ME 594 - Numerical Methods: Homework 3

Four Problems: (10 pts each) Note: A * next to a problem number indicates you must electronically submit a ".m" file.

1*. (10 pts) Write a program to find the inverse of a matrix using Gaussian Elimination with back substitution, as discussed in lecture. The algorithm should be able to handle pivoting, if necessary. Use the same algorithm to find the inverse of (a)

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 3 & -1 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 2 & 3 \end{bmatrix},$$

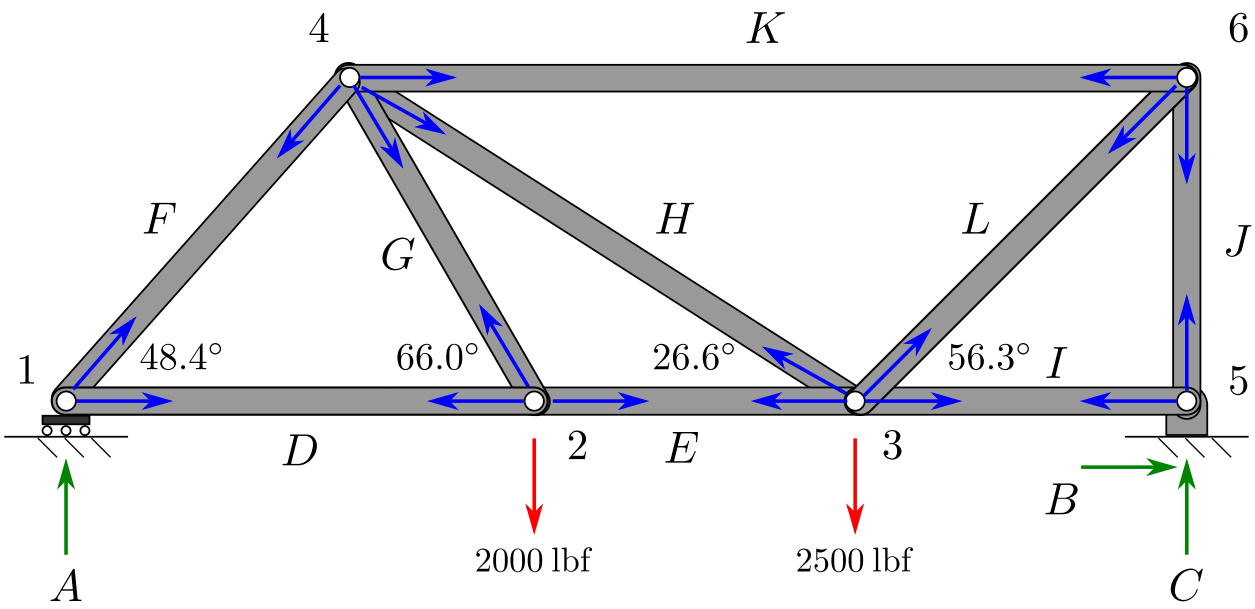
and (b) the matrix given in the file (located on Canvas) `Prob1-InputMatrix.m`. Confirm your answer in both cases by computing the matrix product $A\,A^{-1} = I$.

2*. (10 pts) Textbook problem 4.23: Write a user-defined MATLAB function that decomposes an $n \times n$ matrix $[A]$ into a lower triangular matrix $[L]$ and an upper triangular matrix $[U]$ (such that $[A] = [L][U]$) using the Gauss elimination method (without pivoting). For the function name and arguments, use `[L,U]=LUdecompGauss(A)`, where the input argument `A` is the matrix to be decomposed and the output arguments `L` and `U` are the corresponding upper and lower triangular matrices. Use `LUdecompGauss` to determine the LU decomposition of the following matrix:

$$\begin{bmatrix} 4 & -1 & 3 & 2 \\ -8 & 0 & -3 & -3.5 \\ 2 & -3.5 & 10 & 3.75 \\ -8 & -4 & 1 & -0.5 \end{bmatrix}$$

Note: No credit will be given for codes using Crout's method as there is source code in the textbook.

3*. (10 pts) Write an original program to find the forces necessary to hold the truss in equilibrium. The solution of the resulting system of equations requires partial pivoting.



$$\text{ans. } x = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{bmatrix} = \begin{bmatrix} 1718.8 \\ 0 \\ 2781.2 \\ 1526.0 \\ 2416.5 \\ -2298.4 \\ 2189.3 \\ -628.08 \\ 0 \\ -2781.2 \\ -1854.8 \\ 3343.0 \end{bmatrix}$$

4*. (10 pts) Write an original program that uses the Thomas Algorithm to solve the tridagonal system

$$\begin{aligned} x_1 + x_2 &= 5 \\ 2x_1 - x_2 + 5x_3 &= -9 \\ 3x_2 - 4x_3 + 2x_4 &= 19 \\ 2x_3 + 6x_4 &= 2. \end{aligned}$$

The program should accept four vectors, the main diagonal; the subdiagonal; the superdiagonal; and the right hand side vector.