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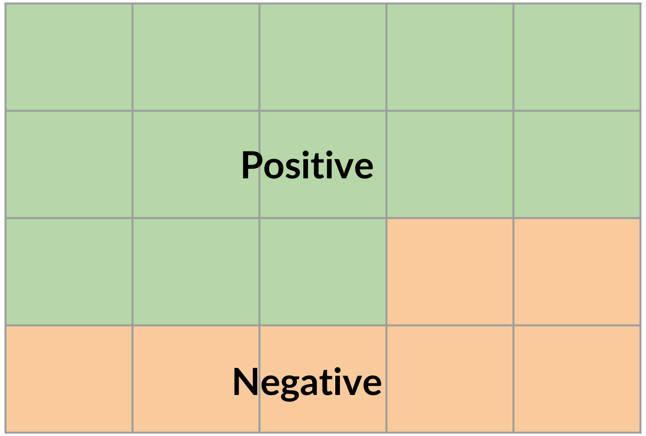
Probability and Bayes' Rule

Outline

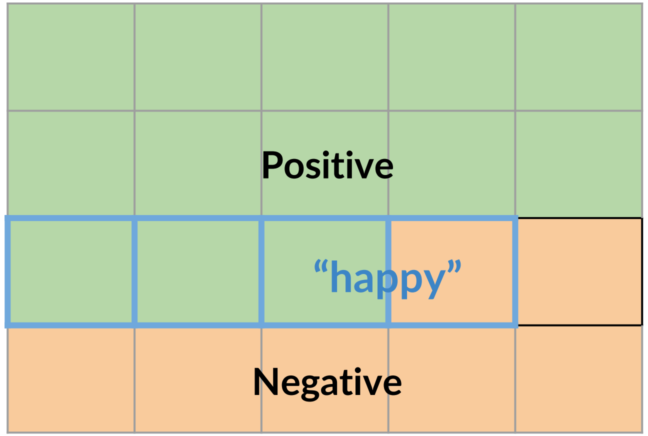
- Probabilities
- Bayes' rule (Applied in different fields, including NLP)
- Build your own Naive-Bayes tweet classifier!

Introduction

Corpus of tweets

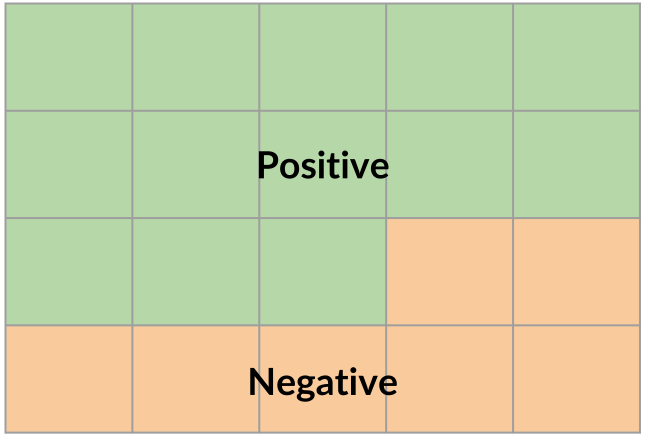


Tweets containing the word "happy"



