E.S. Gopi

Algorithm Collections for Digital Signal Processing Applications using Matlab

Springer

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This book is dedicated to

my Wife G.Viji

and my Son V.G.Vasig

Contents

Preface

The Algorithms such as SVD, Eigen decomposition, Gaussian Mixture Model, PSO, Ant Colony etc. are scattered in different fields. There is the need to collect all such algorithms for quick reference. Also there is the need to view such algorithms in application point of view. This Book attempts to satisfy the above requirement. Also the algorithms are made clear using MATLAB programs. This book will be useful for the Beginners Research scholars and Students who are doing research work on practical applications of Digital Signal Processing using MATLAB.

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Thanks

E.S.Gopi

Chapter 1

ARTIFICIAL INTELLIGENCE
Algorithm Collections

1. PARTICLE SWARM ALGORITHM

Consider the two swarms flying in the sky, trying to reach the particular destination. Swarms based on their individual experience choose the proper path to reach the particular destination. Apart from their individual decisions, decisions about the optimal path are taken based on their neighbor's decision and hence they are able to reach their destination faster. The mathematical model for the above mentioned behavior of the swarm is being used in the optimization technique as the Particle Swarm Optimization Algorithm (PSO).

For example, let us consider the two variables 'x' and 'y' as the two swarms. They are flying in the sky to reach the particular destination (i.e.) they continuously change their values to minimize the function $(x-10)^2+(y-5)^2$. Final value for 'x' and 'y' are 10.1165 and 5 respectively after 100 iterations.

The Figure 1-1 gives the closed look of how the values of x and y are changing along with the function value to be minimized. The minimization function value reached almost zero within 35 iterations. Figure 1-2 shows the zoomed version to show how the position of x and y are varying until they reach the steady state.

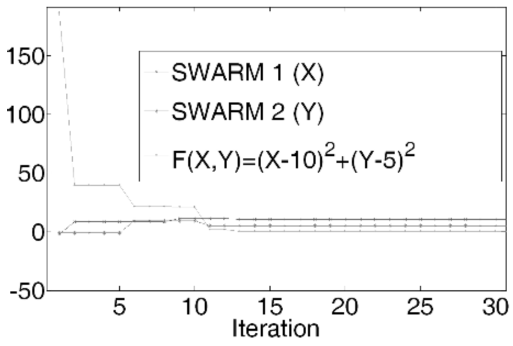


Figure 1-1. PSO Example zoomed version

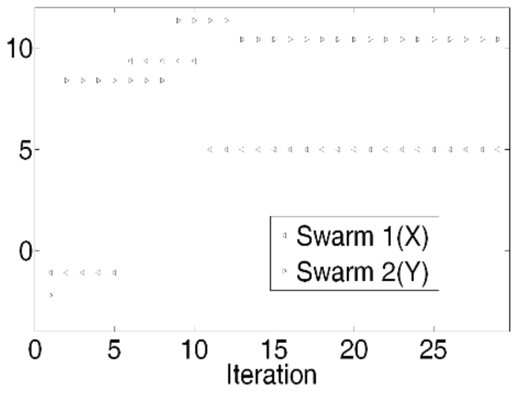


Figure 1-2. PSO Example

1.1 How are the Values of 'x and y' are Updated in Every Iteration?

The vector representation for updating the values for x and y is given in Figure 1-3. Let the position of the swarms be at 'a' and 'b' respectively as shown in the figure. Both are trying to reach the position 'e'. Let 'a' decides to move towards 'c' and 'b' decides to move towards 'd'.

The distance between the position 'c' and 'e' is greater than the distance between 'd' and 'e'. so based on the neighbor's decision position 'd' is treated as the common position decided by both 'a' and 'b'. (ie) the position 'c' is the individual decision taken by 'a', position 'd' is the individual decision taken by 'b' and the position 'd' is the common position decided by both 'a' and 'b'.

Figure 1-3. Vector Representation of PSO Algorithm

‘a’ based on the above knowledge, finally decides to move towards the position ‘g’ as the linear combination of ‘oa’ , ‘ac’ and ‘ad’. [As ‘d’ is the common position decided].The linear combination of ‘oa’ and scaled ‘ac’ (ie) ‘af’ is the vector ‘of’. The vector ‘of’ combined with vector ‘fg’ (ie) scaled version of ‘ad’ to get ‘og’ and hence final position decided by ‘a’ is ‘g’.

Similarly, ‘b’ decides the position ‘h’ as the final position. It is the linear combination of ‘ob’ and ‘bh’(ie) scaled version of ‘bd’. Note as ‘d’ is the common position decided by ‘a’ and ‘b’, the final position is decided by linear combinations of two vectors alone.

Thus finally the swarms ‘a’ and ‘b’ moves towards the position ‘g’ and ‘h’ respectively for reaching the final destination position ‘e’. The swarm ‘a’ and ‘b’ randomly select scaling value for linear combination. Note that ‘oa’ and ‘ob’ are scaled with 1 (ie) actual values are used without scaling. Thus the decision of the swarm ‘a’ to reach ‘e’ is decided by its own intuition along with its neighbor’s intuition.

Now let us consider three swarms (A,B,C) are trying to reach the particular destination point ‘D’. A decides A’, B decides B’ and C decides C’ as the next position. Let the distance between the B’ and D is less compared with A’D and C’ and hence, B’ is treated as the global decision point to reach the destination faster.

Thus the final decision taken by A is to move to the point, which is the linear combination of OA, AA’ and AB’. Similarly the final decision taken by B is to move the point which is the linear combination of OB, BB’. The final decision taken by C is to move the point which is the linear combination of OC, CC’ and CB’.

1.2 PSO Algorithm to Maximize the Function F (X, Y, Z)

1. Initialize the values for initial position a, b, c, d, e
2. Initialize the next positions decided by the individual swarms as a’, b’, c’ d’ and e’
3. Global decision regarding the next position is computed as follows. Compute f (a’, b, c, d, e), f (a, b’, c, d, e), f (a, b, c’, d, e), f (a, b, c, d’, e) and f (a, b, c, d, e’). Find minimum among the computed values. If f (a’, b, c, d, e) is minimum among all, the global position decided regarding the next position is a’. Similarly If f (a, b’, c, d, e) is minimum among all, b’ is decided as the global position regarding the next position to be shifted and so on. Let the selected global position is represented ad ‘global’
4. Next value for a is computed as the linear combination of ‘a’ , (a’-a) and (global-a) (ie)

- nexta = a+ C1 * RAND * (a’ –a) + C2 * RAND * (global –a)
- nextb = b+ C1 * RAND * (b’ –b) + C2 * RAND * (global –b)
- nextc = c+ C1 * RAND * (c’ –c) + C2 * RAND * (global –c)
- nextd = d+ C1 * RAND * (d’ –d) + C2 * RAND * (global –d)
- nexte = e+ C1 * RAND * (e’ –e) + C2 * RAND * (global –e)

5. Change the current value for a, b, c, d and e as nexta, nextb, nextc, nextd and nexte
6. If f (nexta, b, c, d, e) is less than f (a’, b, c, d, e) then update the value for a’ as nexta, otherwise a’ is not changed.

   If f (a, nextb, c, d, e) is less than f (a, b’, c, d, e) then update the value for b’ as nextb, otherwise b’ is not changed
   If f (a, b, nextc, d, e) is less than f (a, b, c’, d, e) then update the value for c’ as nextc, otherwise c’ is not changed

If f (a, b, c, nextd, e) is less than f (a, b, c, d’, e) then update the value for d’ as nextd, otherwise d’ is not changed

If f (a, b, c, d, nexte) is less than f (a, b, c, d, e’) then update the value for e’ as nexte, otherwise e’ is not changed

7. Repeat the steps 3 to 6 for much iteration to reach the final decision.

The values for ‘c1’,’c2’ are decided based on the weightage given to individual decision and global decision respectively.

Let $\Delta a(t)$ is the change in the value for updating the value for ‘a’ in tth iteration, then nexta at (t+1)th iteration can be computed using the following formula. This is considered as the velocity for updating the position of the swarm in every iteration.

$$\text{nexta}\ (t+1) = a\ (t) + \Delta a(t+1)$$

where

$$\Delta a(t+1) = c1 * \text{rand} * (a' - a) + c2 * \text{rand} * (\text{global} - a) + w(t) * \Delta a(t)$$

‘w (t)’ is the weight at t^{th} iteration. The value for ‘w’ is adjusted at every iteration as given below, where ‘iter’ is total number of iteration used.

$$w(t+1)=w(t)-t*w(t)/(\text{iter}).$$

Decision taken in the previous iteration is also used for deciding the next position to be shifted by the swarm. But as iteration increases, the contribution of the previous decision is decreases and finally reaches zero in the final iteration.

1.3 M – program for PSO Algorithm

psogv .m

```
function [value]=psogv(fun,range,ITER)
%psogv.m
%Particle swarm algorithm for maximizing the function fun with two variables x
%and y.
%Syntax
%[value]=psogv(fun,range,ITER)
%example
%fun='f1'
%create the function fun.m
%function [res]=fun(x,y)
%res=sin(x)+cos(x);
%range=[-pi pi;-pi pi];
%ITER is the total number of Iteration
error=[];
vel1=[];
vel2=[];

%Intialize the swarm position
swarm=[];
x(1)=rand*range(1,2)+range(1,1);
y(1)=rand*range(2,2)+range(2,1);
x(2)=rand*range(1,2)+range(1,1);
y(2)=rand*range(2,2)+range(2,1);

%Intialize weight
w=1;
c1=2;
c2=2;
%Initialize the velocity
v1=0;%velocity for x
v2=0;%velocity for y
for i=1:1:ITER
[p,q]=min([f1(fun,x(2),y(1)) f1(fun,x(1),y(2))]);
if (q==1)
   capture=x(2);
else
   capture=y(2);
end
```

Continued…

```
v1=w*v1+c1*rand*(x(2)-x(1))+c2*rand*(capture-x(1));
v2=w*v2+c1*rand*(y(2)-y(1))+c2*rand*(capture-y(1));
vel1=[vel1 v1];
vel2=[vel2 v2];

%updating x(1) and y(1)
x(1)=x(1)+v1;
y(1)=y(1)+v2;

%updating x(2) and y(2)
if((f1(fun,x(2),y(1)))<=(f1(fun,x(1),y(1))))
x(2)=x(2);
else
x(2)=x(1);
end;
if((f1(fun,x(1),y(2)))<=(f1(fun,x(1),y(1))))
y(2)=y(2);
else
y(2)=y(1);
end
error=[error f1(fun,x(2),y(2))];
w=w-w*i/ITER;
swarm=[swarm;x(2) y(2)];
subplot(3,1,3)
plot(error,'-')
title('Error(vs) Iteration');
subplot(3,1,1)
plot(swarm(:,1),'-')
title('x (vs) Iteration');
subplot(3,1,2)
plot(swarm(:,2),'-')
title('y (vs) Iteration');
pause(0.2)
end
value=[x(2);y(2)];
```

f1.m

```
function [res]=f1(fun,x,y);
s=strcat(fun,'(x,y)');
res=eval(s);
```

1.4 Program Illustration

Following the sample results obtained after the execution of the program psogv.m for maximizing the function ‘f1.m’

2. GENETIC ALGORITHM

A basic element of the Biological Genetics is the chromosomes. Chromosomes cross over each other. Mutate itself and new set of chromosomes is generated. Based on the requirement, some of the chromosomes survive. This is the cycle of one generation in Biological Genetics. The above process is repeated for many generations and finally best set of chromosomes based on the requirement will be available. This is the natural process of Biological Genetics. The Mathematical algorithm equivalent to the above behavior used as the optimization technique is called as Artificial Genetic Algorithm.

Let us consider the problem for maximizing the function f(x) subject to the constraint x varies from ‘m’ to ‘n’. The function f(x) is called fitness function. Initial population of chromosomes is generated randomly. (i.e.) the values for the variable ‘x’ are selected randomly between the range ‘m’ to ‘n’. Let the values be x_1, x_2…..x_L, where ‘L’ is the population size. Note that they are called as chromosomes in Biological context.

The Genetic operations like Cross over and Mutation are performed to obtain ‘2*L’ chromosomes as described below.

Two chromosomes of the current population is randomly selected (ie) select two numbers from the current population. Cross over operation generates another two numbers y1 and y2 using the selected numbers. Let the randomly selected numbers be x3 and x9. Y_1 is computed as r*x3+(1-r)*x9. Similarly y_2 is computed as $(1\text{-}r)*x_3+r*x_9$, where ‘r’ is the random number generated between 0 to1.

The same operation is repeated ‘L’ times to get ‘2*L’ newly generated chromosomes. Mutation operation is performed for the obtained chromosomes to generate ‘2*L’ mutated chromosomes. For instance the generated number ‘y_1’ is mutated to give z_1 mathematically computed as r_1*y, where r_1 is the random number generated. Thus the new set of chromosomes after crossover and Mutation are obtained as $[z_1\ z_2\ z_3 \ldots z_{2L}]$.

Among the ‘2L’ values generated after genetic operations, ‘L’ values are selected based on Roulette Wheel selection.

2.1 Roulette Wheel Selection Rule

Consider the wheel partitioned with different sectors as shown in the Figure 1-4. Let the pointer ‘p’ be in the fixed position and wheel is pivoted such that the wheel can be rotated freely. This is the Roulette wheel setup. Wheel is rotated and allowed to settle down.

The sector pointed by the pointer after settling is selected. Thus the selection of the particular sector among the available sectors are done using Roulette wheel selection rule.

In Genetic flow ‘L’ values from ‘2L’ values obtained after cross over and mutation are selected by simulating the roulette wheel mathematically. Roulette wheel is formed with ‘2L’ sectors with area of each sector is proportional to $f(z_1)$, $f(z_2)$ $f(z_3)$… and $f(z_{2L})$ respectively, where ‘$f(x)$’ is the fitness function as described above. They are arranged in the row to form the fitness vector as $[f(z_1), f(z_2)\ f(z_3)\ldots f(z_{2L})]$. Fitness vector is normalized to form Normalized fitness vector as $[f_n(z_1), f_n(z_2)\ f_n(z_3)\ldots f_n(z_{2L})]$, so that sum of the Normalized fitness values become 1 (i.e.) normalized fitness value of $f(z_1)$ is computed as $f_n(z_1) = f(z_1) / [f(z_1) + f(z_2) + f(z_3) + f(z_4)\ldots f(z_{2L})]$. Similarly normalized fitness value is computed for others.

Cumulative distribution of the obtained normalized fitness vector is obtained as

$$[f_n(z_1)\ f_n(z_1)+ f_n(z_2)\ f_n(z_1)+ f_n(z_2)+ f_n(z_3)\ \ldots\ 1].$$

Generating the random number ‘r’ simulates rotation of the Roulette Wheel.

Compare the generated random number with the elements of the cumulative distribution vector. If ‘$r< f_n(z_1)$’ and ‘$r > 0$’, the number ‘z_1’ is selected for the next generation. Similarly if ‘$r< f_n(z_1)+ f_n(z_2)$’ and ‘$r > f_n(z_1)$’ the number ‘z_2’ is selected for the next generation and so on.

Figure 1-4. Roulette Wheel

The above operation defined as the simulation for rotating the roulette wheel and selecting the sector is repeated ‘L’ times for selecting ‘L’ values for the next generation. Note that the number corresponding to the big sector is having more chance for selection.

The process of generating new set of numbers (ie) next generation population from existing set of numbers (ie) current generation population is repeated for many iteration.

The Best value is chosen from the last generation corresponding to the maximum fitness value.

2.2 Example

Let us consider the optimization problem for maximizing the function f(x) = x+10*sin (5*x) +7*cos (4*x) +sin(x) ,subject to the constraint x varies from 0 to 9.

The numbers between 0 and 9 with the resolution of 0.001 are treated as the chromosomes used in the Genetic Algorithm. (ie) Float chromosomes are used in this example. Population size of 10 chromosomes is survived in every generation. Arithmetic cross over is used as the genetic operator. Mutation operation is not used in this example Roulette Wheel selection is made at every generation. Algorithm flow is terminated after attaining the maximum number of iterations. In this example Maximum number of iterations used is 100.

The Best solution for the above problem is obtained in the thirteenth generation using Genetic algorithm as 4.165 and the corresponding fitness function f(x) is computed as 8.443. Note that Genetic algorithm ends up with local maxima as shown in the figure 1-5. This is the drawback of the Genetic Algorithm. When you run the algorithm again, it may end up with Global maxima. The chromosomes generated randomly during the first generation affects the best solution obtained using genetic algorithm.

Best chromosome at every generation is collected. Best among the collection is treated as the final Best solution which maximizes the function f(x).

2.2.1 M-program for genetic algorithm

The Matlab program for obtaining the best solution for maximizing the function f(x) = x+10*sin (5*x) +7*cos (4*x) +sin(x) using Genetic Algorithm is given below.

geneticgv.m

```
clear all, close all
pop=0:0.001:9;
pos=round(rand(1,10)*9000)+1;
pop1=pop(pos);
BEST=[];
for iter=1:1:100
   col1=[];
   col2=[];
   for do=1:1:10
   r1=round(rand(1)*9)+1;
   r2=round(rand(1)*9)+1 ;
   r3=rand;
   v1=r3*pop1(r1)+(1-r3)*pop1(r2);
   v2=r3*pop1(r2)+(1-r3)*pop1(r1);
   col1=[col1 v1 v2];
   end
   sect=fcn(col1)+abs(min(fcn(col1)));
   sect=sect/sum(sect);
   [u,v]=min(sect);
   c=cumsum(sect);
   for i=1:1:10
   r=rand;
   c1=c-r;
   [p,q]=find(c1>=0);
   if(length(q)~=0)
   col2=[col2 col1(q(1))];
   else
   col2=[col2 col1(v)];
   end
   end
pop1=col2;
   s=fcn(pop);
   plot(pop,s)
   [u,v]=max(fcn(pop1));
   BEST=[BEST;pop1(v(1)) fcn(pop1(v(1)))];
   hold on
   plot(pop1,fcn(pop1),'r.');
   M(iter)=getframe;
   pause(0.3)
   hold off
   [iter pop1(v(1)) fcn(pop1(v(1)))]
end
```

```
pop1=col2;
   s=fcn(pop);
   plot(pop,s)
   [u,v]=max(fcn(pop1));
   BEST=[BEST;pop1(v(1)) fcn(pop1(v(1)))];
   hold on
   plot(pop1,fcn(pop1),'r.');
   M(iter)=getframe;
   pause(0.3)
   hold off
   [iter pop1(v(1)) fcn(pop1(v(1)))]
end
for i=1:1:14
   D(:,:,:,i)=M(i).cdata;
end
figure
imshow(M(1).cdata)
figure
imshow(M(4).cdata)
figure
imshow(M(10).cdata)
figure
imshow(M(30).cdata)
```

fcn.m

```
function [res]=fcn(x)
res=x+10*sin(5*x)+7*cos(4*x)+sin(x);
```

Note that the m-file fcn.m may be edited for changing the fitness function

2.2.2 Program illustration

The following is the sample results obtained during the execution of the program geneticgv.m

Figure 1-5. Convergence of population

The graph is plotted between the variable ‘x’ and fitness function ‘f (x)’. The values for the variable ‘x’ ranges from 0 to 9 with the resolution of 0.001. The dots on the graph are indicating the fitness value of the corresponding population in the particular generation. Note that in the first generation the populations are randomly distributed. In the thirteenth generation populations are converged to the particular value called as the best value, which is equal to 4.165, and the corresponding fitness value f (x) is 8.443.

2.3 Classification of Genetic Operators

In genetic algorithm, chromosomes are represented as follows

- Float form

  The raw data itself acts as the chromosomes and can be used without without any modification.

- Binary form

  Binary representation of the number is used as the chromosomes. Number of Bits used to represent the raw data depends upon the resolution required which is measure of accuracy in representation.

➢ Order based form

Suppose the requirement is to find the order in which sales man is traveling among the 'n' places such that total distance traveled by that person is minimum. In this case, the data is the order in which the characters are arranged. Example data = [a b e f g c]. This type of representation is called Order based form of representation.

The tree diagram given above summarizes the different type of Genetic operators. The Examples for every operators are listed below

2.3.1 Simple crossover

Suppose the requirement is to maximize the function f (x, y, z) subject to the constraints x varies from 0.2 to 0.4, y varies from 0.3 to 0.6 and z varies from 0.1 to 0.4.

Let the two chromosomes of the current population be C1 and C2 respectively.

Before crossover

C1 = [0.31 0.45 0.11] and C2 = [0.25 0.32 0.3] (say).

After crossover

```
C1 = [0.3100   0.3200   0.3000]
C2 = [0.2500   0.4500   0.1100]
```

Simple cross over can also be applied for Binary representation as mentioned below

Before Crossover

```
C1 = ['10101', '01010','01011']
C2 = ['11111', '01011', '10100']
```

After Crossover

```
C1 = ['10101','01010','01100']
C2 = ['11111','01011','10011']
```

Note that the crossover point is selected by combining all the binary string into the single string.

2.3.2 Heuristic crossover

Suppose the requirement is to maximize the function f (x, y, z) subject to the constraints x varies from 0.2 to 0.4, y varies from 0.3 to 0.6 and z varies from 0.1 to 0.4.

Let the two chromosomes of the current population be C1 and C2 respectively.

Before crossover

C1 = [0.31 0.45 0.11] and C2 = [0.25 0.32 0.3] (say) and the corresponding fitness value (ie) f(x,y,z)=0.9 and 0.5 respectively

Heuristic crossover identifies the best and worst chromosomes as mentioned below

Best chromosome is one for which the fitness value is maximum. Worst chromosome is one for which the fitness value is minimum. In this example Best chromosome [Bt] is [0.31 0.45 0.11] and Worst Chromosome [Wt] is [0.25 0.32 0.3].

Bt = [0.31 0.45 0.11]
Wt = [0.25 0.32 0.3]

Bt chromosome is returned without disturbance (ie) After cross over,

C1=Bt= [0.31 0.45 0.11]

Second Chromosome after crossover is obtained as follows.

1. Generate the random number 'r'

C2=(Bt-Wt)*r+Bt

2. Check whether C2 is within the required boundary values (i.e.) first value of the vector C2 is within the range from 0.2 to 0.4, second value is within the range from 0.3 to 0.6 and third value is within the range from 0.1 to 0.4.
3. If the condition is satisfied, computed C2 is returned as the second chromosome after crossover.
4. If the condition is not satisfied, repeat the steps 1 to 3.
5. Finite attempts are made to obtain the modified chromosome after cross over as described above. (i.e) If the condition is not satisfied for 'N' attempts, C2=Wt is returned as the second chromosome after crossover.

2.3.3 Arith crossover

Let the two chromosomes of the current population be C1 and C2 respectively.

Before crossover

C1 = [0.31 0.45 0.11] and C2 = [0.25 0.32 0.3] (say)

After Crossover

Generate the random number r=0.3(say)

```
C1=r*C1+(1-r)*C2= [0.2680   0.3590   0.2430]
C2=(1-r)*C1+r*C2= [0.2920   0.4110   0.1670 ]
```

3. SIMULATED ANNEALING

The process of heating the solid body to the high temperature and allowed to cool slowly is called Annealing. Annealing makes the particles of the solid material to reach the minimum energy state. This is due to the fact that when the solid body is heated to the very high temperature, the particles of the solid body are allowed to move freely and when it is cooled slowly, the particles are able to arrange itself so that the energy of the particles are made minimum. The mathematical equivalent of the thermodynamic annealing as described above is called simulated annealing.

The energy of the particle in thermodynamic annealing process can be compared with the cost function to be minimized in optimization problem. The particles of the solid can be compared with the independent variables used in the minimization function.

Initially the values assigned to the variables are randomly selected from the wide range of values. The cost function corresponding to the selected values are treated as the energy of the current state. Searching the values from the wide range of the values can be compared with the particles flowing in the solid body when it is kept in high temperature.

The next energy state of the particles is obtained when the solid body is slowly cooled. This is equivalent to randomly selecting next set of the values.

When the solid body is slowly cooled, the particles of the body try to reach the lower energy state. But as the temperature is high, random flow of the particles still continuous and hence there may be chance for the particles to reach higher energy state during this transition. Probability of reaching the higher energy state is inversely proportional to the temperature of the solid body at that instant.

In the same fashion the values are randomly selected so that cost function of the currently selected random values is minimum compared with the previous cost function value. At the same time, the values corresponding to the higher cost function compared with the previous cost function are also selected with some probability. The probability depends upon the current simulated temperature 'T'. If the temperature is large, probability of selecting the values corresponding to higher energy levels are more. This process of selecting the values randomly is repeated for the finite number of iteration. The values obtained after the finite number of iteration can be assumed as the values with lowest energy state (i.e) lowest cost function Thus the simulated Annealing algorithm is summarized as follow.

3.1 Simulated Annealing Algorithm

The optimization problem is to estimate the best values for 'x', 'y' and 'z' such that the cost function f (x, y, z) is minimized. 'x' varies from 'x_{min} 'to 'x_{max}'. 'y' varies from 'y' varies 'y_{min}' to 'y_{max}' .'z' varies from 'z_{min}' to 'z_{max}'

Step 1: Intialize the value the temperature 'T'.
Step 2: Randomly select the current values for the variables 'x', 'y' and 'z' from the range as defined above. Let it be 'x_c', 'y_c' and 'z_c' respectively.
Step 3: Compute the corresponding cost function value f (x_c, y_c, z_c).
Step 4: Randomly select the next set of values for the variables 'x', 'y' and 'z' from the range as defined above. Let it be 'x_n', 'y_n' and 'z_n' respectively.
Step 5: Compute the corresponding cost function value f (x_n, y_n, z_n).
Step 6: If f (x_n, y_n, z_n)<= f (x_c, y_c, z_c), then the current values for the random variables $x_c = x_n$, $y_c = y_n$ and $z_c = z_n$.
Step 7: If $f(x_n, y_n, z_n) > f(x_c, y_c, z_c)$, then the current values for the random variables $x_c = x_n$, $y_c = y_n$ and $z_c = z_n$ are assigned only when

$$\exp\,[(f\,(x_c, y_c, z_c) - f(x_n, y_n, z_n))/T] > \text{rand}$$

Note that when the temperature 'T' is less, the probability of selecting the new values as the current values is less.

Step 8: Reduce the temperature T=r*T. r' is scaling factor varies from 0 to 1.
Step 9: Repeat STEP 3 to STEP8 for n times until 'T' reduces to the particular percentage of initial value assigned to 'T'

3.2 Example

Consider the optimization problem of estimating the best values for 'x' such that the cost function f(x)= x+10*sin(5*x)+7*cos(4*x)+sin(x) is minimized and x varies from 0 to 5.

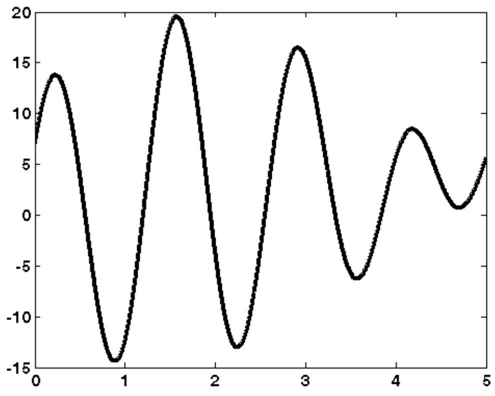


Figure 1-6 Illustration of simulated Annealing

The figure 1-6 shows all possible cost function values corresponding to the ‘x’ values ranges from 0 to 5. The goal is to find the best value of x corresponding to global minima of the graph given above. Note that the global minima is approximately at x=0.8.

Initially the random value is selected within the range (0 to 5). Let the selected value be 2.5 and the corresponding cost function is -3.4382. The variable ‘curval’ is assigned the value 2.5 and the variable ‘curcost’ is assigned the value -3.4382. Intialize the simulated temperature as T=1000.

Let the next randomly selected value be 4.9 and the corresponding cost function is 3.2729. The variable ‘newval’ is assigned the value 4.9 and the ‘newcost’ is assigned the value 3.2729.

Note that ‘newcost’ value is higher than the ‘curcost’ value. As ‘newcost’> ‘curcost’, ‘newval’ is inferior compared with ‘curval’ in minimizing the cost function. As the temperature (T) is large and exp((curcost-newcost)/T)>rand, ‘newcost’ value is assigned to ‘curcost’ and ‘newval’ is assigned to ‘curval’. Now the temperature ‘T’ is reduced by the factor 0.2171=1/log(100), where 100 is the number of iterations used. This is the thumb rule used in this example. This process is said to one complete iteration. Next randomly selected value is selected and the above described process is repeated for 100 iterations.

The order in which the values are selected in the first 6 iterations is not moving towards the local minima point which can be noted from the figure 1-6. This is due to the fact that the initial simulated temperature of the annealing process is high.

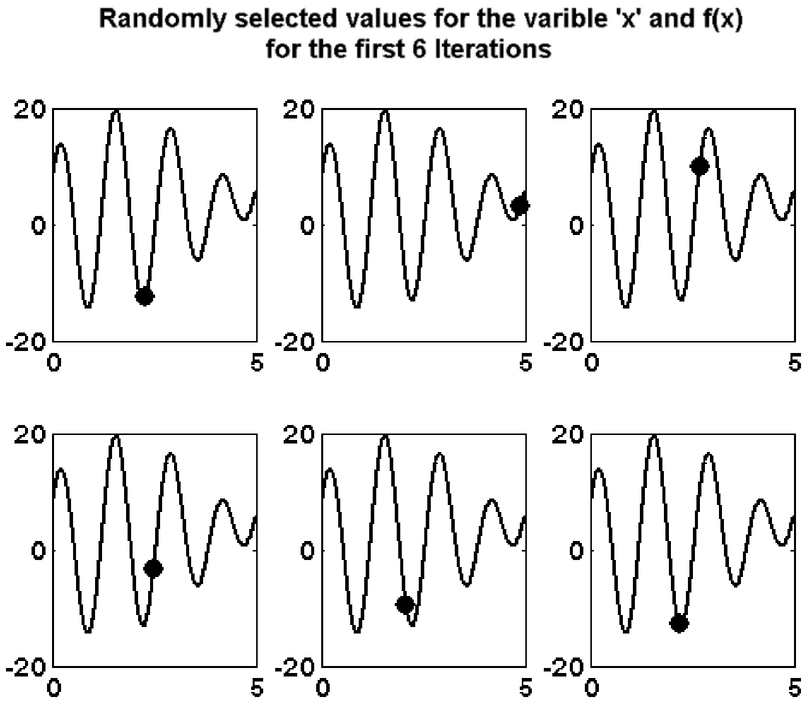


Figure1-7 Illustration of Simulated Annealing 1

As iteration increases the simulated temperature is decreased and the value selected for the variable 'x is moving towards the global minima point as shown in the figure 1-7.

Thus values assigned to the variable 'x' for the first few iterations (about 10 iterations in this example) is not really in decreasing order of f(x). This helps to search the complete range and hence helps in overcoming the problem of local minima. As iteration goes on increasing the values selected for the variable 'x' is moving towards the global minima which can be noted from the figure 1-8.

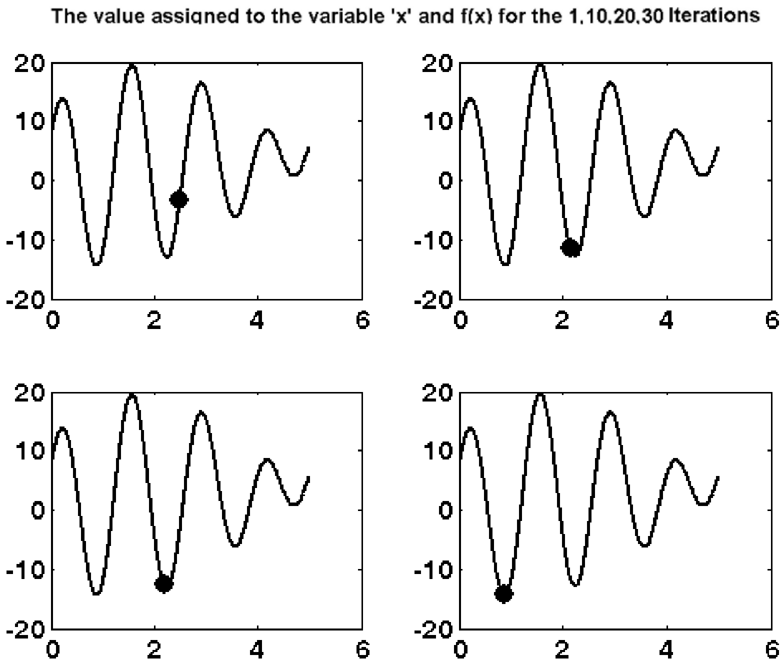


Figure 1-8. Illustration of Simulated Annealing 2

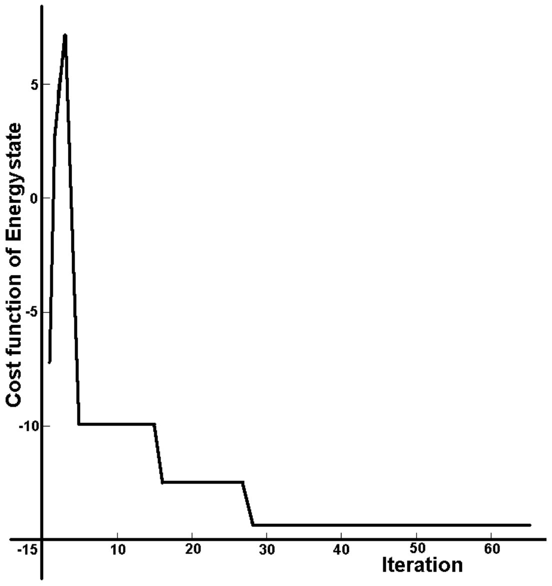


Figure1-9. Illustration of Simulated Annealing 3

3.3 M-program for Simulated Annealing

Matlab program for minimizing the function f(x)= x+10*sin(5*x)+ 7*cos(4*x)+sin(x),where x varies from 0 to 5 .

---

sagv.m

```
x=0:1/300:5
plot(x, minfcn(x))
hold
curval=rand*5;
curcost=minfcn(curval);
T=1000;
for i=1:1:100
    newval=rand*5;
   newcost=minfcn(newval);
   if((newcost-curcost)<=0)
       curval=newval;
       curcost=minfcn(curval);
    else
       if(exp((curcost-newcost)/T)>rand)
           curval=newval;
           curcost=minfcn(curval);
       end
   end
T=T/log(100)
plot(curval,minfcn(curval),'r*');
CURRENTVAL{i}=curval;
M{i}=minfcn(curval);
pause(0.2)
end
```

---

minfcn.m

```
function [res]=minfcn(x)
res=x+10*sin(5*x)+7*cos(4*x)+sin(x)
```

4. BACK PROPAGATION NEURAL NETWORK

The mathematical model of the Biological Neural Network is defined as Artificial Neural Network. One of the Neural Network models which are used almost in all the fields is Back propagation Neural Network.

The model consists of layered architecture as shown in the figure 1-10.

'I' is the Input layer 'O' is the output layer and 'H' is the hidden layer. Every neuron of the input layer is connected to every neuron in the hidden layer. Similarly every neuron in the Hidden layer is connected with every neuron in the output layer. The connection is called weights.

Number of neurons in the input layer is equal to the size of the input vector of the Neural Network. Similarly number of neurons in the output layer is equal to the size of the output vector of the Neural Network. Size of the hidden layer is optional and altered depends upon the requirement

For every input vector, the corresponding output vector is computed as follows

[hidden vector] = func ([Input vector]*[Weight Matrix 1] + [Bias1])
[output vector] = func ([hidden vector]*[Weight Matrix 2] + [Bias2])

If the term [Input vector]*[Weight Matrix 1] value becomes zero, the output vector also becomes zero. This can be avoided by using Bias vectors. Similarly to make the value of the term '[Input vector]*[Weight Matrix 1] + [Bias1]' always bounded, the function 'func' is used as mentioned in the equation given above.

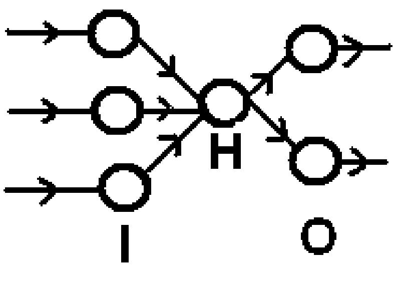


Figure 1-10 BPN Structure

commonly Usually used ‘func’ are

$$\text{logsig}(x) = \frac{1}{[1+\exp(-x)]}$$

$$\text{tansig}(x) = \frac{2}{[1+\exp(-2x)]}$$

The requirement is to find the common Weight Matrix and common Bias vector such that for the particular input vector, computed output vector must match with the expected target vector. The process of obtaining the Weight matrix and Bias vector is called training.

