ME 594 – Numerical Methods - HW01

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Numerical methods -HW01
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Q.1) $f(x) = \frac{1-\cos(x)}{\sin(x)}$

a) $x = 0.007$ ← rad.
$x = 0.401070$ ← deg.

$\therefore f(0.007) = \frac{1-\cos(0.401070)}{\sin(0.401070)}$
$= 0.003428$ ← f_cal

$\therefore \boxed{f(0.007) = 0.003428}$

b) Matlab attached below.

f_Mat = 0.003500

True relative error (TRE) $= \left| \frac{F\_Mat - f\_cal}{F\_cal} \right|$

$= \left| \frac{0.003500 - 0.003429}{0.003429} \right|$

$\boxed{TRE = 0.020706}$

c) $f(x) = \frac{1-\cos(x)}{\sin(x)} \times \frac{1+\cos(x)}{1+\cos(x)}$

$= \frac{1-\cos^2(x)}{\sin(x)(1+\cos(x))} = \frac{\sin^2(x)}{\sin(x)(1+\cos(x))}$

$$\therefore\ f(x) = \frac{\sin(x)}{(1+\cos(x))}$$

$$= \frac{\sin(0.401070)}{1+\cos(0.401070)}$$

$$= \frac{0.006999}{1+0.999976}$$

$$f(x) = 0.003499 \quad // \leftarrow \text{f\_cal 2}$$

$$TRE = \left| \frac{\text{f\_mat} - \text{f\_cal2}}{\text{f\_cal2}} \right|$$

$$= \left| \frac{0.003500 - 0.003499}{0.003499} \right|$$

$$= 0.000286 \quad //$$

$$\therefore \boxed{TRE = 0.000286}$$

Matlab Program:

Q1.B

```
% f_cal --> F using calculator
% f_mat --> F using Matlab
% TSE --> True relative error

format long
x = 0.007;
f_mat = (1 - cos(x))/sin(x);
fprintf('f_mat =');
disp(f_mat)

f_cal = 0.003429;
TRE = abs((f_mat - f_cal)/f_cal);
fprintf('TRE =');
disp(TRE)
```

```
f_mat =   0.003500014291730

TRE =   0.020709913015556
```

Q1.C

```
f_cal2 = 0.003499;
TRE_2 = abs((f_mat - f_cal2)/f_cal2);
fprintf('TRE_2 = ');
disp(TRE_2)
```

```
TRE_2 =      2.898804602291635e-04
```

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Q.2) MatLab program attached.

conv. dec → bin

dec_int / 2 → Remainder 0 / 1 $\xrightarrow{\text{Yes}}$ append in array

↓ No

error

Driver for Q2

function file name - intTObina.m

```
% Test case 1 | d = 81
fprintf('Binary when d = 81 \n');
b = intTObina(81);

% Test case 2 | d = 30952
fprintf('Binary when d = 30952 \n');
b = intTObina(30952);

% Test case 3 | d = 1500000
fprintf('Binary when d = 1500000 \n');
b = intTObina(1500000);
```

Function Program;

```
% Function to convert integer to binary
% Q2
function b = intTObina(d)

b = [];

while d > 0
    r = rem(d,2);
    if (r ==0 || r==1)
        b = [r b];
    else
        fprintf('Error')
    end
    q = floor(d/2);
    d = q;
end
if (length(b) <= 20)
    fprintf('b = ');
    disp(b)
else
   fprintf('Error: Type a smaller number \n');
end
end
```

Matlab Output:

```
Binary when d = 81
b =      1     0     1     0     0     0     1

Binary when d = 30952
b =   Columns 1 through 13

     1     1     1     1     0     0     0     1     1     1     0     1     0

  Columns 14 through 15

     0     0

Binary when d = 1500000
Error: Type a smaller number
```

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Q.3) Matlab program attached.

$$R = (d_1 \times 2^{-1}) + (d_2 \times 2^{-2}) + \dots + (d_n \times 2^{-n}),$$

$R \leftarrow$ User input

$d \leftarrow$ digits (1 to 7)

$(d \leq 7)$ && $(R > 0)$

$R = 2R;$

$d = \text{floor}(R)$

$R = R - d_i$

~~teo~~ loop back

Chopping (No changes)

Rounding (increase digits and apply rounding if digit == 1)

→ Program attached.