Probabilities

Corpus of tweets

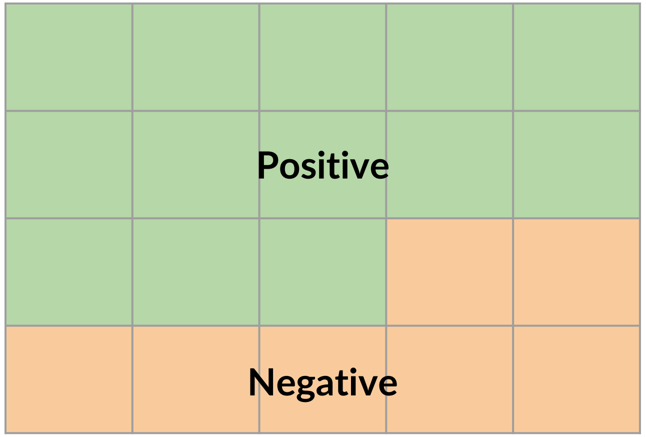


A → Positive tweet

$P(A) = P(\text{Positive}) = N_{pos} / N$

Probabilities

Corpus of tweets



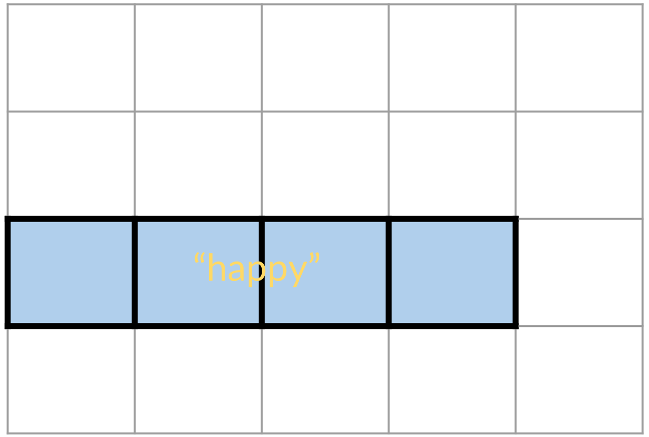
A → Positive tweet

$P(A) = N_{pos} / N = 13 / 20 = 0.65$

$P(Negative) = 1 - P(Positive) = 0.35$

Probabilities

Tweets containing the word “happy”

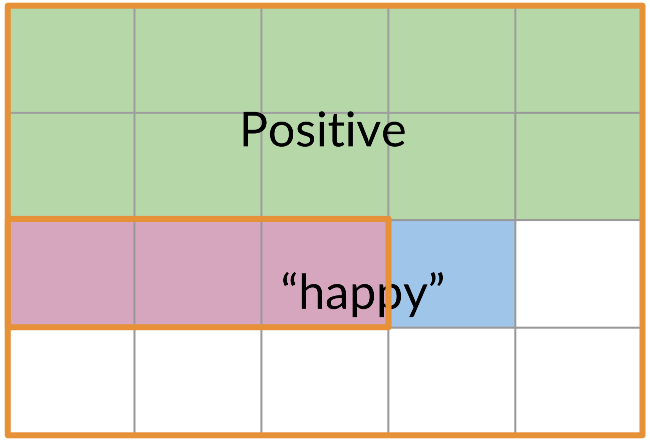


B → tweet contains “happy”.