Consider the architecture shown in the figure 1-9. Number of neurons in the input layer is 3 and number of neurons in the output layer is 2. Number of neurons in the hidden layer is 1.The weight connecting the ith neuron in the first layer and jth neuron in the hidden layer is represented as W_{ij} The weight connecting ith neuron in the hidden layer and jth neuron in the output layer is represented as W_{ij}’ .

Let the input vector is represented as $[i_1\ i_2\ i_3]$. Hidden layer output is represented as $[h_1]$ and output vector is represented as $[o_1\ o_2]$. The bias vector in the hidden layer is given as $[b_h]$ and the bias vector in the output layer is given as $[b_1\ b_2]$. Desired ouput vector is represented is [t1 t2]

The vectors are related as follows.

$h1=func1\ (w_{11}*i_1+w_{21}*i_2+w_{31}*i_3 + b_h)$

$o1= func2\ (w_{11}'*h_1 +b_1)$

$o2= func2\ (w_{12}'*h_1 +b_2)$

t1~=o1

t2~=o2

4.1 Single Neuron Architecture

Consider the single neuron input layer and single neuron output layer.

Output= func (input * W +B)

Desired ouput vector be represented as ‘Target’

Requirement is to find the optimal value of W so that Cost function = $(\text{Target-Output})^2$ is reduced. The graph plotted between the weight vector ‘W’ and the cost function is given in the figure 1-11.

Optimum value of the weight vector is the vector corresponding to the point 1 which is global minima point, where the cost value is lowest.

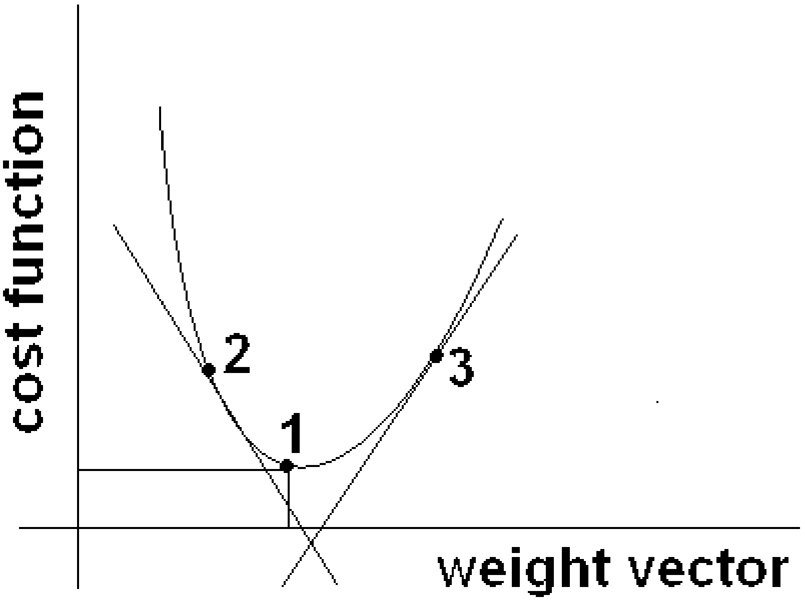


Figure 1-11 ANN Algorithm Illustration

The slope computed at point 2 in the figure is negative. Suppose if the weight vector corresponding to the point 2 is treated as current weight vector assumed, the next best value (i.e) global point is at right side of it. Similarly if the slope is positive, next best value is left side of the current value. This can be seen from the graph. The slope at point 3 is a positive. The best value (i.e) global minimum is left of the current value.

Thus to obtain the best value of the weight vector. Initialize the weight vector. Change the weight vector iteratively. Best weight vector is obtained after finite number of iteration. The change in the weight vector in every iteration is represented as 'ΔW'.

(i.e.) $W(n+1) = W(n) + \Delta W(n+1)$

$W(n+1)$ is the weight vector at $(n+1)^{th}$ iteration $W(n)$ is the weight vector at n^{th} iteration and $\Delta W(n+1)$ is the change in the weight vector at $(n+1)^{th}$ iteration.

The sign and magnitude of the 'ΔW' depends upon the direction and magnitude of the slope of the cost function computed at the point corresponding to the current weight vector.

Thus the weight vector is adjusted as following

New weight vector = Old weight vector - μ*slope at the current cost value.

Cost function(C) = $(\text{Target-Output})^2$

$\Rightarrow$ C= $(\text{Target- func (input * W +B)})^2$

Slope is computed by differentiating the variable C with respect to W and is approximated as follows.

2((Target- func (input * W +B))* (-input)

=2*error*input

Thus W(n+1) =W(n) - 2*μ*error*(-input)

$\Rightarrow$ W(n+1) = W(n) + γ error * input

This is back propagation algorithm because error is back propagated for adjusting the weights. The Back propagation algorithm for the sample architecture mentioned in figure 1-9 is given below.

4.2 Algorithm

Step 1: Initialize the Weight matrices with random numbers.
Step 2: Initialize the Bias vectors with random numbers.
Step 3: With the initialized values compute the output vector corresponding to the input vector [i_1 i_2 i_3] as given below. Let the desired target vector be [t_1 t_2]

h_1=func1 (w_{11}*i_1+w_{21}*i_2+w_{31}*i_3 +b_h)
o_1= func2 (w_{11}'*h_1 +b_1)
o_2= func2 (w_{12}'*h_1 +b_2)

Step 4: Adjustment to weights

a) Between Hidden and Output layer.

w_{11}'(n+1) = w_{11}'(n) + γ_O (t_1-o_1) *h_1
w_{12}'(n+1) = w_{12}'(n) + γ_O (t_2-o_2) *h_1

b) Between Input layer and Hidden layer

$w_{11}(n+1) = w_{11}(n) + \gamma_H * e * i_1$
$w_{21}(n+1) = w_{21}(n) + \gamma_H * e * i_2$
$w_{31}(n+1) = w_{31}(n) + \gamma_H * e * i_3$

'e' is the estimated error at the hidden layer using the actual error computed in the output layer

(i.e.) $e = h_l * (1-h1) * [(t_1-o_1) + (t_2-o_2)]$

Step 5: Adjustment of Bias

$$b_h(n+1) = b_h(n) + e * \gamma_H$$

$$b_1(n+1) = b_1(n) + (t_1-o_1) * \gamma_O$$
$$b_2(n+1) = b_2(n) + (t_2-o_2) * \gamma_O$$

Step 6: Repeat step 3 to step 5 for all the pairs of input vector and the corresponding desired target vector one by one.

Step 7: Let d_1, d_2 d_3 …d_n be the set of desired vectors and y_1, y_2 y_3 y_n be the corresponding output vectors computed using the latest updated weights and bias vectors. The sum squared error is calculated as

$$SSE = (d_1 - y_1)^2 + (d_2 - y_2)^2 + (d_3 - y_3)^2 + (d_4 - y_4)^2 + \dots (d_n - y_n)^2$$

Step 8: Repeat the steps 3 to 7 until the particular SSE is reached.

Note if momentum is included during training updating equations are modified as follows.

$$w_{11}'(n+1) = w_{11}'(n) + \underbrace{\gamma_O (t_1-o_1) * h_l}_{\Delta w_{11}'(n+1)} + \rho o * \Delta w_{11}'(n)$$

$$w_{12}'(n+1) = w_{12}'(n) + \Delta w_{12}'(n+1) + \rho o * \Delta w_{12}(n)$$
$$w_{11}(n+1) = w_{11}(n) + \Delta w_1(n+1) + \rho_H * \Delta w_{11}(n)$$
$$w_{21}(n+1) = w_{21}(n) + \Delta w_{21}(n+1) + \rho_H * \Delta w_{21}(n)$$

$$w_{31}(n+1) = w_{31}(n) + \Delta w_{31}(n+1) + \rho_H * \Delta w_{31}(n)$$

$$b_h(n+1) = b_h(n) \underbrace{+ e * \gamma_H}_{\Delta b_h(n+1)} + \rho_H * \Delta b_h(n)$$

$$b_1(n+1) = b_1(n) + \Delta b_1(n+1) + \rho_O * \Delta b_1(n)$$

$$b_2(n+1) = b_2(n) + \Delta b_2(n+1) + \rho_O * \Delta b_2(n)$$

ρ_H and ρ_O are the momentums used in the hidden and output layer respectively.

4.3 Example

Consider the problem for training the Back propagation Neural Network with Hetero associative data as mentioned below

Input	**Desired Output**
0 0 0	0 0
0 0 1	0 1
0 1 0	0 1
0 1 1	0 1
1 0 0	0 1
1 0 1	0 1
1 1 0	0 1
1 1 1	1 1

ANN Specifications

Number of layers = 3
Number of neurons in the input layer = 3
Number of neurons in the hidden layer =1
Number of neurons in the output layer = 2
Learning rate =0.01
Transfer function used in the hidden layer = ‘logsig’
Transfer function used in the output layer = ‘linear’ (i.e) output is taken as obtained without applying non-linear function like ‘logsig’,’ tansig’.

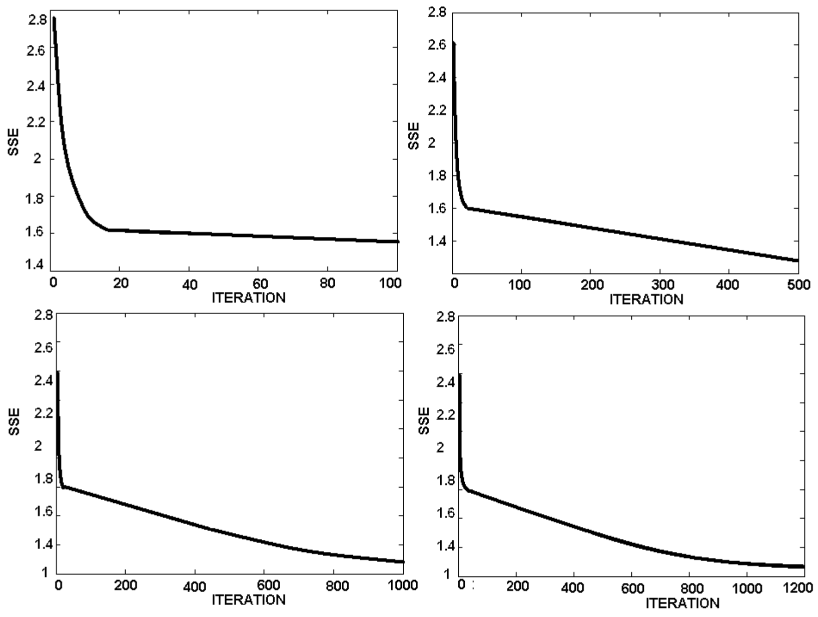


Figure 1-12. Illustration of SSE decreases with Iteration

(See Figure 1-10)

The Figure 1-12 describes how the Sum squared error is decreasing as iteration increases. Note that sum squared error is gradually decreasing from 2.7656 as iteration increases and reaches 1.0545 after 1200 Iteration.

Trained Weight and Bias Matrix

$$W1 = \begin{pmatrix} 1.0374 \\ 0.9713 \\ 0.8236 \end{pmatrix}$$

B1= [-0.5526]
W2= [0.8499 1.3129]
B2= [-0.4466 -0.0163]

OUTPUT obtained after training the neural network

```
 -0.1361   0.4632
  0.0356   0.7285
  0.0660   0.7756
  0.2129   1.0024
  0.0794   0.7962
```

```
   0.2225    1.0172
   0.2426    1.0483
   0.3244    1.1747
```

After rounding OUTPUT exactly matches with the Desired Output as shown below

```
     0     0
     0     1
     0     1
     0     1
     0     1
     0     1
     0     1
     0     1
```

4.4 M-program for Training the Artificial Neural Network for the Problem Proposed in the Previous Section

anngv.m

```
 SSE=[ ];  INDEX=[ ];
 INPUT=[0 0 0;0 0 1;0 1 0;0 1 1;1 0 0;1 0 1;1 1 0;1 1 1];
 DOUTPUT=[0 0;0 1;0 1 ;0 1;0 1;0 1;0 1 ;1 1];
 W1=rand(3,1);W2=rand(1,2);
  B1=rand(1,1);  B2=rand(1,2);
   for i=1:1:1200
          for j=1:1:8
    H=logsiggv(INPUT*W1+repmat(B1,[8 1]));
    OUTPUT=H*W2 +repmat(B2,[8 1]);
    ERR_HO=DOUTPUT - OUTPUT;
    ERR_IH=H.*(1-H).*(sum(ERR_HO')');
    lr_ho=0.01;
    lr_ih=0.01;
    W2(1,1)=W2(1,1)+ERR_HO(j,1)*H(j,1)*lr_ho;
    W2(1,2)=W2(1,2)+ERR_HO(j,2)*H(j,1)*lr_ho;
    W1(1,1)=W1(1,1)+ERR_IH(j,1)*INPUT(j,1)*lr_ih;
    W1(2,1)=W1(2,1)+ERR_IH(j,1)*INPUT(j,2)*lr_ih;
    W1(3,1)=W1(3,1)+ERR_IH(j,1)*INPUT(j,3)*lr_ih;
    B1=B1+ERR_IH(j,1)*lr_ih;
    B2(1,1)=B2(1,1)+ERR_HO(j,1)*lr_ho;
    B2(1,2)=B2(1,2)+ERR_HO(j,2)*lr_ho;
            end
     INDEX=[INDEX i];
    SSE=[SSE sum(sum(ERR_HO.^2))];
     plot(INDEX,SSE,'r');
     xlabel('ITERATION')
     ylabel('SSE')
     pause(0.2)
     end
 H=logsig(INPUT*W1+repmat(B1,[8 1]));
OUTPUT=H*W2 +repmat(B2,[8 1]);
```

---

logsiggv.m

```
function [res]=logsiggv(x)
res=1/(1+exp(-x));
```

5. FUZZY LOGIC SYSTEMS

Fuzzy is the set theory in which the elements of the set are associated with fuzzy membership. Let us consider the set of days = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday}. This is the set in which elements belongs to the set with 100% membership.

Let us consider the set of rainy days ={Sunday (0.6), Monday (0.4), Tuesday (0.4), Wednesday (0.6), Thursday (0.2), Friday (0.6), Saturday (0.3)}. This is the set in which elements are associated with fuzzy membership. Sunday (0.6) indicates that the Sunday belongs to the set of rainy days with membership value 0.6. This is different from probability assignment because in case of probability assignment, Sunday belongs to non-rainy days with membership value 0.4. (ie) (1-0.6). But in case of fuzzy sets Sunday belongs to non-rainy days with membership value of not necessarily 0.4. It can be any value between 0 to 1.

5.1 Union and Intersection of Two Fuzzy Sets

Let us consider the example of computing union and intersection of fuzzy sets as described below.

A={a (0.3), b (0.8), c (0.1), d (0.3)}

B={a (0.1), b (0.6), c (0.9), e (0-.1)}

The elements of the set AUB are the unique elements collected from both the sets A and B. The membership value associated with the elements of the set AUB is the maximum of the corresponding membership values collected from the set A and B.

Thus AUB = {a (0.3), b (0.8), c (0.9), d (0.3), e (0.1)}

Similarly the elements of the set A∩B are the common elements collected from the sets A and B. The membership value associated with the elements of the set AUB is the minimum of the corresponding membership values collected from the set A and B.

Thus A∩B= {a (0.1), b (0.6), c (0.1)}

5.2 Fuzzy Logic Systems

Let us consider the problem of obtaining the optimum value of the variable 'Z' obtained as the output of the fuzzy logic system decided by the values of the variable 'X' and 'Y' taken as the input to the fuzzy logic system using fuzzy logic algorithm.

The values of the variable 'X' vary from X_{min} to $X_{max.}$ Actual value of the variable is called crisp value. The group of the values ranges from X_{min} to X_{max} is grouped into 3 sets XSMALL, XMEDIUM, XLARGE. The relationship between the crisp value and fuzzy membership value is given in the figure 1-13.

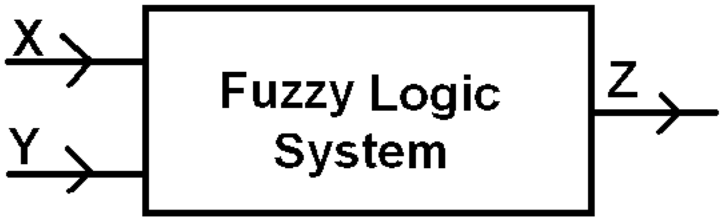


Figure 1-13. Fuzzy system

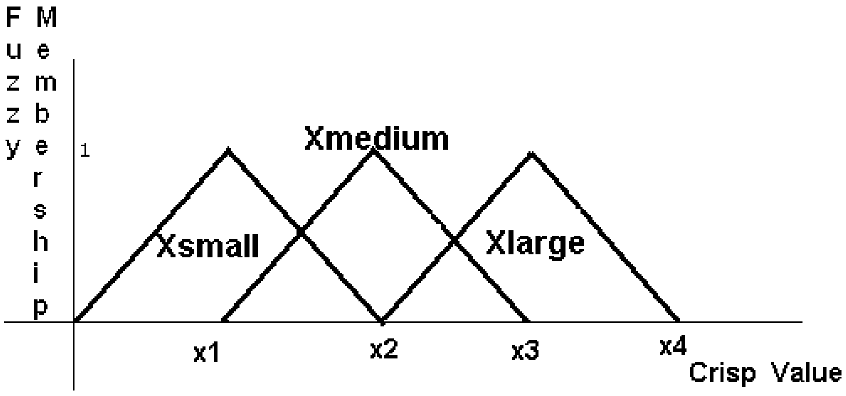


Figure 1-14. Relaionship between crisp value and Fuzzy membership value for the variable X

The crisp values ranges from 0 to x2 belong to the set Xsmall. The crisp values ranges from x1 to x3 belong to the set Xmedium. The crisp value ranges from x2 to x4 belongs to the set Xlarge.

The crisp value from x1 to x2 of the Xsmall set and Xmedium set with different membership value as mentioned in the Figure 1-14. Corresponding membership value decreases gradually from 1 to 0 in the Xsmall set. But the corresponding membership value increases gradually from 0 to 1 in the Xmedium set.

Similarly the relationship between the crisp value and Fuzzy member ship value for the variable 'Y' and the variable 'Z' are mentioned in the Figure 1-15 and Figure 1-16 respectively.

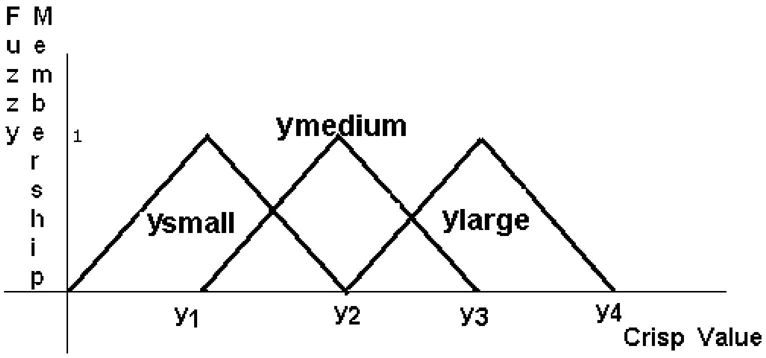


Figure 1-15. Relaionship between crisp value and Fuzzy membership value for the variable Y

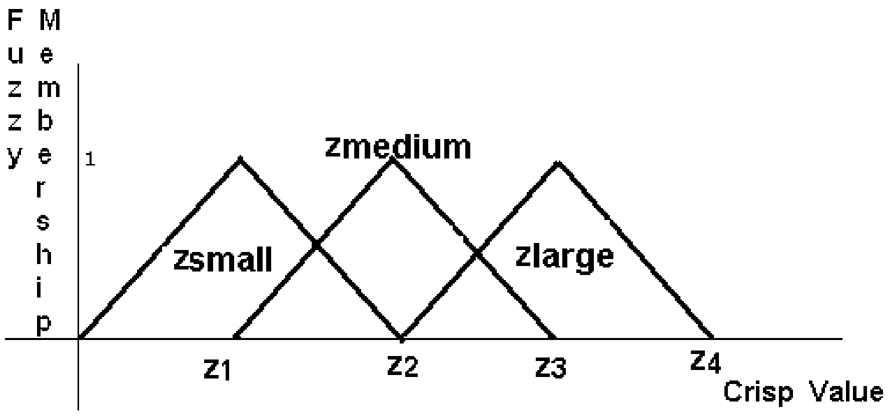


Figure 1-16. Relaionship between crisp value and Fuzzy membership value for the variable Z

The identical relationship between the crisp value and fuzzy value of all the variables X, Y and Z are chosen for the sake of simplicity (see figure). Steps involved in obtaining the crisp value of the output variable for the particular crisp values for the variable X and Y are summarized below.

5.2.1 Algorithm

Step 1: Crisp to fuzzy conversion for the input variable 'X' and 'Y'

Let us consider the crisp value of the input variable X be Xinput the crisp value of the input variable Y be Yinput. Let Xinput belongs to the set Xsmall with fuzzy membership 0.7 (say) belongs to set Xmedium 0.3 (say) as shown in the Figure 1-17 and Figure 1-18 given below.

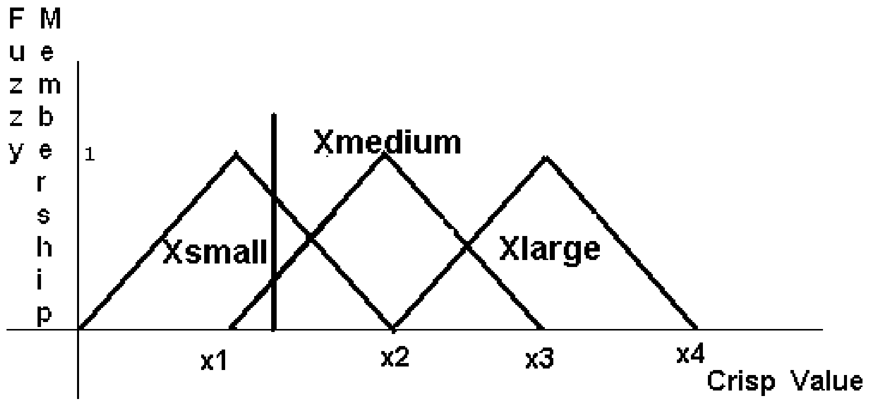


Figure 1-17. Xinput as the crisp input for the variable X

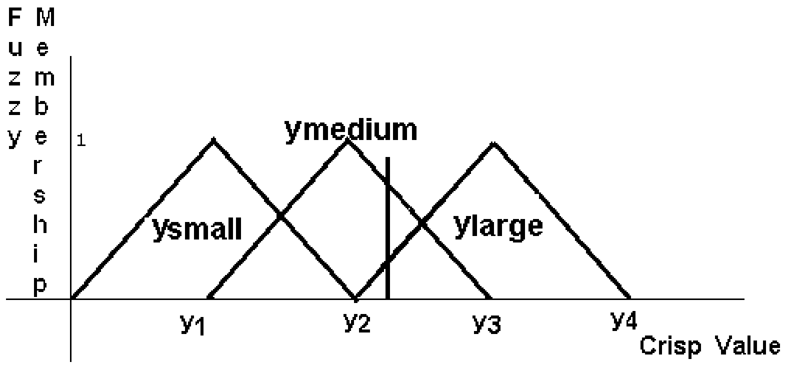


Figure 1-18. Yinput as the crisp input for the variable Y

Similarly the Yinput belongs to the set Ymedium with fuzzy membership 0.7 and belongs to the set Ylarge with 0.2 as shown in the figure given below

Step 2: Fuzzy input to fuzzy output mapping using fuzzy base rule

Fuzzy base rule is the set of rules relating the fuzzy input and fuzzy output of the fuzzy based system. Let us consider the Fuzzy Base Rule as shown in the table given below.

Table 1-1. Fuzzy Base Rule

	Ysmall	**Ymedium**	**Ylarge**
small	Zmedum	Zmedium	Zsmall
Xmedium	Zmedium	Zlarge	Zlarge
Xlarge	Zlarge	Zlarge	Zmedium

Thus Fuzzy input set Xsmall = {Xinput (0.7) }
Xmedium = {Xinput (0.3) }
Xlarge = {Xlarge(0)

Ysmall = {Yinput (0) }
Ymedium = {Yinput (0.7) }
Ylarge = {Xlarge(0.2)
Fuzzy output set is obtained in two stages.

1. Intersection of Fuzzy sets

- Xsmall $\cap$ Ysmall = Zmedium ={Zoutput(0)}
- Xsmall $\cap$ Ymedium = Zmedium ={Zoutput(0.7)}
- Xsmall $\cap$ Ylarge = Zsmall ={Zoutput(0.2)}
- Xmedium $\cap$ Ysmall = Zmedium ={Zoutput(0)}
- Xmedium $\cap$ Ymedium = Zlarge={Zoutput(0.3)}

- Xmedium ∩ Ylarge = Zlarge ={Zoutput(0.2)}
- Xlarge ∩ Ysmall = Zlarge ={Zoutput(0)}
- Xlarge ∩ Ymedium = Zmedium ={Zoutput(0)}
- Xlarge ∩ Ylarge = Zsmall ={Zoutput(0)}

2. Union of Fuzzy sets

- Zsmall = U(Zoutput(0.2), Zoutput(0))={ Zoutput(0.2) }
- Zmedium = U(Zoutput(0), Zoutput(0.7), Zoutput(0),Zoutput(0))
  = {Zoutput (0.7)}
- Zlarge = U(Zoutput(0.3), Zoutput(0.2), Zoutput(0))={Zoutput(0.3)}

Thus Fuzzy output based on the framed fuzzy base rule is given as

Zsmall = {Zoutput(0.2)
Zmedium = {Zoutput(0.7}
Zlarge = {Zoutput(0.3)}

Step 3: Fuzzy to Crisp conversion for the output variable Z using Centroid method

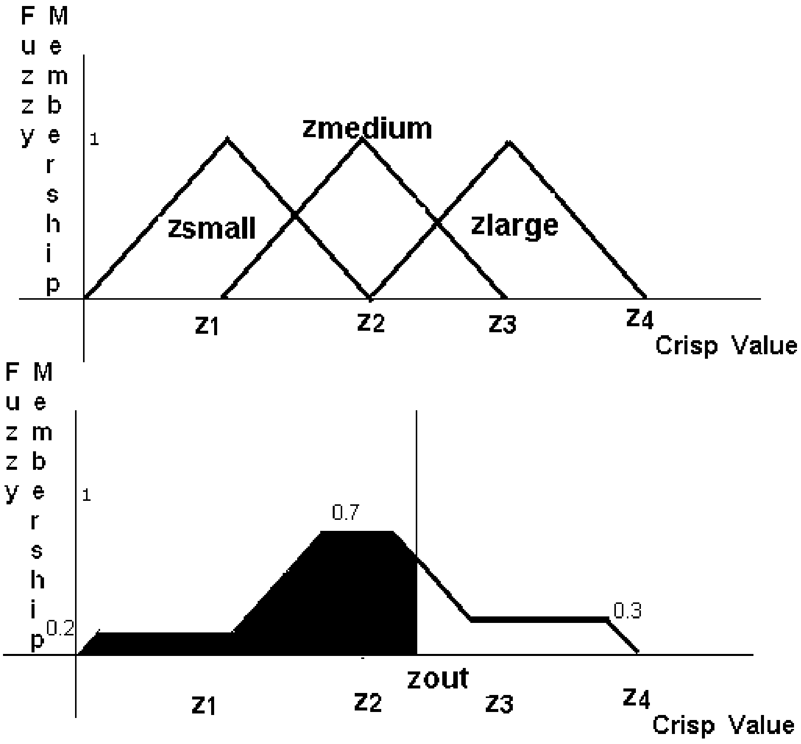


Figure 1-19. Defuzzification

Truncate the portions of the Zsmall, Zmedium and Zlarge sections of the output relationship above the corresponding thresholds obtained from output fuzzy membership as shown in the figure. Remaining portions are combined with overlapping of the individual sections. Combined sections can also be obtained by adding the individual sections instantaneously as illustrated in the example given in the section 5.4 Obtain the crisp value Zout which divides the combined sections into two halves as shown in the figure. Thus crisp value Zout is obtained for the corresponding input crisp value Xinput and Yinput using Fuzzy logic system.

5.3 Why Fuzzy Logic System?

The Automatic system uses set of rules framed with the input data to decide the output data. For example the automatic system is designed to maintain the temperature of the water heater by adjusting the heat knob based on the water level at that particular instant and temperature at that particular instant. In this example water level measured using the sensor at that particular instant and temperature measured using the sensor at that particular instant are the input to the control system. Control system uses the set of rules framed as given below for deciding the position of the heat knob which is the output of the control system

- If T1<=Temperature <=T2 and W1<=Water level<=W2, Adjust the heat knob position to K1. (Say).
- If T0<=Temperature <=T1 and W0<=Water level<=W1, Adjust the heat knob position to K2. (Say) etc.

Suppose if the temperature is just less than T1, this rule is not selected. The small variation 'ΔT' makes the automatic control system not to select this particular rule. To overcome this problem instead of using crisp value for framing the rule, fuzzy values can be used to frame the rules (i.e) without using the actual values obtained through input sensors This is called fuzzy base rule as given below

- If the temperature of the water is small and water level is high, place the heat knob in the high position. (say)
- If the temperature of the water is medium and water level is less, place the heat knob in the less position. (say) etc.

In this case the problem only fuzzy sets are used for decision making which makes the system to take decision like the human decision.

To design the fuzzy based system, the knowledge about the system behavior is required to find the relationship between crisp value and the fuzzy value (see above) and to frame the fuzzy base rule. This is the drawback of the fuzzy based system when compared to ANN based system which doesn't require any knowledge about the system behavior.

5.4 Example

Let us consider the problem of obtaining the optimum value of the variable 'Z' obtained as the output of the fuzzy logic system decided by the values of the variable 'X' and 'Y' taken as the input to the fuzzy logic system using fuzzy logic algorithm. The crisp –fuzzy relationship is as mentioned in the section 5.2. The fuzzy base rule is as given in the table 1-1.

The values for the variables used in the section 5.2 are assumed as mentioned in the table 1-2.

Table 1-2. Assumed values for the variables used in the section 5.2

X1	X2	X3	X4	Y1	Y2	Y3	Y4	Z1	Z2	Z3	Z4
3	5	7	10	20	45	65	100	35	70	90	100

For the arbitrary input crisp values xinput=6 and yinput=50, zout is obtained as 78.8 using fuzzy logic system described in the section 5.2

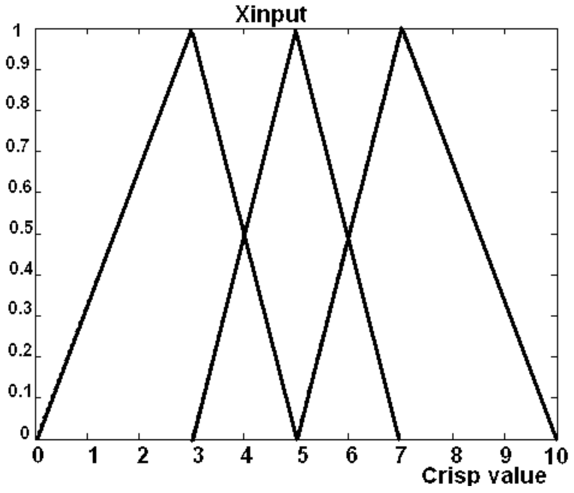


Figure 1-20. Relationship between crisp value and fuzzy membership for the Input variable X in the example

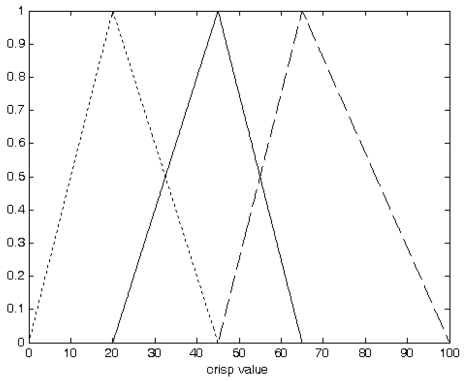


Figure1-21. Relationship between crisp value and fuzzy membership for the Input variable Y in the example

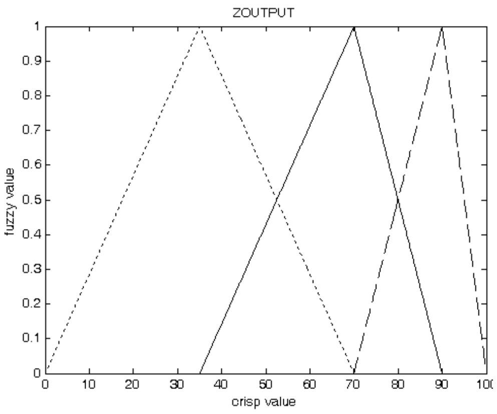


Figure 1-22. Relationship between crisp value and fuzzy membership for the Input variable Z in the example

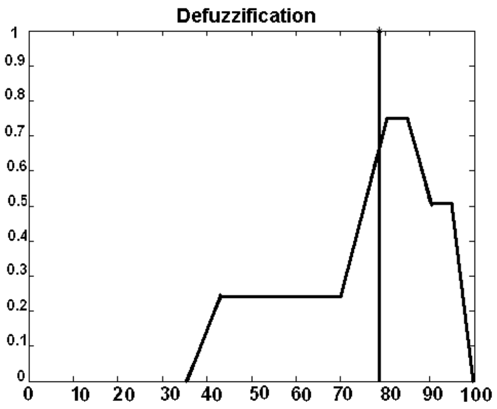


Figure 1-23. Defuzzification in the example

5.5 M-program for the Realization of Fuzzy Logic System for the Specifications given in Section 5.4

fuzzygv . m

```
posx=[0 3 5 7 10];
maxx=max(posx);
posx=posx/max(posx);
posy=[0 20 45 65 100];
maxy=max(posy);
posy=posy/max(posy);
posz=[0 35 70 90 100];
maxz=max(posz);
posz=posz/max(posz);
i=posx(1):1/999:posx(2);
j=(posx(2)):(1/999):posx(3);
k=(posx(3)):(1/999):posx(4);
l=(posx(4)):(1/999):posx(5);
xsmall=[1/posx(2)*i (-1/(posx(3)-posx(2))*(j-posx(3))) zeros(1,length([k l]))];
xmedium =[zeros(1,length([i])) (1/(posx(3)-posx(2))*(j-posx(2))) (-1/(posx(4)-
posx(3))*(k-posx(4))) zeros(1,length([l]))];
xlarge=[zeros(1,length([i j])) (1/(posx(4)-posx(3))*(k-posx(3))) (-1/(posx(5)-posx(4))*(l-
posx(5))) ];

figure
plot([i j k l]*maxx,xsmall,':');
hold
plot([i j k l]*maxx,xmedium,'-');
plot([i j k l]*maxx,xlarge,'--');
xlabel('crisp value')
ylabel('fuzzy value')
title('XINPUT')

i=posy(1):1/999:posy(2);
j=(posy(2)):(1/999):posy(3);
k=(posy(3)):(1/999):posy(4);
l=(posy(4)):(1/999):posy(5);
ysmall=[1/posy(2)*i (-1/(posy(3)-posy(2))*(j-posy(3))) zeros(1,length([k l]))];
```

```
ymedium =[zeros(1,length([i])) (1/(posy(3)-posy(2))*(j-posy(2))) (-1/(posy(4)-posy(3))*(k-posy(4))) zeros(1,length([l]))];
ylarge=[zeros(1,length([i j])) (1/(posy(4)-posy(3))*(k-posy(3))) (-1/(posy(5)-posy(4))*(l-posy(5))) ];

figure
plot([i j k l]*maxy,ysmall,':');
hold
plot([i j k l]*maxy,ymedium,'-');
plot([i j k l]*maxy,ylarge,'--');
xlabel('crisp value')
ylabel('fuzzy value')
title('YINPUT')

i=posz(1):1/999:posz(2);
j=(posz(2)):(1/999):posz(3);
k=(posz(3)):(1/999):posz(4);
l=(posz(4)):(1/999):posz(5);
zsmall=[1/posz(2)*i (-1/(posz(3)-posz(2))*(j-posz(3))) zeros(1,length([k l]))];
zmedium =[zeros(1,length([i])) (1/(posz(3)-posz(2))*(j-posz(2))) (-1/(posz(4)-posz(3))*(k-posz(4))) zeros(1,length([l]))];
zlarge=[zeros(1,length([i j])) (1/(posz(4)-posz(3))*(k-posz(3))) (-1/(posz(5)-posz(4))*(l-posz(5))) ];

figure
plot([i j k l]*maxz,zsmall,':');
hold
plot([i j k l]*maxz,zmedium,'-');
plot([i j k l]*maxz,zlarge,'--');
xlabel('crisp value')
ylabel('fuzzy value')
title('ZOUTPUT')

xinput=[6];
xinput=round(((xinput/maxx)*1000))
yinput=[50];
yinput=round(((yinput/maxy)*1000))
%fuzzy values

xfuzzy=[xsmall(xinput) xmedium(xinput) xlarge(xinput)];
yfuzzy=[ysmall(yinput) ymedium(yinput) ylarge(yinput)];
```

```
%Output fuzzy values obtained using framed fuzzy base rule
zfuzzys=max([(min([xfuzzy(1) yfuzzy(3)]))]);
zfuzzym=max([(min([xfuzzy(1) yfuzzy(2)])) (min([xfuzzy(2) yfuzzy(1)])) (min([xfuzzy(3) yfuzzy(3)]))]);
zfuzzyl=max([(min([xfuzzy(2) yfuzzy(2)])) (min([xfuzzy(2) yfuzzy(3)])) (min([xfuzzy(3) yfuzzy(1)])) (min([xfuzzy(3) yfuzzy(2)]))]);

%Defuzzification
zs=reshape((quantiz(zsmall,zfuzzys)*2-2)*(-1/2),1,size(zsmall,2)).*zsmall+quantiz(zsmall,zfuzzys)'.*zfuzzys;
zm=reshape((quantiz(zmedium,zfuzzym)*2-2)*(-1/2),1,size(zmedium,2)).*zmedium+quantiz(zmedium,zfuzzym)'.*zfuzzym;
zl=reshape((quantiz(zlarge,zfuzzyl)*2-2)*(-1/2),1,size(zlarge,2)).*zlarge+quantiz(zlarge,zfuzzyl)'.*zfuzzyl;

figure
plot([i j k l]*maxz,zs,':')
hold
plot([i j k l]*maxz,zm,'-')
plot(zl,'--')

final=sum([zs;zm;zl]);

figure
plot([i j k l]*maxz ,final)
title('DEFUZZIFICATION')

S=cumsum(final)
[p,q]=find(S>=(sum(final)/2))
zout=(q(1)/1000)*maxz;
hold
plot(ones(1,100)*zout,[0:1/99:1],'*')
```

6. ANT COLONY OPTIMIZATION

Ant colony optimization techniques exploit the natural behavior of ants in finding the shortest path to reach destination from the source.