Driver for Q3

function file name - binfrac.m

```
%Test case 1 | R = 0.40625
fprintf('when R = 0.40625 \n');
[chopped_number_1,rounded_number_1] = binfrac(0.40625)

fprintf('---------------------------------------------------------\n');

%Test case 2 | R = 0.7
fprintf('when R = 0.7 \n');
[chopped_number_2,rounded_number_2] = binfrac(0.7)

fprintf('---------------------------------------------------------\n');

%Test case 3 | R = 0.12109375
fprintf('when R = 0.121090375 \n');
[chopped_number_3,rounded_number_3] = binfrac(0.12109375)
```

Function program:

```
% Binary fractions
% Q3
function [chopped,rounded] = binfrac(R)
format long
i = 1;
i_max = 7;
a = [];

while (i<=i_max) && (R > 0)
    R = 2 * R;
    d = floor(R);
    a = [a d];
    R = R - d;
    i = i + 1;
end

% printing in 0.dddd way
chopped = '0.';
for j = 1:i-1
    chopped = strcat(chopped,(int2str(a(j))));
end

% When the number can be represented exactly
rounded = 'Rounding is not required';

% When the number requires additional unavailable digits
if (R > 0)
    fprintf('Failed to store the entire number \n');
    R = 2 * R;
    d = floor(R);
    b = a;
    c = i_max;
    while (d==1)
        f = d + b(c);
        b(c) = mod(f,2);
        d = f - d;
        c = c - 1;
    end

    % printing the new rounded value in 0.ddddd format
    rounded = '0.';
    for j = 1:i-1
        rounded = strcat(rounded,(int2str(b(j))));
    end

    % Computing the decimal representations after chooping and rounding
    chopped_dec = 0;
    rounded_dec = 0;
    for j = 1:i_max
        chopped_dec = chopped_dec + a(j)/2^j;
        rounded_dec = rounded_dec + b(j)/2^j;
    end

    %printing the decimal values
    fprintf('\n Decimal representation after chopping = ');
    disp(chopped_dec)
    fprintf('\n Decimal representation after rounding = ');
    disp(rounded_dec)
end
```

Matlab Output:

```
when R = 0.40625

chopped_number_1 =

    '0.01101'


rounded_number_1 =

    'Rounding is not required'

---------------------------------------------------------
when R = 0.7
Failed to store the entire number

 Decimal representation after chopping =    0.695312500000000


 Decimal representation after rounding =    0.703125000000000


chopped_number_2 =

    '0.1011001'


rounded_number_2 =

    '0.1011010'

---------------------------------------------------------
when R = 0.121090375
Failed to store the entire number

 Decimal representation after chopping =    0.117187500000000


 Decimal representation after rounding =    0.125000000000000


chopped_number_3 =

    '0.0001111'


rounded_number_3 =

    '0.0010000'
```

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Q.4) Taylor series

$y = \cos x$ $\quad x = 0$

3, 5 & 7 terms of Taylor series

a) To find: $f\left(\frac{\pi}{3}\right)$, $f\left(\frac{2\pi}{3}\right)$ & TRE

for 3, 5 & 7 terms.

- when 3 terms.

$$f_3(x) = \cos x\Big|_{x=0} + (-\sin x)\Big|_{x=0}(x-0) + \frac{1}{2!}(-\cos x)\Big|_{x=0}(x-0)^2$$

$$f_3(x) = 1 - \frac{x^2}{2}, \quad f_3\left(\frac{\pi}{3}\right) = 1 - \frac{(\pi/3)^2}{2} = 0.451689$$

$f_{3\_mat} = 0.5$

$$\therefore TRE = \left|\frac{0.451689 - 0.5}{0.5}\right| = 0.096622$$

$$f_3\left(\frac{2\pi}{3}\right) = 1 - \frac{(2\pi/3)^2}{2} = -1.19325$$

$f_{3\_mat} = -0.5$

$$\therefore TRE = \left|\frac{-1.19325 + 0.5}{-0.5}\right| = 1.3865$$

- when 5 terms

$$f_5(x) = 1 - \frac{x^2}{2} + \frac{1}{3!}(\sin x)\Big|_{x=0}(x-0)^3 + \frac{1}{4!}(\cos x)\Big|_{x=0}(x-0)^4$$

$$f_5(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$$f_5\left(\frac{\pi}{3}\right) = 0.501798, \quad f_{mat} = 0.5$$

