ME 594 - Numerical Methods: Homework 4

Four Problems: (10 pts each) Note: A * next to a problem number indicates you must electronically submit a ".m" file.

1. Textbook problem 4.16 (Solve by hand): Carry out the first three iterations of the solution of the following system of equations using the Gauss-Seidel iterative method. For the first guess of the solution, take the value of all of the unknowns to be zero. Report the final answer using five significant digits.

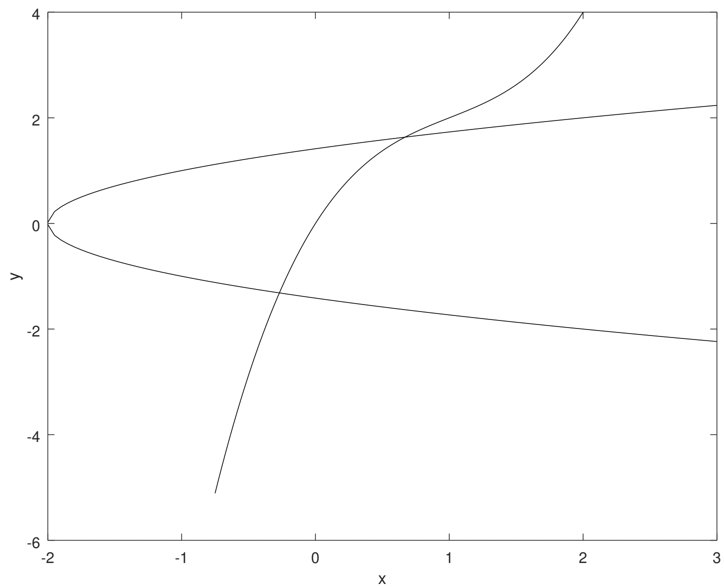
$$\begin{bmatrix} 4 & 0 & 1 & 0 & 1 \\ 2 & 5 & -1 & 1 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 0 & 1 & 0 & 4 & -2 \\ 1 & 0 & -1 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 32 \\ 19 \\ 14 \\ -2 \\ 41 \end{bmatrix}$$

2. Textbook problem 4.19 (Solve by hand): Find the condition number of the matrix in Problem 4.13 using the 1-norm. You may use Matlab to find the inverse matrix, but do not use Matlab to compute the norms or the condition number directly.

$$\begin{bmatrix} 10 & 12 & 0 \\ 0 & 2 & 8 \\ 2 & 4 & 8 \end{bmatrix}$$

3*. Consider the nonlinear system defined by the cubic and parabola:

$$f_1 = y - x^3 + 3x^2 - 4x = 0$$
$$f_2 = y^2 - x - 2 = 0.$$



Use the fixed point iteration method to create iteration functions to find the solutions for the system, i.e. the points of intersection of the curves. Use the *relative error* between iterations as the convergence criterion with a tolerance of 10^{-6}. Report the two solutions, the relative error, and the number of iterations used to converge to each point. You may start the iteration at any point close to the solution. You may use the `norm()` function to calculate the relative error. The answer below is reported using `format short`.

ans. $(x_1, y_1) = (-0.26950, -1.31548)$; $(x_2, y_2) = (0.66995, 1.63400)$

4*. Use Newton's method to solve

$$\begin{aligned} x^2 - x + y^2 + z^2 - 5 = 0 \\ x^2 + y^2 - y + z^2 - 4 = 0 \\ x^2 + y^2 + z^2 + z - 6 = 0 \end{aligned}$$

near the point $X_0^T = [x_0, y_0, z_0] = [0, 0, 0]$. You are <u>not</u> allowed to use any high-level Matlab intrisic functions (e.g. `inv(A)`, `A^-1`, `A\b`, etc.) to find the inverse of a matrix directly as this is unstable and not often done in practice; you may use a program that was developed for a previous homework assignment to solve the linear system. Assume the solution has converged when either the *relative error* or the *function error* is reduced below a tolerance of 10^{-8}. Report the solution, the relative error, the function error, and the number of iterations used to converge to the solution. You may use the `norm()` function to calculate the convergence criteria. The answer below is reported using `format short`.

$$\text{ans.} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -0.84713 \\ 0.15287 \\ 1.84713 \end{bmatrix}$$