HW08.
VIRAL PANCHAL

Q.1) Given:

$$y' = -2xy \qquad y(0) = 1$$

$h = 0.4, 0.2, 0.1, 0.05$

domain $[0, 6]$

a) Using Explicit Euler

$$y_{k+1} = y_k + hf(x_k, y_k)$$

$$= y_k + h[-2x_k y_k]$$

$$y_{k+1} = (1 - 2x_k h)\, y_k$$ ... Solved in Matlab

b) Using Implicit Euler

$$y_{k+1} = y_k + hf(x_{k+1}, y_{k+1})$$

$$= y_k + h[-2x_{k+1}, y_{k+1}]$$

$$\therefore y_{k+1}(1 + 2x_{k+1} h) = y_k$$

$$\therefore y_{k+1} = \frac{y_k}{(1 + 2x_{k+1} h)}$$ ... Matlab program attached.

Q2) Given:

$$y' = -xy^3 \qquad y(0) = 1$$

$h = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ domain $[0, 20]$

Using Implicit Euler method:

$$y_{k+1} = y_k + hf(x_{k+1}, y_{k+1})$$

$$= y_k + h\left[-x_{k+1}\, y_{k+1}^3\right]$$

$$y_{k+1} = y_k - h x_{k+1} y_{k+1}^3$$

← This non-linear equation can be solved using Newton's method.

$$g(y_{k+1}) = h x_{k+1} y_{k+1}^3 - y_k = 0$$

let's say $z = y_{k+1}$

$$g(z) = h x_{k+1} z^3 + z - y_k$$

$$\therefore g'(z) = 3 h x_{k+1} z^2 + 1$$

$$\therefore z^{(k+1)} = z^{(k)} - \frac{g(z)}{g'(z)}$$

↓

Matlab code attached

Q.3) ~~Giving~~ Given:

$$y' = x - \frac{xy}{2} \quad ; \quad y(1) = 1$$

domain $[1, 5]$

a) Using Trapezoid method

$h = 0.8, 0.4, 0.2$

$$y_{K+1} = y_K + \frac{h}{2}\left[f(x_K, y_K) + f(x_{K+1}, y_{K+1})\right]$$

$$y_{K+1} = y_K + \frac{h}{2}\left[\left(x_K - \frac{x_K y_K}{2}\right) + \left(x_{K+1} - \frac{x_{K+1} y_{K+1}}{2}\right)\right]$$

$$= y_K + \frac{h}{2}\left[x_K + x_{K+1} - \frac{x_K y_K}{2}\right] - \frac{h}{4} x_{K+1} y_{K+1}$$

$$\left(1 + \frac{h}{4} y_{K+1}\right) y_{K+1} = y_K + \frac{h}{2}\left[x_K + x_{K+1} - \frac{x_K y_K}{2}\right]$$

$$y_{K+1} = \frac{y_K + h/2\left[x_K + x_{K+1} - x_K y_K/2\right]}{1 + \frac{h}{4} y_{K+1}}$$

b) Using Improve Euler method:

$$y^*_{K+1} = y_K + h f(x_K, y_K)$$

$$= y_K + h\left[x_K - \frac{x_K y_K}{2}\right]$$

$$y_{K+1} = y_K + h/2\left[f(x_K, y_K) + f(x_{K+1}, y^*_{K+1})\right]$$

$$y_{K+1} = y_K + \frac{h}{2}\left[\left(x_K - \frac{x_K y_K}{2}\right) + \left(x_{K+1} - \frac{x_{K+1} y^*_{K+1}}{2}\right)\right]$$

Matlab program attached