$$\therefore TRE = \left|\frac{0.501798 - 0.5}{0.5}\right| = 0.00389$$

$$f_5\left(\frac{2\pi}{3}\right) = -0.391525 \quad f.\text{mat} = -0.5$$

$$TRE = \left|\frac{-0.391525+0.5}{-0.5}\right| = 0.21695$$

- when 7 terms

$$f_7 = 1-\frac{x^2}{2}+\frac{x^4}{24}+\frac{1}{5!}(-\sin x)\Big|_{x=0}(x-0)^5+\frac{1}{6!}(-\cos x)\Big|_{x=0}(x-0)^6$$

$$f_7 = 1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}$$

$$f_7\left(\frac{\pi}{3}\right) = 0.499965.$$

$$TRE = \left|\frac{0.499965-0.5}{0.5}\right| = 0.709 \times 10^{-6}$$

$$f_7\left(\frac{2\pi}{3}\right) = -0.508749$$

$$TRE = \left|\frac{-0.508749+0.5}{-0.5}\right| = 0.017448$$

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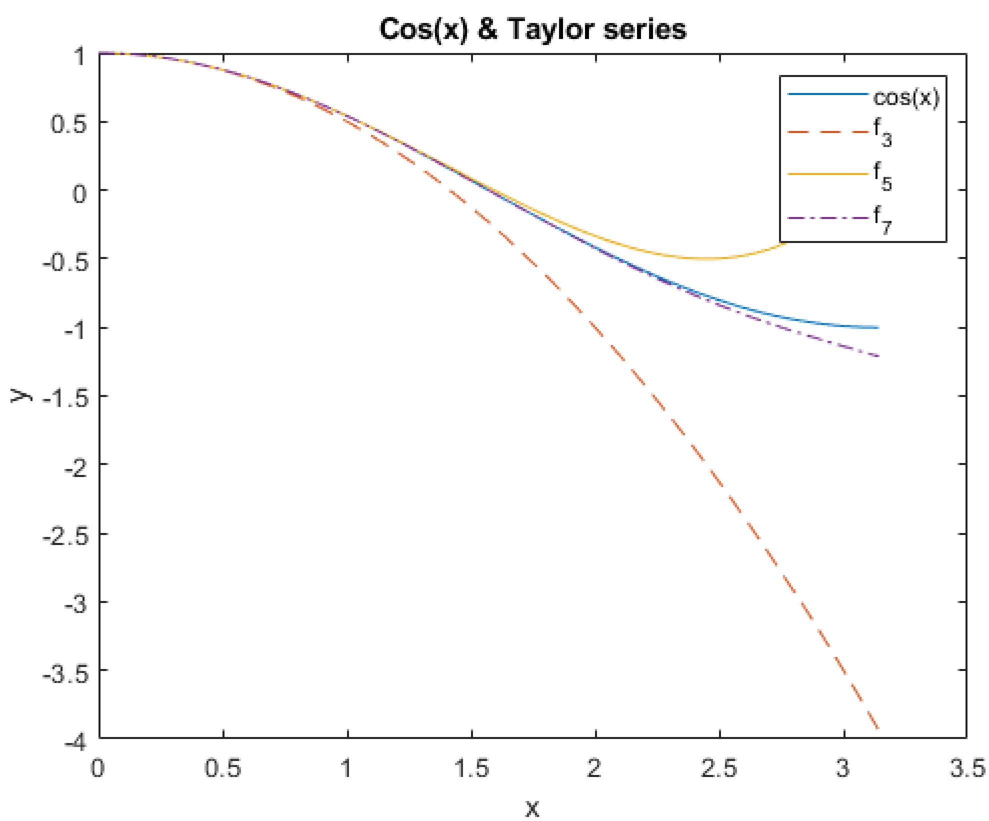
Viral

Q4 - part 2

plotting the three approximations solved in part 1

```
x = linspace(0,pi,1000);
f_x = cos(x);

f_3 = 1 - (x.^2/2);
f_5 = 1 - (x.^2/2) + (x.^4/24);
f_7 = 1 - (x.^2/2) + (x.^4/24) - (x.^6/720);
plot(x,f_x,x,f_3,'--',x,f_5,'-',x,f_7,'-.');
title('Cos(x) & Taylor series')
xlabel('x');
ylabel('y');
legend('cos(x)','f_3','f_5','f_7');
```



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