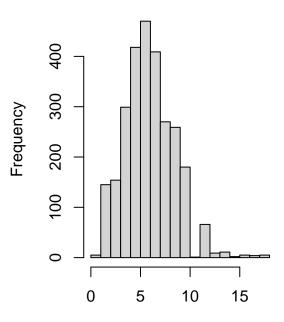


% Coverage Among 1-Year-Olds

General Government Health Expenditure

Histogram of General Government Health Expenditure

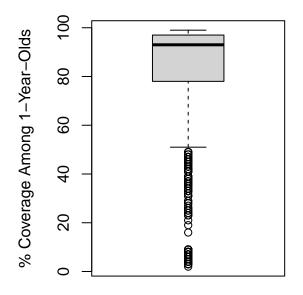


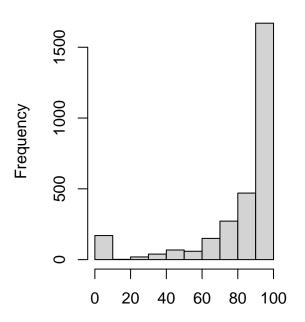


% of Total Government Expenditure

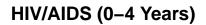
DTP3 Immunization

Histogram of DTP3 Immunization

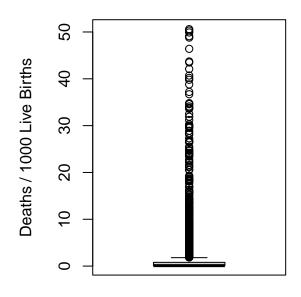


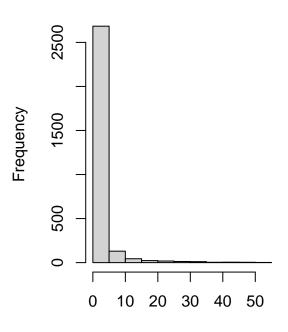


% Coverage Among 1-Year-Olds

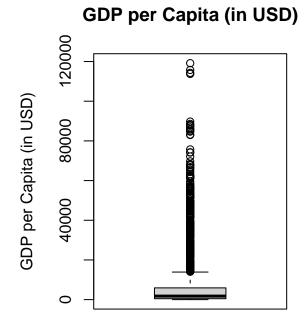


Histogram of HIV/AIDS (0-4 Years)

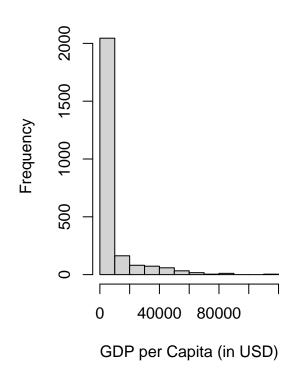




Deaths / 1000 Live Births

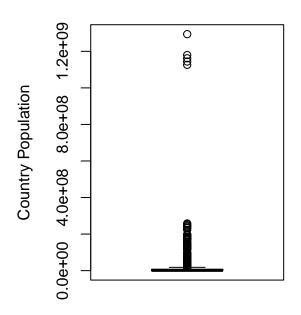


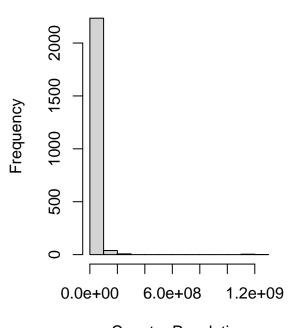
Histogram of GDP per Capita (in USD)



Country Population

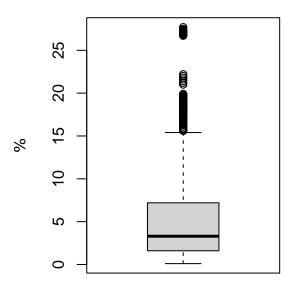
Histogram of Country Population

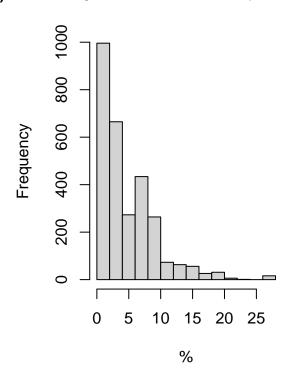




Prevalence of Thinness (10–19 Years)

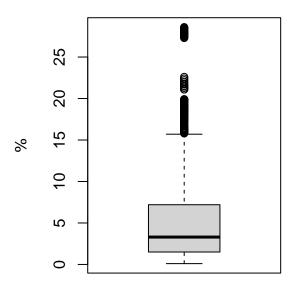
Histogram of Prevalence of Thinness (10-19 Years)

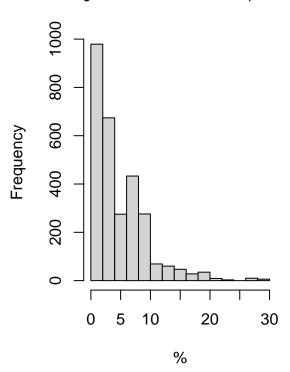


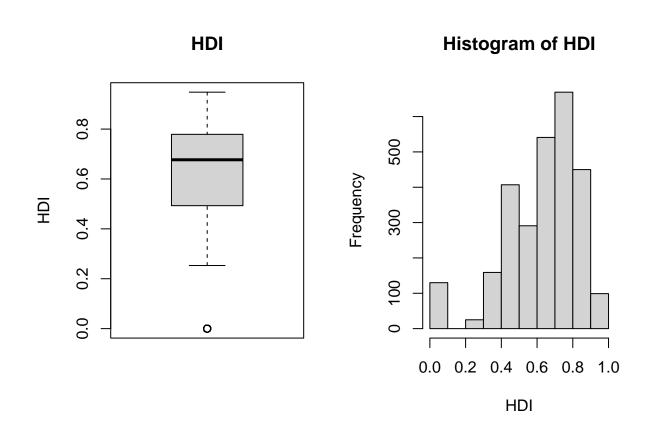


Prevalence of Thinness (5–9 Years)

Histogram of Prevalence of Thinness (5-9 Years)







Schooling

Histogram of Schooling

