ME 594 – Numerical Methods – HW05

Viral Panchal | Due Date: 10/26

"I pledge my honor that I have abided by the stevens honor system"

Q1) Inverse Power Method

MATLAB Program:

- ***Script for Inverse Power Method:***

```
% Function to determine smallest eigen value and its eigen vector using
% inverse power method.
function [lambda_n,v_n,n_iter] = inverse_power(A,x0,tol,max_iter)

[n,n] = size(A);
P = eye(n);
L = zeros(n,n);
L(1,1) = 1;
u = A;

for i = 1:n-1
    [max_element,k] = max(abs(u(i:n,i)));
    if (k > 1)
        temp_row = u(i,:);
        u(i,:) = u(i+k-1,:);
        u(i+k-1,:) = temp_row;

        temp_row = P(i,:);
        P(i,:) = P(i+k-1,:);
        P(i+k-1,:) = temp_row;
    end

    L(i+1,i+1) = 1;

    for j = i+1:n
        pivot = u(j,i)/u(i,i);
        L(j,i) = pivot;
        u(j,i:n) = u(j,i:n) - pivot.*u(i,i:n);
    end
end

x_k = x0/norm(x0);
y=zeros(n,1);
for k = 1:max_iter
    b = P*x_k;
    y(1) = b(1)/L(1,1);
    for i=2:n
        y(i) = (b(i)-L(i,1:i-1)*y(1:i-1))/L(i,i);
    end

    x_k(n) = y(n)/u(n,n);
    for i = n-1:-1:1
```

```
        x_k(i) = (y(i)-u(i,i+1:n)*x_k(i+1:n))/u(i,i);
    end
    x_k = x_k/norm(x_k);
    lambda_n = x_k'*A*x_k;
    error = norm(A*x_k-lambda_n*x_k);
    if (error < tol)
        v_n = x_k;
        n_iter = k;
        break
    end
    v_n = x_k;
    n_iter = -1;
end
```

- ***Script to run the above function:***

```
% Q1 driver
close all
clear all
clc

n = 25;
A = zeros(n);
A(1,1) = -2;
A(1,2) = 1;
A(n,n-1) = 1;
A(n,n) = -2;
for i=2:n-1
    A(i,i-1) = 1;
    A(i,i) = -2;
    A(i,i+1) = 1;
end

x0 = ones(n,1);
tol = 10^(-6);
max_iter = 1000;

[lambda_min,v_min,n_iter] = inverse_power(A,x0,tol,max_iter);
lambda_min
lamda_theoretical = -pi^2/(n+1)^2
v_min = [0 v_min' 0]';
x = linspace(0,1,n+2);
plot(x,v_min)
xlabel("x")
ylabel("y")
grid on
title("Buckling deflection mode")
```

