ME 594 – Numerical Methods – HW06

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"I pledge my honor that I have abided by the Stevens Honor System"

1. Textbook problem 6.2 (Solve by hand): The following data is given:

x	-7	-4	-1	0	2	5	7
y	20	14	5	3	-2	-10	-15

(a) Use linear least-squares regression to determine the coefficients m and b in the function $y = mx + b$ that best fit the data.

ans. $m = -2.53984$; $b = 2.86853$

(b) Use Eq. (6.5) [see eq. below in answer] to determine the overall error.

ans. $E = \sum_{i=1}^{n} [y_i - (mx_i + b)]^2 = 1.62948$

Q.1) a) Linear least-squared regression.

Given:

x	-7	-4	-1	0	2	5	7
y	20	14	5	3	-2	-10	-15

Sol:

$$m = \frac{n S_{xy} - S_x S_y}{n S_{xx} - (S_x)^2} \; ; \; b = \frac{S_{xx} S_y - S_{xy} S_x}{n S_{xx} - (S_x)^2}$$

x	y	xy	xx
-7	20	-140	49
-4	14	-56	16
-1	5	-5	1
0	3	0	0
2	-2	-4	4
5	-10	-50	25
7	-15	-105	49
2	15	-360	144

$$\therefore m = \frac{7(-360) - 2(15)}{7(144) - (2)^2} = -2.53984 \,//$$

$$\therefore b = \frac{144(15) - (-360)(2)}{7(144) - (2)^2} = 2.86852 \,// \text{ Ans}$$

b) $E = \sum_{i=1}^{m} [y_i - (mx_i + b)]^2$

$= [20 - (m(-7) + b)]^2 + [14 - (m(-4) + b)]^2 +$

$[5 - (m(-1) + b)]^2 + [3 - (m(0) + b)]^2 +$

$[-2 - (m(+2) + b)]^2 + [-10 - (m(5) + b)]^2 +$

$[-15 - (m(7) + b)]^2$

Substituting m & b

$E = 0.419140 + 0.945001 + 0.166763 + 0.0172854$
$+ 0.0445866 + 0.0286702 + 0.00803559$

$\therefore E = 1.62948$ // Ans.

2. Textbook problem 6.11 (Solve by hand): Using the method in Section 6.8 (use the section in the notes on "General Linear Least Squares"), determine the coefficients of the equation $y = ax + b/x^2$ that best fit the following data:

x	0.8	1.6	2.4	3.2	4.0
y	6	3.6	4.1	5.1	6.2

ans. $a = 1.49868$; $b = 3.07141$

Q.2) General linear least squares method:

Given:

x	0.8	1.6	2.4	3.2	4.0
y	6	3.6	4.1	5.1	6.2

5 data points , 2^{nd} order

$$y = f(x) = \sum_{j=1}^{m} c_j f_j(x)$$

$$\sum_{j=1}^{M}\left[\sum_{k=1}^{N} f_i(x_k) f_j(x_k)\right] C_j = \sum_{k=1}^{N} f_i(x_k)\, y_k$$

$$\left(\sum_{k=1}^{N} f_1(x_k) f_1(x_k)\right) C_1 + \left(\sum_{k=1}^{N} f_1(x_k) f_2(x_k)\right) C_2 = \sum_{k=1}^{N} f_1(x_k)\, y_k$$

$$\left(\sum_{k=1}^{N} f_2(x_k) f_1(x_k)\right) C_1 + \left(\sum_{k=1}^{N} f_2(x_k) f_2(x_k)\right) C_2 = \sum_{k=1}^{N} f_2(x_k)\, y_k$$

$$\therefore \left(\sum_{k=1}^{N} x_k^2\right) C_1 + \left(\sum_{k=1}^{N} 1/x_k\right) C_2 = \sum_{k=1}^{N} x_k y_k$$

$$\therefore \left(\sum_{k=1}^{N} 1/x_k\right) C_1 + \left(\sum_{k=1}^{N} 1/x_k^4\right) C_2 = \sum_{k=1}^{N} y_k/x_k^2$$