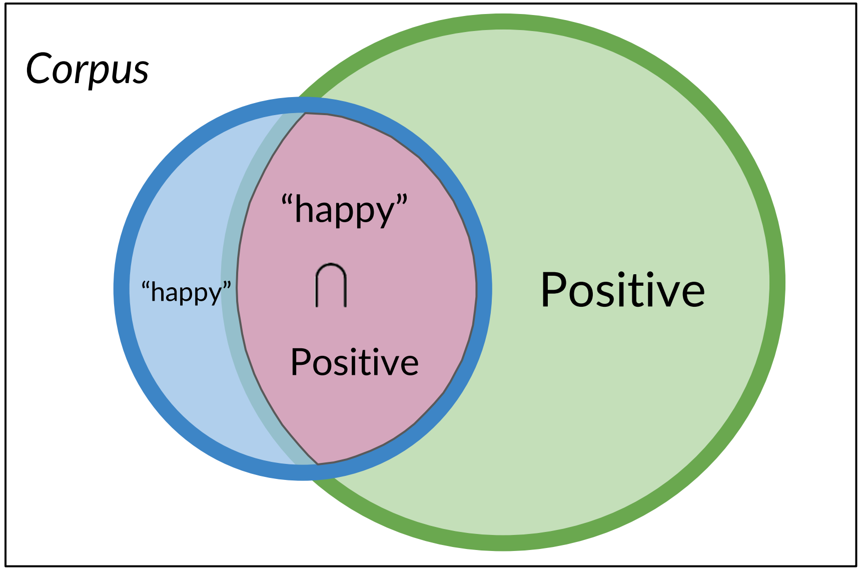
$P(B) = P(happy) = N_{happy} / N$

$P(B) = 4 / 20 = 0.2$

Probability of the intersection



$$P(A \cap B) = P(A, B) = \frac{3}{20} = 0.15$$



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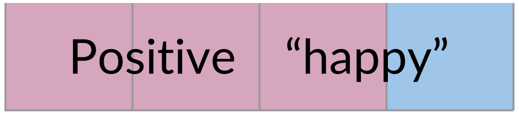
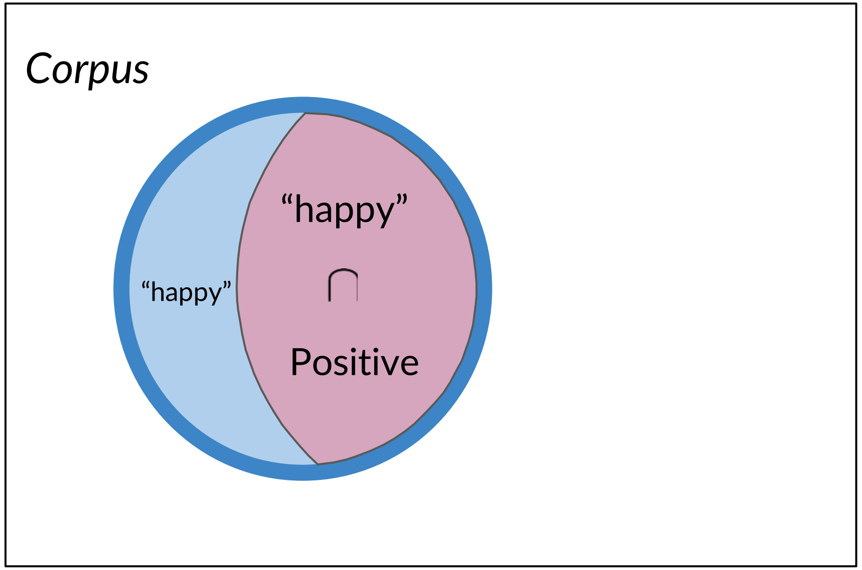
Bayes' Rule

Conditional Probabilities

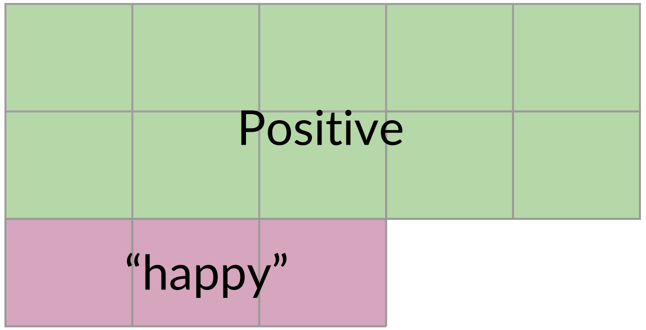


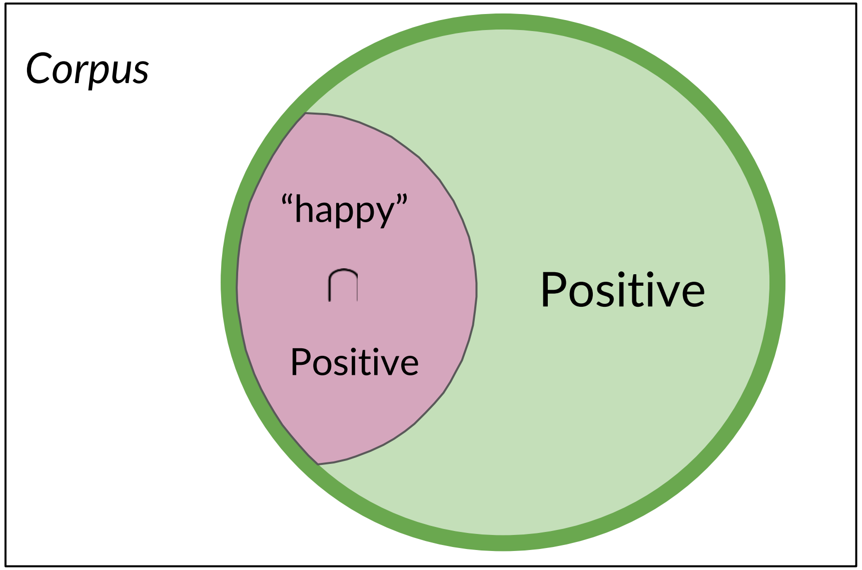
P(A | B) = P(Positive | “happy”)

P(A | B) = 3 / 4 = 0.75



Conditional Probabilities





P(B | A) = P("happy"| Positive)

