ME 594 - Numerical Methods: Homework 8

Four Problems: (10 pts each) Note: A * next to a problem number indicates you must electronically submit a ".m" file.

1*. Solve the ODE $y' = -2xy$ with $y(0) = 1$ using both the explicit and implicit Euler methods. Use step sizes $h = 0.4$, 0.2, 0.1, and 0.05 on the domain $[0, 6]$. Plot the results for all step sizes on separate plots for the implicit and explicit methods. The exact solution is $y(x) = e^{-x^2}$ which should also be included in the plots. There should be two plots submitted, each with five curves on it. Include the code used to solve the problem. You can use a shortcut for the implicit equation here; it is not necessary to use a nonlinear equation solver as part of the code.

2*. Solve the ODE $y' = -xy^3$ with $y(0) = 1$ using the implicit Euler method. For this problem, you must include a nonlinear equation solver. Include your code, and use step sizes $h = 1/2$, $1/4$, and $1/8$ on the interval $[0, 20]$. Plot the results for all step sizes along with the exact solution,

$$y(x) = \frac{1}{(1 + x^2)^{1/2}}.$$

3*. Solve the ODE $y' = x - xy/2$ with $y(1) = 1$ using the trapezoid method and the improved Euler method. Include your code for each method, and use step sizes $h = 0.8$, 0.4, and 0.2 on the interval $[1, 5]$. Plot the results for all step sizes along with the exact solution,

$$y = 2 - e^{\frac{1-x^2}{4}}.$$

4*. In a chemical reaction, one molecule of A combines with one molecule of B to form one molecule of the chemical C. It is found that the concentration $y(t)$ of C at time t is the solution to the I.V.P.

$$y' = k(a - y)(b - y) \quad \text{with} \quad y(0) = 0,$$

where k is a positive constant and a and b are the initial concentrations of A and B, respectively. Suppose that $k = 0.01$ liter/(millimole $\cdot$ s), $a = 70$ millimoles/liter, and $b = 50$ millimoles/liter. Include your code, and use the classical 4th order Runge-Kutta method with $h = 0.5$ to find the solution over $[0, 20]$.

You can compare your numerical solution with the exact solution:

$$y(t) = 350 \frac{(1 - e^{-0.2t})}{(7 - 5e^{-0.2t})}.$$

Observe that the limiting value is 50 as $t \to +\infty$.