y_k	x_k	x_k^2	$1/x_k$	$1/x_k^4$	$x_k y_k$	y_k/x_k^2
6	0.8	0.64	1.25	2.4414	4.8	9.375
3.6	1.6	2.56	0.625	0.1525	5.76	1.40625
4.1	2.4	5.76	0.4166	0.0301	9.84	0.71180
5.1	3.2	10.24	0.3125	0.00954	16.32	0.49804
6.2	4.0	16	0.25	0.00390	24.8	0.3875
25	12	35.2	2.8541	2.63744	61.52	12.37860

$$\therefore\ \cancel{35}(35.2)C_1 + (2.8541)C_2 = 61.52$$

$$(2.8541)C_1 + (2.63744)C_2 = 12.37860$$

$$C_2 = \frac{12.3786 - 2.85417\, C_1}{2.63758}$$

$$35.2C_1 + \frac{2.85417}{2.63758}(12.37860 - 2.85417C_1) = 61.52$$

$C_1 = a = 1.49868$ //

$C_2 = b = 3.07141$ // Ans

3. Textbook problem 6.15 (Solve by hand) The following data is given:

x	1	2.2	3.4	4.8	6	7
y	2	2.8	3	3.2	4	5

(a) Write the polynomial in Lagrange form that passes through the points; then use it to calculate the interpolated value of y at $x = 5.4$.

(b) Write the polynomial in Newton's form that passes through the points; then use it to calculate the interpolated value of y at $x = 5.4$.

You may write a script to evaluate the polynomials after solving for their general form by hand.

ans. $y(5.4) = 3.51159$

Q.3). Given:

x	1	2.2	3.4	4.8	6	7
y	2	2.8	3	3.2	4	5

a) Lagrange form

$$f(x) = \sum_{i=1}^{n} y_i \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{(x-x_j)}{(x_i-x_j)}$$

$$= \frac{2(x-2.2)(x-3.4)(x-4.8)(x-6)(x-7)}{(1-2.2)(1-3.4)(1-4.8)(1-6)(1-7)} +$$

$$\frac{2.8(x-1)(x-3.4)(x-4.8)(x-6)(x-7)}{(2.2-1)(2.2-3.4)(2.2-4.8)(2.2-6)(2.2-7)}$$

$$+ \frac{3(x-1)(x-2.2)(x-4.8)(x-6)(x-7)}{((3.4)-1)(3.4-2.2)(3.4-4.8)(3.4-6)(3.4-7)}$$

$$+ \frac{3.2(x-1)(x-2.2)(x-3.4)(x-6)(x-7)}{(4.8-1)(4.8-2.2)(4.8-3.4)(4.8-6)(4.8-7)}$$

$$+\frac{4(x-1)(x-2.2)(x-3.4)(x-4.8)(x-7)}{(6-1)(6-2.2)(6-3.4)(6-4.8)(6-7)} +$$

$$\frac{5(x-1)(x-2.2)(x-3.4)(x-4.8)(x-6)}{(7-1)(7-2.2)(7-3.4)(7-4.8)(7-4)}$$

when $x = 5.4$

$$\therefore y(5.4) = \frac{7.3728}{(-328.32)} + \frac{14.19264}{73.98144} + \frac{24.33024}{(-37.73952)} + \frac{86.50752}{36.51648} + \frac{(-108.1344)}{(-59.28)} + \frac{(-50.188)}{684.288}$$

$$= -0.02246 + 0.191841 - 0.144689 + 2.36899 + 1.82413 - 0.070476$$

$y(5.4) = 3.5116$ {Used calculator}

(b) Newton's form.

x_i	1	2.2	3.4	4.8	6	7
y_i	2	2.8	3	3.2	4	5

for a_2

① $\frac{2.8-2}{2.2-1} = 0.66667$

② $\frac{3-2.8}{3.4-2.2} = 0.16667$

③ $\frac{3.2-3}{4.8-3.4} = 0.14286$

④ $\frac{4-3.2}{6-4.8} = 0.66667$

⑤ $\frac{5-4}{7-6} = 1$

$\therefore a_2 = [0.66667 \quad 0.16667 \quad 0.14286 \quad 0.66667 \quad 1]$

for a_3.