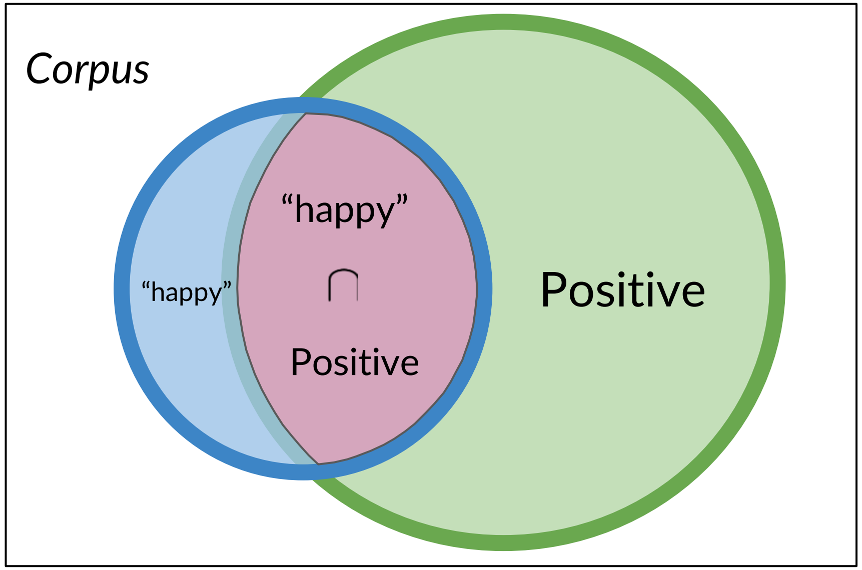
P(B | A) = 3 / 13 = 0.231

Conditional probabilities

Probability of $\underline{B}$, given $\underline{A}$ happened

Looking at the elements of set $\underline{A}$, the chance that one also belongs to set $\underline{B}$

Conditional probabilities



$$P(\text{Positive} \mid \text{“happy”}) = \frac{P(\text{Positive} \cap \text{“happy”})}{P(\text{“happy”})}$$

Bayes' rule

$$P(\text{Positive} \mid \text{``happy''}) = \frac{\boxed{P(\text{Positive} \cap \text{``happy''})}}{P(\text{``happy''})}$$

$$P(\text{``happy''} \mid \text{Positive}) = \frac{\boxed{P(\text{``happy''} \cap \text{Positive})}}{P(\text{Positive})}$$

Quiz

Objective: Derive Bayes' rule from the equations given on the last slide.

Question:

From the equations presented below, express the probability of a tweet being positive given that it contains the word happy in terms of the probability of a tweet containing the word happy given that it is positive

$$P(\text{Positive}|\text{“happy”}) = \frac{P(\text{Positive} \cap \text{“happy”})}{P(\text{“happy”})} \qquad P(\text{“happy”}|\text{Positive}) = \frac{P(\text{“happy”} \cap \text{Positive})}{P(\text{Positive})}$$

Type: Multiple Choice, single answer

Options and solution:

$$\boxed{P(\text{Positive}|\text{“happy”}) = P(\text{“happy”}|\text{Positive}) \times \frac{P(\text{Positive})}{P(\text{“happy”})}}$$

That's right. You just derived Bayes' rule.

$$P(\text{Positive}|\text{“happy”}) = P(\text{“happy”}|\text{Positive}) \times \frac{P(\text{“happy”})}{P(\text{Positive})}$$

The ratio is upside-down in this equation.

$$P(\text{Positive}|\text{“happy”}) = P(\text{“happy”} \cap \text{Positive}) \times \frac{P(\text{Positive})}{P(\text{“happy”})}$$

Your result should not include any intersection probabilities.

$$P(\text{Positive}|\text{“happy”}) = P(\text{“happy”} \cap \text{Positive}) \times \frac{P(\text{“happy”})}{P(\text{Positive})}$$

Your result should not include any intersection probabilities.

Bayes' rule

$$\boxed{P(\text{Positive}|\text{“happy”}) =} \boxed{P(\text{“happy”}|\text{Positive})} \times \boxed{\frac{P(\text{Positive})}{P(\text{“happy”})}}$$

Quiz: Bayes' Rule Applied

Objective: Compute conditional probability using Bayes Rule

Question:

Here, again, is Bayes' rule:

$$\boxed{P(X|Y) = P(Y|X) \times \frac{P(X)}{P(Y)}}$$

Suppose that in your dataset, 25% of the positive tweets contain the word 'happy'. You also know that a total of 13% of the tweets in your dataset contain the word 'happy', and that 40% of the total number of tweets are positive. You observe the tweet: ''happy to learn NLP'. What is the probability that this tweet is positive?

Type: Multiple Choice, single answer

Options and solution:

A: P(Positive | "happy") = 0.77 That's right. You just applied Bayes' rule.

B: P(Positive | "happy") = 0.08 Oops, looks like you might have the ratio of P(X) and P(Y) upside-down.