The figure 1-24 is the top view of the ants moving from source to destination in search of food. There is the obstacle in the middle. There are two paths available so that the ants can choose to move from source to destination. As expected after some time duration ants prefer to choose path1, which is the shortest distance between source and destination. This is due to the fact that ants excrete the pheromones when they are moving. As more ants have crossed the shortest path within the stipulated time when compared to the other path, more pheromones are excreted in the shortest path. This helps the following ants to choose the shortest path to cross the obstacles.

The mathematical equivalent of this behavior is used in optimization technique as the Ant colony optimization.

6.1 Algorithm

Consider the problem of finding the optimum order in which the numbers from 1 to 8 are arranged so that the cost of that order is maximized. Cost of the particular order is computed using the two matrices A and B as described below.

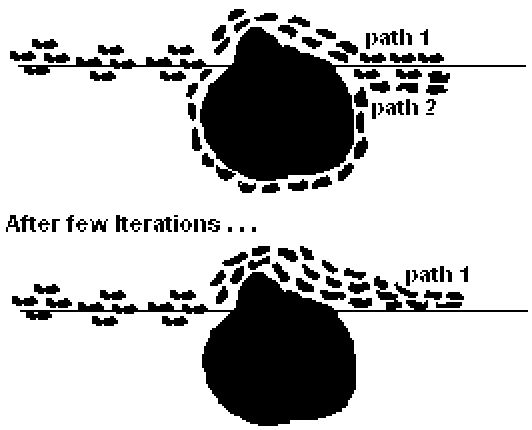
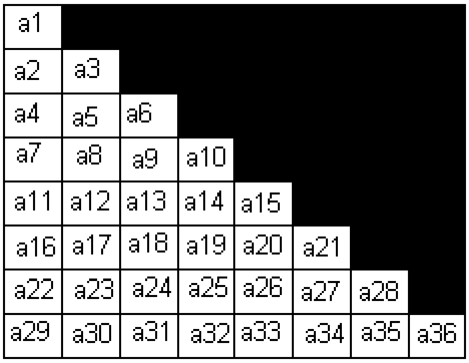
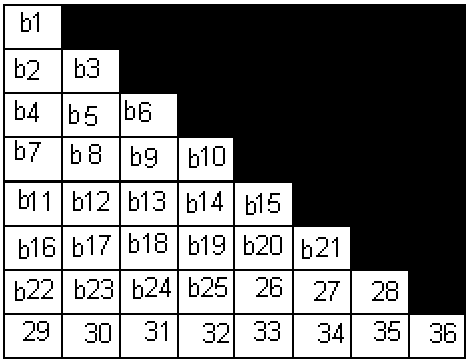


Figure 1-24. Illustration of Ant Colony



a1							
a2	a3						
a4	a5	a6					
a7	a8	a9	a10				
a11	a12	a13	a14	a15			
a16	a17	a18	a19	a20	a21		
a22	a23	a24	a25	a26	a27	a28	
a29	a30	a31	a32	a33	a34	a35	a36

Figure 1-25. Matrix A



b1							
b2	b3						
b4	b5	b6					
b7	b8	b9	b10				
b11	b12	b13	b14	b15			
b16	b17	b18	b19	b20	b21		
b22	b23	b24	b25	26	27	28	
29	30	31	32	33	34	35	36

Figure 1-26. Matrix B

The cost of the order [**3 2 1 5 7 2 4 6 8]** is computed as the sum of the product of values obtained from the positions (**1**,3) or (3,1), (**2**,2), (**3**,1), (**4**,5) or (5,4) , (**5**,7) or (7,5) ,(**6**,2), (**7**,4) and (**8**,6) of the matrices A and B .

Cost ([**3 2 1 5 7 2 4 6 8]**) = **a6*b6 + a3*b3 + a6*a6 + a14*b14 + a26*b26 + a17*b17 +a25*b25 + a36*b36**
(see Matrix A and Matrix B)

The problem of finding the optimal order is achieved using Ant colony optimization technique as described below.

Step 1: Generate the 5 sets of orders randomly which are treated as the initial values selected by the 5 Ants respectively.

The following are the orders selected by the 5 ants along with the corresponding cost measured as described in the previous section.

Table 1-3. Initialization of 5 values to the 5 Ants (say)

ANT	ORDERS	COST
ANT 1	7 1 3 6 2 5 4 8	C1
ANT 2	2 4 8 6 3 7 5 1	C2
ANT 3	7 2 3 5 1 6 4 8	C3
ANT 4	1 5 4 8 7 3 2 6	C4
ANT 5	4 8 5 3 6 2 7 1	C5

Step 2: Pheromone is the value assigned for selecting the ith number for the jth position of the sequence selected by all the ants.(i.e) it is the matrix in which the value at the index (i ,j) is the value assigned for selecting the ith number for the jth position of the order selected by all the ants. Initially the values of the matrix are completely filled up with ones. The values of the Pheromone matrix are updated in every iteration as

Pheromone matrix (n+1) = Pheromone matrix (n) + Updating Pheromone matrix

The value of the updating pheromone matrix must be high if the minimum cost is assigned. Hence the matrix is filled up with the inverse values of the corresponding cost as displayed below.

Table 1-4. Updating Pheromone Matrix

	1	2	3	4	5	6	7	8
1	1/C4	1/C1	0	0	1/C3	0	0	1/C2+1/C5
2	1/C2	1/C3	0	0	1/C1	1/C5	1/C4	0
3	0	0	1/C1+1/C3	1/C5	1/C2	1/C4	0	0
4	1/C5	1/C2	1/C4	0	0	0	1/C1+1/C3	0
5	0	1/C4	1/C5	1/C3	0	1/C1	1/C2	0
6	0	0	0	1/C1+1/C2	1/C5	1/C3	0	1/C4
7	1/C1+1/C3	0	0	0	1/C4	1/C2	1/C5	0
8	0	1/C5	1/C2	1/C4	0	0	0	1/C1+1/C3

Step 3: Next set of orders selected by the ants is as described below.

- Generate the position sequence randomly
  7 8 2 3 6 4 1 5 (say)

Let the order decided by the ant1 be represented as [u1 u2 u3 u4 u5 u6 u7 u8]

- The value for the u7 in the order as mentioned above is selected first as the first number of the randomly generated position sequence is 7.
- To select the value for the u7, 7th column of the pheromone matrix is used .(Note that 7th column belongs to 7th position of the order generated)
- 7th column of the pheromone matrix is rewritten below as Column7 vector arranged in row wise for better clarity.

  Column7 = [0 1/C4 0 1/C1+1/C3 1/C2 0 1/C5 0]

- Normalized value of the 7th column of the pheromone matrix is given below as Normalized vector7 arranged in row wise. This is also called as the probability vector used for selecting the value for the u7 in the order.

  S = [0 + 1/C4 + (1/C1+1/C3) + 1/C2 + 1/C5]

  Normalized vector7 = (Column 7) / S = [p1 p2 p3 p4 p5 p6 p7 p8] (say)

Select the number corresponding to the position of the lowest value of the normalized vector. Suppose if p5 is the lowest among the values in the vector, the number 5 is assigned to the variable u7.

- The next value in the position sequence is 8.Hence the value for the variable u8 is selected. But the value cannot be 5 as it was already assigned to the variable u7. 8th column of the pheromone matrix except the 5th row is used for selecting the value to be assigned to the variable u8 as described below.
- 8th column of the pheromone matrix is rewritten below as Column 8 vector with 5th row filled up **X** indicating 5th row is not consider for selection. The vector is rearranged in row wise as displayed below

  Column 8= [(1/C2 +1/C5) 0 0 0 **X** 1/C4 0 (1/C1+1/C3)]

- Normalized value of the 8th column of the pheromone matrix is

  S= [(1/C2 +1/C5) + 0 + 0 + 0 + 1/C4 + 0 + (1/C1+1/C3)]

  Normalized vector8 = (Column 8) / S = [q1 q2 q3 q4 q6 q7 q8]

Select the number corresponding to the position of the lowest value of the Normalized vector8. Suppose if q3 is the lowest among the values in the vector, the number 3 is assigned to the variable u8.

This process is repeated to obtain the orders selected by the Ant 2, Ant3, An4 and Ant 5.This is called one iteration.

Step 4:

- The updating pheromone matrix for the second iteration is computed as described in the step 2 using the set of orders selected by the five ants in the first iteration. This is called as **Updating pheromone matrix (2)**.
- Pheromone matrix used in the 2nd iteration is computed as

Pheromone matrix (2) = Pheromone matrix (1) +Updating Pheromone matrix (2)

Step 5: Next set of orders selected by the five ants are computed as described in the step 3.

Step 6: The step 3 to step 5 is repeated for the particular number of iterations. Thus best set of orders selected by the five ants are obtained using Ant colony optimization.

Note that best among the best set of orders selected by the five ants in the last iteration is selected as the global best order.

6.2 Example

Consider the problem of finding the optimum order in which the numbers from 1 to 8 are arranged so that the cost of the order is maximized. Cost of the particular order is computed as described in the section 6.1. The matrices A and B are given below.

$$A=\begin{pmatrix} 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{pmatrix}$$

$$B=\begin{pmatrix}9&0&0&0&0&0&0&0&0\\0&9&0&0&0&0&0&0&0\\0&0&9&0&0&0&0&0&0\\0&0&0&9&0&0&0&0&0\\0&0&0&0&9&0&0&0&0\\0&0&0&0&0&9&0&0&0\\0&0&0&0&0&0&9&0&0\\0&0&0&0&0&0&0&9&0\\0&0&0&0&0&0&0&0&9\end{pmatrix}$$

Initial orders selected by the four ants are displayed below.

ANT1:	5	4	2	1	3	6	7	8
ANT2:	3	7	8	5	2	1	4	6
ANT3:	5	1	7	2	4	3	6	8
ANT4:	6	1	7	2	8	5	3	4

Number of ants used to find the optimum order is 4. The orders selected by the four ants after 50 Iterations are displayed below.

ANT1:	1	2	3	4	5	6	7	8
ANT2:	1	2	3	4	5	6	7	8
ANT3:	3	2	7	4	1	6	5	8
ANT4:	4	2	3	7	1	6	5	8

Note that ANT1 and ANT2 found the best order as expected using Ant colony technique.

Initially the cost values are having more changes. After 40th iteration, cost value gradually increases and reaches maximum which is the optimum cost corresponding the optimum order selected using Ant colony technique.

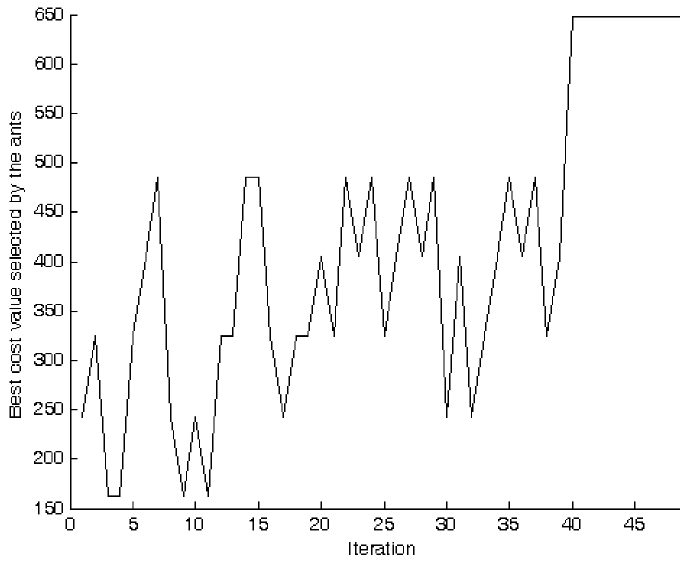


Figure 1-27. Iteration (vs.) Best Cost selected by the 4 ants

6.3 M-program for Finding the Optimal Order using Ant Colony Technique for the Specifications given in the Section 6.2

antcolonygv.m

```
%Consider the problem of finding the optimum order in which the numbers from 1 to 8
   are arranged so that
%the cost of that order is decreased .Cost of the particular order is computed using the
   two matrices A and B as described below.
%The  cost of the order [3 2 1 5 7 2 4 6  8]  is computed as the sum of the product of
   values
%obtained from the  positions (1,3) or (3,1),(2,2),(3,1) ,(4,5) or (5,4) , (5,7) or (7,5) ,(6,2),
   (7,4) and (8,6) of the matrices A and B .
%Cost ([3 2 1 5 7 2 4 6 8] )  = a6*b6 + a3*b3 + a6*a6 + a14*b14 + a26*b26 + a17*b17
   +a25*b25 + a36*b36 see Matrix A and Matrix B

%Form the matrices given below
matA=diag([ones(1,9)*9]);
```

```
matB=diag([ones(1,9)*9]);

%Initializing the orders selected by 4 Ants

[p1,ANT1]=sort(rand(1,8));
[p2,ANT2]=sort(rand(1,8));
[p3,ANT3]=sort(rand(1,8));
[p4,ANT4]=sort(rand(1,8));

%Intializing the Pheromone matrix
%columns are the positions
%rows are the value of the position
ANTS=[ANT1;ANT2;ANT3;ANT4];
P=ones(8,8);
col=[];
figure
hold
for i=1:1:50
   %Updating pheromones
c1=computecost(ANTS(1,:),matA,matB);
c2=computecost(ANTS(2,:),matA,matB);
c3=computecost(ANTS(3,:),matA,matB);
c4=computecost(ANTS(4,:),matA,matB);
col=[col max([c1 c2 c3 c4])];
plot(col)
pause(0.01)
c=[c1 c2 c3 c4];
for i=1:1:8
for j=1:1:8
[x,y] = find(ANTS(:,i)==j);
for k=1:1:length(x)
P(j,i)=P(j,i)+(1/c(x(k)));
end
end
end

ANTS=[];
   for i=1:1:4
      P_cur=P;
      tempant=zeros(1,8);
      notation=[1 2 3 4 5 6 7 8];
      [p,position]=sort(rand(1,8));
```

```
        for i=1:1:8
        temp=P_cur(:,position(i));
        temp=temp/sum(temp);
        [value,index]=min(temp);
        tempant(position(i))=notation(index);
        P_cur(index,:)=[];
        notation(index)=[];
        end
        ANTS=[ANTS;tempant];
      end
    c1=computecost(ANTS(1,:),matA,matB);
    c2=computecost(ANTS(2,:),matA,matB);
    c3=computecost(ANTS(3,:),matA,matB);
    c4=computecost(ANTS(4,:),matA,matB);
    end
```

---

computecost.m

```
    function [s]=computecost(A,P,Q)
      s=0;
      for i=1:1:8
      s=s+P(max(i,A(i)),min(i,A(i))).*Q(max(i,A(i)),min(i,A(i)));
    end
```

Chapter 2

PROBABILITY AND RANDOM PROCESS

Algorithm Collections

1. INDEPENDENT COMPONENT ANALYSIS

Consider 'm' mixed signals $y_1(t)$, $y_2(t)\ldots y_m(t)$ obtained from the linear combination of 'n' independent signals $x_1(t),\ x_2(t)\ \ldots\ x_n(t)$, where n>=m is given below.

$y_1(t) = a_{11}\ x_1(t) + a_{12}\ x_2(t) + a_{13}\ x_3(t) + \ldots a_{1n}x_n(t)$
$y_2(t) = a_{21}\ x_1(t) + a_{22}\ x_2(t) + a_{23}\ x_3(t) + \ldots a_{2n}x_n\ (t)$
$y_3(t) = a_{31}\ x_1(t) + a_{32}\ x_2(t) + a_{33}\ x_3(t) + \ldots a_{3n}x_n(t)$
$y_4(t) = a_{41}\ x_1(t) + a_{42}\ x_2(t) + a_{43}\ x_3(t) + \ldots a_{4n}x_n(t)$
…
$y_m(t) = a_{m1}\ x_1(t) + a_{m2}\ x_2(t) + a_{m3}\ x_3(t) + \ldots a_{mn}x_n(t)$

Independent signals are obtained from the mixed signals using Independent Component Analysis (ICA) algorithm as described below

1.1 ICA for Two Mixed Signals

The samples of the signals $x_1(t)$ and $x_2(t)$ are represented in the vector form as $x_1=[x_{11}\ x_{12}\ x_{13}\ \ldots x_{1n}]$ and $x_2=[x_{21}\ x_{22}\ x_{23}\ \ldots\ x_{2n}]$ respectively. The signals $y_1(t)=a_{11}*x_1(t)+a_{12}*x_2(t)$ and $y_2(t)=a_{21}*x_1(t)+a_{22}*x_2(t)$ are represented in the vector form as $y_1=[y_{11}\ y_{12}\ y_{13}\ \ldots y_{1n}]$ and $y_2=[y_{21}\ \ y_{22}\ y_{23}\ \ldots\ y_{2n}]$

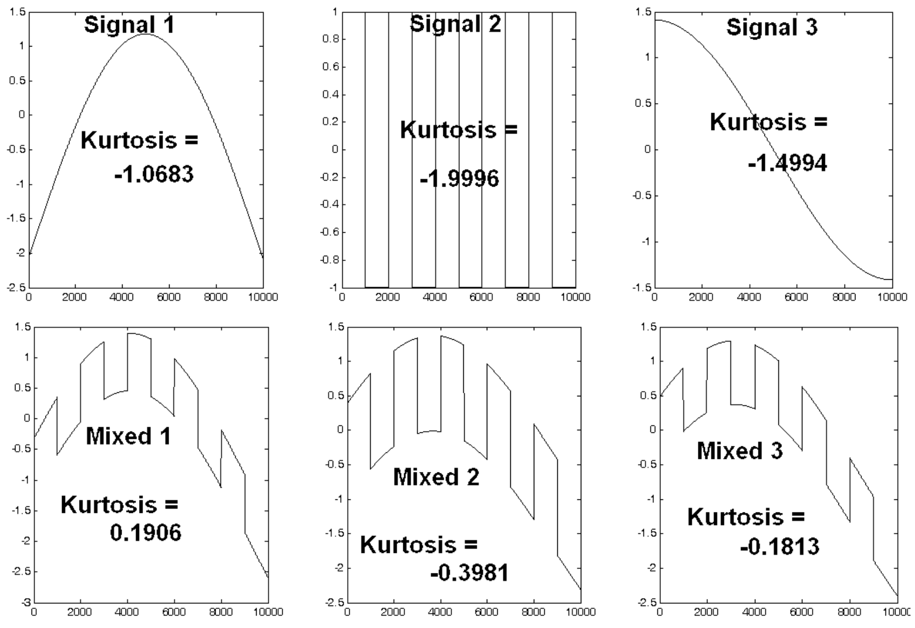
respectively. Thus the ICA problem can be represented in the form of matrix as described below.

$$\begin{pmatrix} y_{11} & y_{21} \\ y_{12} & y_{22} \\ y_{13} & y_{23} \\ y_{14} & y_{24} \\ y_{15} & y_{25} \\ y_{16} & y_{26} \\ \cdot & \\ \cdot & \\ \cdot & \\ y_{1n} & y_{2n} \end{pmatrix} = \begin{pmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \\ x_{13} & x_{23} \\ x_{14} & x_{24} \\ x_{15} & x_{25} \\ x_{16} & x_{26} \\ & \\ & \\ & \\ x_{1n} & x_{2n} \end{pmatrix} \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$$

$$[Y]^T \quad = \quad [X]^T \quad [A]^T$$

Given the matrix [Y], obtaining the matrices [X] and [A] such that the columns of the matrix $[X]^T$ are independent to each other is the ICA problem.

Consider three independent signals and the corresponding mixed signals along with the kurtosis values are displayed below.



Kurtosis is the statistical parameter used to measure the Gaussian nature of the signal. Kurtosis is inversely proportional to the Gaussianity of the signal. Note that the kurtosis values are maximum for independent signals compared to the mixed signals. (i.e.) Independent signals are more non-Gaussian compared with mixed signals. Mathematically kurtosis is computed using the formula as displayed below, where E[X] is the expectation of the vector X

ICA problem is to obtain the matrices [X] and [A] such that column vectors of the matrix $[X]^T$ are independent to each other. (i.e.) Kurtosis values computed for the column vectors are maximum.

$$[Y]^T = [X]^T [A]^T$$

$$(\text{ i.e.}) [Y] = [A] [X]$$

$$[X] = [A]^{-1} [Y]$$

$$[X]=[B][Y]$$

X1 and X2 are the two column vectors of the matrix $[X]^T$

$$(\text{i.e.}) \ [X]^T = [X1 \ X2]$$

Kurtosis measured for the Column vector [X1] is given as

$$E [X1^4]-3\{E [X1^2]\}^2$$

Similarly kurtosis measured for the column vector [X2] is given as

$$E [X2^4]-3\{E [X2^2]\}^2$$

Estimating the best values for the matrix [B] such that kurtosis measured for the column vectors of the matrix $[X]^T = [[B][Y]]^T$ as defined above is maximum

We require another constraint about the matrix [B] to obtain the values of the matrix [B] which is described below

The covariance matrix (COV(Y)) computed for the group of vectors collected row-by-row of the matrix Y^T is computed using the formula as displayed below

$$\text{Let } M = \begin{bmatrix} (y_{11} + y_{12} + y_{13} + \ldots y_{1n}) / n \\ \\ (y_{21} + y_{22} + y_{23} + \ldots y_{2n}) / n \end{bmatrix}$$

$$\text{COV}(Y) =$$

$$\left\{ \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix} \begin{bmatrix} y_{11} & y_{21} \end{bmatrix} + \begin{bmatrix} y_{12} \\ y_{22} \end{bmatrix} \begin{bmatrix} y_{12} & y_{22} \end{bmatrix} + \ldots \begin{bmatrix} y_{1n} \\ y_{2n} \end{bmatrix} \begin{bmatrix} y_{1n} & y_{2n} \end{bmatrix} \right\} / n - MM^T$$

$$\text{(i.e) COV}(Y) = E[YY^T] - MM^T$$

The matrix computed is of size 2x2. The value at the position (1,1) conveys the information about how the first elements of the collected vectors are correlated with itself (variance). The value at the position (1,2) conveys the information about how the first elements are correlated with second elements of the collected vectors.

For example the covariance matrix computed for 2D vectors collected from the particular independent signals and the corresponding mixed signals are given below.

$$\text{COV}_{\text{independent}} = \begin{bmatrix} 0.2641 \times 10^{-006} & -0.1086 \times 10^{-006} \\ -0.1086 \times 10^{-006} & 0.5908 \times 10^{-006} \end{bmatrix}$$

$$\text{COV}_{\text{mixed}} = \begin{bmatrix} 0.1086 & 0.0613 \\ 0.0613 & 0.0512 \end{bmatrix}$$

To compare the covariance matrix, correlation coefficient is used. It is the normalized covariance matrix whose (m, n) element is computed as COV(m, n) / (sqrt(COV(m,m)*COV(n,n))

The correlation co-efficient for the above covariance matrix (COV) is given below

$$CORR_{independent} = \begin{bmatrix} 1 & -0.2750 \\ -0.2750 & 1 \end{bmatrix}$$

$$CORR_{mixed} = \begin{pmatrix} 1 & 0.822 \\ 0.822 & 1 \end{pmatrix}$$

Note that cross correlation value is less for independent signals compared to the mixed signals. This fact is used as the source for second constraint. (i.e.) the correlation coefficients matrix of the independent signals is almost identity matrix. Also if the variances of the signals are unity the covariance matrix and the correlation co-effients are identical.

The mean and variance of the mixed signals are made 0 and 1 respectively so that the mean and variance of the row vectors of the matrix [Y] are 0 and 1 respectively. [Refer Mean and variance Normalization Section 4 of Chapter 2]

The matrix [Y] can be transformed into another matrix [Z] such that the covariance matrix computed for the 2D vectors collected from the matrix [Z] as described above is almost unit vector (diagonal) using KLT (Refer Hotelling transformation). (i.e.) $E[[Z][Z]^T]=[I]$

Let us define the transformation matrix [T], which transforms the matrix [Y] into [Z] as described below.

$$[Z]=[T][Y] \text{ implies}$$

$$[Y]=[T]^{-1} [Z] \text{ implies}$$

$$[Y]=[U] [Z]$$

$$\text{Also, } [X]=[B][Y]$$

$$[X]=[B][U][Z]$$

$$[B][U][Z]=[X]$$

$$[B][U][B][B]^{-1}[Z]=[X]$$

$$[A][Z]=\{[B][U][B]\}^{-1}[X]$$

Note that $A=[B]^{-1}$

Column vectors of the transformation matrix [U] are orthonormal to each other. (ie) $[U]^{T} = [U]^{-1}$

If $E[XX^{T}] = [I]$ (Identity Matrix)
((i.e.) Covariance matrix of the Independent signals with mean zero and variance one is Identity matrix)

Computing Covariance Matrix of the RHS of the equation

$$\mathbf{[A][Z]=\{[B][U][B]\}^{-1}[X]}$$

$$E[(\{[B][U][B]\}^{-1}[X])\ (\{[B][U][B]\}^{-1}[X])^{T}]$$

$$= E[(\{[B][U][B]\}^{-1})[X]\,[X]^{T}(\{[B][U][B]\}^{-1})^{T}]$$

$$= (\{[B][U][B]\}^{-1})\ \mathbf{E\,[XX^{T}]}\ (\{[B][U][B]\}^{-1})^{T}$$

$$= (\{[B][U][B]\}^{-1})(\{[B][U][B]\}^{-1})^{T}$$

$$= ([B]^{-1}[U]^{-1}[B]^{-1})(\,[B]^{-1}[U]^{-1}[B]^{-1})^{T}$$

$$=[B]^{-1}[U]^{-1}[B]^{-1}\,[B^{-1}]^{T}[U^{-1}]^{T}[B^{-1}]^{T}$$

If the columns of the B matrix is orthonormal to each other

(i.e.) $[B]^{-1} = [B]^{T}$

$$[B]^{-1}[U]^{-1}\mathbf{[B]^{-1}\,[B^{-1}]^{T}}[U^{-1}]^{T}[B^{-1}]^{T}$$

$$=[B]^{-1}[U]^{-1}[U^{-1}]^{T}[B^{-1}]^{T}$$

$$= [B]^{-1}[B^{-1}]^{T} \text{ (Because } [U]^{T}=[U]^{-1})$$

$$=[B]^{-1}[B]$$

$$= [I] \text{ (Identity Matrix)}$$

Computing Covariance Matrix of the LHS of the equation

$$\mathbf{[A][Z]=\{[B][U][B]\}^{-1}\,[X]}$$

$$E\,(([A][Z])\,([A][Z])^{T})$$

$$=AE[ZZ^{T}]A^{T}$$

$$= [A][I][A]^{T}$$

$$= [B]^{-1}[B^{-1}]^{T}$$

$$= [I]$$

(Because $[B]^{-1}=[B]^{T}$ is assumed)

Thus Covariance matrix of the LHS and RHS are Identity matrix if

- $E[XX^{T}]$ is the Identity matrix (Assumed)
- $E[ZZ^{T}]$ is the Identity matrix (Property)
- $[B]^{-1}= [B]^{T}$ (Assumed)
- $[U]^{-1}=[U]^{T}$ (Property)

In other words, If the Columns of the matrix [B] are orthonormal to each other (i.e) $[B]^{-1} = [B]^T$, then $E[XX^T]=[I]$ (i.e) Covariance Matrix of the independent signals is the Identity Matrix.(Required).

$$[A][Z]=\{[B][U][B]\}^{-1}[X]$$

When [A] is estimated such that covariance matrix of the LHS and RHS are Identity Matrix, [A][Z]=[X] (ie) $\{[B][U][B]\}^{-1}$ becomes the Identity matrix.

Thus ICA Problem is the optimization problem as mentioned below.

Given the matrix [Y], estimating the best values for the matrix [B] such that kurtosis measured for the row vectors of the matrix $[X] = [A][Z]=[B^T][Z]$ is maximum.

Subject to the constraint $B^TB=[I]$ (i.e) column vectors of the matrix B is orthonormal to each other.

$$\text{Let } [B] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$Z = \begin{bmatrix} z_{11} & z_{12} & z_{13} & \dots & z_{1n} \\ z_{21} & z_{22} & z_{23} & \dots & z_{2n} \end{bmatrix}$$

$$[B]^T[Z] = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} z_{11} & z_{12} & z_{13} & \dots & z_{1n} \\ z_{21} & z_{22} & z_{23} & \dots & z_{2n} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11}z_{11}+b_{21}z_{21} & b_{11}z_{12}+b_{21}z_{22} & \dots & b_{11}z_{1n}+b_{21}z_{2n} \\ b_{12}z_{11}+b_{22}z_{21} & b_{12}z_{12}+b_{22}z_{22} & \dots & b_{12}z_{1n}+b_{22}z_{2n} \end{bmatrix}$$

Kurtosis of the first row $= E[(b_{11}z_{1i}+b_{21}z_{2i})^4] - 3\ \{E[(b_{11}z_{1i}+b_{21}z_{2i})^2]\}^2$

Kurtosis of the second row$= E[(b_{12}z_{1i}+b_{22}z_{2i})^4] - 3\ \{E[(b_{12}z_{1i}+b_{22}z_{2i})^2]\}^2$

Constraint $[B]^T[B]=[I]$

(i.e) $b_{11}\, b_{11}+b_{21}\, b_{21}=1$
$b_{12}\, b_{12}+b_{22}\, b_{22}=1$

Thus Iterative approach to estimate the best values for B matrix using Lagrange's method is described below.

Partial differentiating the function

$$E[(b_{11}z_{1i}+b_{21}z_{2i})^4]-3\ \{E[(b_{11}z_{1i}+b_{21}z_{2i})^2]\}^2\ +\lambda\ (b_{11}\, b_{11}+b_{21}\, b_{21}-1)$$

with respect to the unknown constants $b_{11,}\ b_{21}$ and equate to zero.

With respect to b_{11}

$$E[4(b_{11}z_{1i}+b_{21}z_{2i})^3\ z_{1i}]-3*2\{E[(b_{11}z_{1i}+b_{21}z_{2i})^2]\}E[2(b_{11}z_{1i}+b_{21}z_{2i})\ z_{1i}]$$

$$+\lambda*2*b_{11}=0$$

$E[(b_{11}z_{1i}+b_{21}z_{2i})^2]$ is the variance of the independent signal. This can be assumed to be 1.

Also E $(z_{1i}\ z_{1i})$ is the variance of the first mixed signal and hence it is 1. E $(z_{1i}\ z_{2i})$ is the covariance between the two mixed signals and the value is zero.

Therefore the above equation becomes

$$4*E[(b_{11}z_{1i}+b_{21}z_{2i})^3\ z_{1i}]-6*2*b_{11}+\lambda*2*b_{11}=0$$

Let the Lagrange multiplier λ be (-2). [For More Details Refer Estimation Techniques]

Thus Iteration equation becomes

$$b_{11}(t+1)=E[(b_{11}(t)z_{1i}+b_{21}(t)\ z_{2i})^3\ z_{1i}]-3*b_{11}(t)$$

$b_{11}(t)$ is the value for b_{11} in tth iteration

$b_{11}(t+1)$ is the value for b_{11} in (t+1)th iteration

Similarly iteration equation for other elements of the matrix B are computed and are displayed below

$$b_{11}(t+1) = E[(b_{11}(t)z_{1i}+b_{21}(t)\ z_{2i})^3\ z_{1i}]-3*b_{11}(t)$$

$$b_{21}(t+1) = E[(b_{11}(t)z_{1i}+b_{21}(t)\ z_{2i})^3\ z_{2i}]-3*b_{21}(t)$$

$$b_{12}(t+1) = E[(b_{12}(t)z_{1i}+b_{22}(t)\ z_{2i})^3\ z_{1i}]-3*b_{12}(t)$$

$$b_{22}(t+1) = E[(b_{12}(t)z_{1i}+b_{22}(t)\ z_{2i})^3\ z_{2i}]-3*b_{22}(t)$$

Recall that the columns of the matrix B are made orthonormal to each other. This can be explicitly performed in every iteration, after updating the values using the equation given above.

1.1.1 ICA algorithm

Step 1: Given mixed signals y1(t) and y2(t) are converted into z1(t) and z2(t) using Hotellilng transformation such that the covariance matrix computed using the converted signals z1(t) and z2(t) is the identity matrix.[Use Hotelling Transformation]

Step 2: Initialize the values for the matrix B such that $B^TB=[I]$

Step 3: Update the elements of the matrix B using the Iteration formula displayed above.

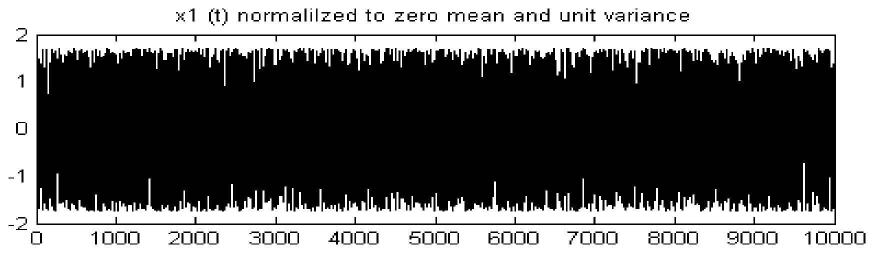
Step 4: Columns of the matrix B is made orthonormal to each other as mentioned below.