① $\frac{0.16667 - 0.66667}{3.4 - 1} = -0.20833$

② $\frac{0.14286 - 0.16667}{4.8 - 2.2} = -0.009158$

③ $\frac{0.66667 - 0.14286}{1 - 3.4} = 0.201465$

④ $\frac{1 - 0.66667}{7 - 4.8} = 0.15152$

$\therefore a_3 = \begin{bmatrix} -0.20833 & -0.009159 & 0.201465 & 0.15152 \end{bmatrix}$

for a_4,

① $\frac{-0.009159 - (-0.20833)}{4.8 - 1} = 0.52414$

② $\frac{0.201465 - (-0.009159)}{6 - 2.2} = 0.055437$

③ $\frac{0.15152 - 0.201465}{7 - 3.4} = -0.013888$

$\therefore a_4 = \begin{bmatrix} 0.52414 & 0.05543 & -0.01388 \end{bmatrix}$

for a_5,

① $\frac{0.05543 - 0.52414}{6 - 1} = 0.000603$

② $\frac{-0.01388 - 0.05543}{7 - 2.2} = -0.014438$

$\therefore a_5 = \begin{bmatrix} 0.000603 & -0.014438 \end{bmatrix}$

for a_6,

① $\dfrac{-0.014438 - 0.000603}{7-1} = -0.002507$ //

$a_6 = -0.002507$

$\therefore f(x) = a_1 + a_2(x-1) + a_3(x-1)(x-2.2) + a_4(x-1)(x-2.2)(x-3.4) + a_5(x-1)(x-2.2)(x-3.4)(x-4.8) + a_6(x-1)(x-2.2)(x-3.4)(x-4.8)(x-6)$

$f(5.4) = 2 + 0.66617(5.4-1) + (-0.208333)(5.4-1)(5.4-2.2) + 0.05241(5.4-1)(5.4-2.2)(5.4-3.4) + 0.000603(5.4-1)(5.4-2.2)(5.4-3.4)(5.4-4.8) + (-0.00251)(5.4-1)(5.4-2.2)(5.4-3.4)(5.4-4.8)(5.4-6)$

$= 2 + 2.9335 + (-2.93329) + 1.4759 + 0.01019 + 0.02545$

$f(5.4) = 3.51175$ ... {Used calculator}

4*. The following data is given:

t	1.3	1.7	1.9	2.3	2.7	3.1	3.6	4.0
y	2.8	3.2	3.1	3.5	4.8	4.2	5.3	4.8

Fit the data using cubic spline interpolation using the boundary conditions for natural splines. Plot the data using large points, a linear spline (for comparison), and the cubic spline. You must write an original program to solve for the spline equations and to make the plots, but previously developed codes may be used, i.e. the Thomas algorithm. Do not use Matlab intrinsic functions like `inv()` or \, etc.

Matlab Program:

- ***Script for Thomas algorithm***

```
% Function to implement thomas algorithm
function x = thomas_alg(a,b,c,rhs)

n = length(rhs);

c(1) = c(1)/b(1);
rhs(1) = rhs(1)/b(1);
b(1) = 1;

for k=2:n

    b(k) = b(k) - a(k-1)*c(k-1);
    rhs(k) = rhs(k) - a(k-1)*rhs(k-1);

    if (k < n)
        c(k) = c(k)/b(k);
    end

    rhs(k) = rhs(k)/b(k);
    b(k) = 1;
end

x(n) = rhs(n);
for k = n-1:-1:1
    x(k) = (rhs(k) - (c(k) * x(k+1)));
end
x = x';
```