C: P(Positive | "happy") = 0.10 Remember to calculate the ratio in the formula for Bayes' rule.

D: P(Positive | "happy") = 1.92 Did you use the probability of a tweet being positive? Remember that a fractional probability must be between 0 and 1.

Summary

- Conditional probabilities $\longrightarrow$ Bayes' Rule
- $P(X|Y) = P(Y|X) \times \frac{P(X)}{P(Y)}$

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Naïve Bayes Introduction

Naïve Bayes for Sentiment Analysis

Positive tweets

I am happy because I am learning NLP

I am happy, not sad.

Negative tweets

I am sad, I am not learning NLP

I am sad, not happy

word	Pos	Neg
I	3	3
am	3	3
happy	2	1
because	1	0
learning	1	1
NLP	1	1
sad	1	2
not	1	2
N_{class}	13	12

$P(w_i \mid \text{class})$

word	Pos	Neg
I	3	3
am	3	3
happy	2	1
because	1	0
learning	1	1
NLP	1	1
sad	1	2
not	1	2
Nclass	13	

12

$$p(I \mid Pos) = \frac{3}{13}$$

$$p(I \mid Neg) = \frac{3}{12}$$

word	Pos	Neg
Sum	1	1

$P(w_i \mid \text{class})$

word	Pos	Neg
I	0.24	0.25
am	0.24	0.25
happy	0.15	0.08
because	0.08	0
learning	0.08	0.08
NLP	0.08	0.08
sad	0.08	0.08
not	0.08	0.17

Naïve Bayes

Tweet: I am happy today; I am learning.

$$\prod_{i=1}^{m} \frac{P(w_i|pos)}{P(w_i|neg)} = \frac{0.14}{0.10} = 1.4 \quad > 1$$

$$\cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.20}{0.20}} * \boxed{\frac{0.14}{0.10}} * \cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.20}{0.20}} * \cancel{\frac{0.10}{0.10}}$$

word	Pos	Neg
I	0.20	0.20
am	0.20	0.20
happy	0.14	0.10
because	0.10	0.05
learning	0.10	0.10
NLP	0.10	0.10
sad	0.10	0.10
not	0.10	0.15

Summary

- Naive Bayes inference condition rule for binary classification

$$\prod_{i=1}^{m} \frac{P(w_i|pos)}{P(w_i|neg)}$$

- Table of probabilities

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Laplacian Smoothing

Laplacian Smoothing

$$P(\mathrm{w_i}|\mathrm{class}) = \frac{\mathrm{freq}(\mathrm{w_i}, \mathrm{class})}{\mathrm{N_{class}}}$$

class ∈ {Positive, Negative}

$$P(\mathrm{w_i}|\mathrm{class}) = \frac{\mathrm{freq}(\mathrm{w_i}, \mathrm{class}) + \boxed{1}}{\mathrm{N_{class}} + \boxed{\mathrm{V_{class}}}}$$

N_{class} = frequency of all words in class

V_{class} = number of unique words in class

Introducing $P(w_i$ | class) with smoothing

word	Pos	Neg
I	3	3
am	3	3
happy	2	1
because	1	0
learning	1	1
NLP	1	1
sad	1	2
not	1	2
Nclass	13	

12

Laplacian Smoothing

$$P(\text{I}|\text{Pos}) = \frac{3+1}{13+8}$$

V = 8

word	Pos	Neg
Sum	1	1

Summary

- Laplacian smoothing to avoid *P(w$_i$|class)* = 0

- Naïve Bayes formula

$$\prod_{i=1}^{m} \frac{P(w_i|pos)}{P(w_i|neg)}$$

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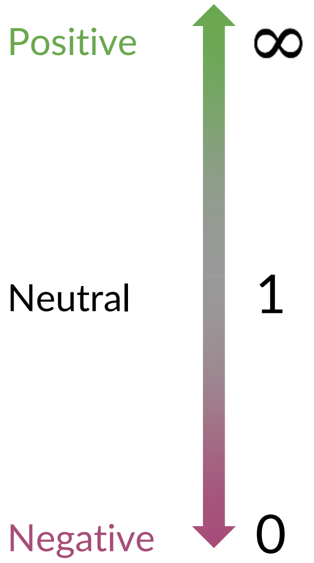
Log Likelihood, Part 1

