Asymptotic analysis of the transmission eigenvalue problem for a Dirichlet obstacle coated by a thin layer of non-absorbing media

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***Abstract*—We consider the transmission eigenvalue problem for an impenetrable obstacle with Dirichlet boundary condition surrounded by a thin layer of non-absorbing in-homogeneous material. We derive a rigorous asymptotic expansion for the first transmission eigenvalue with respect to the thickness of the thin layer. Our convergence analysis is based on a Max–Min principle and an iterative approach which involves estimates on the corresponding eigen functions. We provide explicit expressions for the terms in the asymptotic expansion up to order 3.**

I. Introduction

The main concern of this study is to develop a rigorous asymptotic expansion for transmission eigenvalues as $\delta \to 0$. Our asymptotic analysis is based on an iterative and constructive approach. We restrict ourselves here to the first transmission eigenvalue [1]. As expected this transmission eigenvalue is close to the first Dirichlet eigenvalue up to order δ, result that is proved directly in this paper by using the Max–Min principle.

II. Ease of Use

A. Maintaining the Integrity of the Specifications

A better model to capture both the thickness and the refractive index in the first-order term in the context of electromagnetic scattering is to write the problem in terms of the magnetic field, which would lead to Neumann boundary condition on the boundary of the inclusion. This measurement and others are deliberate, using specifications that anticipate your paper as one part of the entire proceedings, and not as an independent document. [2] Please do not revise any of the current designations.

III. Formal asymptotic expansion

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections III-A–III-E below for more information on proofreading, spelling and grammar.

Identify applicable funding agency here. If none, delete this.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Formulation of the problem

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
  1) The convergence of the zero-order approximation
  2) The convergence of the first-order approximation
  3) The convergence of the second-order approximation
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
- Do not mix complete spellings and abbreviations of units: "Wb/m^2" or "webers per square meter", not "webers/m^2". Spell out units when they appear in text: ". . . a few henries", not ". . . a few H".
- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

C. Elliptic estimates for interior transmission problems in the presence of thin layers

The main ingredient to justify the formal expansion above is to establish explicit a priories estimates with respect to δ for the solutions of the interior transmission problem.

$$a + b = \gamma \tag{1}$$

$$\tau'(x) = -k(s)\gamma(s) \tag{2}$$

$$\alpha(x) = \frac{\sum_{i=1}^{a} x_i}{\sum_{j=1}^{b} x_j} = \frac{x_1 + x_2 + x_3 + ... + x_{a-1} + x_a}{x_1 + x_2 + x_3 + ... + x_{b-1} + x_b} \tag{3}$$

$$y(x) = \begin{cases} 5, \text{ if x is divisible by 5} \\ 10, \text{ if x is divisible by 10} \\ -1, \text{ otherwise} \end{cases} \tag{4}$$

Definite Integral as a limit of Reimann Sum is denoted as

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i-1})\Delta x = \int_a^b f(x)dx \tag{5}$$

where

$$\Delta x = \frac{b-a}{n}$$

The Polynomial Equation can be written as

$$p_n x^n + p_{n-1}x^{n-1} + p_{n-2}x^{n-2} + ... + p_1 x + p_0 = 0$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(2)", not "Eq. (2)" or "equation (2)", except at the beginning of a sentence: "Equation (2) is . . ."

D. Convergence analysis

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line. [3]

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols. Please note that the `{subequations}`

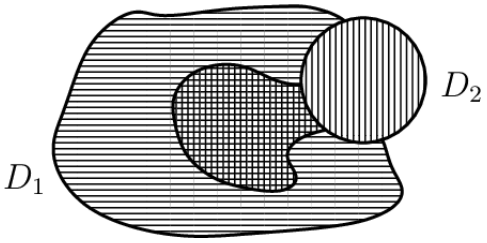

Fig. 1. Factorization method

environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

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Do not use `\nonumber` inside the `{array}` environment. It will not stop equation numbers inside `{array}` (there won't be any anyway) and it might stop a wanted equation number in the surrounding equation.

Time(s)		Athlete 1	Athlete 2
1.00	Distance	4.02	4.02
	Speed	6.5	6.44
2.00	Distance	11.5	11.37
	Speed	8.08	7.9
3.00	Distance	19.82	19.48
	Speed	8.47	8.23

TABLE I
DISTANCE AND SPEED OF TWO ATHLETES AT DIFFERENT TIME

E. Funding

- The word "data" is plural, not singular. [4]
- The subscript for the permeability of vacuum μ_0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".
- Be aware of the different meanings of the homophones "affect" and "effect", "complement" and "compliment", "discreet" and "discrete", "principal" and "principle".
- Do not confuse "imply" and "infer".
- The prefix "non" is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the "et" in the Latin abbreviation "et al.".
- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

An excellent style manual for science writers is [6].

F. An application of Green's identity

The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization). [5]

G. A Coercivity Constant

Run-in heads, such as “Abstract”, will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

H. Dirichlet eigenvalue

a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures. Use the abbreviation “Fig. 2”, even at the beginning of a sentence.

TABLE II
COMPARISON OF LANCZOS AND DAVIDSON METHODS

Davidson Ritz value	Lanczos Ritz value	Residual norm
3.23529	3.23529	5.27
3.17006	1.21302	3.17
1.65718	0.784054	1.80
1.48600	0.476551	1.78

Our asymptotic analysis is based on an iterative and constructive approach. We restrict ourselves here to the first transmission eigenvalue. As expected this transmission eigenvalue is close to the first Dirichlet eigenvalue up to order δ, result that is proved directly in this paper by using the Max– Min principle. As an example, write the quantity “Magnetization”, or “Magnetization, M”, not just “M”. If including units in the label, present them within parentheses. Do not label axes only with units.Then, the main idea of our approach is, roughly speaking, having proved convergence of order k for the asymptotic expansion of the transmission eigenvalue. [4]

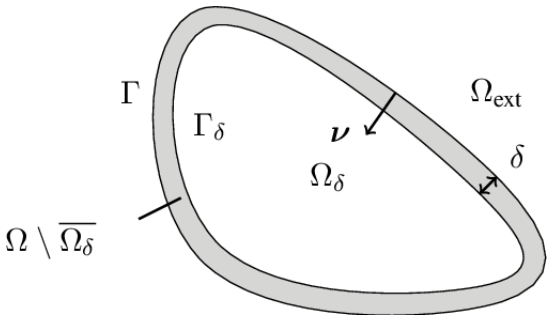

Fig. 2. The Scattering Layered Object.

Figure Labels: Use 8 point Times New Roman for Figure labels. [6]Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. In the example, write “Magnetization (A/m)” or “Magnetization {A[m(1)]}”, not just “A/m”. Do not label axes with a ratio of quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

ACKNOWLEDGMENT

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks . . .”. Instead, try “R. B. G. thanks. . .”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

ABOUT

Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use “Ref. [3]” or “reference [3]” except at the beginning of a sentence: “Reference [3] was the first . . .” do not use “et al.”. Papers that have not been published, even if they have been submitted for publication, should be cited as “unpublished” [4]. Papers that have been accepted for publication should be cited as “in press” [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

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