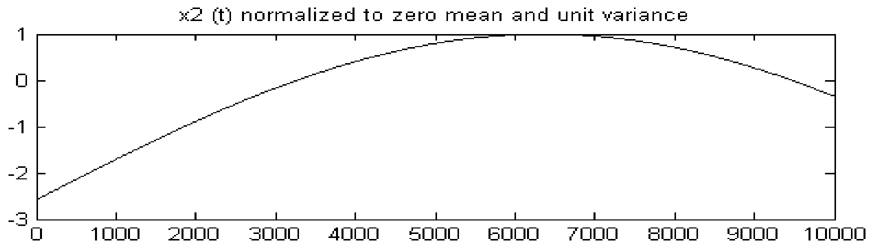
```
B = B * real(inv(B' *B)^(1/2));
```

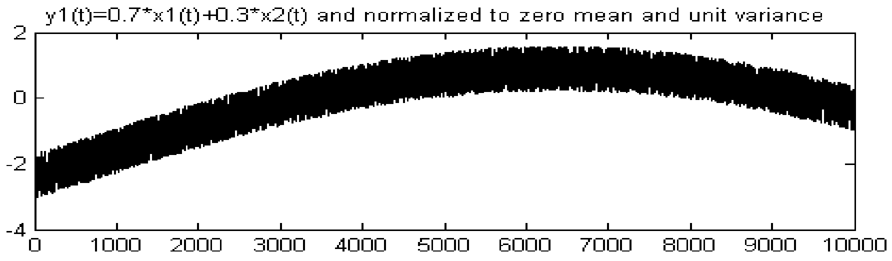
Step 5: Repeat step 3 and step 4 for 'N' Iteration.

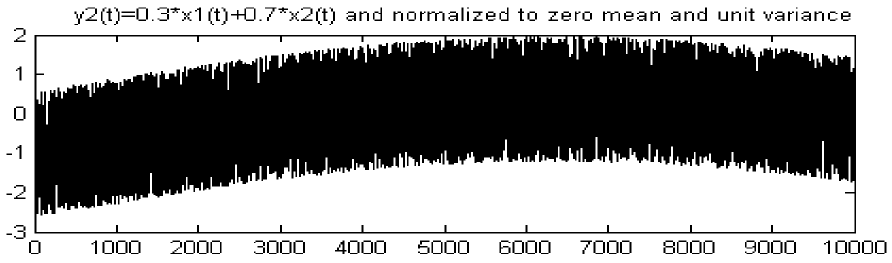
Step 6: Independent signals are obtained by multiplying B^T with the Z matrix

Example: Independent signals x1 (t) and x2 (t) are mixed to get the signals y1 (t)=0.3*x1 (t)+0.7*x2 (t), y2 (t)=0.7*x1 (t)+0.3*x2 (t) as shown in the figure given below

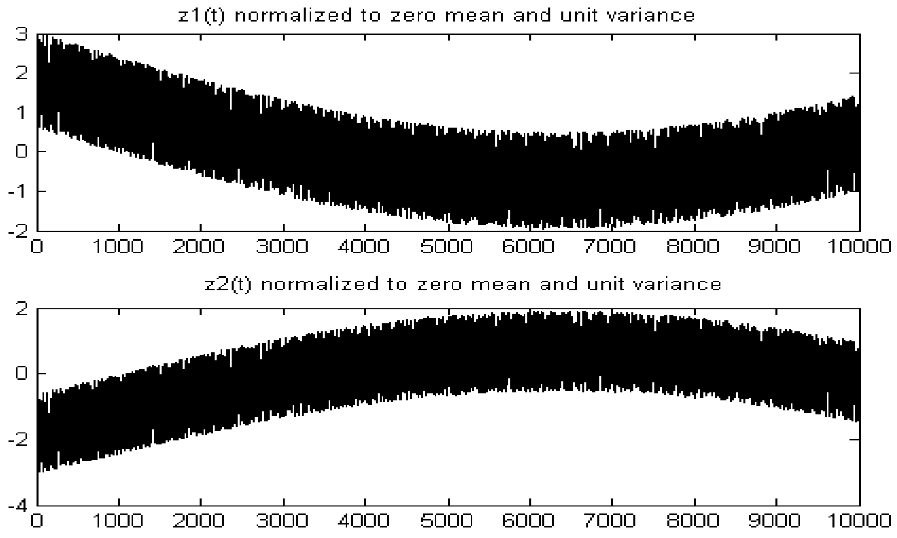


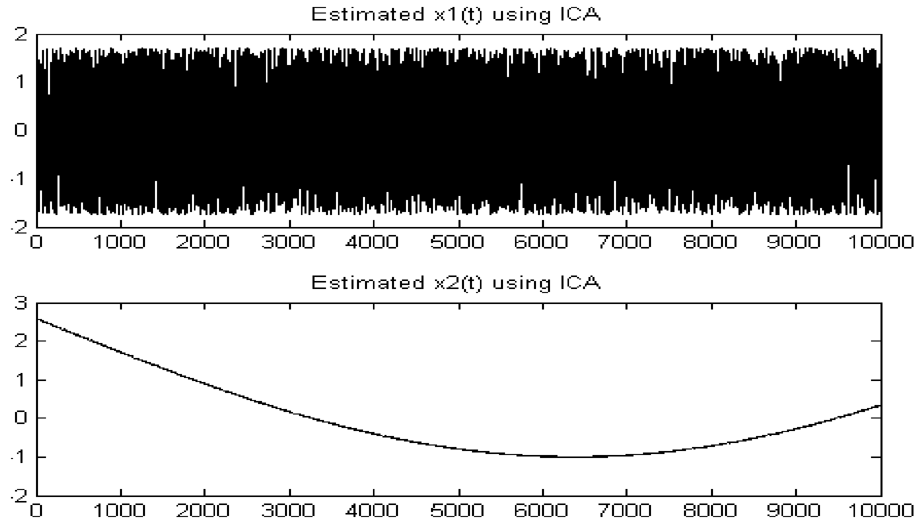






Using ICA independent signals are obtained in two stages with decorrelated signals z1(t) and z2(t) in the first stage and the Independent estimated signals x1(t) and x2(t) in the second stage as displayed below. Number of Iterations (N) used is 10000.





1.2 M-file for Independent Component Analysis

icagv.m

```
%ICA WITH THREE COMPONENTS

close all
clear all
i=0:1/10:1000;
a=sin(0.1*pi*i);
a=a(1:1:10000);
a=meanvarnorm(a,0,1)
b=randn(1,10000);
b=meanvarnorm(b,0,1);
i=0:1/10:1000;
c=exp(0.001*pi*i);
c=meanvarnorm(c,0,1);
c=c(1:1:10000);
s1=0.9*a+0.6*b+0.6*c;
s1=meanvarnorm(s1,0,1);
s2=0.6*a+0.9*b+0.6*c;
s2=meanvarnorm(s2,0,1);
s3=0.6*a+0.6*b+0.9*c;
s3=meanvarnorm(s3,0,1);
figure
subplot(3,1,1)
plot(a);
subplot(3,1,2)
plot(b);
subplot(3,1,3)
plot(c);
figure
subplot(3,1,1)
plot(s1);
subplot(3,1,2)
plot(s2);
subplot(3,1,3)
plot(s3);
%ICA START
data1=[s1;s2;s3];
MEANMAT=repmat(mean(data1')',1,length(data1));
```

```
c1=cov(data1');
[T1,E1]=eig(c1);
data2=T1'*(data1-MEANMAT);
%data2=T1*(data1);
c2=cov(data2');
[T2,E2]=eig(c2);
data=E2^(-1/2)*data2;
T=E2^(-1/2)*T1;
c=cov(data');
figure
subplot(3,1,1)
plot(data(1,:));
subplot(3,1,2)
plot(data(2,:));
subplot(3,1,3)
plot(data(3,:));
%ICA STARTS
w11=rand(1);
w12=rand(1);
w13=rand(1);
w21=rand(1);
w22=rand(1);
w23=rand(1);
w31=rand(1);
w32=rand(1);
w33=rand(1);
W1=[w11 w21 w23]';
W2=[w12 w22 w32]';
W3=[w13 w23 w33]';
W=[W1 W2 W3]';
W = W * real(inv(W' * W)^(1/2));
w11=W(1,1);
w12=W(1,2);
w13=W(1,3);
w21=W(2,1);
w22=W(2,2);
w23=W(2,3);
w31=W(3,1);
w32=W(3,2);
w33=W(3,3);
for i=1:1:40000
   w11=sum(((w11*data(1,:)+w21*data(2,:)+w31*data(3,:)).^3).*data(1,:))/length(data);
```

```
    w21=sum(((w11*data(1,:)+w21*data(2,:)+w31*data(3,:)).^3).*data(2,:))/length(data);
    w31=sum(((w11*data(1,:)+w21*data(2,:)+w31*data(3,:)).^3).*data(3,:))/length(data);
    w12=sum(((w12*data(1,:)+w22*data(2,:)+w32*data(3,:)).^3).*data(1,:))/length(data);
    w22=sum(((w12*data(1,:)+w22*data(2,:)+w32*data(3,:)).^3).*data(2,:))/length(data);
    w32=sum(((w12*data(1,:)+w22*data(2,:)+w32*data(3,:)).^3).*data(3,:))/length(data);
     w13=sum(((w13*data(1,:)+w23*data(2,:)+w33*data(3,:)).^3).*data(1,:))/length(data);
    w23=sum(((w13*data(1,:)+w23*data(2,:)+w33*data(3,:)).^3).*data(2,:))/length(data);
    w33=sum(((w13*data(1,:)+w23*data(2,:)+w33*data(3,:)).^3).*data(3,:))/length(data);
    w11=w11-3*w11;
    w12=w12-3*w12;
    w13=w13-3*w13;
    w21=w21-3*w21;
    w22=w22-3*w22;
    w23=w23-3*w23;
    w31=w31-3*w31;
    w32=w32-3*w32;
    w33=w33-3*w33;
W1=[w11 w21 w23]';
W2=[w12 w22 w32]';
W3=[w13 w23 w33]';
W=[W1 W2 W3]';
W = W * real(inv(W' *W)^(1/2));
w11=W(1,1);
w12=W(1,2);
w13=W(1,3);
w21=W(2,1);
w22=W(2,2);
w23=W(2,3);
w31=W(3,1);
w32=W(3,2);
w33=W(3,3);
W*W';
end
W=[w11 w12 w13;w21 w22 w23;w31 w32 w33];
y=W'*data;
figure
subplot(3,1,1)
plot(y(1,:));
subplot(3,1,2)
plot(y(2,:));
subplot(3,1,3)
plot(y(3,:));
```

2. GAUSSIAN MIXTURE MODEL

Let us consider the data collected from the certain group of people about their age and height are represented in two-dimensional vectors. Let the first element of the vector be the age and the second element is the height of the corresponding person, as described below. $P_1 = [a_1\ h_1]$, $P_2 = [a_2\ h_2]$ ... $P_m = [a_m\ h_m]$. Now let us define the problem for classifying the collected group of vectors into 'n' category called as 'n' clusters, such that the centroids of the clusters are away from each other and the data belonging to the same cluster are nearer to each other as shown in the figure. The dots in the Figure 2-1 belong to the centroid of the individual clusters. The problem defined is called clustering the data, otherwise called as grouping the data.

Let us model the probability of the particular vector 'x' from the collected data as the Linear combination of Multi Variate Gaussian density function.

(ie) $p(x) = p(c_1)$ * Gaussian density function of 'x' with mean vector 'm_1' and covariance matrix 'cov_1' + $p(c_2)$ * Gaussian density function of 'x' with mean vector 'm_2' and covariance matrix 'cov_2' + … $p(c_n)$ * Gaussian density function of 'x' with mean vector 'm_n' and covariance matrix 'cov_n' where $p(c_n)$ is the probability of the clustern n.

(ie) $p(x) = p(c_1)\ p(x/c_1) + p(c_2)\ p(x/c_2) + p(c_3)\ p(x/c_3) + . . .\ p(c_n)\ p(x/c_n)$

The model described above is called as Gaussian Mixture Model. Given the data (x_1, x_2, x_3, x_4... x_m) , obtaining the mean vectors m_1, m_2, … and the covariance matrices c_1, c_2, … and the probability of clusters $p(c_1)$, $p(c_2)$… such that the probability of the collected data is maximized is the task to be performed for modeling. As the collected data are independent to each other, the probability of the collected data (p) is represented as the product of $p(x_1)$, $p(x_2)…p(x_m)$ (ie) $P_D=p(x_1)\ p(x_2)…p(x_m)$

Figure 2-1. Clustering Example with Four Clusters

Estimating the best values for mean vectors and covariance matrices such that the probability P_D is maximized is described below.

Maximization is obtained by differentiating $P_D = P_D = p(x_1)\ p(x_2)\ldots p(x_m)$ with respect to the unknown parameters $m_1, m_2, m_3, \ldots m_n$, cov_1, cov_2, $cov_3 \ldots cov_n$ and equate to zero. Solving the above set of obtained equations gives the best estimate of the unknown parameters, which are listed below

$$m_l = \frac{[(p(c_l/x_1)*(x_1) + p(c_l/x_2)*(x_2) + p(c_l/x_3)*(x_3) + \ldots\ldots\ p(c_l/x_m)*x_m)]}{p(c_l/x_1) + p(c_l/x_2) + p(c_l/x_3) + \ldots\ldots\ p(c_l/x_m)}$$

where $p\ (c_l/x_l)$ is computed as $[p(x_{l/}c_l)\ *\ p\ (c_l)]\ /\ [p(x_l)]$

Note that $p(x_{l/}c_l)$ is the probability of 'x_l' computed using Gaussian density function of 'x' with mean 'm_l' and covariance 'cov_l'. Also $p(x_l) = p(c_1)\ p(x_l/c_1) + p\ (c_2)\ p(x_l/c_2) + p(c_3)\ p(x_l/c_3) + \ldots p(c_n)\ p(x_l/c_n)$

The requirement is to estimate the best value for m1.But for computing the best value requires $p(c_1/x_1)$, which in further requires $p(x_{1/}c_1)$ as discussed above. p(x1/c1) requires m1 for computation. [Surprise!].

To proceed further, an iteration approach called as Expected – Maximization algorithm is used to estimate the best value for m_1 as described below.

2.1 Expectation-maximization Algorithm

1. Initialize the mean vectors randomly m_1, m_2, $m_{3,}$… m_n
2. Find the Euclidean distance between the x_1 vector and the mean vectors m_1 m_2 …m_n are computed as ‘d1’, ‘d2’, … ‘dn’ respectively. If the distance d2 is smaller among all, the x_1 vector is treated as the vector belongs to the 2nd cluster. In general if the dk is smallest among all, x_1 vector belongs to the kth cluster. Thus cluster index assigned to the vector x_1 is ‘k’. Similarly cluster index is obtained for the entire data vector.
3. Let the number of data belongs to the first cluster be ‘k’. (ie) Number of data vectors having cluster index 1 is ‘k. Then p (c1) = probability of the cluster 1 is computed as

$$p(c_1) = \frac{k1}{\text{Total Number of data}}$$

and p(c2) is computed as

$$p(c_2) = \frac{k2}{\text{Total Number of data}}$$

and so on

4. Covariance matrix of the cluster 1 is computed as $cov_1 = E([X-\mu_X][X-\mu_X{}^T]) = E(XX^T)-\mu_X\mu_X{}^T$. For instance, let us consider the vectors $x_{1,}$ x_3, x_7, and x_9 arranged in the column format, belongs to cluster1 and the corresponding covariance matrix is computed as follows.

$$\mu_X = [x_1 + x_3 + x_7 + x_9]/4.$$

$$E(XX^T) = [x_1 x_1{}^T + x_2 x_2{}^T + x_3 x_3{}^T + x_4 x_4{}^T]/4$$

Thus cov_1 is computed as $E(XX^T) - \mu_X\mu_X{}^T$.

5. Similarly, the covariance matrices cov_2, cov_3, cov_4 … cov_n are computed.

2.1.1 Expectation Stage

Using the above initialized values,

$$\begin{matrix}[\ p\ (c_1/x_1), p\ (c_1/x_2)\ p\ (c_1/x_3)\ldots\ p\ (c_1/x_m) \\ p\ (c_2/x_1), p\ (c_2/x_2)\ p\ (c_2/x_3)\ldots\ p\ (c_2/x_m) \\ p\ (c_3/x_1), p\ (c_3/x_2)\ p\ (c_3/x_3)\ldots\ p\ (c_3/x_m) \\ p\ (c_4/x_1), p\ (c_4/x_2)\ p\ (c_4/x_3)\ldots\ p\ (c_4/x_m) \\ \ldots \\ p\ (c_n/x_1), p\ (c_n/x_2)\ p\ (c_n/x_3)\ldots\ p\ (c_n/x_m)]\end{matrix}$$

are computed. This is called Expectation stage of the E-M algorithm.

2.1.2 Maximization stage

Maximization stage of the E-M algorithm belongs to maximizing the probability P_D. That is obtained by differentiating the probability P_D with respect to unknown parameter and equating them to zero. Solving the set of obtained equations gives the best estimate of unknown parameters which are listed below.

$$m_1 = \frac{[(p(c_1/x_1)*(x_1) + p(c_1/x_2)*(x_2) + p(c_1/x_3)*(x_3) + \ldots\ldots\ p(c_1/x_m)*x_m)]}{p(c_1/x_1) + p(c_1/x_2) + p(c_1/x_3) + \ldots\ldots\ p(c_1/x_n)}$$

$$m_2 = \frac{[(p(c_1/x_1)*(x_1) + p(c_1/x_2)*(x_2) + p(c_1/x_3)*(x_3) + \ldots\ldots\ p(c_1/x_m)*x_m)]}{p(c_1/x_1) + p(c_1/x_2) + p(c_1/x_3) + \ldots\ldots\ p(c_1/x_n)}$$

$$m_3 = \frac{[(p(c_1/x_1)*(x_1) + p(c_1/x_2)*(x_2) + p(c_1/x_3)*(x_3) + \ldots\ldots\ p(c_1/x_m)*x_m)]}{p(c_1/x_1) + p(c_1/x_2) + p(c_1/x_3) + \ldots\ldots\ p(c_1/x_n)}$$

$$m_4 = \frac{[(p(c_1/x_1)*(x_1) + p(c_1/x_2)*(x_2) + p(c_1/x_3)*(x_3) + \ldots\ldots\ p(c_1/x_m)*x_m)]}{p(c_1/x_1) + p(c_1/x_2) + p(c_1/x_3) + \ldots\ldots\ p(c_1/x_n)}$$

$$cov_n = \frac{[(p(c_n/x_1)*(\ [x_1-\mu_{X1}]\ [x_1-\mu_{X1}]^T) + \ldots\ldots\ p(c_n/x_m)*\ (\ [x_m-\mu_{Xm}]\ [x_m-\mu_{Xm}]^T)]}{p(c_n/x_1) + p(c_n/x_2) + p(c_n/x_3) + \ldots\ldots\ p(c_n/x_m)}$$

The computed values are the current best estimates of means and covariance matrices. Using this Expectation stage is executed, followed by Maximization stage. Thus Expectation stage and Maximization stage is repeated for N iterations and hence best estimate of the means and covariance matrices are obtained using E-M algorithm.

Using the estimated means $p(c_1)$, $p(c_2)$, $p(c_3)\ldots p(c_n)$ are obtained and hence the Gaussian Mixture Model of the collected is obtained.

2.2 Example

Two dimensional vectors are randomly generated and the Gaussian Mixture Model of the generated data is obtained using the algorithm described above is displayed below.

About 350 vectors are randomly generated. Among which 100 vectors are with mean [0.9 0.8], 100 vectors are with mean [0.7 0.6] and 150 vectors are with mean [0.5 0.4].

The elements of the individual vector are generated independently and hence the estimated covariance matrices are diagonal in nature.

Estimated values of the GMM model after 10 Iterations are given below

```
Mean vector 1 = [0.9124   0.7950]
Mean vector 2 = [0.7125   0.6091]
Mean vector 3 = [0.4994   0.3990]

Covariance matrix 1
                         0.0045  -0.0001
                        -0.0001   0.0047

Covariance matrix 2
                         0.0048  -0.0003
                        -0.0003   0.0048

Covariance matrix 3
                         0.0055  -0.0005
                        -0.0005   0.0054
```

2.3 Matlab Program

---

gmmmodelgv.m

```
%GMM MODEL
cluster1=clustercol([0.9 0.8],100);
cluster2=clustercol([0.7 0.6],100);
cluster3=clustercol([0.5 0.4],150);
clustertot=[cluster1 cluster2 cluster3];
%[p,q]=kmeans(clustertot',3);This may be used for initializing the means
q(1,:)=[0.83 0.79];
q(2,:)=[0.76 0.74];
q(3,:)=[0.6 0.3];

%To Estimate  q ,cov1,cov2,cov3
[p]=clusterno(clustertot,q);
[m1,n1]=find(p==1);
collect1=clustertot(:,n1);
if(length(n1)~=0)
cov1=cov(collect1');
else
   cov1=diag([1 1]);
end
[m1,n1]=find(p==2);
collect2=clustertot(:,n1);
cov2=cov(collect2');
if(length(n1)~=0)
cov2=cov(collect2');
else
   cov2=diag([1 1]);
end
[m1,n1]=find(p==3);
collect3=clustertot(:,n1);
cov3=cov(collect3');
if(length(n1)~=0)
cov3=cov(collect3');
else
   cov3=diag([1 1]);
end
plot(collect1(1,:),collect1(2,:),'r*')
```

```
hold on
plot(collect2(1,:),collect2(2,:),'g*')
hold on
plot(collect3(1,:),collect3(2,:),'b*')
hold on
pause(0.3)
w1=length(find(p==1))/length(p);
w2=length(find(p==2))/length(p);
w3=length(find(p==3))/length(p);
for j=1:1:10
pw1x=[];
pw2x=[];
pw3x=[];
for i=1:1:length(clustertot)
pw1x=[pw1x (w1*pxw(clustertot(:,i),q(1,:),cov1))/...
   (w1*pxw(clustertot(:,i),q(1,:),cov1)+...
    w2*pxw(clustertot(:,i),q(2,:),cov2)+...
    w3*pxw(clustertot(:,i),q(3,:),cov3))];
pw2x=[pw2x (w2*pxw(clustertot(:,i),q(2,:),cov2))/...
   (w1*pxw(clustertot(:,i),q(1,:),cov1)+...
    w2*pxw(clustertot(:,i),q(2,:),cov2)+...
    w3*pxw(clustertot(:,i),q(3,:),cov3))];
pw3x=[pw3x (w3*pxw(clustertot(:,i),q(3,:),cov3))/...
   (w1*pxw(clustertot(:,i),q(1,:),cov1)+...
    w2*pxw(clustertot(:,i),q(2,:),cov2)+...
    w3*pxw(clustertot(:,i),q(3,:),cov3))];
end
q(1,1)=sum(pw1x.*clustertot(1,:))/sum(pw1x);
q(1,2)=sum(pw1x.*clustertot(2,:))/sum(pw1x);
q(2,1)=sum(pw2x.*clustertot(1,:))/sum(pw2x);
q(2,2)=sum(pw2x.*clustertot(2,:))/sum(pw2x);
q(3,1)=sum(pw3x.*clustertot(1,:))/sum(pw3x);
q(3,2)=sum(pw3x.*clustertot(2,:))/sum(pw3x);
cov1(1,1)=sum(pw1x.*[(clustertot(1,:)-q(1,1)).*...
   (clustertot(1,:)-q(1,1))])/sum(pw1x);
cov1(1,2)=sum(pw1x.*[(clustertot(1,:)-q(1,1)).*...
   (clustertot(2,:)-q(1,2))])/sum(pw1x);
cov1(2,1)=sum(pw1x.*[(clustertot(2,:)-q(1,2)).*...
   (clustertot(1,:)-q(1,1))])/sum(pw1x);
cov1(2,2)=sum(pw1x.*[(clustertot(2,:)-q(1,2)).*...
   (clustertot(2,:)-q(1,2))])/sum(pw1x);
```

```
cov2(1,1)=sum(pw2x.*[(clustertot(1,:)-q(2,1)).*...
   (clustertot(1,:)-q(2,1))])/sum(pw2x);
cov2(1,2)=sum(pw2x.*[(clustertot(1,:)-q(2,1)).*...
   (clustertot(2,:)-q(2,2))])/sum(pw2x);
cov2(2,1)=sum(pw2x.*[(clustertot(2,:)-q(2,2)).*...
   (clustertot(1,:)-q(2,1))])/sum(pw2x);
cov2(2,2)=sum(pw2x.*[(clustertot(2,:)-q(2,2)).*...
   (clustertot(2,:)-q(2,2))])/sum(pw2x);

cov3(1,1)=sum(pw3x.*[(clustertot(1,:)-q(3,1)).*...
   (clustertot(1,:)-q(3,1))])/sum(pw3x);
cov3(1,2)=sum(pw3x.*[(clustertot(1,:)-q(3,1)).*...
   (clustertot(2,:)-q(3,2))])/sum(pw3x);
cov3(2,1)=sum(pw3x.*[(clustertot(2,:)-q(3,2)).*...
   (clustertot(1,:)-q(3,1))])/sum(pw3x);
cov3(2,2)=sum(pw3x.*[(clustertot(2,:)-q(3,2)).*...
   (clustertot(2,:)-q(3,2))])/sum(pw3x);

[p]=clusterno(clustertot,q);
w1=length(find(p==1))/length(p);
w2=length(find(p==2))/length(p);
w3=length(find(p==3))/length(p);
[m1,n1]=find(p==1);
collect1=clustertot(:,n1);
[m1,n1]=find(p==2);
collect2=clustertot(:,n1);
[m1,n1]=find(p==3);
collect3=clustertot(:,n1);
close all
plot(collect1(1,:),collect1(2,:),'r*')
hold on
plot(q(1,1),q(1,2),'co')
plot(collect2(1,:),collect2(2,:),'g*')
plot(q(2,1),q(2,2),'mo')
plot(collect3(1,:),collect3(2,:),'b*')
plot(q(3,1),q(3,2),'yo')
pause(0.3)
totcol1{j}=collect1;
totcol2{j}=collect2;
totcol3{j}=collect3;
qcol{j}=q;
end
```

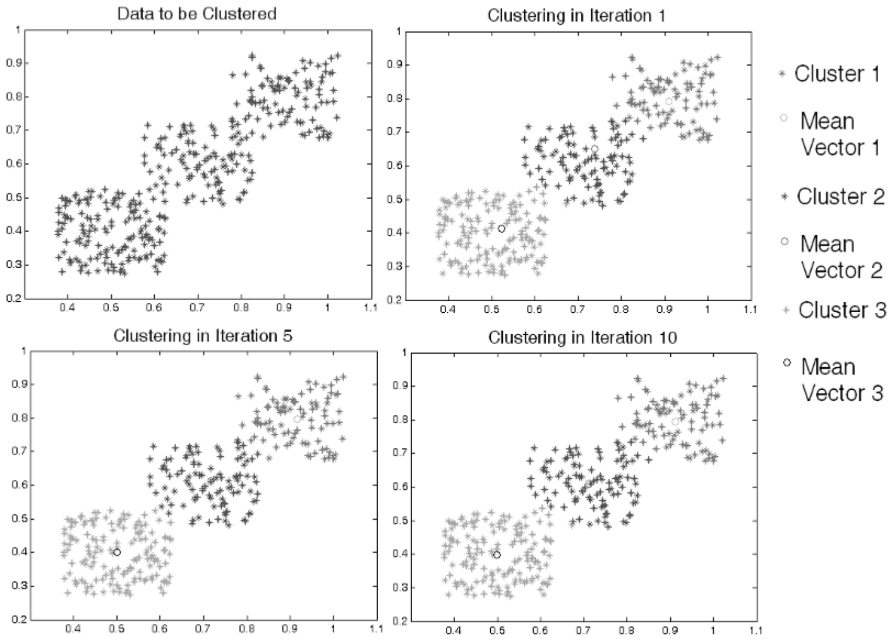
clustercol.m

```
function [res] =clustercol(pos,N)
x1=rand(1,N)/4-(0.25)/2;
y1=rand(1,N)/4-(0.25)/2;
res=[x1+pos(1);y1+pos(2)];
```

clusterno.m

```
function [q]=clusterno(clustertot,q)
d1=(clustertot'-repmat(q(1,:),length(clustertot),1)).^2;
d1=sum(d1');
d2=(clustertot'-repmat(q(2,:),length(clustertot),1)).^2;
d2=sum(d2');
d3=(clustertot'-repmat(q(3,:),length(clustertot),1)).^2;
d3=sum(d3');
[p,q]=min([d1;d2;d3]);
```

2.4 Program Illustration



3. K-MEANS ALGORITHM FOR PATTERN RECOGNITION

If the groups of n-dimensional vectors are given, there is the need to classify the vectors into ‘k’ category. The centroid vectors of each group are used to represent the identity for ‘k’ category. (i.e) If the new vector is to be classified among the ‘k’ category, the distance between the new vector with all the centroids are computed. The centroid corresponding to the smallest distance is treated as the identified group. The centroids of all the groups are obtained using K-means algorithm as described below.

Consider the set of normalized marks (i.e. the mark ranges from 0 to 1) scored by the 100 students in the class for particular subject. Each mark is treated as the vector with one-dimensional. There is the need to classify the collected marks into 6 groups for allocating 6 grades. This is done using k-means algorithm as described below.

3.1 K-means Algorithm

Step 1: Initialize the 6 centroids randomly corresponding to 6 grades .

Step 2: Classify the marks and identify the grade for the individual marks using the initialized 6 centroids as described in the section 3.

Step 3: Find the mean of the marks collected in the particular grade say ‘1’. This is treated as the centroid corresponding to the grade 1 used for the next iteration. This process is repeated to compute the new sets of centroids corresponding to the individual grade.

Step 4: Go to step 2.

Step 5: The process involved from Step 2 to Step 4 is considered as the one iteration and this process is repeated for finite number of iterations to obtain the final centroids corresponding to each grades.

3.2 Example

The particular sets of marks (data) are subjected to k-means algorithm Final clusters along with clusters obtained at every iteration are displayed below. Note that after 6th iteration, clusters are not changed.

Figure 2-2. Illustration of K-Means Algorithm

Figure 2-3. Data along with the final centroids obtained by k-means algorithm

In the above-mentioned example, the data ranges from 0.0067 to 0.9925 and the six centroids obtained in the 10th iteration are displayed below. 0.0743 0.2401 0.4364 0.6115 0.7763 0.9603 (see the Figure 2-3).

3.3 Matlab Program for the K-means Algorithm Applied for the Example given in Section 3.2

kmeansgv.m

```
a=rand(1,100);
plot(a,zeros(1,100),'*');
b=rand(1,6);
for iter=1:1:10
hold off
M=[(a-b(1)).^2;(a-b(2)).^2;(a-b(3)).^2;(a-b(4)).^2;(a-b(5)).^2;(a-b(6)).^2;];
[P,CLUSTERNO]=min(M);
u=['r','g','b','c','m','y'];
figure
```

```
for i=1:1:6
[x,y]=find(CLUSTERNO==i);
col=[];
for k=1:1:length(x)
col= [col a(x(k),y(k))];
end
plot(col,zeros(1,length(col)),strcat(u(i),'*'))
hold on
b(i)=mean(col);
end
pause(0.01)
end
```

---

4. FUZZY K-MEANS ALGORITHM FOR PATTERN RECOGNITION

Consider the problem described in the section 3 for classifying the normalized marks into 6 clusters for the assignment of grades. In fuzzy k-means technique, fuzzy set theory is used to obtain the optimal values of the centroid.

In this technique the particular vector (In this problem it is the normalized mark scored by the student) belongs to all the 6 clusters with different membership values. For instant the vector 0.3 belongs to the different clusters with different membership values as given below

Cluster 1 = {0.3 **(0.0001)**}
Cluster 2 = {0.3 **(0.0448)**}
Cluster 3 = {0.3 **(0.0022)**}
Cluster 4 = {0.3 **(0.0031)**}
Cluster 5 = {0.3 **(0.9492)**}
Cluster 6 = {0.3 **(0.0007)**}

The numbers in bold letters are the corresponding membership values. (i.e.) 0.3 belongs to the cluster 1 with membership value 0.0001 and belongs to the cluster 2 with membership value 0.0448 and so on. Note that sum of the membership values is 1.

Consider the vector x1 belongs to the cluster 1 whose centroid is c1, then (x1-c1)^2 must be minimized. But if the x1 belongs to cluster 1 with the membership value m1, then m1*(x1-c1)^2 has to be minimized. Similarly x1 belongs to the cluster 2 whose centroid is c2. Also x1 belongs to the cluster 2 with membership value m2. Thus the term m2*(x1-c2)^2 also has to be minimized.

Thus the fuzzy k-means problem is treated as the optimization technique for obtaining the membership values along with the centroids of the individual clusters such that

$$\sum_{i=1}^{100} \sum_{k=1}^{6} M_{ik}{}^{2} * (x_i - c_k)^2 \text{ is minimized}$$

M_{ij} is membership value of the vector xi belongs in the cluster k. Xi is the ith vector. c_k is the kth centroid.

The algorithm for obtaining the centroids along with membership values is displayed below.

4.1 Fuzzy K-means Algorithm

Step 1: Intialize the membership values (M_{ij}) randomly.
Step 2: Compute the centroids of the 6 clusters.

C1=[Vector1* (the member ship value of the vector 1 belongs to the cluster 1)^2 (i.e.) $M_{11}{}^{2}$ + Vector2* (the member ship value of the vector 2 belongs to the cluster 1)^2 (i.e.) $M_{12}{}^{2}$ +… Vector100* (the member ship value of the vector 100 belongs to the cluster 1)^2 (i.e.) $M_{1,100}{}^{2}$]/

Sum of the squared values of the membership values belonging to the cluster 1.

Similarly the centroids C2, C3, C4, C5 and C6 are obtained.

Step 3: Update the membership values M using the current values of the 6 centroids as given below.

$$\begin{aligned} M_{11} = 1/\ [&((\text{vector } 1\text{-c}1)^2 / (\text{vector}1 - \text{c}1)^2)^2 + \\ &((\text{vector } 1\text{-c}1)^2 / (\text{vector}1 - \text{c}2^2)^2 + \\ &((\text{vector } 1\text{-c}1)^2 / (\text{vector}1 - \text{c}3^2)^2 + \\ &((\text{vector } 1\text{-c}1)^2 / (\text{vector}1 - \text{c}4^2)^2 + \\ &((\text{vector } 1\text{-c}1)^2 / (\text{vector}1 - \text{c}5^2)^2 + \\ &((\text{vector } 1\text{-c}1)^2 / (\text{vector}1 - \text{c}6^2)^2\ ] \end{aligned}$$

$$
\begin{aligned}
M_{45} = 1/\ [&((\text{vector } 4\text{-c}5)^2 / (\text{vector}4 - \text{c}1)^2)^2 + \\
&((\text{vector } 4\text{-c}5)^2 / (\text{vector}4 - \text{c}2^2)^2 + \\
&((\text{vector } 4\text{-c}5)^2 / (\text{vector}4 - \text{c}3^2)^2 + \\
&((\text{vector } 4\text{-c}5)^2 / (\text{vector}4 - \text{c}4^2)^2 + \\
&((\text{vector } 4\text{-c}5)^2 / (\text{vector}4 - \text{c}5^2)^2 + \\
&((\text{vector } 4\text{-c}5)^2 / (\text{vector}4 - \text{c}6^2)^2\ ]
\end{aligned}
$$

Similarly other membership values are computed.

Step 4: Compute the sum of the squared difference between the previous membership value and the current membership value. If the computed value is not less than the threshold value go to step 2 to compute the next set of centroids and followed by next set of membership values. If the threshold value is less than the threshold value, stop the iteration. Thus the centroids are obtained using fuzzy k-means algorithm. Using the computed centroids, clustering can be obtained as described in the section 3.

4.2 Example

The particular sets of marks (data) are subjected to Fuzzy k-means algorithm. Final clusters along with clusters obtained at every iteration are displayed below.

Figure 2-4. Illustration of Fuzzy K-means algorithm

Figure 2-5. Data and the final centroids obtained using Fuzzy k-means algorithm

In the above mentioned example data ranges from 0.0099 to 0.9883. The final centroids obtained after 10 iteration using fuzzy k-means algorithm is displayed below 0.0313 0.2188 0.3927 0.5317 0.6977 0.8850 (see figure 2-5). Also the graph between the sum of the squared difference between the previous membership value and the current membership value (vs.) Iteration number is displayed below.