Ratio of probabilities



word	Pos	Neg	ratio
I	0.19	0.20	
am	0.19	0.20	
happy	0.14	0.10	[illegible]
because	0.10	0.05	
learning	0.10	0.10	
NLP	0.10	0.10	1
sad	0.10	0.15	
not	0.10	0.15	0.6

$$\text{ratio}(w_i) = \frac{P(w_i \mid \text{Pos})}{P(w_i \mid \text{Neg})}$$

$$\approx \frac{freq(w_i, 1) + 1}{freq(w_i, 0) + 1}$$

Naïve Bayes' inference

class $\in$ *{pos, neg}*
w *-> Set of m words in a tweet*

$$\boxed{\frac{P(pos)}{P(neg)}} \boxed{\prod_{i=1}^{m} \frac{P(w_i|pos)}{P(w_i|neg)}} > 1$$

- A simple, fast, and powerful baseline
- A probabilistic model used for classification

Log Likelihood

$$\frac{P(pos)}{P(neg)} \prod_{i=1}^{m} \frac{P(w_i|pos)}{P(w_i|neg)}$$

- Products bring risk of underflow
- $log(a * b) = log(a) + log(b)$
- $log(\frac{P(pos)}{P(neg)} \prod_{i=1}^{n} \frac{P(w_i|pos)}{P(w_i|neg)}) \Rightarrow log\frac{P(pos)}{P(neg)} + \sum_{i=1}^{n} log\frac{P(w_i|pos)}{P(w_i|neg)}$

log prior + **log likelihood**

Calculating Lambda

tweet: I am happy because I am learning.

$$\lambda(w) = log \frac{P(w|pos)}{P(w|neg)}$$

$$\lambda(I) = log \frac{0.05}{0.05}$$

$$\lambda(am) = log \frac{0.04}{0.04} = log(1) = 0$$

word	Pos	Neg	λ
I	0.05	0.05	
am	0.04	0.04	
happy	0.09	0.01	
because	0.01	0.01	
learning	0.03	0.01	
NLP	0.02	0.02	
sad	0.01	0.09	
not	0.02	0.03	-0.4

Summing the Lambdas

doc: I am happy because I am learning.

$$\lambda(w) = log\frac{P(w|pos)}{P(w|neg)}$$

$$\lambda(\text{happy}) = log\frac{0.09}{0.01} \approx 2.2$$

word	Pos	Neg	λ
I	0.05	0.05	0
am	0.04	0.04	0
happy	0.09	0.01	
because	0.01	0.01	
learning	0.03	0.01	
NLP	0.02	0.02	
sad	0.01	0.09	
not	0.02	0.03	-0.4

Summary

- Word sentiment

$$\left\{ \begin{array}{l} ratio(w) = \dfrac{P(w|pos)}{P(w|neg)} \\ \\ \lambda(w) = \; log\dfrac{P(w|pos)}{P(w|neg)} \end{array} \right.$$

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Log Likelihood, Part 2

Log Likelihood

doc: I am happy because I am learning.

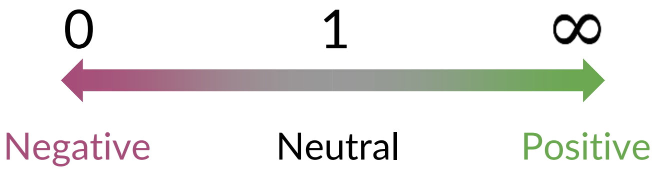
$$\sum_{i=1}^{m} log \frac{P(w_i|pos)}{P(w_i|neg)} = \sum_{i=1}^{m} \lambda(w_i)$$

log likelihood = 0 + 0 + 2.2 + 0 + 0 + 0 + 1.1 = **3.3**

word	Pos	Neg	λ
I	0.05	0.05	0
am	0.04	0.04	0
happy	0.09	0.01	2.2
because	0.01	0.01	0
learning	0.03	0.01	1.1
NLP	0.02	0.02	0
sad	0.01	0.09	-2.2
not	0.02	0.03	-0.4

