

HWO6

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Q.1) a) Linear least-squares regression.

Given:

x	-7	-4	-1	0	2	5	7
y	20	14	5	3	-2	-10	-15

Sol:

$$m = \frac{nS_{xy} - S_x S_y}{nS_{xx} - (S_x)^2}; \quad b = \frac{S_{xx} S_y - S_x S_y S_x}{nS_{xx} - (S_x)^2}$$

x	y	xy	xx
-7	20	-140	49
-4	14	-56	16
-1	5	-5	1
0	3	0	0
2	-2	-4	4
5	-10	-50	25
<u>2</u>	<u>-15</u>	<u>-105</u>	<u>49</u>
<u>2</u>	<u>15</u>	<u>-360</u>	<u>144</u>

$$\therefore m = \frac{7(-360) - 2(15)}{7(144) - (2)^2} = -2.53984 //$$

$$\therefore b = \frac{144(15) - (-360)(2)}{7(144) - (2)^2} = 2.86852 //$$

Ans.

$$b) E = \sum_{i=1}^n [y_i - (mx_i + b)]^2$$

$$= [20 - (m(-7) + b)]^2 + [14 - (m(-4) + b)]^2 +$$

$$[5 - (m(-1) + b)]^2 + [3 - (m(0) + b)]^2 +$$

$$[-2 - (m(2) + b)]^2 + [-10 - (m(5) + b)]^2 +$$

$$[-15 - (m(7) + b)]^2$$

Substituting  $m$  &  $b$

$$E = 0.419140 + 0.945001 + 0.166763 + 0.0172854$$

$$+ 0.0445866 + 0.0286702 + 0.00803559$$

$$\therefore E = 1.62948 \quad // \text{ Ans.}$$

Q.2) General linear least squares method:

Given:

$x$	0.8	1.6	2.4	3.2	4.0
$y$	6	3.6	4.1	5.1	6.2

5 data points, 2<sup>nd</sup> order

$$y = f(x) = \sum_{j=1}^n c_j f_j(x)$$



$$\sum_{j=1}^M \left[ \sum_{k=1}^N f_i(x_k) f_j(x_k) \right] c_j = \sum_{k=1}^N f_i(x_k) y_k$$

$$\left( \sum_{k=1}^N f_1(x_k) f_1(x_k) \right) c_1 + \left( \sum_{k=1}^N f_1(x_k) f_2(x_k) \right) c_2 = \sum_{k=1}^N f_1(x_k) y_k$$

$$\left( \sum_{k=1}^N f_2(x_k) f_1(x_k) \right) c_1 + \left( \sum_{k=1}^N f_2(x_k) f_2(x_k) \right) c_2 = \sum_{k=1}^N f_2(x_k) y_k$$

$$\therefore \left( \sum_{k=1}^N x_k^2 \right) c_1 + \left( \sum_{k=1}^N 1/x_k \right) c_2 = \sum_{k=1}^N x_k y_k$$

$$\therefore \left( \sum_{k=1}^N 1/x_k \right) c_1 + \left( \sum_{k=1}^N 1/x_k^4 \right) c_2 = \sum_{k=1}^N y_k / x_k^2$$

$y_k$	$x_k$	$x_k^2$	$1/x_k$	$1/x_k^4$	$x_k y_k$	$y_k / x_k^2$
6	0.8	0.64	1.25	2.4414	4.8	9.375
3.6	1.6	2.56	0.625	0.15625	5.76	1.40625
4.1	2.4	5.76	0.4166	0.0301	9.84	0.71180
5.1	3.2	10.24	0.3125	0.00954	16.32	0.49804
6.2	4.0	16	0.25	0.00390	24.8	0.3675
<u>25</u>	<u>12</u>	<u>35.2</u>	<u>2.8541</u>	<u>2.63744</u>	<u>61.52</u>	<u>12.37860</u>

$$25(35.2) c_1 + (2.8541) c_2 = 61.52$$

$$(2.8541) c_1 + (2.63744) c_2 = 12.37860$$

$$c_2 = \frac{12.3786 - 2.8541 c_1}{2.63744}$$

$$35.2 C_1 + \frac{2 \cdot 85417}{2 \cdot 63758} (12.37860 - 2 \cdot 85417 C_1) = 61.52$$

$$C_1 = a = 1.49868$$

$$C_2 = b = 3.07141$$

Q.3). Given:

