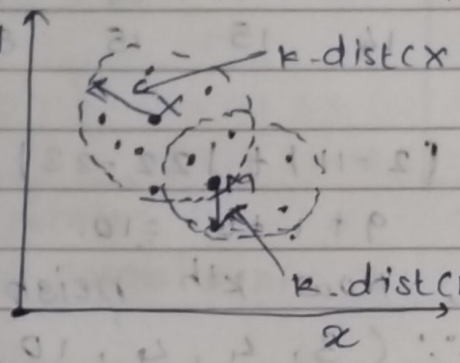
* Local outlier Factor (LOF) :- (unsupervised)
  - density based technique of outliers in ml.
  - LOF compares the local density of point with the densities of its neighbours.
  - points that have a substantially lower density than their neighbors are considered to be outliers.

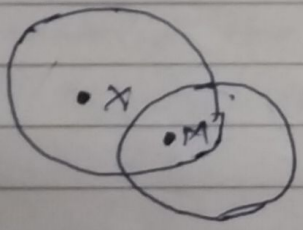
1) Reacability_distance :-

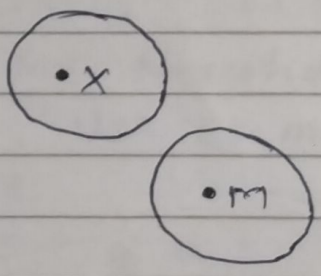
$$reach\_dist_k(M,X) = max(\{k\_dist(X), dist(M,X)\})$$

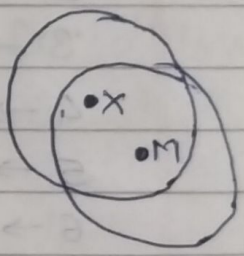
Here,
(M,X) = distance of X from M.



* cases:-

1) 

2) 

3) 

2) Local Rechability density (LRD) :-

$$LRD_k(M) = \frac{1}{\left[\frac{\sum_{X \in N_k(M)} reach\_dist(X)}{|N_k(M)|}\right]}$$

3) $LOF_k(M) = \frac{1}{|N_k(M)|} \cdot \sum_{X \in N_k(M)} \frac{LRD_k(X)}{LRD_k(M)}$

* Example :-

index	x	y
0	2	3
1	4	5
2	1	2
3	2	4
4	3	4
5	7	5
6	3	5
7	8	9
8	2	1
9	3	6

use distance matrix given in ex. pdf. $k = 2$

Finding k-dist & N_k.

index	k_dist	$N_k(M)$	$\|N_k(M)\|$
0	1.41	(2, 3, 4)	3
1	1.41	(4, 6, 9)	3
2	2.24	(0, 3, 8)	3
3	1.41	(0, 4, 6)	3
4	1.41	(0, 1, 3, 6)	4
5	4	(1, 6)	2
6	1.41	(1, 3, 4, 9)	4
7	5.66	(1, 5)	2
8	2	(0, 2)	2
9	1.41	(1, 6)	2

* Reachability distance:-

reach-dist$_k$(0,2) = max(k-dist(2), dist(0,2))

= max(2.24, 1.41)

= 2.24

∴ LRD$_k$(0):-

X	k-dist(x)	d(x,m)	reach-dist(m,x)
2	2.24	1.41	2.24
3	1.41	1	1.41
4	1.41	1.41	1.41

NOW, LRD$_k$(0) = $\frac{1}{\frac{\text{reach-dist}_k(0,2) + \text{reach-dist}(0,3) + \text{reach-dist}(0,4)}{3}}$

= $\frac{1}{\frac{2.24 + 1.41 + 1.41}{3}}$

= 0.59

LRD$_k$(2) =

X	k-dist(x)	d(x,m)	reach-dist(m,x)
0	1.41	1.41	1.41
3	1.41	2.24	~~1.41~~ 2.24
8	2	1.41	2

LRD$_k$(2) = $\frac{1}{\frac{1.41 + 2.24 + 2}{3}}$

= ~~0.62~~ 0.53

LRD(2)

Rc(2,x)

LRDk(3) :-

x	k-dist(x)	d(x,m)	reach.dist(m,x).
0	1.41	1	1.41
4	1.41	1	1.41
6	1.41	1.41	1.41

$$LRDk(3) = \frac{1}{\frac{1.41+1.41+1.41}{3}}$$

$$= 0.70$$

LRDk(4) :-

x	k-dist(x)	d(x,m)	reach.dist(m,x)
0	1.41	1.41	1.41
1	1.41	1.41	1.41
3	1.41	1	1.41
6	1.41	1	1.41

$$LRDk(4) = \frac{1}{\frac{1.41+1.41+1.41+1.41}{4}}$$

$$= 0.70$$

$$LOFk(0) = \frac{1}{|Nk(m)|} \cdot \sum_{x \in Nk(m)} \frac{LRDk(x)}{LRDk(m)}$$

$$= \frac{1}{3} \times \left(\frac{LRDk(2)}{LRDk(0)} + \frac{LRDk(3)}{LRDk(0)} + \frac{LRDk(4)}{LRDk(0)} \right)$$

$$= \frac{1}{3} \times \left(\frac{0.53}{0.59} + \frac{0.70}{0.59} + \frac{0.70}{0.59} \right)$$

$$= 1.09$$