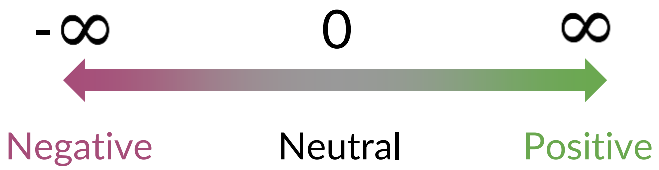
Log Likelihood

$$\prod_{i=1}^{m} \frac{P(w_i|pos)}{P(w_i|neg)} > 1$$



$$\sum_{i=1}^{m} log \frac{P(w_i|pos)}{P(w_i|neg)} > 0$$

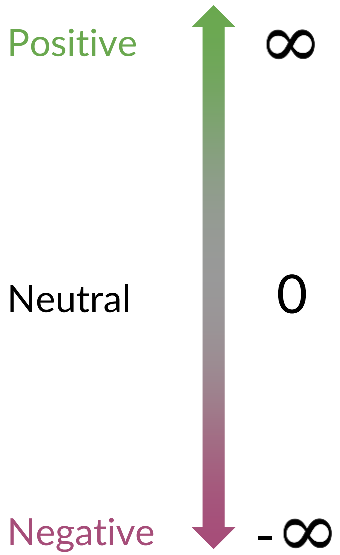
3.3 > 0



Summary

Tweet sentiment:

$$log\prod_{i=1}^{m} ratio(w_i) = \sum_{i=1}^{m} \lambda(w_i) \quad > 0$$



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Training Naïve Bayes

Outline

- Five steps for training a Naïve Bayes model

Training Naïve Bayes

Step 0: Collect and annotate corpus

Positive tweets

I am happy because I am learning NLP

I am happy, not sad. @NLP

Negative tweets

I am sad, I am not learning NLP

I am sad, not happy!!

Step 1: Preprocess

- Lowercase
- Remove punctuation, urls, names
- Remove stop words
- Stemming
- Tokenize sentences

Positive tweets

[happi, because, learn, NLP]

[happi, not, sad]

Negative tweets

[sad, not, learn, NLP]

[sad, not, happi]

Training Naïve Bayes

Positive tweets

[happi, because, learn, NLP]

[happi, not, sad]

Negative tweets

[sad, not, learn, NLP]

[sad, not, happi]

Step 2: Word count

freq(w, class)

word	Pos	Neg
happi	2	1
because	1	0
learn	1	1
NLP	1	1
sad	1	2
not	1	2
N_{class}	7	7

Training Naïve Bayes

freq(w, class)

word	Pos	Neg
happi	2	1
because	1	0
learn	1	1
NLP	1	1
sad	1	2
not	1	2
N_{class}	7	7

Step 3:
$P(\mathrm{w}|\mathrm{class})$

$$V_{\mathrm{class}} = 6$$

$$\frac{\mathrm{freq(w, class)} + 1}{N_{\mathrm{class}} + V_{\mathrm{class}}}$$

$$\lambda(w) = log\frac{P(\mathrm{w}|\mathrm{pos})}{P(\mathrm{w}|\mathrm{neg})}$$

Step 4:
Get
lambda

word	Pos	Neg	λ
happy	0.23	0.15	0.43
because	0.15	0.07	0.6
learning	0.08	0.08	0
NLP	0.08	0.08	0
sad	0.08	0.17	-0.75
not	0.08	0.17	-0.75

Training Naïve Bayes

Step 5:
Get the
log prior

D_{pos} = Number of positive tweets
D_{neg} = Number of negative tweets

$$\text{logprior} = log\frac{D_{pos}}{D_{neg}}$$

If dataset is balanced, D_{pos} = D_{neg} and logprior = 0.

Summary

1. Get or annotate a dataset with positive and negative tweets
2. Preprocess the tweets: process_tweet(tweet) ➞ $[w_1, w_2, w_3, \ldots]$
3. Compute freq(w, class)
4. Get P(w | pos), P(w | neg)
5. Get λ(w)
6. Compute logprior = log(P(pos) / P(neg))

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Testing Naïve Bayes

Outline

- Predict using a Näive Bayes Model
- Using your validation set to compute model accuracy

Predict using Naïve Bayes

- log-likelihood dictionary $\lambda(w) = log\frac{P(w|pos)}{P(w|neg)}$

$$logprior = log\frac{D_{pos}}{D_{neg}} = 0$$

- Tweet: [I, pass, the, NLP, interview] 👍

word	λ
I	-0.01
the	-0.01
happi	0.63
because	0.01
pass	0.5
NLP	0
sad	-0.75
not	-0.75

$$score = -0.01 + 0.5 - 0.01 + 0 + logprior = 0.48$$

$$pred = score > 0$$

Testing Naïve Bayes

- $X_{val}\ Y_{val}\ \lambda\ logprior$

$$score = predict(X_{val}, \lambda, logprior)$$

$$pred = score > 0$$

$$\begin{bmatrix} 0.5 \\ -1 \\ 1.3 \\ \vdots \\ score_m \end{bmatrix} > 0 = \begin{bmatrix} 0.5 > 0 \\ -1 > 0 \\ 1.3 > 0 \\ \vdots \\ score_m > 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ \vdots \\ pred_m \end{bmatrix}$$