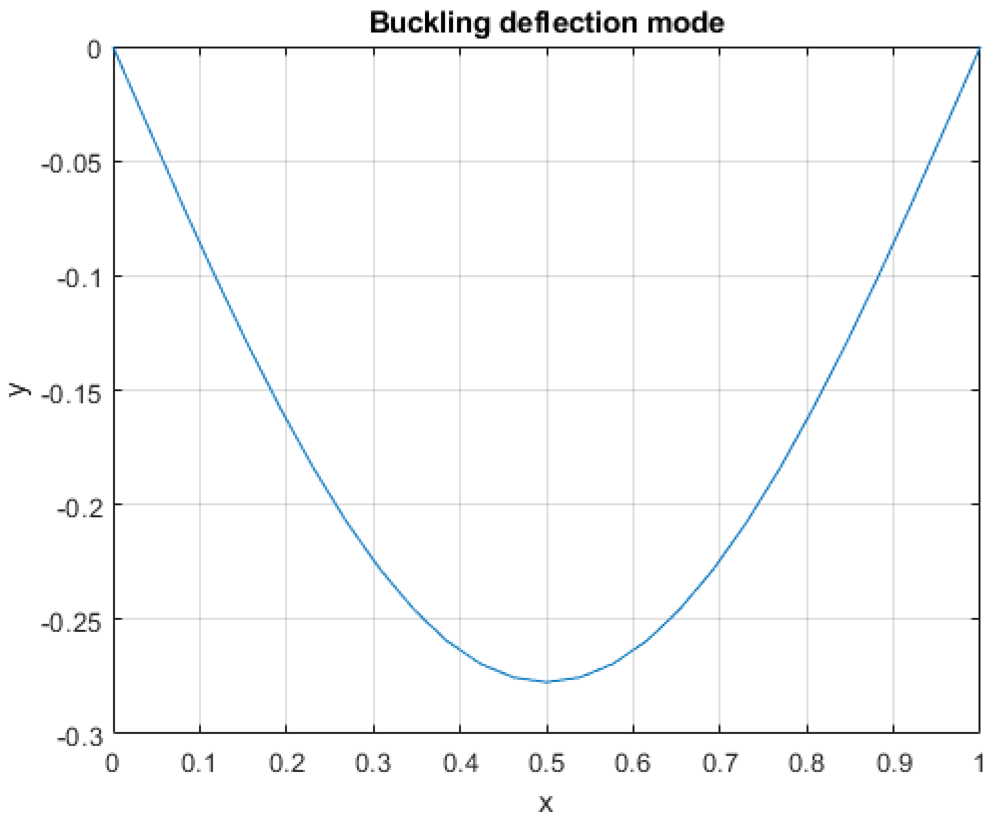
- ***MATLB Output:***

```
lambda_min =

   -0.0146


lamda_theoretical =

   -0.0146
```



[Published with MATLAB® R2021a](#)

Q2) Givens Rotation – QR Iteration

MATLAB Program:

- ***Script for matrix decomposition using givens method***

```
% Function to decompose A using Givens method
function [Q,R] = givens_qr(A)

[n,n] = size(A);
Q = eye(n);

for i = 1:n-1
    for j=i+1:n
        [c,s] = givens_params(A(i,i),A(j,i));
        A = givens_mul(A,i,j,c,s);
        Q = givens_mul(Q,i,j,c,s);
    end
end

R = A;
Q = Q';
```

- ***Script to determine givens parameters***

```
% Functions to determine the givens parameters (c,s)
function [c,s] = givens_params(x_i,x_j)

if (x_j==0)
    c = 1;
    s = 0;
elseif (abs(x_j)>abs(x_i))
    t = x_i/x_j;
    s = 1/sqrt(1+t^2);
    c = s*t;
else
    t = (x_j)/(x_i);
    c = 1/sqrt(1+t^2);
    s = c*t;
end
```

- ***Script to generate the givens multiple matrix***

```
% Function to make a givens multiple matrix
function A = givens_mul(A,i,j,c,s)

a = A(i,:);
b = A(j,:);

A(i,:) = (c*a)+(s*b);
A(j,:) = (-s*a)+(c*b);
```

- ***Script to perform QR iteration***

```
% Function to perform QR iteration and determine the eigen values
function [eig_vals] = qr_iter(A)

for i=1:40
    [Q,R] = givens_qr(A);
    A = R * Q;
end

eig_vals = diag(A);
```

- ***Driver script to run Q2***

```
% Q2 driver
close all
clear all
clc

A = [2.5 -2 2.5 0.5
    0.5 5 -2.5 -0.5
    -1.5 1 3.5 -2.5
    2 3 -5 3]

[eig_vals] = qr_iter(A)
```

MATLAB Output:

```
A =

    2.5000   -2.0000    2.5000    0.5000
    0.5000    5.0000   -2.5000   -0.5000
   -1.5000    1.0000    3.5000   -2.5000
    2.0000    3.0000   -5.0000    3.0000


eig_vals =

    6.0000
    4.0000
    3.0000
    1.0000
```