- ***Script for plotting splines***

```
% generalised function to plot splines
function spline_plot(t,y)

n = length(t) -1;

a = ones(n,1);
```

```
b = ones(n+1,1);
b = b*4;
b(1) = 2;
b(n+1) = 2;
c = ones(n,1);

rhs(1) = 3*(y(2) - y(1));
rhs(n+1) = 3* (y(n+1)- y(n));
for i = 2:n
    rhs(i) = 3*(y(i+1) - y(i-1));
end

d = thomas_alg(a,b,c,rhs);

aa = zeros(n,1);
bb = zeros(n,1);
cc = zeros(n,1);
dd = zeros(n,1);

for i = 1:n
    aa(i) = y(i);
    bb(i) = d(i);
    cc(i) = 3*(y(i+1)-y(i)) - 2*d(i) - d(i+1);
    dd(i) = 2*(y(i)-y(i+1))+d(i)+d(i+1);
end

u=linspace(0,1,101);

plot(t,y,'*-');
xlabel("t");
ylabel("y");
hold on
grid on
axis padded

for i=1:n
    tt = u*(t(i+1) - (t(i))) + t(i);
    yy = aa(i) + bb(i) * u(:) + cc(i) * u(:).^2 + dd(i) * u(:).^3;
    plot(tt,yy,'-k')
end

legend('Linear spline','Cubic spline')
```

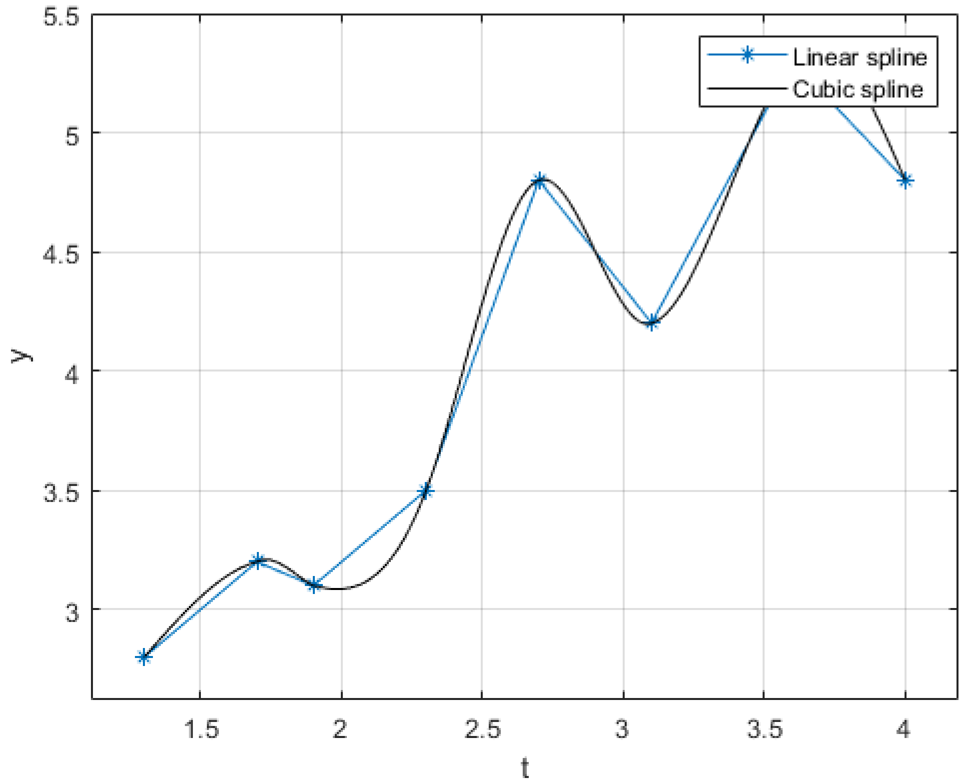
- ***Driver to run the above functions***

```
% Q4 driver
close all
clear all
clc

t = [1.3 1.7 1.9 2.3 2.7 3.1 3.6 4.0];
y = [2.8 3.2 3.1 3.5 4.8 4.2 5.3 4.8];

spline_plot(t,y)
```

Matlab Output:



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