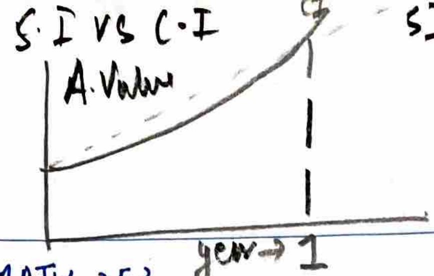

$A(t+s) = AV_{t+s}$

MATH 253

Simple Interest and Discount.

Interest $= A(t+s) - A(t)$ or $AV_{t+s} - AV_t$

" over a p.o.t $= A(t+s) - A(t) / A(t)$.

" earned in n-th period $= A(n) - A(n-1)$ or $AV_n - AV_{n-1}$

* $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$
* $(a^x)' = a^x \ln(a)$
* $(e^x)' = e^x$
* $S_N = \frac{a(1-r^{N+1})}{1-r}$ for $|r|<1$, $-1<r<1$

Nth partial sum

* $\frac{a}{1-r}$ if $|r|<1$

if $r \geq 1$, series doesnt converge

Effective

* $i = \frac{a(1)-a(0)}{a(0)=1} = \frac{1+i-1}{1} = \frac{A(1)-A(0)}{A(0)} = \frac{I_1}{A(0)}$

$i_n = \frac{A(n)-A(n-1)}{A(n-1)} = \frac{I_n}{A(n-1)}$

effective.

* $i = \frac{d}{1-d}$, $d = \frac{i}{1+i}$

* $a(t) = 1+it$
* $A(t) = k(1+it)$
* $V = 1/1+i$ [V = discount factor]
* $a^{-1}(t) = 1/1+it$ [discount func.]

$FV = PV(1+it)$

$a(t) = 1/1-dt$

$a^{-1}(t) = 1-dt$

Arithmetic Sum

* $S_N = \sum_{i=0}^{N-1} (a+id)$

* Discount $= A(n+s) - A(n)$
* Discount rate $= \frac{A(n+s)-A(n)}{A(n+s)}$
* Annual discount rate $= \frac{A(n)-A(n-1)}{A(n)}$

AP+GP

* $A = Pv + (P+Q)v^2 + (P+2Q)v^3 + \cdots + (P+(n-1)Q)v^n$

A = Annuity → Series of payment at equal intervals

A is annuity which starts w/ initial payment P which is increased by Q

Q is usually added to account for inflation

Compound Interest + Discount

* $a(t) = (1+i)^t$
* $FV = PV(1+i)^t \to CI$
* $PV = AV_t(1-d)^t \to CD$
* $i = \frac{d}{1-d}$

$FV = PV(1+i)^t = PV\left(1+\frac{i^{M}}{M}\right)^{Mt}$

$V = \frac{1}{1+i}$, where i is the interest rate

* $A = \frac{1}{i}\left(P\cdot(1-v^n) + Qv\cdot\frac{1-v^n}{1-v} - Qnv^n\right)$

* nth Taylor Polynomial of f at a is

$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \cdots$

$f(x) = \sum_{i=0}^{\infty} \frac{f^{n}(a)}{i!}(x-a)^i$

Effective discount rate

$\boxed{\begin{array}{l} d = 1-\left(1-\frac{d^{M}}{M}\right)^{M} \\ d = 1-(1-d)^{1/t} \end{array}}$

$\frac{i^{M}}{M} = \frac{\frac{d^{M}}{M}}{1-\frac{d^{M}}{M}}$

$\frac{d^{M}}{M} = \frac{i^{M}/M}{1+i^{M}/M}$

$\int u\,dv = uv - \int v\,du =$

$\int f'(x)g(x)dx = f(x)g(x) - \int g'(x)\cdot f(x)dx$

Constant force of Interest

$\delta_t = \frac{A'(t)}{A(t)} = \frac{a'(t)}{a(t)}$ $a(t) = e^{\int_0^t \delta_r dr} \sim e^{\delta} = \left(1+\frac{i^{m}}{m}\right)^{m}$

delta

Amount of interest earned over n periods $= A(n) - A(0)$.

$\delta'_t = -\frac{\frac{d}{dt} a^{-1}(t)}{a^{-1}(t)}$ | $\delta'_t = \delta_t$

Force of discount

$\delta_t = \frac{a'(t)}{a(t)} = \frac{i}{1+it}$

$\delta'_t = -\frac{\frac{d}{dt}(a^{-1}(t))}{a^{-1}(t)} = \frac{d}{1-dt}$

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* $i = e^{\delta} - 1$

* $\delta = \ln(1+i)$

$(e^{\delta})^t = (i+1)^t$ | $d < d^p < \delta < i^m < i$

$\frac{d^m}{m} = \frac{i^m}{1+i^m/m}$

$i_k = (1+r)^k(1+i) - 1 \Rightarrow$ accumulated value of 1 at the end of n-periods where effective r.o.i for kth period.

Annuity

Present Value $= a_{\overline{n}|} = v + v^2 + \cdots + v^n = \frac{v - v^{n+1}}{1-v} = \frac{1-v^n}{i}$

Future Value $= s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$

$s_{\overline{n}|} = a_{\overline{n}|}(1+i)^n$

$a_{\overline{m+n}|} = a_{\overline{m}|} + v^m a_{\overline{n}|}$

$\frac{1}{a_{\overline{n}|}} = \frac{1}{s_{\overline{n}|}} + i$

Similar to annuity immediate, we can calculate annuity-due

Present Value $\ddot{a}_{\overline{n}|i} = 1 + v + \cdots + v^{n-1} = \frac{1-v^n}{d}$

Future Value $\ddot{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{d}$

$\frac{1}{\ddot{a}_{\overline{n}|}} = \frac{1}{\ddot{s}_{\overline{n}|}} + d.$

Relationship b/w annuity-due & immediate

$\ddot{a}_{\overline{n}|} = a_{\overline{n}|}(1+i)$, $\ddot{a}_{\overline{n}|} = 1 + a_{\overline{n-1}|}$

$\ddot{s}_{\overline{n}|} = s_{\overline{n}|}(1+i)$, $\ddot{s}_{\overline{n}|} = s_{\overline{n+1}|} - 1$