Numerical Methods - HW 01

VIRAL PANCHAL

Q.1) $f(x) = \dfrac{1-\cos(x)}{\sin(x)}$

a) $x = 0.007 \leftarrow$ rad.

$x = 0.401070 \leftarrow$ deg.

$\therefore f(0.007) = \dfrac{1-\cos(0.401070)}{\sin(0.401070)}$

$= 0.003428 \leftarrow f\_cal$

$\therefore \boxed{f(0.007) = 0.003428}$

b) Matlab attached below.

$f\_Mat = 0.003500$

True relative error (TRE) $= \left| \dfrac{F\_Mat - f\_cal}{F\_cal} \right|$

$= \left| \dfrac{0.003500 - 0.003429}{0.003429} \right|$

$\therefore \boxed{TRE = 0.020706}$

c) $f(x) = \dfrac{1-\cos(x)}{\sin(x)} \times \dfrac{1+\cos(x)}{1+\cos(x)}$

$= \dfrac{1-\cos^2(x)}{\sin(x)(1+\cos(x))} = \dfrac{\sin^2(x)}{\sin(x)(1+\cos(x))}$

$$\therefore f(x) = \frac{\sin(x)}{(1+\cos(x))}$$

$$= \frac{\sin(0.401070)}{1+\cos(0.401070)}$$

$$= \frac{0.006999}{1+0.999976}$$

$$f(x) = 0.003499 \quad // \leftarrow f\_cal2$$

$$TRE = \left| \frac{f\_Mat - f\_cal2}{f\_cal2} \right|$$

$$= \left| \frac{0.003500 - 0.003499}{0.003499} \right|$$

$$= 0.000286 \quad //$$

$$\therefore \boxed{TRE = 0.000286}$$

Q.2) Matlab program attached.

Conv. dec → bin

dec_int / 2 → Remainder 0 / 1 → Yes → append in array

↓ No

error

Q.3) Matlab program attached.

$$R = (d_1 \times 2^{-1}) + (d_2 \times 2^{-2}) + \dots + (d_n \times 2^{-n}),$$

R ← User input
d ← digits (1 to 7)
(d ≤ 7) && (R > 0)
R = 2R ;
d = floor(R)
R = R - di

→ loop back

↓ Chopping (No changes)

↘ Rounding (increase digits and apply rounding if digit == 1)

→ Program attached.

Q.4) Taylor series

$y = \cos x$ $\quad x = 0$

3, 5 & 7 terms of Taylor series

a) To find: $f\left(\frac{\pi}{3}\right)$, $f\left(\frac{2\pi}{3}\right)$ & TRE

for 3, 5 & 7 terms.

- When 3 terms.

$$f_3(x) = \cos x\big|_{x=0} + (-\sin x)\big|_{x=0}(x-0) + \frac{1}{2!}(-\cos x)\big|_{x=0}(x-0)^2$$

$$f_3(x) = 1 - \frac{x^2}{2}, \quad f_3\left(\frac{\pi}{3}\right) = 1 - \frac{(\pi/3)^2}{2}$$

$$= 0.451689$$

$f_3\_mat = 0.5$

$$\therefore TRE = \left|\frac{0.451689 - 0.5}{0.5}\right| = 0.091622 //$$

$$f_3\left(\frac{2\pi}{3}\right) = 1 - \frac{(2\pi/3)^2}{2} = -1.19325 //$$

$f_3\_mat = -0.5$

$$\therefore TRE = \left|\frac{-1.19325 + 0.5}{-0.5}\right| = 1.3865 //$$

- When 5 terms

$$f_5(x) = \left| -\frac{x^2}{2} + \frac{1}{3!}(\sin x)\right|_{x=0}(x-0)^3 + \frac{1}{4!}(\cos x)\big|_{x=0}(x-0)^4$$

$$f_5(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$$f_5\left(\frac{\pi}{3}\right) = 0.501798, \quad f\_mat = 0.5$$

$$\therefore TRE = \left|\frac{0.501798 - 0.5}{0.5}\right| = 0.003\,89$$

$f_5\left(\frac{2\pi}{3}\right) = -0.391525 \quad f.\text{val} = -0.5$

$$TRE = \left|\frac{-0.391525 + 0.5}{-0.5}\right| = 0.21695 //$$

- When 7 terms

$$f_7 = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \frac{1}{5!}(-\sin x)\Big|_{x=0}(x-0)^5 + \frac{1}{6!}(-\cos x)\Big|_{x=0}(x-0)^6$$

$$f_7 = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$$

$$f_7\left(\frac{\pi}{3}\right) = 0.499965$$

$$TRE = \left|\frac{0.499965 - 0.5}{0.5}\right| = 0.709 \times 10^{-6} //$$

$$f_7\left(\frac{2\pi}{3}\right) = -0.508749$$

$$TRE = \left|\frac{-0.508749 + 0.5}{-0.5}\right| = 0.017448 //$$

"I pledge my honor that I have abided by the Stevens Honor System".

Virali