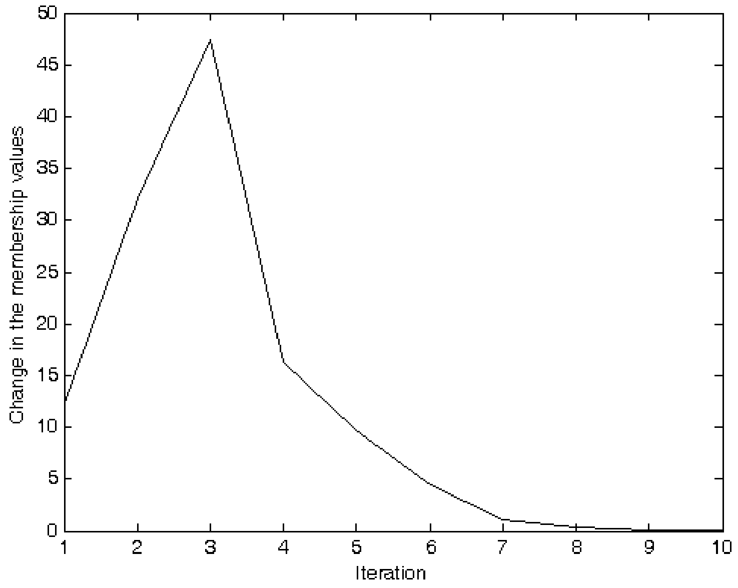


Figure 2-6. Illustration of change in the membership value in every iteration

4.3 Matlab Program for the Fuzzy k-means Algorithm Applied for the Example given in the Section 4-2

fuzzykmeansgv.m

```
%Fuzzy k-means algorithm
a=rand(1,100);
s=[];
fm=[];
for i=1:1:100
   r=rand(1,6);
   r=r/sum(r);
fm=[fm;r];
end
%Computation of centroids using fuzzy membership

for l=1:1:10
for i=1:1:6
   cen{i}=sum((fm(:,i).^2).*a')/sum(fm(:,i).^2);
end
%Updating fuzzy membership value (fm)
fm1=[];
for p=1:1:6
   for q=1:1:100
      temp=0;
      for r=1:1:6
      temp =temp+(((a(q)-cen{p})^2)/((a(q)-(cen{r}))^2))^2;
      end
      fm1(q,p)=1/temp;
   end
end
s=[s sum(sum((fm-fm1).^2))];
fm=fm1;
b=cell2mat(cen);
M=[(a-b(1)).^2;(a-b(2)).^2;(a-b(3)).^2;(a-b(4)).^2;(a-b(5)).^2;(a-b(6)).^2;];
[P,CLUSTERNO]=min(M);
u=['r','g','b','c','m','y'];
figure
for i=1:1:6
[x,y]=find(CLUSTERNO==i);
col=[];
```

```
for k=1:1:length(x)
col= [col a(x(k),y(k))];
end
plot(col,zeros(1,length(col)),strcat(u(i),'*'))
hold on
end
pause(0.01)
end
figure
plot(s)
xlabel('Iteration')
ylabel('Change in the membership values
```

5. MEAN AND VARIANCE NORMALIZATION

Suppose in the speech recognition system the two speech signals corresponding to the same word are to be compared. Two signals are not recorded exactly with the same volume (i.e.) the two signals are not with the same energy. Thus before extracting features from the speech signals, there is the need to normalize the speech signal in terms of mean and variance so that two speech signals are made recorded with the same volume.

Mean and variance normalization of the signal is obtained as described below. Consider the speech signal ‘A(t)’ with mean ‘m_d’ and variance ‘v_d’ and speech signal ‘B(t)’ with mean m and variance ‘v’. The requirement is to normalize the speech signal B(t) so that the mean and variance are changed to the desired mean ‘m_d’ and desired variance ‘v_d’ respectively. The procedure for the mean and variance normalization is given below.

5.1 Algorithm

Step 1: Create the vector completely filled up desired mean ‘m_d’. Number of elements of the vector is equal to the number of samples in the speech signal.

Step 2: Each and every element of the vector is added with $+\Delta$ or $-\Delta$. If the sample value in the original vector is greater than mean ‘m’, add $+\Delta$ to the corresponding sample in the vector created. Otherwise the value is added with ‘$-\Delta$’. The value of the ‘Δ’ is computed using the sample value, mean ‘m’, variance ‘v’ of the signal ‘A (t)’ and the desired variance ‘v_d’ .

Let Δ_i be the value calculated for the i^{th} sample of the signal ‘A’ (original signal) and is computed as described below.

The value A (i) is deviated from 'm' with (A (i)-m) when the standard deviation of the signal is $v^{(1/2)}$, then the deviation of the signal from the desired mean 'm_d' with the standard deviation of the signal $v_d^{(1/2)}$ is given by

$$\Delta_i = (v_d / v)^{(1/2)} * (A(i) - m)$$

5.2 Example 1

Consider the speech signal A(t) with mean=0.0062 and variance=0.0057 which is kept as the reference signal. Consider speech signal B(t) corresponding to the same word with mean=0.0061 and variance=0.0094. The speech signal B(t) is subjected to mean and variance normalization as described above to obtain B(t) with desired mean and variance as shown in the figure below.

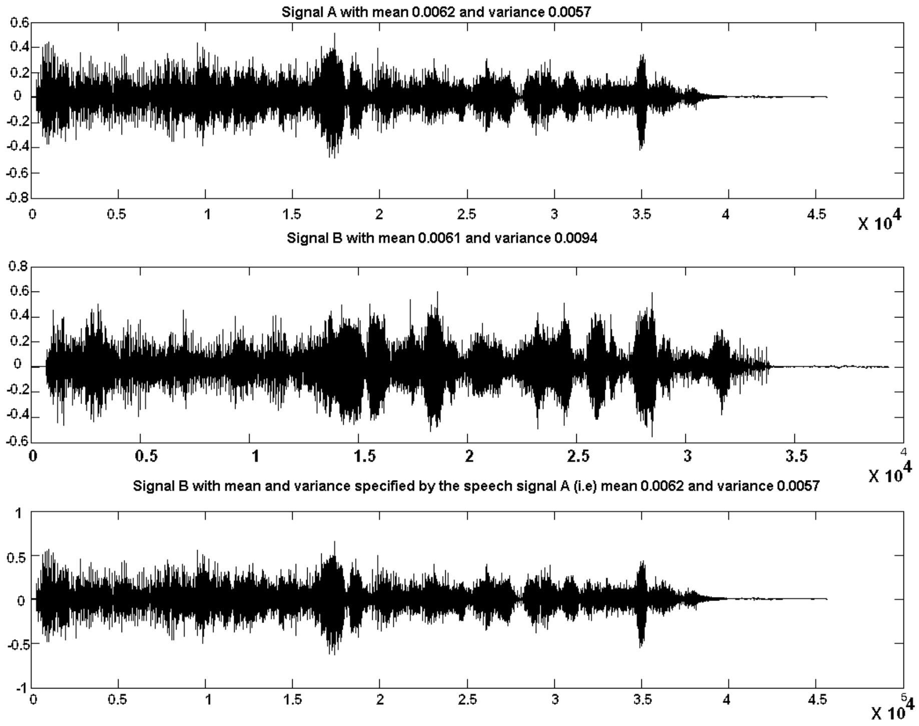


Figure 2-7. Illustration of Mean and Variance Normalization of the speech signal

Note that the signal at the top and the bottom of the figure 2-7 are almost identical. This preprocessing stage is normally used before extracting features from the signal for pattern recognition.

5.3 M-program for Mean and Variance Normalization

meanvarnormgv.m

```
%a is the speech signal to be modified
%b is the reference signal with desired mean and variance.
% res is the normalilzed version of signal ‘a’ with desired mean and variance
m=mean(a);
v=sqrt(var(a));
md=mean(b);
vd=sqrt(var(b));
delta=abs((a-m)*vd/v);
res=ones(1,length(a))*md;
[p]=quantiz(a,m);
p=p';  p=p*2-1;
res=res+p.*delta;
```

Chapter 3

NUMERICAL LINEAR ALGEBRA

Algorithm Collections

1. HOTELLING TRANSFORMATION

Consider the set of n-dimensional vectors collected in which the elements of the vector are dependent to each other (i.e.) the covariance matrix computed for the collected vectors are not the diagonal matrix. The co-variance matrix is computed as follows.

Let x_1, x_2,…x_m be the collection of 'm', 'n-dimensional vectors. The co-variance matrix is computed using $E[(X-\mu_x)\ (X-\mu_x)^T] = E(XX^T)- \mu_x\ \mu_x^T$, where μ_x is the mean vector computed using the collected vectors (ie) $\mu_x=[x_1+x_2+x_3+\ldots x_m]/m$.

Hotelling Transformation transforms the set of vectors x_1, x_2,…x_m into another set of vectors (ie) y_1, y_2,…y_m, such that the covariance matrix computed for the transformed vectors is diagonal in nature.

$$\text{Covariance Matrix (CM)} \quad \begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ b_1 & b_2 & c_2 & d_2 \\ c_1 & c_2 & c_3 & d_3 \\ d_1 & d_2 & d_3 & d_4 \end{pmatrix}$$

The element $\mathbf{a_1}$ in the above matrix gives the information about how the first elements of the collected vectors are correlated to each other. Similarly $\mathbf{c_1}$ gives the information about how the first elements are correlated with third element. Thus if the matrix is the diagonal matrix, then the elements of the vectors collected are made independent to each other.

1.1 Diagonalization of the Matrix 'CM'

1. Compute the Eigen values and the corresponding Eigen vectors of the matrix 'CM'

Eigen values are computed by solving the determinants $| CM-\lambda I| = 0$. Number of Eigen values obtained is equal to the size of the matrix. For instance the size of the above mentioned matrix is 4 and hence number of Eigen values = 4. Let it be λ_1, λ_2, λ_3 and λ_4

2. Compute the Eigen vector 'X_1'corresponding to the Eigen value 'λ_1' by solving the equation $[CM. - \lambda_1] X_1=0$.The size of the Eigen vector 'X_1' is 1X4 for the above example.

3. Similarly Eigen vectors X_2, X_3 and X_4 are computed.

4. Eigen vectors are arranged rowwise in the matrix to form Transformation Matrix (say 'TM')

5. Transformed Vector 'Y_1' is obtained using the transformation matrix as mentioned below.

Y_1= TM*(X_1- μ_x).Similarly Y_2,Y_3, …Y_m are obtained. If covariance matrix is computed for the transformed set of vectors [(i.e.) Y_1, Y_2, Y_3 … Y_m] the covariance matrix is almost diagonal.

1.2 Example

Let us consider the Binary image of size 50x50. Let us collect the positions of the Black pixel in the image as the two dimensional vectors. Xposition and Yposition are the elements of the vector considered. Hotelling transformation is applied as described above to get another set of vectors called as transformed positions. The Binary image with all pixels valued '1' (i.e.) white is created. Replace the pixel value of the transformed positions in

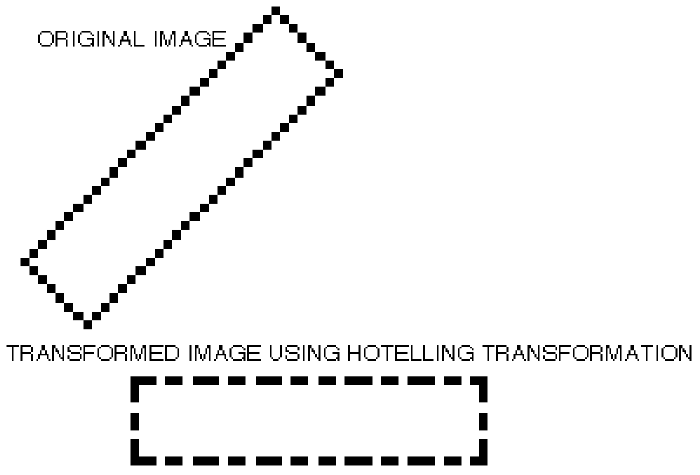


Figure 3-1. Hotelling transformation for binary image rotation

the Binary image created with '0' (i.e.) Black. The image obtained is called Transformed binary image using Hotelling transformation. Realize that the Xposition and Yposition of Black pixels in the Transformed binary image are independent to each other.

Note in the original Binary image, the Xpositions and Ypositions are dependent to each other and hence the co-variance matrix is not the diagonal matrix as mentioned below

```
CM=

  102.2500   -80.9500
  -80.9500   102.2500
```

But in the transformed Binary image, Xpositions and Ypositions are independent to each other and hence the covariance matrix computed is the diagonal matrix as given below

```
CT =

   21.3000     0.0000
    0.0000   183.2000
```

Note that the Diagonal values are Eigen values of the covariance matrix C.

Note that CM = TM’ * CT * TM

As the pixel positions of the image are represented as positive integers, are transformed vectors are properly biased and rounded off.

1.3 Matlab Program

hotellinggv.m

```
A=imread('Binimage','bmp');
A=double(A);
[x,y]=find(A==0);
vector=[x y];
Meanvector=sum(vector)/length(vector);
CM=(vector'*vector)/length(vector);
CM=CM-Meanvector'*Meanvector;
%Covariance matrix for the original set of vectors is CM
[E,V]=eig(C);
%Transformation Matrix
TM=E';
transvector=TM*(vector'-repmat(Meanvector',1,length(vector)));
xposition=fix(transvector(1,:)+abs(min(transvector(1,:)))+1);
yposition=fix(transvector(2,:)+abs(min(transvector(2,:)))+1);
B=zeros(50,50);
for i=1:1:length(xposition)
   B(xposition(i),yposition(i))=1;
end
%Covariance matrix computed for transformed vectors is computed as CT
MeanvectorT=sum(transvector')/length(transvector');
CT=(transvector*transvector')/length(transvector');
CT=CT-MeanvectorT'*MeanvectorT;
figure
subplot(2,1,1)
imshow(A)
title('ORIGINAL IMAGE')
subplot(2,1,2)
imshow(uint8(B*255))
title('TRANSFORMED IMAGE USING HOTELLING TRANSFORMATION')
```

2. EIGEN BASIS

Group of vectors are described as the vector space. All the vectors in the space are spanned by the particular set of orthonormal basis known as Eigen basis as described below. (i.e) Any vector in the space is represented as the linear combination of Eigen basis.

2.1 Example 1

The two dimensional vector are randomly generated and are plotted as given below. The vectors mentioned in the diagram are the Eigen vectors. It is computed as follows.

- Compute the co-variance matrix of the generated two dimensional vectors
- Compute the Eigen values and the corresponding Eigen vectors.
- Direction of the Eigen vectors computed are parallel to the vectors mentioned in the diagram. Note that they are orthonormal to each other.

In the above described example, Eigen vectors are E_1= [-0.2459 -0.9693] and E_2 =[-0.9693 0.2459].

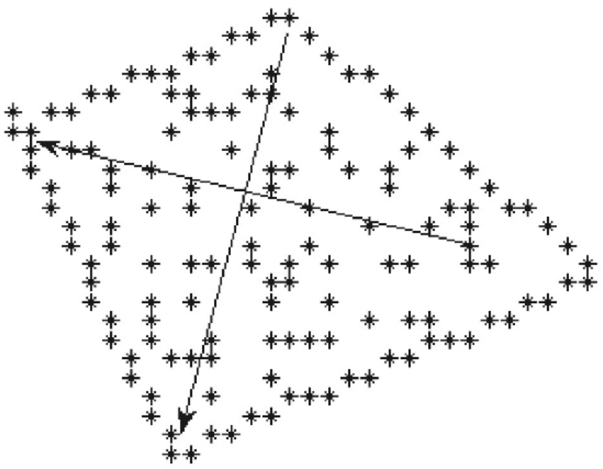


Figure 3-2. Vector space along with Eigen vectors

Also, any vector in the generated vector space is represented as the linear combination of E1 and E2. The co-efficients are obtained as the inner product of vector and the corresponding Eigen vector.

For instance, the vector V_1=[20 16] is represented as

$C_1E_1+C_2E_2$

C_1 is obtained as the inner product of E_1 and V_1
C_2 is obtained as the inner product of E_2 and V_2

C_1 = -20.4269
C_2 = -15.4513

Thus $V_1 = C_1E_1+C_2E_2$
= -20.4269 *[-0.2459 -0.9693]+ -15.4513 *[-0.9693 0.2459].

Eigen vectors corresponding to the insignificant Eigen values are contributing less to the vector representation. In such situation number of orthornormal Eigen basis to represent the particular vector is reduced.

Note that the vectors mentioned in the figure 3-3 is obtained by applying KLT transformation to the vector space mentioned in the Fig 3-2. The Eigen vectors for this space are E_1=[1 0] and E_2=[0 1]

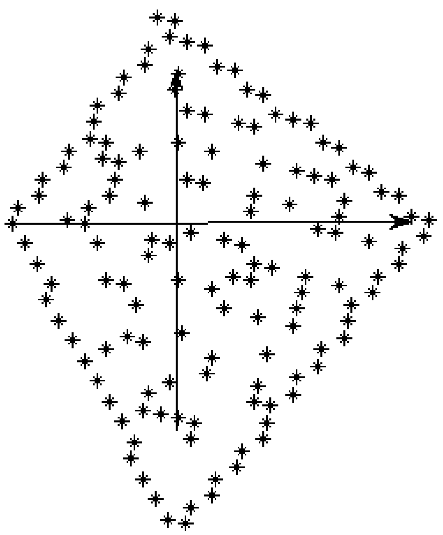


Figure 3-3. Vector space after KLT and the corresponding eigen vectors

3. SINGULAR VALUE DECOMPOSITION (SVD)

As discussed earlier any real square matrix 'A' can be diagonalized using eigen matrix 'E' (every column vector is the eigen vector of the matrix 'A')

$A=E\ D\ E^T$

where 'D' is the diagonal matrix filled with Eigen values in the diagonal

Suppose if the matrix 'A' is the rectangular matrix of size mxn, then the matrix 'A' can be represented as the follows

$A=U \wedge V^T$. This is called as Singular Value Decomposition.

AA^T is the square matrix with size mxm. Using Eigen decomposition
$AA^T = U\ D_1\ U^T$

Similarly A^TA of size nxn can be reprersented using Eigen decomposition as

$A^TA= V\ D_2\ V^T$

If $A = U \wedge V^T$
$A^T=V \wedge^T U^T$

Note that $\wedge$ and $\wedge^T$ are the same as the matrix is the diagonal matrix.

$AA^T = U \wedge V^T V \wedge^T U^T$

$= U \wedge\wedge^T U^T$ [Expected Result]

Similarly

$A^TA = V \wedge^T U^T U \wedge V^T$

$= V \wedge^T\wedge V^T$ [Expected Result]

The diagonal elements of the matrix $\wedge$ is obtained as the positive square root of the diagonal elements of the matrix D_1 or D_2. Note that the diagonal elements of the matrix D_1 and D_2 are same except the addition or deletion of zeros.

Thus the matrix A is represented as the product of the Eigen matrix obtained from the matrix AA^T, Eigen matrix obtained from the matrix A^TA

and the Diagonal matrix ^ obtained as the positive square root of the elements of the eigen values computed for the matrix AA^T (or) A^TA

(i.e.) The Eigen values are same for both the matrices.

3.1 Example

The matrix A is applied to SVD as follows.

```
A =

   1   2   3
   4   5   6

U =

  -0.3863  -0.9224
  -0.9224   0.3863

V =

  -0.4287   0.8060   0.4082
  -0.5663   0.1124  -0.8165
  -0.7039  -0.5812   0.4082

^ =
   9.5080   0         0
   0        0.7729    0

U*^*V' =

   1.0000   2.0000   3.0000
   4.0000   5.0000   6.0000
```

Eigen values of (A'*A) = E_1

```
 -0.0000
  0.5973
 90.4027
```

Eigen values of (A*A’) = E_2

```
  0.5973
 90.4027
```

Note that the diagonal elements of the ^ is obtained as the square root of E_1 or E_2 .

4. PROJECTION MATRIX

4.1 Projection of the Vector ‘a’ on the Vector ‘b’

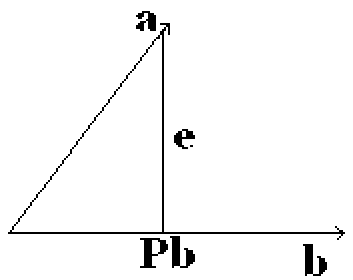


Figure 3-4. Projection Illustration

Consider two vectors a and b. The projection of the vector ‘a’ on ‘b’ is the scaled version of the vector ‘b’ (i.e) Pb. The vector e is orthogonal to the vector b as shown in the figure 3-3.

$e=a-Pb$

$\Rightarrow b^T (a-Pb) =0$

$\Rightarrow b^Ta=Pb^Tb$

$\Rightarrow P = (b^Ta) / (b^Tb)$

Therefore the projection of the vector ‘a’ on ‘b’ is given as $bP = [b(b^T a)] / [(b^Tb)]$.

The Projection matrix (In this case, ,it is the single element matrix) is given as $(bb^T)/(b^T b) =$ **[P$_M$]**

Thus the projection of the vector ‘a’ on ‘b’ is given as **[P$_M$] a**

4.2 Projection of the Vector on the Plane Described by Two Column Vectors of the Matrix ‘X’

Figure 3-5. Projection of the vector on the plane

The Linear combination of certain number of n-dimensional independent vectors forms the vector space. Thus vector space is defined as the space spanned by the set of independent vectors.

Consider the vector ‘a’ projected on the plane described by the vectors ‘x1’ and ‘x2’. Note that the matrix X =[x1 x2]. The vector ‘p’ can be represented as the linear combination of the vector ‘x1’ and ‘x2’ (i.e) column vectors of the matrix X as illustrated in the figure 3-4

$$p=u1*x1+u2*x2$$

$$p=[X]\begin{bmatrix} u1 \\ u2 \end{bmatrix}$$

$$p=[X][s]$$

Also the error vector [a-p] is orthogonal to x1 and x2 as illustrated in the figure.

$$\Rightarrow (a-Xs)^T x1 = 0 \quad \text{and} \quad (a-Xs)^T x2 = 0$$

Jointly represented as $[a-Xs]^T X = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow a^T X = (Xs)^T X$$

$\Rightarrow X^T a = X^T (Xs)$

$\Rightarrow s=(X^T X)^{-1} X^T a$

Therefore the projected vector p=[X][s]=$\mathbf{X(X^T X)^{-1}X^T}$a

Thus projected matrix $\mathbf{P_M}$ is defined as $X(X^T X)^{-1}X^T$ .Also the projected vector p is represented as **[$\mathbf{P_M}$] a**

In general projection of the vector 'a' on the vector space spanned by the column vectors of the matrix 'A' is given as $\mathbf{A(A^T A)^{-1}A^T}$ a

4.2.1 Example

Consider the vector space spanned by the column vectors of the matrix

$$A=\begin{pmatrix} 1 & 3 & 7 & 6 \\ 4 & 6 & 2 & 4 \\ 1 & 7 & 6 & 4 \end{pmatrix}$$

The projection of the vector a= [1 2 3] ^{T}

on the vector space spanned by the column vectors of the matrix A is given using the formula $\mathbf{A(A^T A)^{-1}A^T}$ a is given below

Projection matrix

$\mathbf{P_M = A(A^T A)^{-1}A^T}$ =

$$\begin{pmatrix} -0.2813 & -0.5625 & -0.6563 \\ -0.1875 & 1.1250 & 0.0625 \\ 0.0313 & 0.1875 & 1.5313 \end{pmatrix}$$

Projected vector is computed as $\mathbf{A(A^T A)^{-1}A^T}$ a

$$= \begin{pmatrix} -3.3750 \\ 2.2500 \\ 5.0000 \end{pmatrix}$$

4.2.2 Example 2

The column vectors A, B and C are displayed below.

$A=[10 \quad 2 \quad 6 \quad 5 \quad 9 \quad 8 \quad 5 \quad 0 \quad 8 \quad 4]^T$

$B= [6 \quad 8 \quad 9 \quad 7 \quad 2 \quad 4 \quad 9 \quad 9 \quad 4 \quad 9]^T$

$C=[43 \quad 26 \quad 44 \quad 29 \quad 32 \quad 34 \quad 35 \quad 24 \quad 35 \quad 32]^T$

The column vector C is related to the column vectors A and B as the linear combination as displayed below C = m*A+ n*B. The requirement is to find the optimal value for the scaling constant m and n.

If C is in the space spanned by the column vectors of A and B, unique m and n values can be computed easily. But if C is not in the space spanned by the column vectors of A and B, the constants 'm' and 'n' are the best estimated values such that C'=m*A+n*B is in the space spanned by the column vectors A and B.

Also the mean squared error (i.e) E {(C-C') ^{2}] is minimized. This is obtained using projection matrix as described below.

The column vector C' is the projection of the vector 'C' on the space spanned by the vectors A and B.

Representing the vectors A and B in the matrix column wise to form the matrix 'P'.

$$\text{Thus } P=\begin{pmatrix} 10 & 6 \\ 2 & 8 \\ 6 & 9 \\ 5 & 7 \\ 9 & 2 \\ 8 & 4 \\ 5 & 9 \\ 0 & 9 \\ 8 & 4 \\ 4 & 9 \end{pmatrix}$$

The projection matrix is computed as PM=P*inv $((P*P^T))*P^T$. Using the projection matrix, the C' column vector is obtained as the projection of the column vector C as C'=PM*C.

$$C' = \begin{pmatrix} 44.3140 \\ 25.6450 \\ 39.9154 \\ 32.0289 \\ 31.4915 \\ 33.4768 \\ 36.9648 \\ 22.2118 \\ 33.4768 \\ 34.0142 \end{pmatrix}$$

Thus the best estimated values of the variable 'm' and 'n' are computed as

$$\begin{pmatrix} 10 & 6 \\ 2 & 8 \end{pmatrix}^{-1} \begin{pmatrix} 44.3140 \\ 25.6450 \end{pmatrix}$$

$$= \begin{pmatrix} 2.9506 \\ 2.4680 \end{pmatrix}$$

MSE is computed as $E[(C-C')^2] = 4.1678$. Note that it is not possible to find the alternate values for the variables 'm' and 'n' which gives MSE less than the 4.1678.

5. ORTHONORMAL VECTORS

The vectors a, b, c are defined as the orthogonal vectors if their inner product matrix is the diagonal matrix. (i.e) dot product of the vector a with itself is some constant, whereas the dot product of the vector a with b is 0.

If the diagonal matrix obtained is the identity matrix, then the vectors are called as orthonormal vectors.

5.1 Gram-Schmidt Orthogonalization Procedure

Consider the set of independent vectors v1, v2 and v3 which spans the particular vector space 'S'. (i.e) All the vectors in the space 'S' are represented as the linear combination of the independent vectors (v2, v2, v3). They are called basis.

It is possible to identify the another set of independent vectors o1, o2 and o3 which spans the space S such that the vectors o1, o2 and o3 are orthonormal to each other. The steps involved in obtaining the orthornormal vectors corresponding to the independent vectors are described below.

Let 'v1' and 'v2' be the independent column vectors. The vector 'v1' is treated as the reference vector. (i.e) Normalized 'v1' is treated as the one of the orthonormal vector 'o1'= 'v1/||v1||'. The orthogonal vector 'p' is computed using the projection of the vector 'v2' on the vector 'v1'.

From the figure 3-5 $p=v2- [(o1^T v2) / (o1^T o1)] o1$. The orthonormal vector is the normalized form of the vector 'p'.

$o2 = p / ||p||$

Similarly the orthonormal vector 'o3' corresponding to the vector 'v3' is obtained as follows.

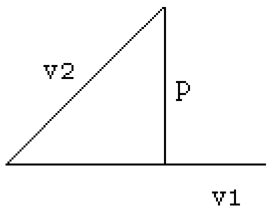


Figure 3-6. Gram-Schmidt Orthogonalization procedure

$$q = v3 - [(o1^T v3) / (o1^T o1)]\, o1] - [(o2^T v3) / (o2^T o2)]\, o2]$$

$$o3 = q / \|q\|$$

5.2 Example

Consider the set of independent column vectors v1, v2 and v3 as displayed below.

$$v1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad v2 = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad v3 = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$$

The corresponding set of orthonormal vectors computed using Gram-Schmidt orthogonalization procedure is given below.

$$o1 = \begin{pmatrix} 0.2673 \\ 0.5345 \\ 0.8018 \end{pmatrix} \quad o2 = \begin{pmatrix} 0.5203 \\ -0.7804 \\ 0.3468 \end{pmatrix} \quad o3 = \begin{pmatrix} 0.8111 \\ 0.3244 \\ -0.4867 \end{pmatrix}$$

The dot product of the vectors o1, o2 and o3 is displayed in the matrix form for illustration.

```
 1.0000  -0.0000   0.0000
-0.0000   1.0000   0.0000
 0.0000   0.0000   1.0000
```

Note that the matrix obtained is the identity matrix as expected.

5.3 Need for Orthonormal Basis

Suppose the vector ‘V’ in the vector space ‘S’ is represented as the linear combination of the orthonormal basis computed in the section 5.1

$$V = \begin{pmatrix} 9 \\ 6 \\ 14 \end{pmatrix}$$

The co-efficients are obtained as the inner product of the vector ‘V’ and the corresponding orthonormal basis.

The co-efficients are

$c1=V^T*o1=16.8375$
$c2=V^T*o2=\ 4.8558$
$c3=V^T*o3=\ 2.4333$

Thus the vector V is represented as

$V = c1*o1+c2*o2+c3*o3$

Computing the co-efficients become easier if the basis are orthonormal to each other and hence orthonormal basis is required.

If adding few more independent vectors increases the number of independent vectors spanning the space, the space spanned by these vectors also increases. The corresponding orthonormal vectors obtained using Gram-Schmidt orthogonalization procedure shows that the orthonormal basis are the same as that of the earlier except the inclusion of additional few orthogonal basis corresponding to the additional independent vectors. Hence the co-efficients of the already existing basis remains the same.

In the previous example if the vector chosen in the space spanned by the vectors v1 and v2.

$$\begin{pmatrix} 6 \\ 6 \\ 14 \end{pmatrix} \text{(say)}$$

The vector can be represented as the linear combination of 'o1' and 'o2' with the corresponding co-efficients 16.0357 and 3.2950 respectively. They are computed using inner product technique. Suppose the same vector is represented as the linear combination of 'o1' 'o2' and 'o3', the corresponding co-efficients are obtained as 16.0357, 3.2950 and 4.4409e-015 respectively. Note that first two co-efficients remains unchanged and also note that the value of the third co-efficient is almost insignificant for representing the vector. This property is used in compression techniques in which the significant co-efficients are stored rather than the vector itself.

5.4 M-file for Gram-Schmidt Orthogonalization Procedure

gramgv.m

```
%Given the independent vectors as the input,Orthonormal vectors as the
%output computed using Gram-Schmidt Orthogonalization procedure
%Input vectors are arranged rowwise

function [res]=gramgv(x)

o{1}=x(1,:)/sqrt(sum(x(1,:).^2));
for k=2:1:size(x,1)
   s=x(k,:);
   for  m=1:1:k-1
         s=s - ((o{m}*x(k,:)')/(o{m}*o{m}'))*o{m} ;
   end
   o{k}=s/sqrt(sum(s.^2));
end
res=o;
```

6. COMPUTATION OF THE POWERS OF THE MATRIX 'A'

Consider the matrix A = $\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$. The matrix A^{100} is computed as described below.

The matrix 'A' can be diagonalized using eigen matrix 'E' (every column vector is the eigen vector of the matrix 'A') as described in the section 3

$A=E\,D\,E^{-1}$

Where 'D' is the diagonal matrix filled with Eigen values in the diagonal.

Consider the computation of the matrix A^2 $=A\,A=EDE^{-1}\,EDE^{T}=EDDE^{T}$ $=ED^2E^T$

In general A^{100} is using $ED^{100}E^T$

For the given matrix 'A', eigen matrix 'E' is computed as

$E=\begin{pmatrix} 0.9315 & -0.8421 \\ 0.3637 & 0.5393 \end{pmatrix}$ and the diagonal matrix D is given as follows

$$D=\begin{pmatrix} 4.5616 & 0 \\ 0 & 0.4384 \end{pmatrix}$$

Therefore A^{100} is computed as $ED^{100}E^{-1}$ =

$$\begin{pmatrix} 5.06471.0e+065 & 7.90871.0e+065 \\ 1.97721.0e+065 & 3.08751.0e+065 \end{pmatrix}$$

Note that if all the eigen values in the Diagonal matrix described above is less than 1, the matrix A^n converges to the value zero when n tends to infinity.

7. DETERMINATION OF KTH ELEMENT IN THE SEQUENCE

Let us consider the sequence 0, 1, 2, 3, 6, 11, 20, 37… in which fourth element is the summation of three previous numbers. Fifth value is the summation of 2, 3 and 4 value of the sequence and so on.

The problem is to obtain the value of the 100th element of the above mentioned sequence. This is achieved using Eigen decomposition as described below.

Let P_k be the value of the k^{th} position of the sequence.

$P_k = P_{k-1} + P_{k-2} + P_{k-3}$

Let us define the matrix $U_k = \begin{pmatrix} P_{k+2} \\ P_{k+1} \\ P_k \end{pmatrix}$

$$U_{k+1} = \begin{pmatrix} P_{k+3} \\ P_{k+2} \\ P_{k+1} \end{pmatrix}$$

$$U_{k+1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} U_K$$

$$U_0 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

Let the matrix A be $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Represent the vector U_0 as the linear combinations of Eigen vectors of the matrix A.

The Eigen values and the corresponding Eigen vectors are computed and displayed below.

Eigen values = $[\lambda_1\ \lambda_2\ \lambda_3]$
= [1.8393 -0.4196 + 0.6063i -0.4196 - 0.6063i]

Eigen vectors arranged in columnwise =[E1 E2 E3] =

$$\begin{pmatrix} -0.8503 & -0.1412 - 0.3752i & -0.1412 + 0.3752i \\ -0.4623 & -0.3094 + 0.4471i & -0.3094 - 0.4471i \\ -0.2514 & 0.7374 & 0.7374 \end{pmatrix}$$

Represent the matrix U_o as the linear combinations of Eigen vectors.

$$\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} =$$

$$\begin{pmatrix} -0.8503 & -0.1412 - 0.3752i & -0.1412 + 0.3752i \\ -0.4623 & -0.3094 + 0.4471i & -0.3094 - 0.4471i \\ -0.2514 & 0.7374 & 0.7374 \end{pmatrix} \begin{pmatrix} c1 \\ c2 \\ c3 \end{pmatrix}$$

$$\begin{pmatrix} c1 \\ c2 \\ c3 \end{pmatrix} = \begin{pmatrix} -2.0649 + 0.0000i \\ -0.3520 + 0.1929i \\ -0.3520 - 0.1929i \end{pmatrix}$$

$U_1 = A\ U_o$
$U_2 = A^2\ U_o$
...
$U_k = A^k U_o$

$U_o = c1*E1+c2*E2+c3*E3$

$AU_o = c1*A*E1+c2*A*E2+c3*A*E3$

$= c1*\lambda_1*E1+c2*\lambda_2*E2+c3*\lambda_3*E3$

Similarly

$U_k = A^k U_o = c1*\lambda_1{}^k*E1+c2*\lambda_2{}^k*E2+c3*\lambda_3{}^k*E3$

Therefore U_{100} is computed as

$U_{100} =$ 1.0e+026

$$\begin{pmatrix} 5.1220 - 0.0000i \\ 2.7848 - 0.0000i \\ 1.5140 - 0.0000i \end{pmatrix}$$

Therefore the 100th term in the sequence is given as 1.51401.0e+026
Let us compute 6th term using the method given above

```
U6=68.0000 - 0.0000i
   37.0000 - 0.0000i
   20.0000 - 0.0000i
```

Therefore 6th term is 20 as expected (see the sequence).

8. COMPUTATION OF EXPONENTIAL OF THE MATRIX

Let A be the matrix, which can be diagonalizable using Eigen matrix as $A=EDE^{-1}$. e^A can be computed using series expansion as given below.

$$e^A = I + A + (A^2)/2! + (A^3)/3! + \dots..$$

$$= I + EDE^{-1} + (EDE^{-1}\, EDE^{-1})/2! + (EDE^{-1}\, EDE^{-1}\, EDE^{-1})/3! + \dots$$

$$= ED^0E^{-1} + EDE^{-1} + (ED^2E^{-1})/2! + (ED^3E^{-1})/3! + \dots$$

$$= E[D^0 + (D/1!) + (D^2/2!) + (D^3/3!) + (D^4/4!) + \dots]\, E^{-1}$$

$$= E\, e^D\, E^{-1}$$

8.1 Example

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = EDE^{-1}$$

$$\begin{pmatrix} 0.5257 & -0.8507 \\ -0.8507 & -0.5257 \end{pmatrix} \begin{pmatrix} -0.6180 & 0 \\ 0 & 1.6180 \end{pmatrix} \begin{pmatrix} 0.5257 & -0.8507 \\ -0.8507 & -0.5257 \end{pmatrix}$$

Therefore e^A is computed as

$$E\, e^D\, E^{-1} =$$

$$\begin{pmatrix} 0.5257 & -0.8507 \\ -0.8507 & -0.5257 \end{pmatrix} \begin{pmatrix} e^{-0.6180} & 0 \\ 0 & e^{1.6180} \end{pmatrix} \begin{pmatrix} 0.5257 & -0.8507 \\ -0.8507 & -0.5257 \end{pmatrix}$$

$$= \begin{pmatrix} 3.7982 & 2.0143 \\ 2.0143 & 1.7839 \end{pmatrix}$$

9. SOLVING DIFFERENTIAL EQUATION USING EIGEN DECOMPOSITION

Consider the problem of solving the third order differential equation as given below.

