

Day	Weather	Temperature	Humidity	Wind	Play?
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Cloudy	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Cloudy	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Cloudy	Mild	High	Strong	Yes
13	Cloudy	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

A] Calculate IG of Weather

Step 1: Entropy of entire dataset

$$S\{9, 5\} = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

Step 2: Entropy of all attributes

$$\text{Entropy of sunny } \{2, 3\} = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$\text{Entropy of Cloudy } \{4, 0\} = -\frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4} = 0$$

$$\text{Entropy of Rain } \{3, 2\} = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97$$

$$\begin{aligned} \text{Information Gain} &= \text{Entropy(whole)} - \frac{5}{14} \text{Ent}(S) - \frac{4}{14} \text{Ent}(C) - \frac{5}{14} \text{Ent}(R) \\ &= 0.94 - \frac{5}{14} \times 0.97 - \frac{4}{14} \times 0 - \frac{5}{14} \times 0.97 \\ &= 0.246 \end{aligned}$$

B] Calculate IG of Temperature

Step 1 : 0.94

Step 2 : Entropy of all attributes

$$\text{Entropy of Hot } \{+2, -2\} = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1.0$$

$$\text{Entropy of Mild } \{+4, -2\} = -\frac{4}{6} \log_2 \frac{4}{6} - \frac{2}{6} \log_2 \frac{2}{6} = 0.91$$

$$\text{Entropy of Cold } \{+3, -1\} = -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.81$$

$$\text{I.G.} = E(\text{whole data}) - \frac{4}{14} \times 1.0 - \frac{6}{14} \times 0.91 - \frac{4}{14} \times 0.81$$

$$= 0.029$$

C] Calculate IG of Humidity

Step 1 : 0.94

Step 2 : Entropy of all attributes

$$\text{Entropy of High } \{+8, -4\} = -\frac{8}{7} \log_2 \frac{8}{7} - \frac{4}{7} \log_2 \frac{4}{7} = 0.98$$

$$\text{Entropy of Normal } \{+6, -1\} = -\frac{6}{7} \log_2 \frac{6}{7} - \frac{1}{7} \log_2 \frac{1}{7} = 0.59$$

$$\text{I.G.} = 0.94 - \frac{7}{14} \times 0.98 - \frac{7}{14} \times 0.59$$

$$= 0.15$$

0] Calculate IG of Wind

Step 1 : 0.94

Step 2 : Entropy of all attributes:

$$\text{Entropy of Strong } \{+3, -3\} = -\frac{3}{6} \log_2 \frac{3}{6} - \frac{3}{6} \log_2 \frac{3}{6} = 1.0$$

$$\text{Entropy of Normal } \{+6, -2\} = -\frac{6}{8} \log_2 \frac{6}{8} - \frac{2}{8} \log_2 \frac{2}{8} = 0.81$$

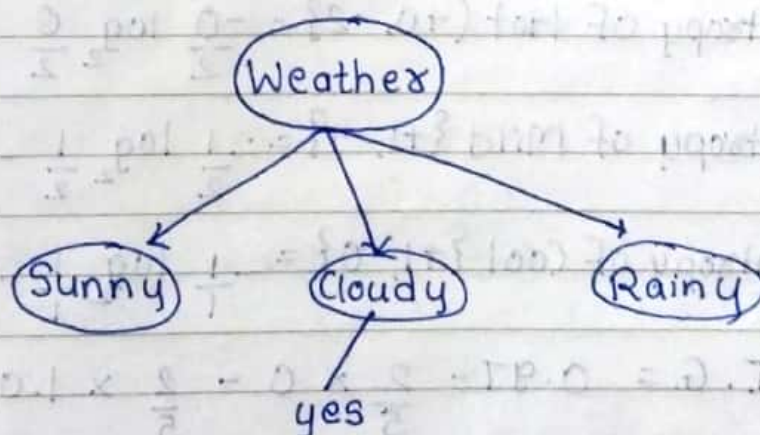
$$\begin{aligned} \text{I.G} &= 0.94 - \frac{6}{14} \times 1.0 - \frac{8}{14} \times 0.81 \\ &= 0.0478 \end{aligned}$$

$$\text{Gain (S, Weather)} = 0.246 \quad \leftarrow \text{max (root node)}$$

$$\text{Gain (S, Temp)} = 0.029$$

$$\text{Gain (S, Humidity)} = 0.15$$

$$\text{Gain (S, Wind)} = 0.0478$$



> Calculate for Sunny

Day	Weather	Temperature	Humidity	Wind	Play
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Step 1: Entropy of whole dataset

$$S\{+2, -3\} = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

Step 2: Entropy of all attributes

a) Temperature

$$\cdot \text{Entropy of Hot } \{+0, -2\} = -\frac{0}{2} \log_2 \frac{0}{2} - \frac{2}{2} \log_2 \frac{2}{2} = 0$$

$$\cdot \text{Entropy of Mild } \{+1, -1\} = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1.0$$

$$\cdot \text{Entropy of Cool } \{+1, -0\} = -\frac{1}{1} \log_2 \frac{1}{1} - \frac{0}{1} \log_2 \frac{0}{1} = 0$$