Testing Naïve Bayes

- $X_{val}\ Y_{val}\ \lambda\ logprior$

$$score = predict(X_{val}, \lambda, logprior)$$

$$pred = score > 0$$

$$\frac{1}{m}\sum_{i=1}^{m}(pred_i == Y_{val_i})$$

$$\begin{bmatrix} \underline{0} \\ \underline{1} \\ 1 \\ \vdots \\ pred_m \end{bmatrix} == \begin{bmatrix} \underline{0} \\ \underline{0} \\ 1 \\ \vdots \\ Y_{val_m} \end{bmatrix}$$

$$\begin{bmatrix} \underline{1} \\ \underline{0} \\ 1 \\ \vdots \\ pred_m == Y_{val_m} \end{bmatrix}$$

Summary

- $X_{val}\ Y_{val} \longrightarrow$ Performance on unseen data
- Predict using λ and $logprior$ for each new tweet
- Accuracy $\longrightarrow \displaystyle \frac{1}{m}\sum_{i=1}^{m}(pred_i == Y_{val_i})$
- What about words that do not appear in λ(w)?

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Applications of Naïve Bayes

Applications of Naïve Bayes

$$P(pos|tweet) \approx P(pos)P(tweet|pos)$$

$$P(neg|tweet) \approx P(neg)P(tweet|neg)$$

$$\frac{P(pos|tweet)}{P(neg|tweet)} = \frac{P(pos)}{P(neg)} \prod_{i=1}^{m} \frac{P(w_i|pos)}{P(w_i|neg)}$$

Applications of Naïve Bayes

Author identification:

$$\frac{P(\quad|\text{book})}{P(\quad|\text{book})}$$

Spam filtering:

$$\frac{P(\text{spam}|\text{email})}{P(\text{nonspam}|\text{email})}$$

Applications of Naïve Bayes

Information retrieval:

$$P(\mathrm{document_k}|\mathrm{query}) \propto \prod_{i=0}^{|query|} P(\mathrm{query_i}|\mathrm{document_k})$$

Retrieve document if $P(\mathrm{document_k}|\mathrm{query}) > \mathrm{threshold}$

"Icon made by Vector Market from www.flaticon.com"

Applications of Naïve Bayes

Word disambiguation:

$$\frac{P(\text{river}|\text{text})}{P(\text{money}|\text{text})}$$

Bank:

"Pictures with CC"

Naïve Bayes Applications

- Sentiment analysis
- Author identification
- Information retrieval
- Word disambiguation
- Simple, fast and robust!

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Naïve Bayes Assumptions

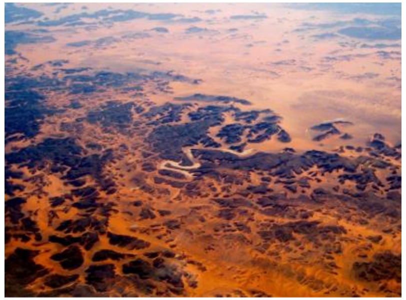
Outline

- Independence
- Relative frequency in corpus

Naïve Bayes Assumptions

- Independence

“It is sunny and hot in the Sahara desert.”



Naïve Bayes Assumptions

“It’s always cold and snowy in ___ .”

spring?? summer? fall?? $\boxed{\text{winter}}$??

Naïve Bayes Assumptions

- Relative frequencies in corpus

Summary

- Independence: Not true in NLP
- Relative frequency of classes affect the model

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Error Analysis

Outline

- Removing punctuation and stop words
- Word order
- Adversarial attacks

Processing as a Source of Errors: Punctuation

Tweet: My beloved grandmother$\cancel{:}$(

processed_tweet: [belov, grandmoth]

Processing as a Source of Errors: Removing Words

Tweet: This is not good, because your attitude is not even close to being nice.

processed_tweet: [good, attitude, close, nice]

Processing as a Source of Errors: Word Order

Tweet: I am happy because I do not go.

Tweet: I am not happy because I did go.

Adversarial attacks

Sarcasm, Irony and Euphemisms

Tweet: This is a ridiculously powerful movie. The plot was gripping and I cried right through until the ending!

processed_tweet: [ridicul, power, movi, plot, grip, cry, end]

Summary

- Removing punctuation
- Removing words
- Word order
- Adversarial attacks