$\partial^3 u / \partial t^3 + 2\, \partial^2 u / \partial t^2 - \partial u / \partial t + u = 0$ with the initial condition given $U''(0)=3$, $U'(0)=1$, $U(0)=-1$.

Representing the above equation in the form of matrix representation.

$$V = \begin{pmatrix} U'' \\ U' \\ U \end{pmatrix}$$

$$V' = \begin{pmatrix} U''' \\ U'' \\ U' \end{pmatrix}$$

$$V' = \begin{pmatrix} -2 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} V$$

$$V = c1\, e^{\lambda 1\, t}\, E_1 + c2\, e^{\lambda 2\, t}\, E_2 + c3\, e^{\lambda 3\, t}\, E_3$$

Where c1, c2 and c3 are the constants obtained using initial conditions. E1, E2 and E3 are the eigen vectors of the matrix A corresponding to the eigen values $\lambda_1, \lambda_2, \lambda_3$ respectively.

The Eigen vectors and the eigen values of the matrix A are displayed below.

$\lambda_1 = -2.2470$ $\lambda_2 = 0.8019$ $\lambda_3 = -0.5550$

$E1 = [-0.8990 \quad 0.4001 \quad -0.1781]^T$

E2= $[-0.4484 \quad -0.5592 \quad -0.6973]^T$

E3= $[0.2600 \quad -0.4686 \quad 0.8443]^T$

Thus $V=\begin{pmatrix} U'' \\ U' \\ U \end{pmatrix}$ is obtained as follows

$U''= c1\, e^{-2.2470\, t} (-0.8990) + c2\, e^{0.8019\, t} (-0.4484) + c3\, e^{-0.5550\, t} (0.2600)$

$U'= c1\, e^{-2.2470\, t} (0.4001) + c2\, e^{0.8019\, t} (-0.5592) + c3\, e^{-0.5550\, t} (-0.4686)$

$U= c1\, e^{-2.2470\, t} (-0.1781) + c2\, e^{0.8019\, t} (-0.6973) + c3\, e^{-0.5550\, t} (0.8443)$

The values of the constants can be solved using the initial conditions using the set of equations given below.

$U''(0) =3 =(-0.8990)\, c1 + (-0.4484)\, c2 + (0.2600)\, c3$

$U'(0) =1 = (0.4001)\, c1 + (-0.5592)\, c2 + (-0.4686)\, c3$

$U(0) = -1 = (-0.1781)c1 + (-0.6973)c2 + (0.8443)\, c3$

Thus c1= -3.4815, c2= -1.5790, c3= -3.2227

10. COMPUTATION OF PSEUDO INVERSE OF THE MATRIX

Let us consider the matrix $A = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

Decompose the matrix 'A' using Singular Value Decomposition.

A= [P][Q][R^T]

$$A= \begin{pmatrix} -0.6057 & -0.7957 \\ -0.7957 & 0.6057 \end{pmatrix} \begin{pmatrix} 8.8839 & 0 & 0 \\ 0 & 0.2757 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -0.4051 & -0.5628 & -0.7205 \\ 0.8181 & 0.1288 & -0.5605 \\ 0.4082 & -0.8165 & 0.4082 \end{pmatrix}$$

Pseudo inverse matrix A^+ is given as

$A^+ = \{[R]\ [Q+][P^T]\}$

$$[Q^+] = \begin{pmatrix} 1/\ 8.8839 & 0 \\ 0 & 1/\ 0.2757 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.1126 & 0 \\ 0 & 3.6268 \\ 0 & 0 \end{pmatrix}$$

$$QQ^+ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^+ = \begin{pmatrix} -2.3333 & 1.8333 \\ -0.3333 & 0.3333 \\ 1.6667 & -1.6667 \end{pmatrix}$$

$$AA^+ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

11. COMPUTATION OF TRANSFORMATION MATRICES

If the vectors in the space1 are spanned by the independent vectors u1, u2, u3, … un. Also if the vectors in the space2 are spanned by the independent vectors v1, v2 …vn . They are called basis of the respective spaces.

Suppose the vector ‘u’ in the space 1 is mapped to the vector v in the space 2.

(i.e.) T (u) = v. The vector ‘v’ can be represented as the linear combinations of the independent vectors v1, v2, …vn.

Similarly the transformation of the basis vector u1 in the space1 can be represented as the linear combinations of v1, v2, v3…vn.

Therefore T (u1) =a_{11} **v1+** $\mathbf{a_{12}}$v2+ a_{13} **v3+** $\mathbf{a_{14}}$v4 + … a_{1n} vn

Similarly T (u2) =a_{21} **v1+** $\mathbf{a_{22}}$v2+ a_{23} **v3+** $\mathbf{a_{24}}$v4 + … a_{2n} vn

…

T (un) =an_1 **v1+** $\mathbf{a_{n2}}$v2+ a_{n3} **v3+** $\mathbf{a_{n4}}$v4 + … a_{nn} vn

The co-efficients of the vector v1, v2, v3 .. vn in the above mentioned equation are arranged in the vector form and are called co-efficient vector as given below.

The vector [a_{11} $\mathbf{a_{12}}$ a_{13} $\mathbf{a_{14}}$ ... a_{1n}] is the co-efficient vector associated with the vector T(u1) which is in the space 2. Similarly the vector [a_{21} $\mathbf{a_{22}}$ a_{23} $\mathbf{a_{24}}$ ... a_{2n}] is the co-efficient vector associated with the vector T(u2).

Consider the vector u1 in the space 1 which can also be represented as the linear combinations of the basis vectors u1, u2, u3, …un as given below.

u1=1*u1+0*u2+0*u3+….0*un.

In this case [1 0 0 0 …0] is the co-efficient vector associated with the vector u1 in the space1.

As mentioned earlier the any vector ‘u’ in the space 1 and the corresponding mapped vector v in the space 2 can also have their associated coefficient vectors. Let [p1, p2, p3, …pn] be the co-efficient vector associated with the vector ‘u’ and [q1, q2, q3…qn] be the co-efficient vector associated with the vector ‘v’.

The matrix relating the co-efficient vector of the particular vector in the space 1 and the co-efficient vector of the corresponding mapped vector in the space 2 is called transformation matrix and is computed as described below.

(i.e.)

$[p1\ p2\ p3\ p4\ \ldots pn]^T\ \mathbf{[TM]} = [q1\ q2\ q3\ q4\ \ldots\ qn]^T$

The transformation matrix is obtained using the co-efficient vectors computed for the T(u1), T(u2),…T(un) in the space 2 where u1,u2 u3,..un are the independent vectors which spans the space 1.

$$\text{Thus TM} = \begin{pmatrix} a11 & a21 & a31 & a41 & a51 & \ldots an1 \\ a12 & a22 & a32 & a42 & a52 & \ldots an2 \\ a13 & a23 & a33 & a43 & a53 & \ldots an3 \\ a14 & a24 & a34 & a44 & a54 & \ldots an4 \\ \ldots & & & & & \\ a1n & a2n & a3n & a4n & a5n & \ldots ann \end{pmatrix}$$

For example the vector u1 with co-efficient vector [1 0 0 0 0 0…0] is mapped to the vector T(u1) whose co-efficient vector is obtained using TM as

$$\begin{pmatrix} a11 & a21 & a31 & a41 & a51 & \ldots an1 \\ a12 & a22 & a32 & a42 & a52 & \ldots an2 \\ a13 & a23 & a33 & a43 & a53 & \ldots an3 \\ a14 & a24 & a34 & a44 & a54 & \ldots an4 \\ \ldots & & & & & \\ a1n & a2n & a3n & a4n & a5n & \ldots ann \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \\ 0 \end{pmatrix} = \begin{pmatrix} a11 \\ a12 \\ a13 \\ a14 \\ \\ a1n \end{pmatrix}$$

The co-efficient vector obtained is as expected.

11.1 Computation of Transformation Matrix for the Fourier Transformation

Consider that the vector space 1 is spanned by the 4 basis as given below.

$$v1=\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad v2=\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad v3=\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad v4=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Every vector in the space spanned by the basis vector given above is mapped to the vector in vector space 2 called as Fourier space. Fourier transformation of the basis vectors v1, v2, v3, v4 are given as

$T(v1) = [1 \quad 1 \quad 1 \quad 1]^T$

$T(v2) = [1.0000 \quad 0 - 1.0000i \quad -1.0000 \quad 0 + 1.0000i]^T$

$T(v3) = [1 \quad -1 \quad 1 \quad -1]^T$

$T(v4) = [1.0000 \quad 0 + 1.0000i \quad -1.0000 \quad 0 - 1.0000i]^T$

The basis vectors of the Fourier space is given as

$f1 = (1/2)* [1 \quad 1 \quad 1 \quad 1]^T$

$f2 = (1/2) * [1.0000 \quad 0 - 1.0000i \quad -1.0000 \quad 0 + 1.0000i]^T$

$f3 = (1/2) * [1 \quad -1 \quad 1 \quad -1]^T$

$f4 = (1/2)* [1.0000 \quad 0 + 1.0000i \quad -1.0000 \quad 0 - 1.0000i]^T$

The transformed vector T(v1) is represented as the linear combination of f1, f2, f3, f4. Note that the basis are orthonormal basis

The co-efficient vector is obtained as the inner product of the vector T(v1) with the corresponding basis.

$[T(v1)]^T f1^* = [2]$ [Note that the inner product complex numbers ‘a’ and ‘b’ is computed as $a^T b^*$]

$[T(v1)]^T f2^* = [0]$

$[T(v1)]^T f1^* = [\ 0]$

$[T(v1)]^T f1^* = [0]$

Therefore the co-efficient vector associated with the vector T(v1) is given as $[2\ 0\ 0\ 0]^T$

Similarly the co-efficient vector associated with the vector T(v2), T(v3) and T(v4) is given as $[0\ 2\ 0\ 0]^T$, $[0\ 0\ 2\ 0]^T$ and $[0\ 0\ 0\ 2]^T$ respectively.

Thus the transformation matrix for the Fourier transform is given as

$$\begin{pmatrix} 2\,0\,0\,0 \\ 0\,2\,0\,0 \\ 0\,0\,2\,0 \\ 0\,0\,0\,2 \end{pmatrix}$$

As the transformation matrix is the diagonal matrix with ‘2’ as the diagonal elements, the co-efficient vector for any vector in the Vector space 1 is ‘coef’ then the co-efficient vector for the corresponding mapped vector in the Vector space 2 (Fourier domain) is given be 2*c.

Thus the Fourier transformation of the vector [2 3 4 1] is given as the

$$(1/2)\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix} \begin{pmatrix} 2*2 \\ 3*2 \\ 4*2 \\ 1*2 \end{pmatrix} = \begin{pmatrix} 10 \\ -2-i \\ 2 \\ -2+2i \end{pmatrix}$$

In the same fashion, DCT, DST the transformation matrix is the diagonal matrix and hence the transformation values can be easily obtained using simple matrix multiplication as described above.

11.2 Basis Co-efficient Transformation

Consider the 4-dimensional vector space which are spanned by the basis u1=$[1\ 0\ 0\ 0]^T$, u2=$[0\ 1\ 0\ 0]^T$ and u3=$[0\ 0\ 1\ 0]^T$ and u4=$[0\ 0\ 0\ 1]^T$. Consider another set of orthonormal basis which spans the space.

v1 = (1/2)* $[1 \quad 1 \quad 1 \quad 1\,]^T$

v2 = (1/2) * $[1.0000 \quad 0 - 1.0000i \quad -1.0000 \quad 0 + 1.0000i\,]^T$

v3 = (1/2) * $[1 \quad -1 \quad 1 \quad -1]^T$

v4 = (1/2)* $[1.0000 \quad 0 + 1.0000i \quad -1.0000 \quad 0 - 1.0000i\,]^T$

Any vector in the space can be represented as the linear combination of u1, u2, u3 and u4. The same vector in the space can be represented as the linear combination of v1, v2, v3 and v4.

Consider the transformation T which transforms the vector v is represented as T(v) and is equal to v.

v is represented as the linear combination of u1 u2 u3 and u4.Let the co-efficient vector be [p1 p2 p3 p4]

T(v)=v is represented as the linear combination of v1,v2,v3,v4.Let the co-efficient vector be [q1 q2 q3 q4].

The transformation matrix which converts the co-efficient vector [p1 p2 p3 p4] into the co-efficient vector [q1 q2 q3 q4] is transformation matrix for the change of basis. It is obtained as described below.

T([1 0 0 0])=[1 0 0 0] is represented as 0.5*v1+0.5*v2+0.5*v3+0.5*v4

T([0 1 0 0])=[0 1 0 0] is represented as
0.5*v1 + (0.5 i)*v2+ (-0.5)*v3+ (-0.5 i)*v4

T([0 0 1 0])=[0 0 1 0] is represented as
0.5*v1 + (- 0.5)*v2+ (0.5)*v3+ (- 0.5) *v4

T([0 0 0 1])=[0 0 0 1] is represented as
0.5*v1 + (- 0.5 i)*v2+ (- 0.5)*v3+ (0.5 i) *v4

There the transformation matrix which converts the co-efficients of the particular basis into the co-efficients of the another set of basis belongs to the same space is given as

$$\begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5i & -0.5 & -0.5i \\ 0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & -0.5i & -0.5 & 0.5i \end{pmatrix}$$

For instant consider the vector [2 3 4 5] can be represented using the basis u1, u2, u3, u4 with co-efficients [2 3 4 5]. The vector [2 3 4 5] can be represented using the basis v1, v2, v3, v4 using the co-efficients computed as

$$\begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5i & -0.5 & -0.5i \\ 0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & -0.5i & -0.5 & 0.5i \end{pmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{pmatrix} 7 \\ -1-i \\ -1 \\ -1+i \end{pmatrix}$$

(i.e.) 7*(v1)+(-1-I)*v2+(-1)*v3+(-1+i)*v4 = [2 3 4 5]

Thus the transformation matrix which converts the co-efficients of the particular vector represented using the basis 'u' into the co-efficients of the same vector represented using the basis 'v'.

If there are only few significant co-efficients obtained when represented using the particular set of basis, data compression is achieved. (See chapter 4). This property is called data compaction property of the transformation. Discrete Cosine Transformation (DCT) is having very high data compaction property. Hence JPEG (Joint Photographic expert group) is using DCT for image compression. Also JPEG2000 standard is using DWT (Discrete Wavelet Transformation) for image compression.

11.3 Transformation Matrix for Obtaining Co-efficient of Eigen Basis

Consider the group of two dimensional vectors collected randomly forms the vector space. This is consider as the subspace spanned by the independent vectors u1 = $[1\ 0]^T$ and u2=$[0\ 1]^T$. The subspace is spanned by the Eigen basis E1=$[-0.2459\ \ -0.9693]^T$ and E_2 =$[-0.9693\ \ 0.2459]^T$ (see section 2-1).

The vector [1 0] is represented as the linear combination u1 and u2 as [1 0] = 1*u1+0*u2.The transformed vector of [1 0] is the same vectors itself (i.e.)[1 0].The vector [1 0] is represented as the linear combinations of Eigen basis given by -0.2459*E1+(-0.9693)*E2 =[1 0]. Similarly the vector [0 1] is represented as the linear combination of u1 and u2 as $[0\ 1]^T$ = 0*u1+1*u2 and its transformed vector [0 1] is represented as the linear combination of E1 and E2 as given by -0.9693*E1 +0.2459*E2.

Thus the transformation matrix which converts the co-efficients of the basis u1 and u2 into the co-efficients of the Eigen basis E1 and E2 for the particular vector in the space.

$$TM = \begin{pmatrix} -0.2459 & -0.9693 \\ -0.9693 & -0.2459 \end{pmatrix}$$

Note that the transformation matrix consists of Eigen basis arranged row wise.

11.4 Transformation Matrix for Obtaining Co-efficient of Wavelet Basis

Consider the 8-dimensional space spanned by the 8 independent vectors

u1= $[1\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$ u2= $[0\ 1\ 0\ 0\ 0\ 0\ 0\ 0]^T$ u3= $[0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T$
u4= $[0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]^T$ u5= $[0\ 0\ 0\ 0\ 1\ 0\ 0\ 0]^T$ u6= $[0\ 0\ 0\ 0\ 0\ 1\ 0\ 0]^T$
u7= $[0\ 0\ 0\ 0\ 0\ 0\ 1\ 0]^T$ u8= $[0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$

Consider any vector in the space can also be represented as the linear combination of Wavelet basis as described below. Note that orthonormal basis

w1= (1/sqrt(8)) $[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]^T$
w2= (1/sqrt(8)) $[1\ 1\ 1\ 1\ -1\ -1\ -1\ -1]^T$
w3= (1/sqrt(4)) $[1\ 1\ -1\ -1\ 0\ 0\ 0\ 0]^T$
w4= (1/sqrt(4)) $[0\ 0\ 0\ 0\ 1\ 1\ -1\ -1]^T$
w5= (1/sqrt(2)) $[1\ -1\ 0\ 0\ 0\ 0\ 0\ 0]^T$
w6= (1/sqrt(2)) $[0\ 0\ 1\ -1\ 0\ 0\ 0\ 0]^T$
w7= (1/sqrt(2)) $[0\ 0\ 0\ 0\ 1\ -1\ 0\ 0]^T$
w8= (1/sqrt(2)) $[0\ 0\ 0\ 0\ 0\ 0\ 1\ -1]^T$

As described in the section 11.3 the transformation matrix which converts the co-efficient of the basis u1,u2,…u8 into the co-efficient of the basis w1, w2, w3,…w8 for the particular vector is computed and is displayed below.

$$\begin{pmatrix}
0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\
0.3536 & 0.3536 & 0.3536 & 0.3536 & -0.3536 & -0.3536 & -0.3536 & -0.3536 \\
0.5000 & 0.5000 & -0.5000 & -0.5000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5000 & 0.5000 & -0.5000 & -0.5000 \\
0.7071 & -0.7071 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.7071 & 0.7071 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.7071 & 0.7071 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.7071 & 0.7071
\end{pmatrix}$$

Note that the transformation matrix consists of the wavelet basis arranged row wise.

12. SYSTEM STABILITY TEST USING EIGEN VALUES

Input signal and output signal of the system are usually related with differential equation. The output signal is solved using the eigen decomposition as described in the section 9. The general expression of the output signal is of the form output (t)=c1 $e^{\lambda 1\ t}$ E_1 +c2 $e^{\lambda 2\ t}$ E_2 + c3 $e^{\lambda 3\ t}$ E_3. where λ1, λ2 and λ3 are the eigen values. E_1, E2 and E3 are the eigen vectors of the matrix described in the section 9.

From the above equation it is observed that Output (t) is bounded only when all the eigen values are less than 0 (i.e.) negative values. This is the test for stability of the system using eigen values.

13. POSITIVE DEFINITE MATRIX TEST FOR MINIMA LOCATION OF THE FUNCTION F (X1, X2, X3, … XN)

Consider the function $2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$. To find the minima of the function f(x), partial differentiate the function with respect to x1, x2, x3 and equate to zero and solve for x1, x2, x3 to get (x_0, y_0, z_0). The slope at this point (x_0, y_0, z_0) is zero. To confirm that the points so obtained are minima, all the partial second derivative values are positive. This is tested using matrix technique as described below.

Represent the equation as the product of three matrices

$$[x_1\ x_2\ x_3] \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

If the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ is the positive definite matrix, then the

function $f(x) = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$ is the minima function and the minima occurs at the point where the slope is zero (i.e.) first derivative is zero. If all the eigen values of the matrix are positive, then the matrix is positive definite matrix. Thus sign of all the eigen values of the matrix defined above decides whether the point (x_0, y_0, z_0) at which the slope is zero is minima or not.

14. WAVELET TRANSFORMATION USING MATRIX METHOD

The 1D signal can be analyzed by using wavelet transformation. This can be used to compress the data and to denoise the signal. Wavelet transformation of the 1D signal contains the information regarding the low frequency content of the signal, which is called approximation co-efficients and the information regarding high frequency content of the signal, which is called detail co-efficients. The wavelet transformation of the signal using matrix method is described below.

14.1 Haar Transformation

Consider the signal with number of samples N. Form the Haar matrix with size NXN and with the diagonal matrices filled up with the matrix given below.

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

For N=8, the matrix obtained is

$$TM1 \quad \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Multiply the matrix TM1 with the is the signal data [d0 d1 d2 d3 d4 d5d6 d7] to obtain first level decomposition of Haar transformation.

I level Haar decomposition of the signal

[½ (d0+d1) ½ (d2-d3) ½ (d4+d5) ½ (d6-d7)]

The samples ½ (d0+d1) ½ (d4+d5) are the average samples. They are called approximated co-efficients of the signal at the first level. The samples ½ (d2-d3) ½ (d6-d7) are called detail co-efficients of the signal at the first level.

Thus Approximation 1 = [½ (d0+d1) ½(d4+d5)]

Detail 1= [½ (d2-d3) ½(d6-d7)]

The approximated co-efficients of the signal obtained in the first level (Approximation 1) is further decomposed using Haar transformation to obtain approximation and detail co-efficients of the second level using the transformation matrix as given below.

$$TM2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Multiply the matrix [TM2] with the approximated data obtained in the first level decomposition to obtain approximation and detail co-efficient of the signal at the second level.

Thus second level approximation is given as

$$\text{Approximation 2} = [\tfrac{1}{2}\,(\,\tfrac{1}{2}\,(d0+d1) + \tfrac{1}{2}\,(d4+d5)\,)]$$

$$= \tfrac{1}{4}\,[\,d0+d1+d4+d5]$$

$$\text{Detail 2} = [\tfrac{1}{2}\,(\,\tfrac{1}{2}\,(d0+d1) - \tfrac{1}{2}\,(d4+d5)\,)]$$

$$= \tfrac{1}{4}\,[d0 + d\,1 - d4 - d5]$$

Note that approximation co-efficient in the first level and second level are the low frequency information derived from the signal. Similarly the detail co-efficient in the first level and the second level are the high frequency information derived from the signal.

The Approximation 2, Detail 1 and Detail 2 completely describes the signal. It is possible to reconstruct the signal using Approximation 2, Detail2 and Detail 2 co-efficients of the signal.

Reconstruction of the signal is obtained using the inverse Haar matrix Formed with the diagonal matrices filled up with the matrix

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

For instance steps involved in reconstructing the signal for N=8 is given below.

- Form the vector with the elements filled up with approximation 2 and detail 2

  [¼ [d0+d1+d4+d5] ¼ [d0 + d 1 - d4 - d5]]

- Form the Inverse transformation matrix 1

$$[ITM1] = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

- Multiply the vector with the matrix to obtain Approximation 1 and detail 1 in the jumbled order as [½ (d0+d1) ½ (d2-d3) ½ (d4+d5) ½(d6-d7)]

- Form the inverse transformation matrix ITM2

$$[ITM2] = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

- Multiply the vector obtained in jumbled order as mentioned above with the matrix [ITM2] to obtain the original signal [d0 d1 d2 d3 d4 d5 d6 d7]

14.1.1 Example

Consider the signal x(n) =a=sin(2*pi*n)+sin(2*pi*100*n) with number of samples =512 and sampling frequency Fs =512. The Haar transformation is applied to the signal. The approximation and detail co-efficients are obtained as described above and is displayed below for illustration. Note that approximation co-efficients is the low frequency

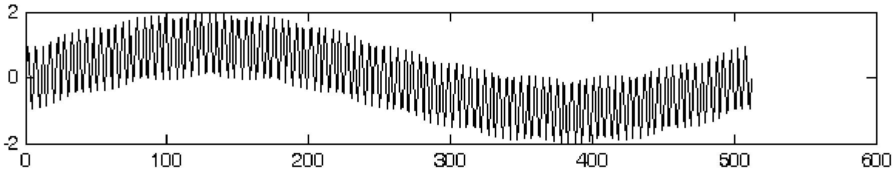


Figure 3-7. Original signal used for Haar decomposition

information derived from the signal and detail co-efficients is the high frequency information derived form the signal.

The signal is reconstructed back using inverse Haar transformation and the original signal along with reconstructed signal is displayed below.

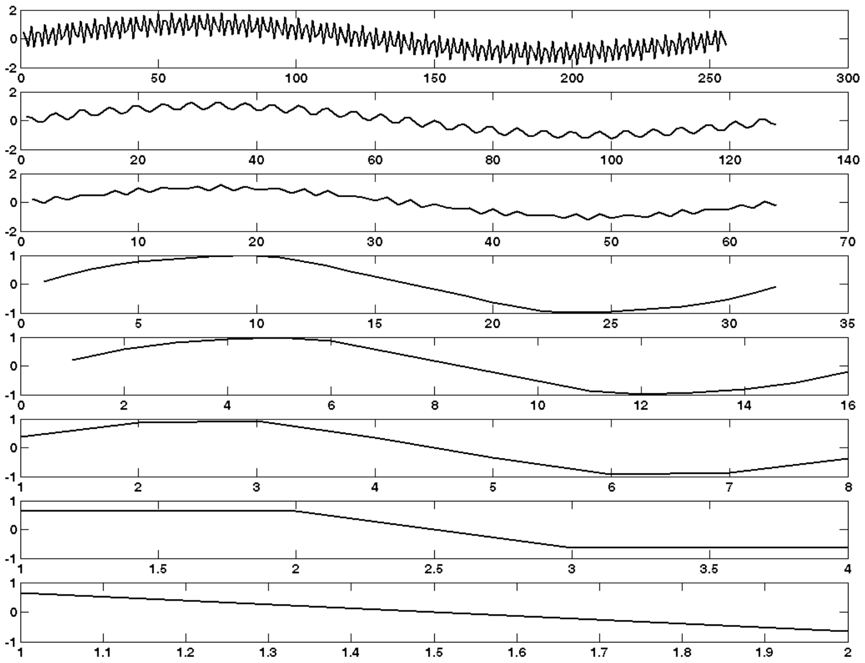


Figure 3-8. Approximation co-efficients obtained using Haar transformation

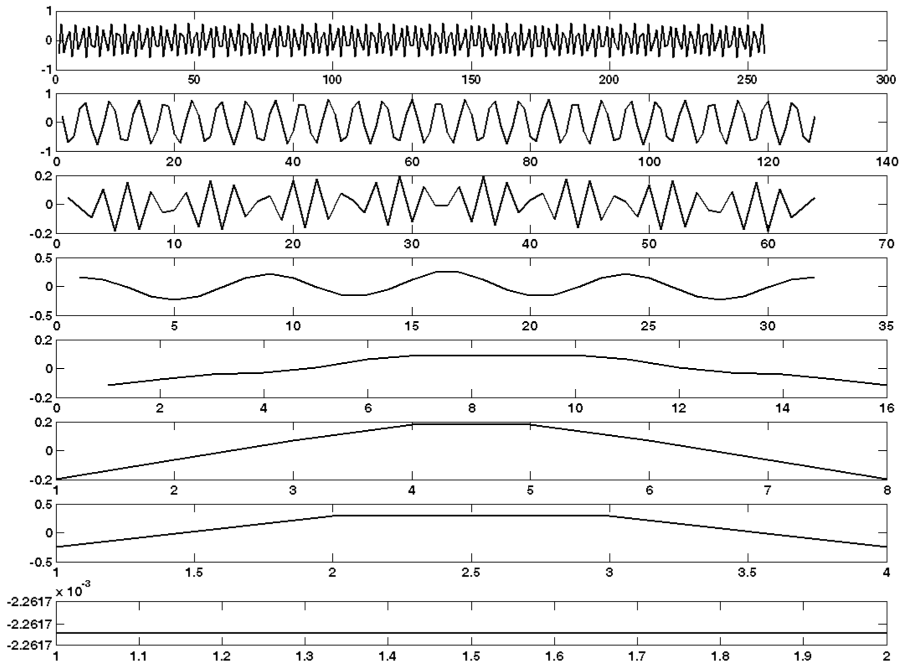


Figure 3-9. Detail co-efficients obtained using Haar transformation

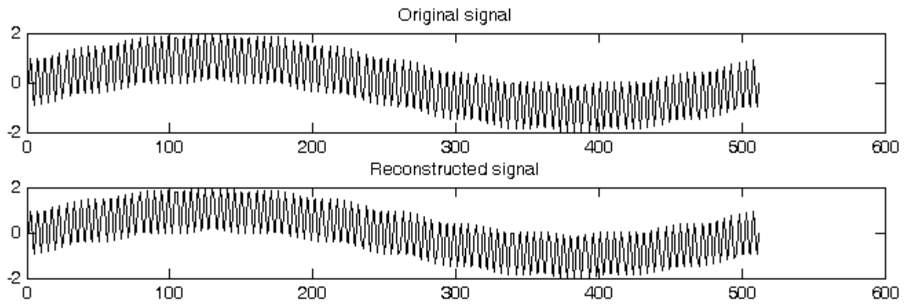


Figure 3-10. Illustration of Inverse Haar Transformation

14.1.2 M-file for haar forward and inverse transformation

haartrans.m

```
i=0:1/511:1;
a=sin(2*pi*i)+sin(2*pi*100*i);
plot(a)
figure
s=length(i);
for i=1:1:log2(s)
  temp=createhaarmatrix(length(a))*a';
  temp=reshape(temp,2,length(temp)/2);
  approx{i}=temp(1,:);
  det{i}=temp(2,:);
  a=approx{i};
end
for k=1:1:(length(approx)-1)
subplot((length(approx)-1),1,k)
plot(approx{k})
title(strcat('approx',num2str(k)))
end
subplot(
figure
for k=1:1:(length(approx)-1)
subplot((length(approx)-1),1,k)
plot(det{k})
title(strcat('detail',num2str(k)))
end
```

createhaarmatrix.m

```
function [res]=createhaarmatrix(m)
temp1=[ones(1,m/2);(-1)*ones(1,m/2)];
temp1=reshape(temp1,1,size(temp1,1)*size(temp1,2));
d1=temp1.*(1/2);
temp2=[ones(1,m/2);zeros(1,m/2)];
temp2=reshape(temp2,1,size(temp2,1)*size(temp2,2));
d2=(1/2)*temp2(1:1:length(temp2)-1);
res=diag(d1)+diag(d2,1)+diag(d2,-1);
```

haarinvtrans.m

```
app=approx{8};
for i=(log2(s)-1):-1:1
   a=[app;det{i}];
   a=reshape(a,1,size(a,1)*size(a,2));
   app=createinvhaarmatrix(length(a))*a';
   app=app';
end
figure
subplot(2,1,1)
i=0:1/511:1;
a=sin(2*pi*i)+sin(2*pi*100*i);
plot(a)
title('Original signal');
subplot(2,1,2)
plot(app)
title('Reconstructed signal');
```

---

createinvhaarmatrix.m

```
function [res]=createinvhaarmatrix(m)
temp1=[ones(1,m/2);(-1)*ones(1,m/2)];
temp1=reshape(temp1,1,size(temp1,1)*size(temp1,2));
d1=temp1;
temp2=[ones(1,m/2);zeros(1,m/2)];
temp2=reshape(temp2,1,size(temp2,1)*size(temp2,2));
d2=temp2(1:1:length(temp2)-1);
res=diag(d1)+diag(d2,1)+diag(d2,-1);
```

14.2 Daubechies- 4 Transformation

Consider the signal consists of N samples. Form the Daubechies Transformation Matrix [DTM 1] with diagonal matrices filled up with the matrix 'D' given below.

$$\begin{pmatrix} [(1+\sqrt{3}) / 4\sqrt{2}] & [(3+\sqrt{3}) / 4\sqrt{2}] & [(3-\sqrt{3}) / 4\sqrt{2}] & [(1-\sqrt{3}) / 4\sqrt{2}] \\ [(1-\sqrt{3}) / 4\sqrt{2}] & -[(3-\sqrt{3}) / 4\sqrt{2}] & [(3+\sqrt{3}) / 4\sqrt{2}] & -(1+\sqrt{3}) / 4\sqrt{2}] \end{pmatrix}$$

For simplicity matrix D is represented as follows.

$$D = \begin{pmatrix} a0\ a1\ a2\ a3 \\ b0\ b1\ b2\ b3 \end{pmatrix}$$

For N=8, the matrix 'DTM1' is formed as given below.

$$\begin{pmatrix} a0 & a1 & a2 & a3 & 0 & 0 & 0 & 0 & 0 & 0 \\ b0 & b1 & b2 & b3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a0 & a1 & a2 & a3 & 0 & 0 & 0 & 0 \\ 0 & 0 & b0 & b1 & b2 & b3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a0 & a1 & a2 & a3 & 0 & 0 \\ 0 & 0 & 0 & 0 & b0 & b1 & b2 & b3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a0 & a1 & a2 & a3 \\ 0 & 0 & 0 & 0 & 0 & 0 & b0 & b1 & b2 & b3 \end{pmatrix}$$

Note that the matrix 'D' are arranged with overlapping in the diagonal of the matrix 'DTM 1'. Compare with the Haar transformation in which the matrix are arranged without overlapping. Also note that the size of the matrix is of size 8X10 for the signal of size 1x8. So 1x8 sized signal is extended to the size 1x10 with 9th and 10th samples are filled up with 1st and 2nd samples respectively. [Assuming that the signals are repeated cyclically].

The steps involved in obtaining the approximation and detail co-efficients are as described in the section 14.1 for Haar transformation.

Similarly to reconstruct the signal, inverse Daubechies matrix is formed with the diagonal matrices filled up with the matrix [ID] as given below.

$$[ID] = \begin{pmatrix} a3 & b3 & a1 & b1 \\ a4 & b4 & a2 & b2 \end{pmatrix}$$

For N=8 the second level Inverse Daubechies matrix DITM2 is given as

$$\begin{pmatrix}
a3 & b3 & a1 & b1 & 0 & 0 & 0 & 0 & 0 & 0 \\
a4 & b4 & a2 & b2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a3 & b3 & a1 & b1 & 0 & 0 & 0 & 0 \\
0 & 0 & a4 & b4 & a2 & b2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a3 & b3 & a1 & B1 & 0 & 0 \\
0 & 0 & 0 & 0 & a4 & b4 & a2 & B2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a3 & B3 & a1 & b1 \\
0 & 0 & 0 & 0 & 0 & 0 & a4 & B4 & a2 & b2
\end{pmatrix}$$

Similar to the DTM 1,size of the matrix is 8x10. Also similar to forward transformation, 1x8 sized wavelet co-efficients obtained in the forward transformation is extended to 1x10-sized co-efficients with 1st and 2nd co-efficients filled up with the last two co-efficients of the wavelet co-efficients.

The steps involved in reconstructing the signal are same as that of the Haar transformation except that in Daubechies matrix the diagonal matrices are arranged with overlapping whereas in Haar matrix there is no overlapping.

14.2.1 Example

Consider the signal x(n) =a=sin (2*pi*n)+sin(2*pi*100*n) with number of samples = 128 and sampling frequency Fs = 128. The Daubechies transformation is applied to the signal The approximation and detail co-efficients are obtained as described above and is displayed below for illustration. Note that approximation co-efficients is the low frequency information derived from the signal and detail co-efficients is the high frequency information derived from the signal.