Published with MATLAB® R2021a

Q3) Householder Reflections – QR iteration

MATLAB Program

- ***Script to decompose given matrix using HHR method:***

```
% Function to decompose given matrix using HHR method
function [Q,R] = hhr_qr(A)

% Input - A matrix (3*3)
% Output - Q --> dialgonal matrix | R-->Upper triangular matrix

R = A;
[n,n] = size(R);
I = eye(n);
Q=I;

for i = 1:n-1
    col=R(:,i);
    if(i>1)
        col(1:i-1)=0;
    end
    max_col = max(abs(col));
    col = col/max_col;
    e(1:n,1)=0;
    e(i)=1;

    mag_col = sqrt(col'*col);

    if(col(i) >= 0)
        u = col + mag_col*e;
    else
        u = col - mag_col*e;
    end

    beta = 2/(u'*u);
    H = I - (beta*u*u');

    Q = Q*H;
    R = H*R;

end
```

- ***Script to perform QR iteration:***

```
% Function to perform QR iteration
function [eig_vals] = qr_iter(A)

for i=1:20
    [Q,R] = hhr_qr(A);
    A = R*Q;
end

eig_vals = diag(A);
end
```

- ***Driver to run Q3***

```
% Q3 driver
close all
clear all
clc

A = [7 6 -3
    -12 -20 24
    -6 -12 16]

[eig_vals] = qr_iter(A)
```

MATLAB Output:

```
A =

     7     6    -3
   -12   -20    24
    -6   -12    16


eig_vals =

   -2.0000
    4.0000
    1.0000
```

Published with MATLAB® R2021a

Q4) Jacobi Method to determine eigen pairs:

MATLAB Program:

- ***Function for Jacobi method***

```
% Function to determine eigen pairs using Jacobi method.
function [V,D,num_sweeps] = jacobi_cyclic(A,eps,max_sweeps)
 %Input - Matrix A (4*4)
 %Output - V-->Eigen vectors | D-->Diagonal matrix with eigen values

D=A;
[n,n] = size(A);
V = eye(n);
num_sweeps = 0;
tol_reached = false;

while((num_sweeps <= max_sweeps) && (~tol_reached))
    for i = 1:n-1
        for j = i+1:n
           tau = (D(j,j) - D(i,i))/2/D(i,j);
           if (tau >= 0)
               t = 1/(tau + sqrt(tau^2 + 1));
           else
               t = -1/(-tau + sqrt(tau^2 + 1));
           end
           c = 1/sqrt(1+t^2);
           s = c*t;

           R = [c s; -s c];
           D([i j],:) = R'*D([i j],:);

           D(:,[i j]) = D(:,[i j])*R;

           V(:,[i j]) = V(:,[i j])*R;
        end
    end

    off_A = 0;
    for i = 1:n-1
        for j = i+1:n
            off_A = off_A + D(i,j)^2;
        end
    end

    off_A = sqrt(2*off_A);
    if (off_A < eps)
        tol_reached = true;
    end
    num_sweeps = num_sweeps + 1;
end

D = diag(diag(D));
```

- ***Driver to run Q4***

```
% Q4 driver

close all
clear all
clc

A = [-8 16 23 -13
    16 9 2 3
    23 2 1 -23
    -13 3 -23 -7]

eps = 10^(-6);
max_sweeps = 100;

[V,D,num_sweeps] = jacobi_cyclic(A,eps,max_sweeps)
```

MATLAB Output:

```
A =

    -8    16    23   -13
    16     9     2     3
    23     2     1   -23
   -13     3   -23    -7


V =

    0.6928    0.5461   -0.1499    0.4464
   -0.2301    0.2943   -0.8784   -0.2980
   -0.6473    0.6355    0.2742    0.3193
   -0.2191   -0.4596   -0.3615    0.7811


D =

  -30.6917         0         0         0
         0   38.3264         0         0
         0         0   12.3412         0
         0         0         0  -24.9760


num_sweeps =

     4
```

Published with MATLAB® R2021a