$$\cdot I.G = 0.97 - \frac{2}{5} \times 0 - \frac{2}{5} \times 1.0 - \frac{1}{5} \times 0 = 0.57$$

b) Humidity

Step 2:

$$\cdot \text{Entropy of High } \{+0, 3\} = -\frac{0}{3} \log_2 \frac{0}{3} - \frac{3}{3} \log_2 \frac{3}{3} = 0$$

$$\cdot \text{Entropy of Normal } \{+2, 0\} = -\frac{2}{2} \log_2 \frac{2}{2} - \frac{0}{2} \log_2 \frac{0}{2} = 0$$

$$\begin{aligned} \cdot \text{I.G} &= 0.97 - \frac{2}{5} \times 0 - \frac{3}{5} \times 0 \\ &= 0.97 \end{aligned}$$

c) Wind

$$\cdot \text{Entropy of Strong } \{+1, -1\} = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

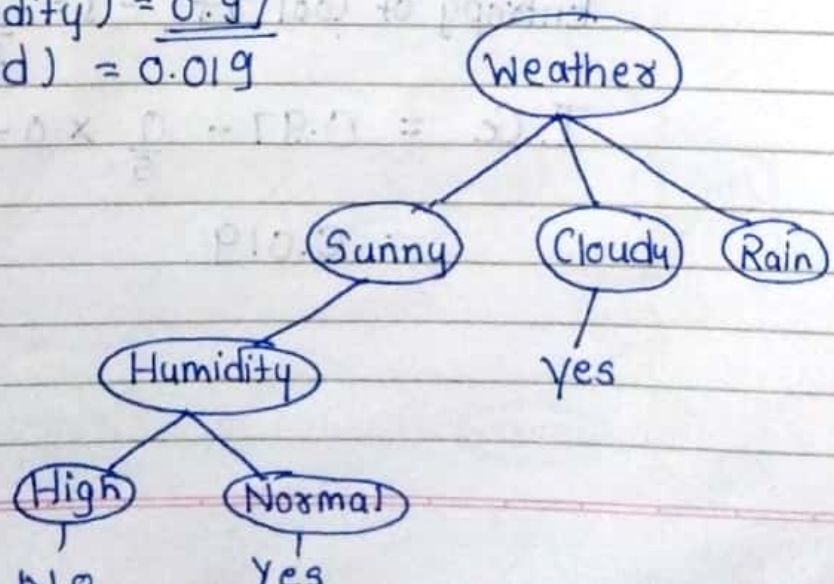
$$\cdot \text{Entropy of Weak } \{+1, -2\} = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = 0.918$$

$$\begin{aligned} \cdot \text{I.G} &= 0.97 - \frac{2}{5} \times 1 - \frac{3}{5} \times 0.918 \\ &= 0.019 \end{aligned}$$

$$\text{Gain (Sunny, Temp)} = 0.57$$

$$\text{Gain (Sunny, Humidity)} = \underline{0.97}$$

$$\text{Gain (Sunny, Wind)} = 0.019$$



7 Calculate for Rainy

Day	Weather	Temperature	Humidity	Wind	Play?
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Entropy of whole dataset

$$\text{Entropy of Rain } \{+3, -2\} = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97$$

Entropy of all attributes

a) Temperature

$$\text{Entropy of Hot } \{+0, -0\} = -\frac{0}{2} \log_2 \frac{0}{2} - \frac{0}{2} \log_2 \frac{0}{2} = 0$$

$$\text{Entropy of Mild } \{+2, -1\} = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.918$$

$$\text{Entropy of Cool } \{+1, -1\} = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1.0$$

$$\begin{aligned} I.G. &= 0.97 - \frac{0}{5} \times 0 - \frac{3}{5} \times 0.918 - \frac{2}{5} \times 1.0 \\ &= 0.019 \end{aligned}$$

b) Humidity

$$\text{Entropy of High } \{+1, -1\} = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

$$\text{Entropy of Normal } \{+2, -1\} = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.98$$

$$\begin{aligned} \text{I. G} &= \text{Entropy (Rain)} - \frac{2}{3} \times 1 - \frac{3}{5} \times 0.98 \\ &= 0.019 \end{aligned}$$

c) Wind

$$\text{Entropy of Strong } \{+0, -2\} = -\frac{0}{2} \log_2 \frac{0}{2} - \frac{2}{2} \log_2 \frac{2}{2} = 0$$

$$\text{Entropy of Weak } \{+3, -0\} = -\frac{3}{3} \log_2 \frac{3}{3} - \frac{0}{3} \log_2 \frac{0}{3} = 0$$

$$\begin{aligned} \text{I. G} &= 0.97 - \frac{2}{5} \times 0 - \frac{3}{5} \times 0 \\ &= 0.97 \end{aligned}$$

$$\text{Gain (SRain, Temp)} = 0.019$$

$$\text{Gain (SRain, Humidity)} = 0.019$$

$$\text{Gain (SRain, Wind)} = 0.97$$