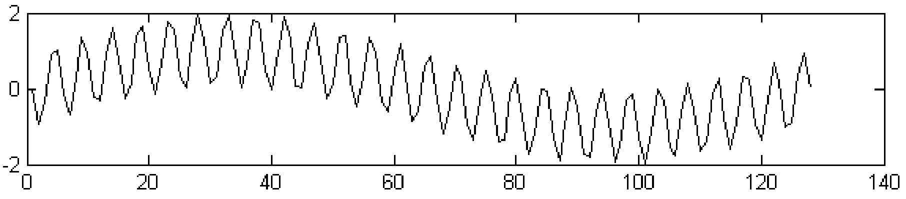


Figure 3-11. Original signal used for Daubechies 4 wavelet decomposition

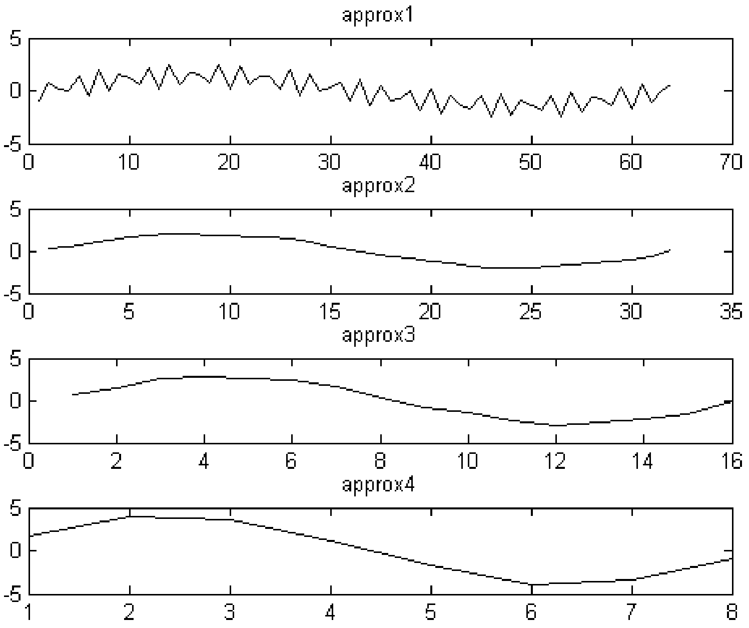


Figure 3-12. Approximation co-efficients at different levels

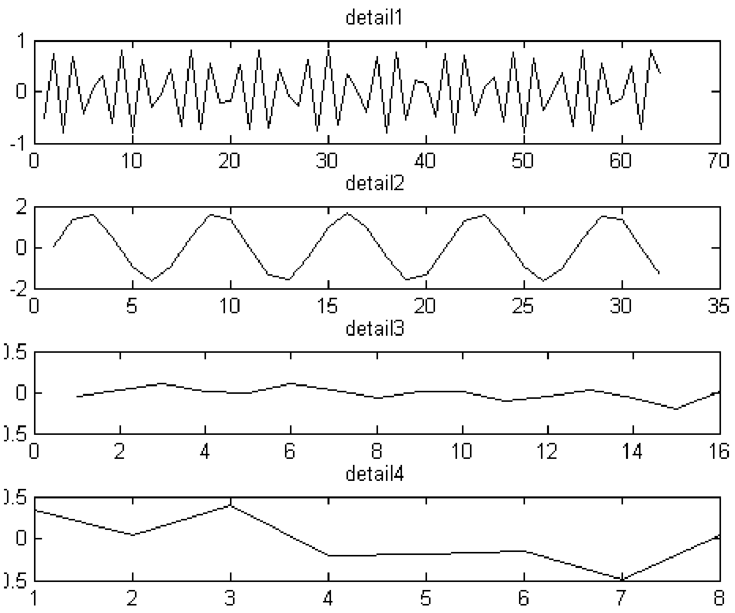


Figure 3-13 Detail Co-efficients at different levels

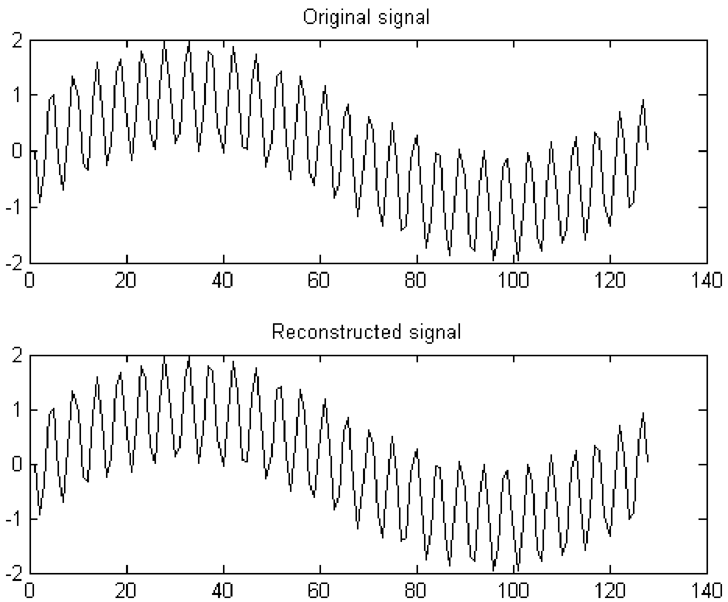


Figure 3-14 Illustration of Inverse Daubechies 4 transformation

14.2.2 M-file for daubechies 4 forward and inverse transformation

daub4trans.m

```
i=0:1/127:1;
a=sin(2*pi*i)+sin(2*pi*100*i);
plot(a)
figure
s=length(a);
for i=1:1:log2(s) -2
   a=[a a(1:2)];
   i
   temp=createdaubmatrix(length(a)-2)*a';
   temp=temp(1:1:length(temp));
   temp=reshape(temp,2,length(temp)/2);
   approx{i}=temp(1,:);
   det{i}=temp(2,:);
   a=approx{i};
end
for k=1:1:(length(approx)-1)
subplot((length(approx)-1),1,k)
plot(approx{k})
title(strcat('approx',num2str(k)))
end
figure
for k=1:1:(length(approx)-1)
subplot((length(approx)-1),1,k)
plot(det{k})
title(strcat('detail',num2str(k)))
end
```

createdaubmatrix.m

```
function [res]=createdaubmatrix(m)
a=[(1+sqrt(3))/(4*sqrt(2))         (3+sqrt(3))/(4*sqrt(2))                 (3-sqrt(3))/(4*sqrt(2))
(1-sqrt(3))/(4*sqrt(2))];
b=[(1-sqrt(3))/(4*sqrt(2))         -(3-sqrt(3))/(4*sqrt(2))                (3+sqrt(3))/(4*sqrt(2))
-(1+sqrt(3))/(4*sqrt(2))];
t1=repmat([a(2) b(3)],1,(m/2));
t2=repmat([a(3) b(4)],1,m/2);
t3=repmat([a(4) 0],1,m/2);
t4=repmat([b(1) 0],1,m/2);
res=diag(repmat([a(1) b(2)],1,m/2)) + diag(t1(1:1:m-1),1)
+diag(t2(1:1:m-2),2)+diag(t3(1:1:m-3),3)+diag(t4(1:1:m-1),-1) ;
res=[res   [zeros(1,m-2) res(1,3) res(2,3)]'  [zeros(1,m-2) res(1,4) res(2,4)]'];
```

daub4invtrans.m

```
app=approx{log2(s)-2};
for i=(log2(s)-2):-1:1
   a=[app;det{i}];
   a=reshape(a,1,size(a,1)*size(a,2));
   a=[ a(length(a)-1:1:length(a))  a];
   app=createdaubinvmatrix(length(a)-2)*a';
   app=app';
end
figure
subplot(2,1,1)
i=0:1/127:1;
a=sin(2*pi*i)+sin(2*pi*100*i);
plot(a)
title('Original signal');
subplot(2,1,2)
plot(app)
title('Reconstructed signal');
```

createdaubinvmatrix.m

```
function [res]=createdaubinvmatrix(m)
a=[(1+sqrt(3))/(4*sqrt(2))      (3+sqrt(3))/(4*sqrt(2))          (3-sqrt(3))/(4*sqrt(2))
(1-sqrt(3))/(4*sqrt(2))];
b=[(1-sqrt(3))/(4*sqrt(2))      -(3-sqrt(3))/(4*sqrt(2))          (3+sqrt(3))/(4*sqrt(2))
-(1+sqrt(3))/(4*sqrt(2))];
t1=repmat([b(3) a(2)],1,(m/2));
t2=repmat([a(1) b(2)],1,m/2);
t3=repmat([b(1) 0],1,m/2);
t4=repmat([a(4) 0],1,m/2);
res=diag(repmat([a(3)          b(4)],1,m/2))          +          diag(t1(1:1:m-1),1)
+diag(t2(1:1:m-2),2)+diag(t3(1:1:m-3),3)+diag(t4(1:1:m-1),-1) ;
res=[res   [zeros(1,m-2) res(1,3) res(2,3)]' [zeros(1,m-2) res(1,4) res(2,4)]'];
```

Chapter 4

SELECTED APPLICATIONS

Algorithm Collections

1. EAR PATTERN RECOGNITION USING EIGEN EAR

Ear pattern recognition is the process of classifying the unknown ear image as one among the finite category. The following is the report on the experiment done on ear pattern recognition with the small database. The experiment uses twelve ear images collected from four persons. Three images are collected from each person. Among them, eight images are used to train the classifier. Remaining four images are used to test the classifier. The steps involved in Ear pattern recognition using Eigen ears are summarized below.

1.1 Algorithm

Step 1: Mean and variance of the collected ear images are made almost equal using mean and variance normalization technique described in the section 3-5. Mean and variance of one of the image from the collections is treated as desired mean and desired variance. (See figure 4-1)

Step 2: Reshape the matrix into the vector whose elements are collected column by column from the matrix.

Step 3: Co-variance matrix of the collected vectors is computed. Eigen values of the co-variance matrix and the Eigen vectors corresponding to the significant Eigen vectors are computed (In this example 6 Eigen vectors are computed)

Step 4: Eigen vectors are reshaped into the matrix of the original size. They are called Eigen ears as given in the figure 4-2

Step 5: The Eigen ears are orthogonal to each other. They can be made orthonormal to each other by normalizing the vectors.

Step 6: For every ear image matrix, feature vectors are obtained as the inner product of eigen basis vectors and the reshaped ear image matrix .

Step 7: Mean vector of the feature vectors collected from the same person is treated as the template assigned to that corresponding person. This is repeated for other persons also. Thus one template is assigned to every person and they are stored in the database.

Step 8: To classify the unknown ear image as one among the four categories, template is computed as the inner product of Eigen basis vectors (Eigen ears) with reshaped normalized unknown ear image. The template thus obtained is compared with the group of templates stored in the database using Euclidean distance.

Step 9: Template corresponding to the minimum Euclidean distance is selected and the person corresponding to that template is declared as the identified person.

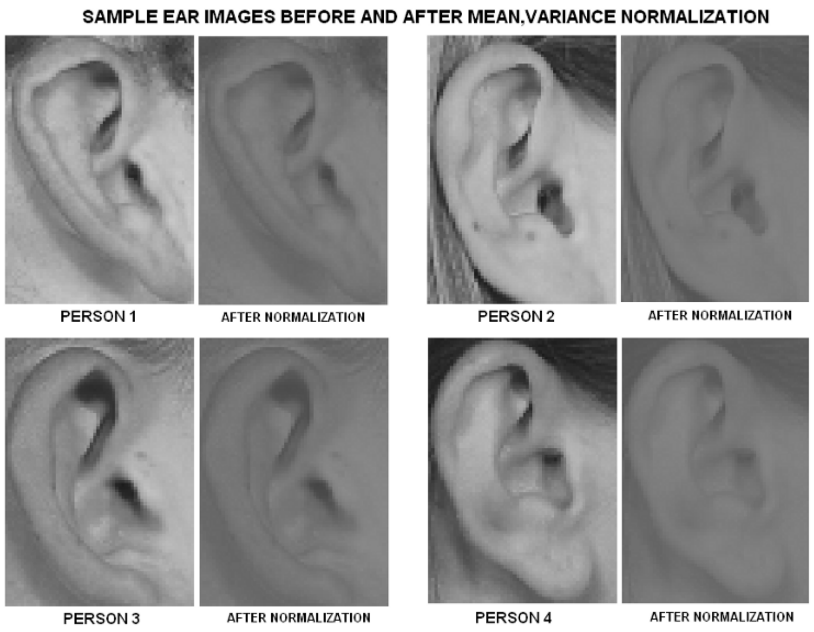


Figure 4-1. Sample Ear Images before and after normalization

Figure 4-2. Eigen Ears corresponding to the largest Eigen values

1.2 M-program for Ear Pattern Recognition

earpatgv.m

```
clear all
close all
k=1;
md=117;
vd=139;
for i=1:1:4
   for j=1:1:1
      s=strcat('ear',num2str(i),num2str(j),'.bmp');
      temp=imread(s);
      temp=rgb2gray(temp);
      temp1=reshape(temp,1,size(temp,1)*size(temp,2));
      temp1=double(temp1);
      temp1=meanvarnorm(temp1,md,vd);
      final{k}=temp1;
   k=k+1;
   end
end
f=cell2mat(final');
f=double(f);
c=cov(f);
[E,V]=eigs(c);
for i=1:1:6
F=reshape(E(:,i),85,60);
subplot(2,3,i)
colormap(gray)
imagesc(F)
end

%Feature vector

for i=1:1:4
   s=strcat('ear',num2str(i),num2str(1),'.bmp');
   s=imread(s);
   s=rgb2gray(s);
   s=double(s);
   s1=reshape(s,1,size(s,1)*size(s,2));
   s1=meanvarnorm(s1,md,vd);
```

```
    F=[];
    for j=1:1:6
    F=[F sum(s1.*E(:,j)')];
    end
    FEA{i}=F;
end
save FEA FEA E md vd
```

---

testinggv.m

```
load FEA
%Testing
[filename, pathname, filterindex] = uigetfile('*.bmp', 'Pick an BMP-file');
s=imread(strcat(pathname,filename));
s=rgb2gray(s);
s=double(s);
s1=reshape(s,1,size(s,1)*size(s,2));
s1=meanvarnorm(s1,md,vd);
F=[];
for j=1:1:6
F=[F sum(s1.*E(:,j)')];
end
S=[sum((FEA{1}-F).^2) sum((FEA{2}-F).^2) sum((FEA{3}-F).^2) sum((FEA{4}-F).^2)];
[P,Q]=min(S);
switch Q
    case 1
    A=imread('ear11.bmp');
    A=rgb2gray(A);
    g=gray(256);
    msgbox('First person','Pattern Recognition','custom',A,g)
    case 2
    A=imread('ear21.bmp');
    A=rgb2gray(A);
    g=gray(256);
    msgbox('Second person','Pattern Recognition','custom',A,g)
    case 3
    A=imread('ear31.bmp');
    A=rgb2gray(A);
    g=gray(256);
    msgbox('Third person','Pattern Recognition','custom',A,g)
    case 4
    A=imread('ear41.bmp');
    A=rgb2gray(A);
    g=gray(256);
    msgbox('Fourth person','Pattern Recognition','custom',A,g)
end
```

1.3 Program Illustration

2. EAR IMAGE DATA COMPRESSION USING EIGEN BASIS

Collected ear images as mentioned in the section 1 are represented as the group of 85x60 sized matrices. The elements of the matrices are stored with the numbers ranging from 0 to 255.8 bits are required to store every number. (i.e.) 8*85*60=40800 bits are required to store the single gray image. Therefore there is the need to identify the technique for storing the image data with reduced number of bits. This technique of representing the data with reduced number of bits by removing the redundancy from the data is called Image data compression.

Ear image data compression using eigen basis is the compression technique which exploits psycho visual property of the eye. This technique is used to compress the similar images belong to the particular group. In the experiment described below, the group considered is the ear images collected from many persons. The steps involved in Image data compression using Eigen basis are described below.

2.1 Approach

Step 1: 8x8 sized subblocks of the ear image matrix are collected randomly from the ear image database.

Step 2: Reshape the subblock matrix into the vector of size 1x64.

Step 3: Thus set of 100 vectors are collected randomly as described in step1 and step2.

Step 4: Co-variance matrix is computed for the collected vectors.

Step 5: Eigen values are computed for the covariance matrix. Eigen vectors corresponding to the significant Eigen values are computed. They are called Eigen basis, which spans the Ear vector space, which consists of all the vectors available as the reshaped sub blocks in the ear images. Note that Eigen vectors thus obtained are orthonormal to each other.

Step 6: Number of Eigen basis obtained as described in the step 4 is less than 64 (vector size). This number is called as the dimension of the Ear vector space.

Step 7: To compress the ear image, ear image matrix is divided into subblocks of size 8x8. Reshape every sub block into the vector of size 1x64. Represent this vector as the linear combination of Eigen basis obtained as described in the step 4. The co-efficients are obtained as the product of transformation matrix with the vector. Transformation matrix is the matrix filled up with Eigen basis arranged in the row wise. Number of Eigen co-efficients obtained is equal to the dimension of the Ear vector space.

Step 8: Thus every 1x64 sized vector obtained from every sub blocks of the ear image is mapped into the vector of size [1Xb], where ‘b’ is the dimension of the ear vector space which is less than 64. Thus every sub blocks of the ear image is stored with ‘b’ values which is very much less than 64. In this experiment, the value of ‘b’ is 20<<64. Thus compression with the ratio 3.2:1 is achieved using eigen basis technique.

Step 9: Every sub blocks of the decompressed image are obtained as the linear combinations of Eigen basis with the corresponding co-efficients associated with that sub block. The compressed and decompressed image of the sample ear image is displayed in the figure 4-3.

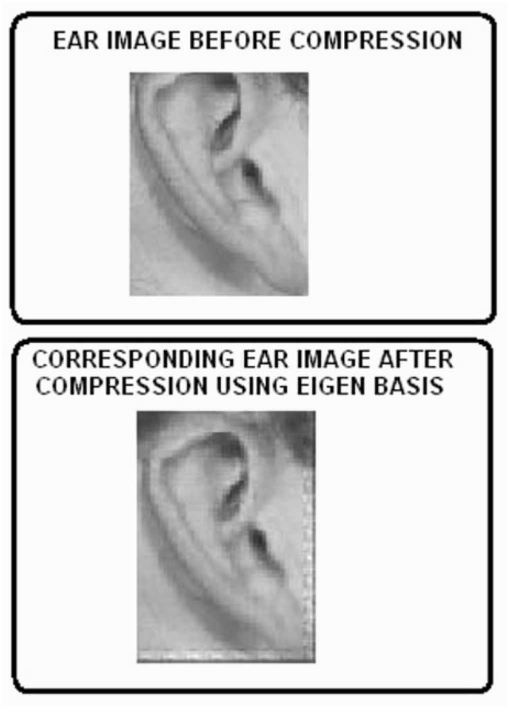

EAR IMAGE BEFORE COMPRESSION
CORRESPONDING EAR IMAGE AFTER COMPRESSION USING EIGEN BASIS

Figure 4-3. Ear image compression with the ratio 3.2:1 using Eigen basis

2.2 M-program for Ear Image Data Compression

earcompgv.m

```
m=1;
for i=1:1:4
   for j=1:1:3
      for k=1:1:20
      s=strcat('ear',num2str(i),num2str(j),'.bmp');
      temp=imread(s);
      temp=rgb2gray(temp);
      x=round(rand*((size(temp,1)-8)))+1;
      y=round(rand*((size(temp,2)-8)))+1;
      t=temp(x:1:(x+7),y:1:(y+7));
      final{m}=double(reshape(t,1,64));
   m=m+1;
   end
end
end
data=cell2mat(final');
c=cov(data);
[E,V]=eig(c);
%Collecting significant Eigen values
E=E(:,45:1:64);
%Transformation matrix
TM=E';
save TMEIG TM
%Image to be compressed
A=imread('ear11.bmp');
B=rgb2gray(A);
C=blkproc(B,[8 8],'compresseig(x)');
D=blkproc(C,[size(E,2), 1],'decompresseig(x)');
figure
subplot(2,1,1)
imshow(B)
title('EAR IMAGE BEFORE COMPRESSION')
subplot(2,1,2)
D=D(1:1:size(B,1),1:1:size(B,2));
imshow(uint8(D))
title('CORRESPONDING EAR IMAGE AFTER COMRESSION USING EIGEN BASIS')
```

compresseig.m

```
function [res]=compresseig(x)
x=double(reshape(x,1,64));
load TMEIG
res=TM*x';
```

---

decompresseig.m

```
function [res]=decompresseig(x)
load TMEIG
res=reshape(TM'*x,8,8);
```

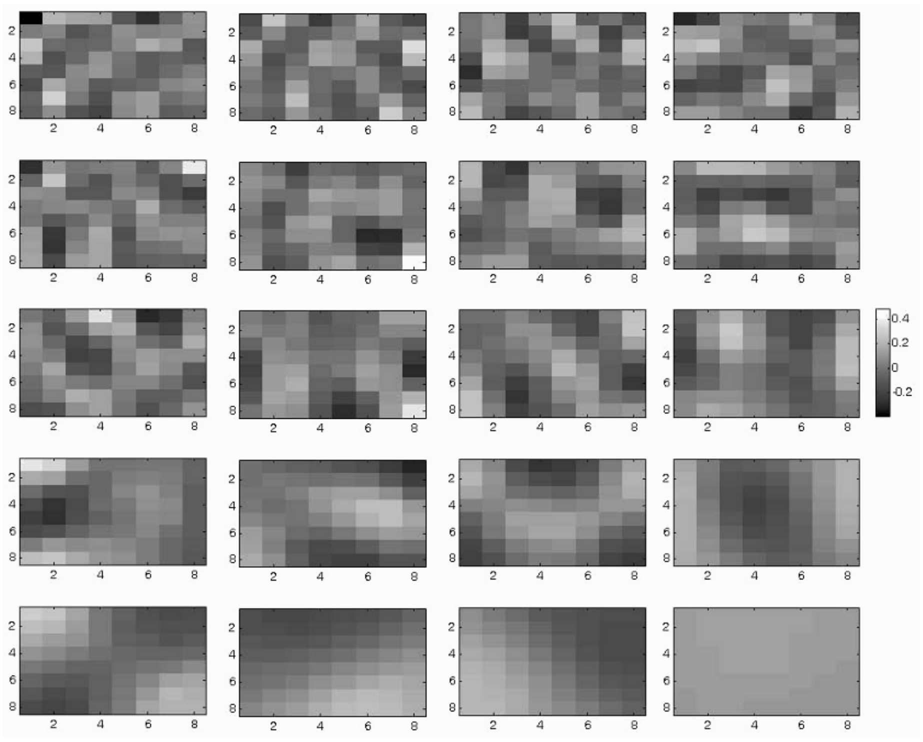


Figure 4-4. Basis used for Ear Image Compression

3. ADAPTIVE NOISE FILTERING USING BACKPROPAGATION NEURAL NETWORK

Consider the signal transmitted through the noisy channel gets corrupted with the noise. The corrupted signal is given as the input to the FIR filter as given below to filter the noisy part of the signal.

Let x(n) be the noisy corrupted input signal to the system, y(n) be the output signal of the system which is the filtered signal and h(n) is the impulse response of the system. They are related mathematically as given below.

$$y(n) = x(n)*h(n)$$

$$\Rightarrow$$

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$$

'*' is the convolution operator. 'N' is the order of the filter. For instance if the order of the filter is 11,

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(3)x(n-3) + h(4)x(n-4) + \ldots h(10)\,x(n-10)$$

The h(0), h(1), … h(10) are the impulse response of the system, otherwise called as the filter co-efficients of the system.

Obtaining the values of the filter co-efficients is the task involved in designing the digital filter to do specific operation. To obtain the values there is the need to study about the nature of the noise of the channel.

In practical situations the reference signal is sent through the channel and the corresponding corrupted signal obtained as the output of the channel is stored. These are used for determining the filter co-efficients of the FIR filter. Once filter co-efficients are obtained, they are used to filter the real time noisy signal corrupted due to channel transmission at the receiver side.

Figure 4-5. FIR FILTER

FIR Filter block as described above can also be replaced with the Back propagation Neural Network block as mentioned in the chapter 1 to perform filtering operation as described below. In this experiment the reference signal and its corresponding corrupted signal are assumed to be known.

3.1 Approach

Step 1: Back propagation neural network (see chapter 1) is constructed with 11 input Neurons and 1 output neuron and 5 Hidden neurons (say)

Step 2: In signal processing point of view, input of the neural network is the corrupted signal and the output of the neural network is the filtered signal.

Step 3: During the training stage, the elements of the Input vectors are the samples collected from the corrupted reference signal. Similarly element of the output vector is the corresponding sample collected from the reference signal.

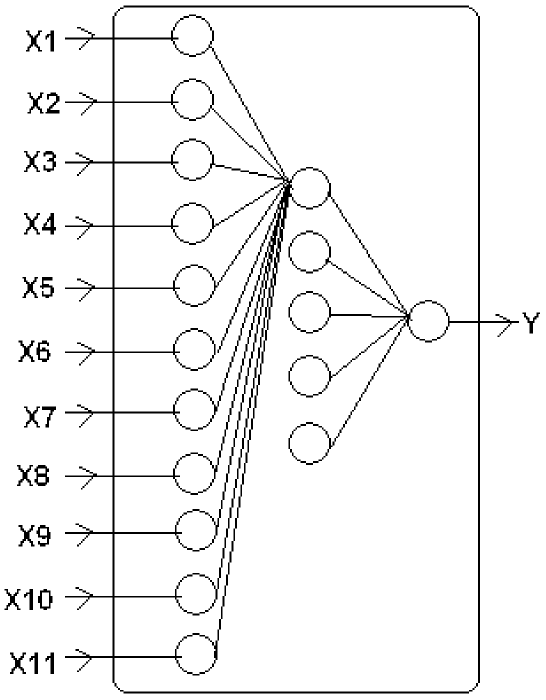


Figure 4-6. BPNN Filter Structure

Table 4-1. Sample Hetero Association table relating input vectors and output vectors of the BPNN

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	Y
X(10)	X(9)	X(8)	X(7)	X(6)	X(5)	X(4)	X(3)	X(2)	X(1)	X(0)	Y(0)
X(11)	X(10)	X(9)	X(8)	X(7)	X(6)	X(5)	X(4)	X(3)	X(2)	X(1)	Y(1)
X(12)	X(11)	X(10)	X(9)	X(8)	X(7)	X(6)	X(5)	X(4)	X(3)	X(2)	Y(2)
…											
X(16)	X(15)	X(14)	X(13)	X(12)	X(11)	X(10)	X(9)	X(8)	X(7)	X(6)	Y(16)
X(17)	X(16)	X(15)	X(14)	X(13)	X(12)	X(11)	X(10)	X(9)	X(8)	X(7)	Y(17)
X(18)	X(17)	X(16)	X(15)	X(14)	X(13)	X(12)	X(11)	X(10)	X(9)	X(8)	Y(18)
X(19)	X(18)	X(17)	X(16)	X(15)	X(14)	X(13)	X(12)	X(11)	X(10)	X(9)	Y(19)
X(20)	X(19)	X(18)	X(17)	X(16)	X(15)	X(14)	X(13)	X(12)	X(11)	X(10)	Y(20)
…											

The Hetero association table relating the sample input vector and the corresponding output vector used for training the constructed ANN is given below. Note that x is the corrupted reference signal and y is the reference signal

Step 4: Train the Artificial Backpropagation Neural Network as described in the chapter 1. Store the Weights and Bias.

Step 5: Now the BPNN filter is ready to filter the original corrupted signal.

3.2 M-file for Noise Filtering Using ANN

noisefiltuanngv.m

```
%Noise filtering using ANN
A=wavread('C:\Program Files\NetMeeting\Testsnd');
B=A(:,2);
B=B(6001:1:7000,1);
B=B';
B=(B+1)/2;
```

```
%Assumed channel characteristics which is disturbing the signal.

H=[0.7071  -0.7071];
C=conv(H,B);

%Collections

INPUTVECTOR=[ ];
OUTPUTVECTOR=[ ];

for i=1:1:5000
   x=round(rand*(100-12))+1;
   INPUTVECTOR=[INPUTVECTOR;C(x:1:x+10)];
   OUTPUTVECTOR=[OUTPUTVECTOR;B((x+11))];
end

W1=rand(5,11);
B1=rand(5,1);
W2=rand(1,5);
B2=rand(1,1);
nntwarn off
[W1,B1,W2,B2]=trainbpx
(W1,B1,'logsig',W2,B2,'logsig',INPUTVECTOR',OUTPUTVECTOR',[250     100000     0.02
0.01]);
save WEIGHTSNOISEFILT W1 B1 W2 B2
D=blkproc(C,[1 1],[0 5],'dofilter(x)');
D=(D-0.5)*2;
figure
subplot(3,1,1)
plot(B)
title('Original signal')
subplot(3,1,2)
plot(C)
title('Corrupted signal')
subplot(3,1,3)
plot(D)
title('Retrieved signal')
```

dofilter.m

```
function [res]=dofilter(x)
load WEIGHTSNOISEFILT
res=simuff(x',W1,B1,'logsig',W2,B2,'logsig');
```

3.3 Program Illustration

Note that low frequency information is lost and that is not retrieved in the filtered signal. This is due to the fact that first 100 samples are used to train the ANN. If the training sequence is increased, low frequency information can also be preserved in the filtered signal. Also note that MATLAB Neural Network toolbox is used to train the network.

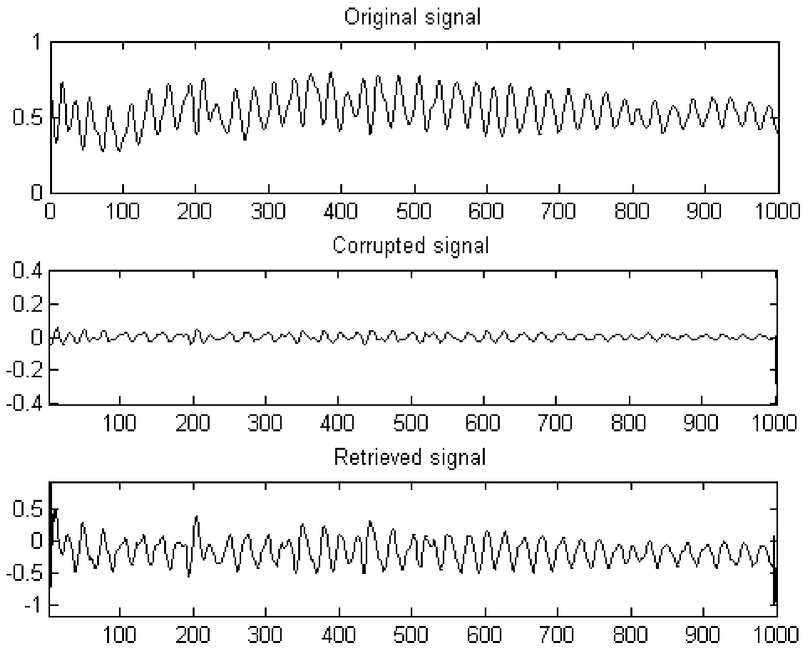


Figure 4-7 Noise Filtering using BPNN

4. BINARY IMAGE ROTATION USING TRANSFORMATION MATRIX

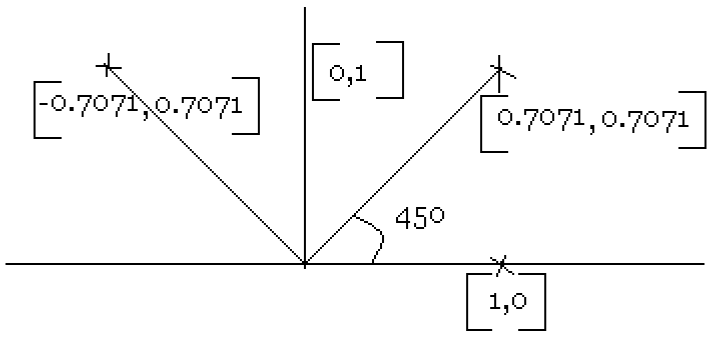


Figure 4-8. Vector basis for Binary Image Rotation

Consider the vector space spanned by the basis $[1\ 0]^T$ and $[0\ 1]^T$. Transformation of the vector [1 0] is the vector [0.7071 0.7071] that is in the same space. Transformed vector is represented as the linear combination of the basis $[1\ 0]^T$ and $[0\ 1]^T$ with vector co-efficient 0.7071 and 0.7071 respectively. Similarly the transformation of the vector $[0\ 1]^T$ is the vector [-0.7071 0.7071] that is in the same space.

The transformed vector [-0.7071 0.7071] is represented as the linear combination of the basis $[1\ 0]^T$ and $[0\ 1]^T$ with co-efficient –0.7071 and 0.7071 respectively.

The transformation described above rotates the vector counter clockwise by 45^0 as mentioned in the figure 4-7.

Thus the transformation matrix used to rotate the vector counter clockwise by 45^0 is computed using the method described in the chapter 3 is given below.

$$\begin{pmatrix} 0.7071 & 0.7071 \\ 0.7071 & -0.7071 \end{pmatrix}$$

4.1 Algorithm

Step 1: Read the Binary image.

Step 2: Find the position vectors of the black pixels of the Binary image.

Step 3: Transform the position vectors using the transformation matrix to obtain the new sets of positions.

Step 4: Create the blank image completely filled up with white pixels. Fill the selected pixels of the blank image described by the positions obtained in step 3 with black to obtain the rotated image.

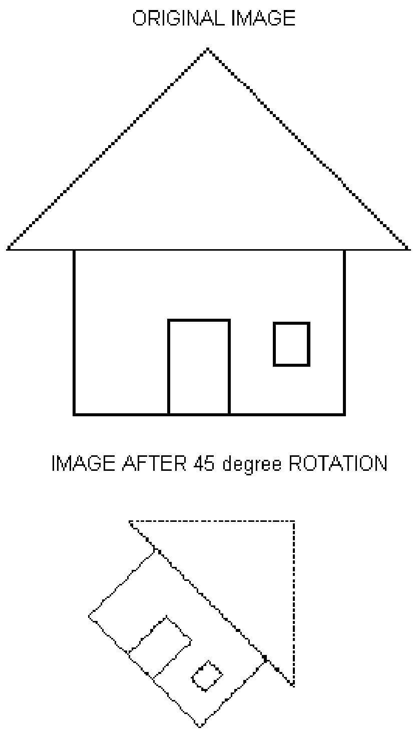


Figure 4-9. Binary Image Rotation

4.2 M-program for Binary Image Rotation with 45 Degree Anticlockwise Direction

---

binimrotationgv.m

```
A=imread(selectfig8','bmp');
TM=[0.7071 0.7071;-0.7071 0.7071];
[X Y]=find(A==0);
VECT=[X Y];
TRANSVECT=TM*VECT';
TRANSVECT=round(TRANSVECT-min(min(TRANSVECT))+1);
m=max(max(TRANSVECT));
B=ones (m, m);
for k=1:1:length(TRANSVECT)
   B(TRANSVECT(1,k),TRANSVECT(2,k))=0;
end
figure
subplot(2,1,1)
imshow(A)
title('ORIGINAL IMAGE')
subplot(2,1,2)
imshow(B)
title('IMAGE AFTER 45 degree ROTATION')
```

---

5. CLUSTERING TEXTURE IMAGES USING K-MEANS ALGORITHM

To retrieve the similar images from the database, sample image looking like the same is used as the key. The low level features extracted from the key image is used as the index for searching. So the data base has to be maintained based on the low level feature vectors derived from the images.

Consider 23 texture images are stored in the database. They are indexed using the low level features collected from the image itself, so that 23 texture images are stored under 4 categories. K-means algorithm is used to classify the collected texture into 4 categories so that each image in the database is associated with the number indicating the category. (i.e.) 1 indicates first category, 2 indicates second category and so on. This is called texture grouping and is done as described below.

5.1 Approach

Step 1: Every texture image in the database is divided into sub blocks of size 8x8. The variance of every sub blocks are calculated. They are arranged in the vector form of size 1x64. This is called Low-level feature vector extracted from the image itself. Note that elements of the low level feature vector can also be any other statistical measurements measured from the image.

Step 2: The collected Low-level feature vectors are subjected to k-means algorithm to classify the images into four categories as described in the chapter 2

Step 3: Thus the index number is identified for every image in the database. Also the centroids of the individual category are also stored. The centroid is the vector which is of the same size as that of the feature vector. [Refer chapter 2]

Step 4: To retrieve the images from the indexed database that looks like the image which is given as the key image is described below.

