Numerical methods - HW02

VIRAL PANCHAL

Q.1)

$\sqrt{x} + x^2 = 7$

Using Newton's method:

$f(x) = \sqrt{x} + x^2 - 7$, $x_0 = 7$

$f'(x) = \dfrac{1}{2\sqrt{x}} + 2x$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 7 - \dfrac{(44.6458)}{(14.189)} = 3.85349$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 3.85349 - \dfrac{(9.81243)}{(7.96169)} = 2.62103$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 2.62103 - \dfrac{(1.48879)}{(5.55091)} = 2.35283$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)} = 2.35283 - \dfrac{(0.0697)}{(5.031627)} = 2.3389$

$x_5 = x_4 - \dfrac{f(x_4)}{f'(x_4)} = (2.3389) - \left(\dfrac{1.852 \times 10^{-4}}{5.004886}\right) =$

2.33894 //

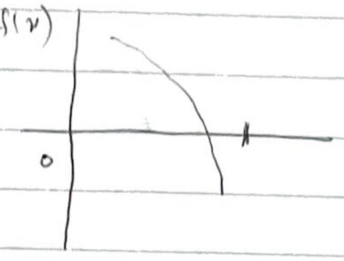
$\therefore x_5 = 2.33894$ // Ans

Q.2) $\cos x - 0.8x^2 = 0$

Positive root by fixed point method:

for fixed point method: $x = g(x)$

Assuming $x_0 = 1$



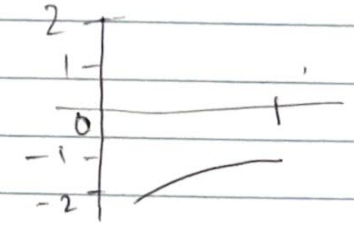
$\therefore 0.8x^2 = \cos x$

$$x = \frac{\cos x}{0.8x}$$

$$\therefore x = 1.25 \frac{\cos x}{x} = g_1(x)$$

$$g_1'(x) = 1.25\left(\frac{x(-\sin x)-(\cos x.1)}{x^2}\right)$$

$$= \frac{-1.25}{x^2}\{x\sin x + \cos x\}$$



$|g_1'(x)| > 1$

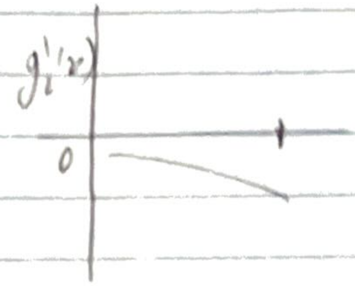
$\Downarrow$

diverging

Alternate solution;

$$x = \sqrt{1.25\cos x} = g_2(x)$$

$$g_2'(x) = \frac{1}{2\sqrt{1.25\cos x}} \cdot (-1.25\cos x) = \frac{-0.625\sin x}{\sqrt{1.25\cos x}}$$



$|g_1'(x)| < 1$
$\downarrow$
may converge

$x_0 = 1$

$x_1 = \sqrt{1.25\cos(1)} = 0.82181$

$x_2 = \sqrt{1.25\cos(0.82181)} = 0.92256$

$x_3 = \sqrt{1.25\cos(0.92256)} = 0.86875$

$x_4 = \sqrt{1.25\cos(0.86875)} = 0.89846$

$x_5 = \sqrt{1.25\cos(0.89846)} = 0.88234$ //

$x_5 = 0.88234$ // Ans

Q 3) Van der waals equation.

$$P = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}$$

$R = 0.08206 \, L/(\text{mole K})$

$n = 1.5 \, \text{Mol}$

$a = 1.39 \, \frac{L^2 \, \text{atm}}{\text{mol}^2}$ $\quad b = 0.03913 \, \frac{L}{\text{mol}}$

$T = 25^\circ C \quad = 298 K$
$P = 13.5$ atm

Based on the Matlab plot:

roots $\in (2, 3)$ ... {plot attached below}

i) Bisection method
ii) Secant metho.

} {Matlab program attached}

Q.4) Quasi-one-dimensional isentropic flow

$$\varepsilon = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/(2(\gamma-1))}$$

$$f(M) = \frac{1}{M}\left[\frac{2}{(\gamma+1)}\left(1+\frac{(\gamma-1)M^2}{2}\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} - \varepsilon = 0$$

$\varepsilon = 10.0$
$\gamma = 1.4$

To find: $M_1 < 1$
& $M_2 > 1$

... Matlab program attached.