- Extract the low level feature vector from the key image as described in the step 1.
- Compute the Euclidean distance between the computed feature vector and all the centroids corresponding to the individual category. The category corresponding to smallest Euclidean distance is selected.
- All the images in that category is retrieved using the index number associated with every images

Figure 4-10. Texture images - Cluster 1

Figure 4-11. Texture images - Cluster 2

Figure 4-12. Texture images - Cluster 3

Figure 4-13. Texture images - Cluster 4

5.2 M – program for Texture Images Clustering

patclasgv.m

```
for i=1:1:23
S=strcat(num2str(i),'.bmp');
temp1=rgb2gray(imread(S));
A{i}=temp1(1:1:40,1:1:48);
temp2=blkproc(A{i},[8 8],'var2(x)');
FEA{i}=reshape(temp2,1,size(temp2,1)*size(temp2,2));
end
FEAVECT=cell2mat(FEA');
P=kmeans(FEAVECT,4);
for i=1:1:4
figure
[x,y]=find(P==i);
for j=1:1:length(x)
subplot(1,length(x),j)
imshow(A{x(j)})
end
end
```

var2.m

```
function [res]=var2(x)
res=var(reshape(x,1,64));
```

6. SEARCH ENGINE USING INTERACTIVE GENETIC ALGORITHM

Consider the small Image database stored with Low-level feature vectors associated with every image. Low-level feature vector associated with every image is the 1D vector whose elements are the statistical measurements like variance, kurtosis, and higher order moments etc., computed from the sub blocks of the image. Low-level feature vector associated with every image in the database acts as the index for searching the particular image from the database. Interactive Genetic algorithm based searching procedure is described below.

6.1 Procedure

1. The front panel of the search engine consists of finite number of images displayed. (Say 8 images) as shown in the figure given below.
2. The user is asked to assign the rank for the individual images. The value ranges from 0 to100 based on the user's interest. (i.e.) User assigns higher

Figure 4-14. Sample front panel

rank if user likes that image and expecting the similar images in the next iteration.

3. 4 Images are selected from the 8 images using Roulette wheel selection (Refer chapter 1) as described below.

- Let the rank values assigned for the eight images are represented as the vector as given below.

  [10 90 10 10 10   90 10 90]

- Normalized vector is computed as

  [0.0313 0.2813 0.0313   0.0313 0.03133 0.2813 0.0313 0.2813]

- Simulate Roulette wheel

Cumulative summation of the above-normalized vector is displayed below.
[0.0313   0.3125   0.3438   0.3750   0.4063   0.6875   0.7188   1]

Random number is generated as 'r'. If r is in between 'a' and 'b' in the cumulative summation vector song corresponding to the value 'b' is selected as mentioned below.

Generated random number = 0.9501(say)
Therefore select 8th image.

Generated number =0.2311(say)
Therefore select 2nd image.

Similarly.
0.6068-----------------------------------6th image
0.0185-----------------------------------1st image.

4. Thus the selected images are 8,2,6,5

5. Low level features corresponding to the image number 8,2 6 and 5 are computed. The vectors thus obtained are randomly chosen and is subjected to cross over and mutation operation as described below.

Low level feature (LLF) vector before and after crossover

Vector 1 = [a b c d e f]

Vector 2 = [A B C D E F] (say)

Note that the number of elements shown in the vector is considered as 6. [Actually the size of the vector is around 200]

Low level feature (LLF) vector after crossover

Vector 1'= [a b c d E F]
Vector 2' = [A B C D e f]

6. Thus 8 Low level feature vectors for the second generation are obtained as mentioned below.
   LLF1, LLF2, LLF3, LLF4, LLF5, LLF6, LLF7 and LLF8 (say)

7. Compute the Euclidean distance between the feature vector LLF1 with all the low level feature vectors in the database. Retrieve the image corresponding to the LLF vector in the database whose Euclidean distance is smallest. Repeat the procedure for the feature vectors LLF2, LLF3, … LLF8

8. Thus the image selected for the next iteration is listed below. 9, 4, 16, 3, 13, 18, 45, 43 (say). Note that 9 in the selected list indicates 9th image in the database.

9. Repeat the step from 1 to 8 until the user is satisfied with the images obtained in the latest iteration.

6.2 Example

Consider the Image database consists of 94 Natural sceneries. Every image is truncated to the standard size 200x200. The feature vector for the particular image is computed as described below.

The image is divided into sub blocks of size 50x50 with overlapping size of 8x8.The first feature namely variance of the histogram of the hue content, are computed for the all the overlapping sub blocks of the image. This is treated as first part of the feature vector corresponding to the image. Similarly other features are computed for all the overlapping sub blocks of the image to obtain other parts of the feature vector. All the parts belonging to the individual features are combined to obtain the feature vector corresponding to that particular image.

Thus feature vectors are computed for all the images in the data base.

6.2.1 List of low level features computed in every sub blocks of the image

1. Variance of the histogram of the hue content of the particular subblock is computed.

2. Measurement of uniformity

$p(z_i)$ is the probability of gray value z_i in the particular sub block of the image. The uniformity of sub block of the image is measured using the formula given below.

$$U = \sum_{i=1}^{N} p^2(z_i)$$

3. Measurement of Average Entropy

Average entropy is computed as described below.

$$e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$$

4. Measurement of Relative smoothness

$$R = 1 - \frac{1}{1+\sigma^2(z)}$$

where, $\sigma(z)$ is the standard deviation of the gray values z in the particular sub block of the image.

5, 6, 7, 8. Variance of the approximation Wavelet co-efficient

The image is subjected to discrete wavelet transformation (DWT) to obtain approximation co-efficient (app), horizontal detail co-efficient (hor), vertical detail co-efficient (ver) and diagonal (both horizontal and vertical) detail co-efficient(dia) The variance of the approximation co-efficient 'app' is computed and considered as 5th feature. Similarly the variance of the detail co-efficient 'hor','ver' and 'dia' are considered as the 6th, 7th and 8th features respectively.

9. Skewness

The Skew ness of the gray values is computed for every sub block of the image as mentioned below.

$$\mu_3(z) = \sum_{i=1}^{L-1} (z_i - m)^3 p(z_i)$$

10. Kurtosis

The Kurtosis of the gray values is computed for every sub block of the image as mentioned below.

$$\mu (z) = \sum_{i=1}^{L-1} (z_i - m)^4 p(z_i)$$

6.3 M-program for Interactive Genetic Algorithm

igagv.m

```
r=round(rand(1,8)*94);
figure
for k=1:1:8
subplot(4,2,k)
title('Assign the rank')
s=strcat(num2str(r(k)),'.jpg');
A=imread(strcat('E:\Naturepix\',s));
imshow(A)
title(strcat('Image',num2str(k)));
end
for iter=1:1:10
s1=input('Enter the rank for image 1(Between 0 to 100)');
s2=input('Enter the rank for image 2(Between 0 to 100)');
s3=input('Enter the rank for image 3(Between 0 to 100)');
s4=input('Enter the rank for image 4(Between 0 to 100)');
```

```
s5=input('Enter the rank for image 5(Between 0 to 100)');
s6=input('Enter the rank for image 6(Between 0 to 100)');
s7=input('Enter the rank for image 7(Between 0 to 100)');
s8=input('Enter the rank for image 8(Between 0 to 100)');
vect=[s1 s2 s3 s4 s5 s6 s7 s8]/100;
normvect=vect/sum(vect);
c=cumsum(normvect);
u=[ ];
for i=1:1:4
r1=rand;
c2=c-r1;
[x,y]=find(c2>0);
if(y(1)==1)
   u1=1
else
   u1=y(1)-1
end
u=[u u1];
end

load NATUREFEATURE
for i=1:1:4
feature{i}=FEATURE(r(u(i)),:);
end
%Cross over to obtain 8 feature vectors
k=1;
for i=1:1:4
p1=round(rand*159)+1
r=round((rand*3)+1);
d1=feature{r};
r=round((rand*3)+1);
d2=feature{r}
nextfeature{k}=[d1(1:1:p1)   d2(p1+1:1:160)];
k=k+1;
nextfeature{k}=[d1(1:1:p1)   d2(p1+1:1:160)];
k=k+1;
end
r=[];
for l=1:1:8
      n=nextfeature{l};
      d=(FEATURE-repmat(n,94,1)).^2;
      s=sum(d')
```

```
        [m1,n1]=min(s);
        r=[r  n1];
end

for k=1:1:8
subplot(4,2,k)
title('Assign the rank')
s=strcat(num2str(r(k)),'.jpg');
A=imread(strcat('E:\Naturepix\',s));
imshow(A)
title(strcat('Image',num2str(k)));
end
end
```

---

igaimagegv.m

```
for i=1:1:94
   s=strcat(num2str(i),'.jpg');
   A=imread(strcat('E:\Naturepix\',s));
   B{i}=naturefeature(A);
end
FEATURE=cell2mat(B');
save NATUREFEATURE  FEATURE
```

---

naturefeature.m

```
function [res]=naturefeature(x)
x=x(100:1:size(x,1),1:1:size(x,2)-100,:,:);
x=imresize(x,[200 200]);
y=rgb2hsv(x);
h=y(:,:,1);
res=blkproc(h,[32 32],[8 8],'huevar(x)');
res1=reshape(res,1,size(res,1)*size(res,2));
y=rgb2gray(x);
res=blkproc(y,[32 32],[8 8],'unif(x)');
res2=reshape(res,1,size(res,1)*size(res,2));
res=blkproc(y,[32 32],[8 8],'entropy(x)');
res3=reshape(res,1,size(res,1)*size(res,2));
res=blkproc(y,[32 32],[8 8],'relativesmooth(x)');
res4=reshape(res,1,size(res,1)*size(res,2));
res=blkproc(y,[32 32],[8 8],'wavelet(x)');
```

```
res5=reshape(res,1,size(res,1)*size(res,2));
res=blkproc(y,[32 32],[8 8],'skew(x)');
res6=reshape(res,1,size(res,1)*size(res,2));
res=blkproc(y,[32 32],[8 8],'kurt(x)');
res7=reshape(res,1,size(res,1)*size(res,2));
res=[res1 res2  res3 res4  res5 res6 res7 ];
```

---

huevar.m

```
function [res]=huevar(x)
x=hist(reshape(x,1,size(x,1)*size(x,2),500);
res=var(x);
```

---

unif.m

```
function [res]=unif(x)
z=reshape(x,1,size(x,1)*size(x,2));
h=hist(double(z),256);
h=h/sum(h);
res=sum(h.^2);
```

---

entropy.m

```
function [res]=entropy(x)
z=reshape(x,1,size(x,1)*size(x,2));
h=hist(double(z),256);
h=h/sum(h);
res=(-1)*sum(h.*log2 (h+eps));
```

---

relativesmooth.m

```
function [res]=relativesmooth(x)
z=reshape(x,1,size(x,1)*size(x,2));
res=1-(1/(1+var(z)));
```

---

wavelet.m

```
function [res]=wavelet(x)
x=double(x);
[c,l]=wavedec2(x,1,'db6');
a=appcoef2(c,l,'db6',1);
v1=var(reshape(a,1,size(a,1)*size(a,2)));
d1=detcoef2('h',c,l,1);
v2=var(reshape(d1,1,size(d1,1)*size(d1,2)));
d2=detcoef2('v',c,l,1);
v3=var(reshape(d2,1,size(d2,1)*size(d2,2)));
d3=detcoef2('d',c,l,1);
v4=var(reshape(d3,1,size(d3,1)*size(d3,2)));
res=[v1 v2 v3 v4];
```

---

skew.m

```
function [res]=skew(x)
x=double(x);
z=reshape(x,1,size(x,1)*size(x,2));
h=hist(double(z),256);
h=h/sum(h);
m=mean(z);
res=sum(((z-m).^3).*h(z+1));
```

---

kurt.m

```
function [res]=kurt(x)
x=double(x);
z=reshape(x,1,size(x,1)*size(x,2));
h=hist(double(z),256);
h=h/sum(h);
m=mean(z);
res=sum(((z-m).^4).*h(z+1));
```

6.4 Program Illustration

7. SPEECH SIGNAL SEPARATION AND DENOISING USING INDEPENDENT COMPONENT ANALYSIS

Consider the two speakers talking in the meeting simultaneously with no background noise. Two microphones are used for recording. One kept very close to the first person. Another microphone is kept near to the second person. The recorded signals of both the microphones can be treated as the linear combinations of independent sources. [Two speech signals] Hence ICA algorithm can be used to separate the two independent signals [Refer chapter 2].

Consider the second situation in which single speaker is talking with the background noise. Two microphones are used to record the signal. One microphone kept near to the speaker. Another microphone kept near to the assumed noise source. The recorded signals of both the microphones can be treated as the linear combinations of independent sources [Speech signal + Noise].Similar to the above ICA algorithm can be used to separate the two independent signals. Hence the ICA algorithm can be used to separate the two independent signals and hence denoising is achieved.

7.1 Experiment 1

Two speech signals x1(t) and x2(t) are linearly mixed to get two mixed signals y1(t) and y2(t) as given below.

y1(t)=0.7*x1(t) +0.3*x2(t)
y2(t)=0.3*x1(t)+0.7*x2(t)

The mixed signals are subjected to ICA algorithm. The independent signals x1(t) and x2(t) are obtained as shown below. Note that **FASTICA** toolbox is used to run the ICA algorithm.

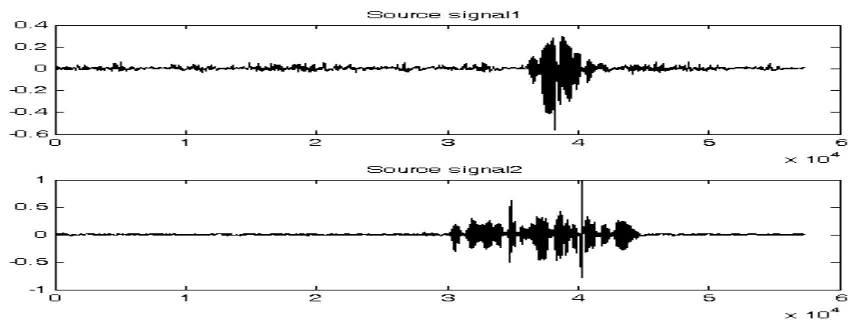


Figure 4-15. Original speech signals

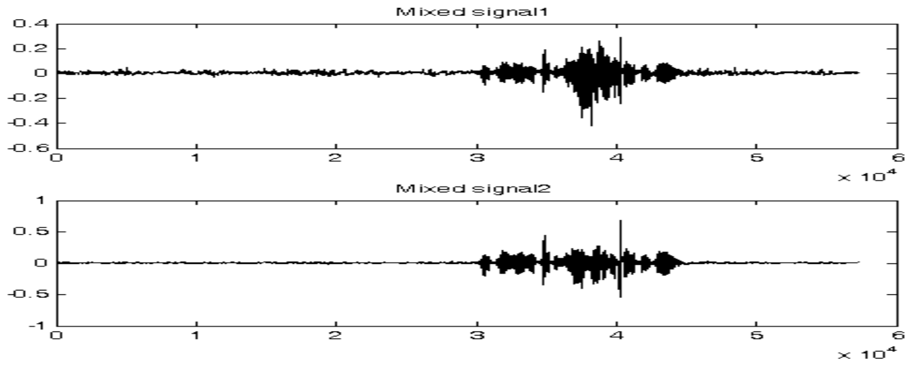


Figure 4-16. Mixed signals

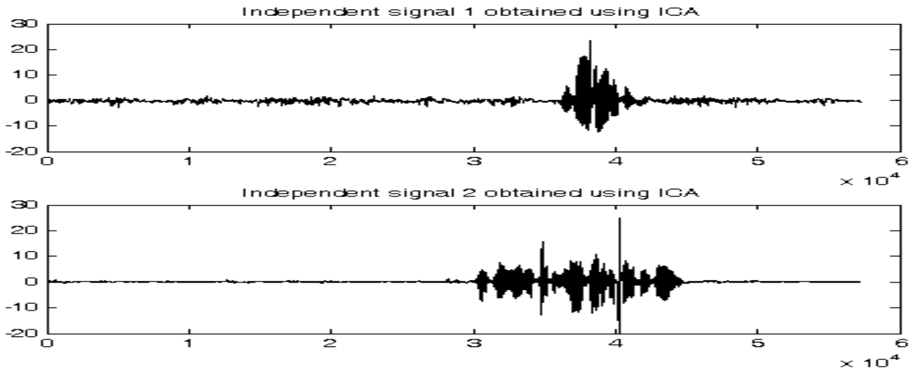


Figure 4-17 Separated speech signals using ICA

7.2 Experiment 2

Speech signal x(t) and noise n(t) are linearly mixed to get two mixed signals y1(t) and y2(t) as given below.

y1(t) = 0.7*x(t) + 0.3*n(t)
y2(t) = 0.3*x(t)+ 0.7*n(t)

The mixed signals are subjected to ICA algorithm to obtain the speech signal and noise signal separately.

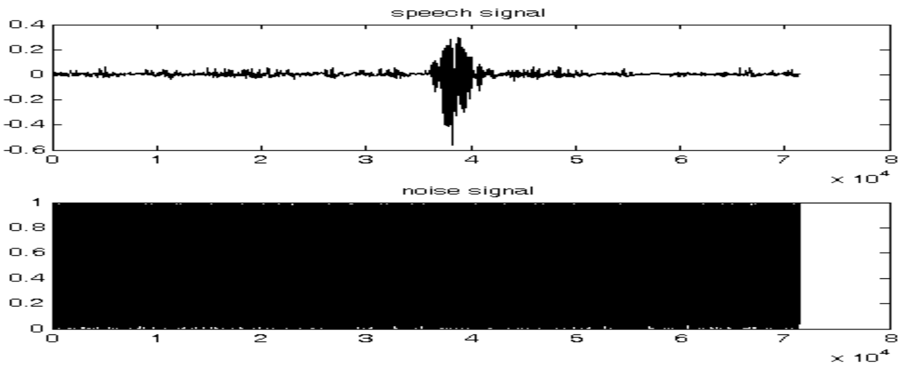


Figure 4-18. Speech signal and Noise signal before mixing

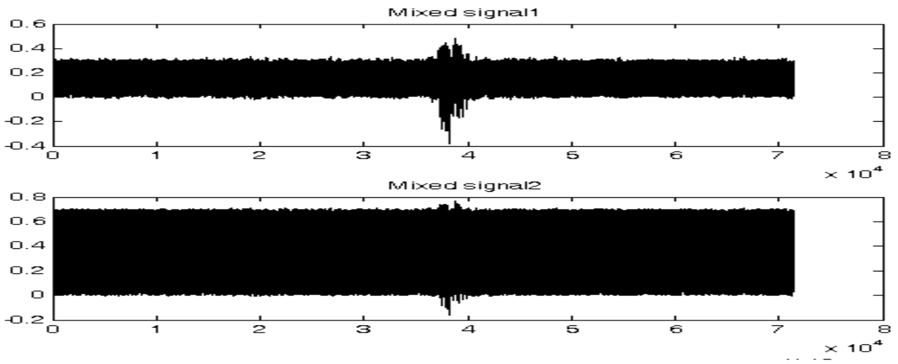


Figure 4-19. Mixed signals

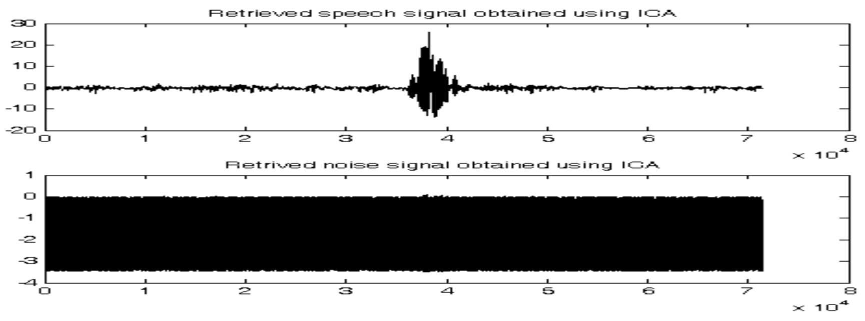


Figure 4-20. Retrieved speech and noise signals using ICA

7.3 M-program for Denoising

---

noiseicagv.m

```
a=wavread('C:\MATLAB7\work\audio\revsam\ram1.wav');
b=rand(length(a),1);
m=min(length(a),length(b));
a=a(1:1:m);
b=b(1:1:m);
mixed1=0.7*a+0.3*b;
mixed2=0.3*a+0.7*b;
mixed=[mixed1'; mixed2'];
[signal]=fastica(mixed);
figure
subplot(2,1,1)
plot(mixed1)
title('Mixed signal1')
subplot(2,1,2)
plot(mixed2)
title('Mixed signal2')
figure
subplot(2,1,1)
plot(a)
title('speech signal')
subplot(2,1,2)
plot(b)
title('noise signal')
figure
subplot(2,1,1)
plot(signal(1,:))
title('Retrieved speech signal obtained using ICA')
subplot(2,1,2)
plot(signal(2,:))
title('Retrived noise signal obtained using ICA')
```

Note that the signal b(n) in the above program is another speech signal in case of speech signal separation. The rests of the program remains same.

8. DETECTING PHOTOREALISTIC IMAGES USING INDEPENDENT COMPONENT ANALYSIS BASIS

The digital images captured using the camera are called as photographic images and the images which are not captured using the camera are called as photorealistic images. ICA basis are used in classifying the produced image into photo graphic or photo realistic images as described below.

Figure 4-21. Sample photo realistic images

Figure 4-22. Sample photographic images

8.1 Approach

- The 10 photographic images and the 10 photo realistic images are collected (see figure 4-3 and figure 4-4). The sub blocks of the image sized 16x16 are randomly collected from both the categories. 500 sub blocks are collected from photographic images and 500 subblocks are collected from the photorealistic images.

- The subblock thus obtained is reshaped into the size 1x256. Auto Regressive (AR) co-efficients of size 1x10 are obtained from the reshaped subblock. This is repeated for all the collected sub blocks. The set of AR vectors collected from photographic images and the photo realistic images are subjected to ICA analysis and 10 ICA basis each of size 1x10 are obtained. [Refer chapter 2]

- Every AR co-efficient vector obtained from the particular subblock of the images is represented as the linear combination of 10 corresponding ICA Basis. The co-efficient of the ICA basis are obtained using the inner product of ICA basis with the corresponding AR co-efficients. This is called feature e vector of that particular subblock of the image.

- Thus 500 feature vectors are collected from photographic images and the 500 feature vectors are collected from the photorealistic images. The centroid of the feature vectors collected from the photographic images is computed as C1. Similarly the centroid of the feature vectors collected from the photo realistic images are computed as C2.

8.1.1 To classify the new image into one among the photographic or photorealistic image

The image is divided into sub blocks. Feature vectors are extracted form every sub blocks using ICA basis as described above. The Euclidean distance between the feature vector obtained from the particular subblock and the centroids 'C1' and 'C2' are computed as 'd1' and 'd2' respectively. Assign the number 1 to that particular subblock if 'd1' is lowest Otherwise 2 is assigned to that subblock. This is repeated for all the sub blocks of the image to be classified.

Count the number of '1's and the number of '0's assigned to the subblocks of the image to be classified. If the number of 1's is greater than

0’s, decide that the image is photographic image. Otherwise the image is classified as photo realistic image.

8.2 M-program for Detecting Photo Realistic Images Using ICA Basis

---

photorealisticdetgv.m

```
k=1;
for  i=1:1:20
   s=strcat('C:\Documents and Settings\user1\Desktop\Photorealistic\',num2str(i),'.jpg');
for  j=1:1:50
   temp=imread(s);
   temp=rgb2gray(temp);
   temp=temp(101:1:200,101:1:200);
   r1=round(rand*(size(temp,1)-8))+1;
   r2=round(rand*(size(temp,2)-8))+1;
   temp=reshape(temp(r1:1:r1+7,r2:1:r2+7),1,64);;
   temp=double(temp);
   A{k}=lpc(temp,16);
   k=k+1;
end
end


temp=cell2mat(A');
temp=temp(:,2:1:17);
I=fastica(temp);
save  I   I

k=1 ;
for i=1:1:20
   s=strcat('C:\Documents and Settings\user1\Desktop\Photorealistic\',num2str(i),'.jpg');
for j=1:1:100
   temp=imread(s);
   temp=rgb2gray(temp);
   temp=temp(101:1:200,101:1:200);
   r1=round(rand*(size(temp,1)-8))+1;
   r2=round(rand*(size(temp,2)-8))+1;
```

```
    temp=reshape(temp(r1:1:r1+7,r2:1:r2+7),1,64);
    temp=double(temp);
    temp=lpc(temp,16);
    temp=temp(2:1:17);
    a=[ ];
    for n=1:1:size(I,1)
     a= [a  sum(temp.*I(n,:)) ];
    end
    feaext1{k}=a;
     k=k+1;
    end
end

FEA=cell2mat(feaext1');
C1=mean(FEA(1:1:1000,:));
save C1 C1

C2=mean(FEA(1001:1:2000,:));
save C2 C2
```

---

phototest.m

```
for i=1:1:20
s=strcat('C:\Documents and Settings\user1\Desktop\Photorealistic\',num2str(i),'.jpg');
A=imread(s);
B=rgb2gray(A);
B=B(101:1:300,101:1:300);
imshow(B)
C=blkproc(B,[8 8],'assigncenno(x)');
if(length(find(C==1))>length(find(C==0)))
    disp('The image is photorealistic image')
else
    disp('The image is photographic image')
end
end
```

---

assigncenno.m

```
function [res]=assigncenno(x)
x=double(x);
load C1
load C2
load  I
x=reshape(x,1,64);
p=lpc(x,16);;
temp=p(2:1:17);
a=[ ];
for n=1:1:size(I,1)
    a=  [a  sum(temp.*I(n,:)) ];
end
d1=sum((C1-a).^2);
d2=sum((C2-a).^2);
if(d1<d2)
   res=0;
else
   res=1;
end
```

8.3 Program Illustration

9. BINARY IMAGE WATERMARKING USING WAVELET DOMAIN OF THE AUDIO SIGNAL

The process of hiding the information like text, binary image, audio etc. into another signal source like image, audio etc. is called watermarking. The approach involved in watermarking the binary image signal in the wavelet domain of the audio signal is described in the example given below.

9.1 Example

Step 1: Consider the audio signal sampled with the sampling rate of 11025 Hz.

Step 2: The audio signal is divided into frames with 1024 samples. Decompose every frame of the speech signal using 'db4' wavelet transformation [See wavelet transformation in chapter 3]. Hide the first bit of the binary data collected from the binary image into the third level detail co-efficients obtained using wavelet decomposition of the first audio frame. This is done by changing the values of the third level detail co-efficients such that mean of the third level detail co-efficients is modified to 'm+k' if the binary value is '1' or to 'm-k' if the binary value is '0'.Note that variance remains same.[See Mean and variance normalization in Chapter 2]. The variable 'm' is the mean of the fifth level detail co-efficients. The value for the constant 'k' is chosen as $1/5^{th}$ of the energy of the third level detail co-efficients.

Step 3: This is repeated for the other frames of the speech signal for hiding the entire bits obtained from the binary image.

Step 4: Thus Binary image is hidden in the wavelet domain of the audio signal.

Step 5: The hidden binary data is retrieved by comparing the mean of the corresponding detail co-efficients computed before and after hiding. If the mean of the third level detail co-efficient of the particular frame corresponding to the original signal is greater than the mean of the third level detail co-efficients of the respective frame corresponding to the watermarked signal, the bit stored is considered as '1',otherwise '0'.

Step 6: Note that the duration of the audio signal is chosen such that all the binary data collected from the binary image are hidden in the audio signal.

In this example length of the audio signal used for hiding binary image data is 11488 samples. It is repeated 100 times so that the length of the audio signal becomes 1148800 samples. The size of the Binary image used is 45X45.Binary image is stored at the rate of 1 bit in 256 samples of the audio signal.

9.2 M-file for Binary Image Watermarking in the Wavelet Domain of the Audio Signal

audiowatermarkdb4gv.m

```
A= wavread ('C:\WINDOWS\Media\Microsoft Office 2000\glass.wav');
A=repmat(A,100,1);
 A=A';
len=fix(length(A)/256);
for k=1:1:len-1
   temp=A((k-1)*256+1:1:(k-1)*256+256);
   col{k}=daub4water(temp);
end
I=imread('BINIMAGE.bmp');
I=I(1:1:45,1:1:45);
I=reshape(I,1,45*45);

for k=1:1:length(I)
     switch I(k)
       case 0
          temp=col{k}{2}{3};
          e=sum(temp.^2)/5;
          m=mean(temp);
          v=var(temp);
          col{k}{2}{3}=meanvarnorm(temp,(m-e),v);
       case 1
          temp=col{k}{2}{3};
          e=sum(temp.^2)/5;
          m=mean(temp);
          v=var(temp);
         col{k}{2}{3}=meanvarnorm(temp,(m+e),v);
   end
end
```

```
s=256;
signal=[];
for k=1:1:length(I)
   approx=col{k}{1};
det=col{k}{2};
app=approx{log2(s)-2};
for i=(log2(s)-2):-1:1
   a=[app;det{i}];
   a=reshape(a,1,size(a,1)*size(a,2));
   a=[ a(length(a)-1:1:length(a))  a];
   app=createdaubinvmatrix(length(a)-2)*a';
   app=app';
end
signal=[signal app];
end
figure
subplot(3,1,1)
plot(signal(1:1:5520));
title('Original signal')
subplot(3,1,2)
plot(A(1:1:5520),'r');
title('Watermarked signal');
subplot(3,1,3)
plot(signal(1:1:5520)-A(1:1:5520))
title('Difference signal')

save signal signal
```

---

daub4water.m

```
function [res]=daub4water(x)
a=x;
s=length(a);
for i=1:1:log2(s) -2
   a=[a a(1:2)];
      temp=createdaubmatrix(length(a)-2)*a';
   temp=temp(1:1:length(temp));
   temp=reshape(temp,2,length(temp)/2);
   approx{i}=temp(1,:);
   det{i}=temp(2,:);
   a=approx{i};
```

```
end
res{1}=approx;
res{2}=det;
```

---

gethiddenimage.m

```
% size of the Binary hidden image is 45x45
A= wavread ('C:\WINDOWS\Media\Microsoft Office 2000\glass.wav');
A=repmat(A,100,1);
A=A';
len=fix(length(A)/256);
for k=1:1:len-1
   temp=A((k-1)*256+1:1:(k-1)*256+256);
   col1{k}=daub4water(temp);
end

load signal
A=signal;
len=fix(length(A)/256);
for k=1:1:len
   temp=A((k-1)*256+1:1:(k-1)*256+256);
   col2{k}=daub4water(temp);
end

temp=[];
for   k=1:1:45*45
   m1=mean(col1{k}{2}{3});
   m2=mean(col2{k}{2}{3});
   if(m1>m2)
      temp=[temp 0];
   else
      temp=[temp 1];
   end
end
temp1=reshape(temp,45,45);
figure
imshow(temp1);
```

---

meanvarnorm.m

```
function [res]=meanvarnorm(a,md,vd)
m=mean(a);
v=sqrt(var(a));
vd=sqrt(vd);
delta=abs((a-m)*vd/v);
res=ones(1,length(a))*md;
[p]=quantiz(a,m);
p=p';
p=p*2-1;
res=res+p.*delta;
```

---

createdaubmatrix.m

```
function [res]=createdaubmatrix(m)

a=[(1+sqrt(3))/(4*sqrt(2)) (3+sqrt(3))/(4*sqrt(2))  (3-sqrt(3))/(4*sqrt(2))  …
(1-sqrt(3))/(4*sqrt(2))];

b=[(1-sqrt(3))/(4*sqrt(2)) -(3-sqrt(3))/(4*sqrt(2))  (3+sqrt(3)) /(4*sqrt(2)) …
-(1+sqrt(3))/(4*sqrt(2))];

t1=repmat([a(2) b(3)],1,(m/2));
t2=repmat([a(3) b(4)],1,m/2);
t3=repmat([a(4) 0],1,m/2);
t4=repmat([b(1) 0],1,m/2);
res=diag(repmat([a(1) b(2)],1,m/2)) + diag(t1(1:1:m-1),1)+…
diag(t2(1:1:m-2),2)+diag(t3(1:1:m-3),3)+diag(t4(1:1:m-1),-1) ;

res=[res   [zeros(1,m-2) res(1,3) res(2,3)]'  [zeros(1,m-2) res(1,4) res(2,4)]'];
```

---

createdaubinvmatrix.m

```
function [res]=createdaubinvmatrix(m)
a=[(1+sqrt(3))/(4*sqrt(2)) (3+sqrt(3))/(4*sqrt(2)) (3-sqrt(3))/(4*sqrt(2)) (1-sqrt(3))/(4*sqrt(2))];
b=[(1-sqrt(3))/(4*sqrt(2)) -(3-sqrt(3))/(4*sqrt(2)) (3+sqrt(3))/(4*sqrt(2)) -(1+sqrt(3))/(4*sqrt(2))];
t1=repmat([b(3) a(2)],1,(m/2));
t2=repmat([a(1) b(2)],1,m/2);
t3=repmat([b(1) 0],1,m/2);
t4=repmat([a(4) 0],1,m/2);
res=diag(repmat([a(3) b(4)],1,m/2)) + diag(t1(1:1:m-1),1)+ …
diag(t2(1:1:m-2),2)+diag(t3(1:1:m-3),3)+diag(t4(1:1:m-1),-1) ;

res=[res   [zeros(1,m-2) res(1,3) res(2,3)]' [zeros(1,m-2) res(1,4) res(2,4)]'];
```

9.3 Program Illustration

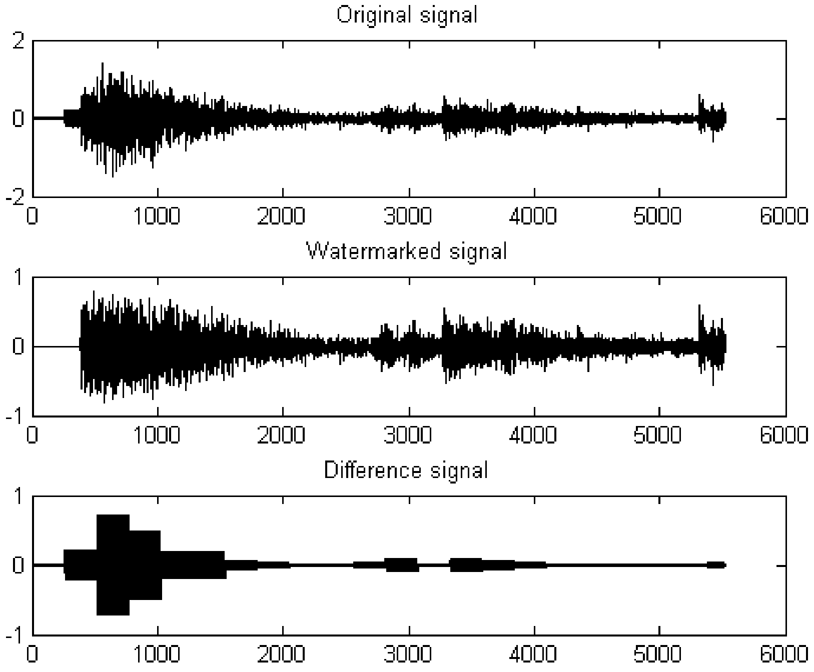


Figure 4-23. Audio signal before and after watermarking

Figure 4-24. Original and watermarked signal in zoomed level

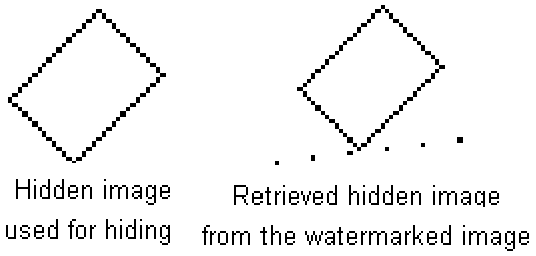


Figure 4-25. Hidden and Retrieved Binary image

Appendix

List of Figures

List of m-files

psogv.m
f1.m

geneticgv.m
fcn.m

sagv.m
minfcn.m

anngv.m
logsiggv.m

fuzzygv.m

antcolonygv.m
computecost.m

icagv.m

gmmmodelgv.m
clustercol.m
clusterno.m

kmeansgv.m
fuzzykmeansgv.m

meanvarnormgv.m

hotellinggv.m

gramgv.m

haartransgv.m
createhaarmatrix.m
haarinvtransgv.m
creatinvhaarmatrix.m

daub4trans.m
createdaubmatrix.m
daub4invtrans.m
createdaubinvmatrix.m

earpatgv.m
testinggv.m

earcompgv.m
compresseig.m
decompresseig.m

noisefiltanngv.m
dofilter.m

binimrotationgv.m

patclassgv.m
var2.m

igagv.m
igaimagegv.m
naturefeature.m
huevar.m
unif.m
entropy.m
relativesmooth.m
wavelet.m
skew.m
kurt.m

noiseicagv.m

photorealisticdetgv.m
phototest.m
assigncenno.m

audiowatermarkdb4gv.m
daub4water.m
gethiddenimage.m

Index