$x$	1	2.2	3.4	4.8	6	7
$y$	2	2.8	3	3.2	4	5

a) Lagrange form

$$f(x) = \sum_{i=1}^n y_i \prod_{\substack{j=1 \\ j \neq i}}^n \frac{(x-x_j)}{(x_i-x_j)}$$

$$= \frac{2(x-2.2)(x-3.4)(x-4.8)(x-6)(x-7)}{(1-2.2)(1-3.4)(1-4.8)(1-6)(1-7)} +$$

$$\frac{2.8(x-1)(x-3.4)(x-4.8)(x-6)(x-7)}{(2.2-1)(2.2-3.4)(2.2-4.8)(2.2-6)(2.2-7)} +$$

$$\frac{3(x-1)(x-2.2)(x-4.8)(x-6)(x-7)}{((3.4-1)(3.4-2.2)(3.4-4.8)(3.4-6)(3.4-7)} +$$

$$\frac{3.2(x-1)(x-2.2)(x-3.4)(x-6)(x-7)}{(4.8-1)(4.8-2.2)(4.8-3.4)(4.8-6)(4.8-7)} +$$



$$+ \frac{4(x-1)(x-2.2)(x-3.4)(x-4.8)(x-7)}{(6-1)(6-2.2)(6-3.4)(6-4.8)(6-7)} +$$

$$\frac{5(x-1)(x-2.2)(x-3.4)(x-4.8)(x-6)}{(7-1)(7-2.2)(7-3.4)(7-4.8)(7-7)}$$

When  $x = 5.4$

$$\begin{aligned} \therefore y(5.4) &= \frac{7.3728}{(-328.32)} + \frac{14.19264}{73.98144} + \frac{24.33024}{(-57.73952)} \\ &+ \frac{86.50752}{36.51648} + \frac{(-108.1344)}{(-59.28)} + \frac{(-50.688)}{684.288} \end{aligned}$$

$$= -0.02246 + 0.191841 - 0.44689 + 2.36899 + 1.82413 - 0.070476$$

$$y(5.4) = 3.5716 \quad \text{Used calculator}$$

(b) Newton's form.

$x_i$	1	2.2	3.4	4.8	6	7
$y_i$	2	2.8	3	3.2	4	5

for  $a_2$

$$\textcircled{1} \frac{2.8-2}{2.2-1} = 0.66667$$

$$\textcircled{4} \frac{4-3.2}{6-4.8} = 0.66667$$

$$\textcircled{2} \frac{3-2.8}{3.4-2.2} = 0.16667$$

$$\textcircled{5} \frac{5-4}{7-6} = 1$$

$$\textcircled{3} \frac{3.2-3}{4.8-3.4} = 0.14286$$

$$\therefore a_2 = [0.66667 \quad 0.16667 \quad 0.14286 \quad 0.66667 \quad 1]$$

for  $a_3$ .

$$\textcircled{1} \frac{0.16667 - 0.66667}{3.4 - 1} = -0.20833$$

$$\textcircled{2} \frac{0.14286 - 0.16667}{4.8 - 2.2} = -0.009159$$

$$\textcircled{3} \frac{0.66667 - 0.14286}{1 - 3.4} = 0.201465$$

$$\textcircled{4} \frac{1 - 0.66667}{7 - 4.8} = 0.15152$$

$$\therefore a_3 = \begin{bmatrix} -0.20833 & -0.009159 & 0.201465 & 0.15152 \end{bmatrix}$$

for  $a_4$ ,

$$\textcircled{1} \frac{-0.009159 - (-0.20833)}{4.8 - 1} = 0.52414$$

$$\textcircled{2} \frac{0.201465 - (-0.009159)}{6 - 2.2} = 0.05543$$

$$\textcircled{3} \frac{0.15152 - 0.201465}{7 - 3.4} = -0.01388$$

$$\therefore a_4 = \begin{bmatrix} 0.52414 & 0.05543 & -0.01388 \end{bmatrix}$$

for  $a_5$ ,

$$\textcircled{1} \frac{0.05543 - 0.52414}{6 - 1} = -0.000603$$

$$\textcircled{2} \frac{-0.01388 - 0.05543}{7 - 2.2} = -0.014438$$

$$\therefore a_5 = \begin{bmatrix} 0.000603 & -0.014438 \end{bmatrix}$$

for  $a_6$ ,

$$\textcircled{1} \quad \frac{-0.014438 - 0.000603}{7-1} = -0.002507 //$$

$$a_6 = -0.002507$$

$$\therefore f(x) = a_1 + a_2(x-1) + a_3(x-1)(x-2.2) + a_4(x-1)(x-2.2)(x-3.4) + a_5(x-1)(x-2.2)(x-3.4)(x-4.8) + a_6(x-1)(x-2.2)(x-3.4)(x-4.8)(x-6)$$

$$\begin{aligned} f(5.4) &= 2 + 0.66617(5.4-1) + (-0.208333)(5.4-1)(5.4-2.2) \\ &\quad + 0.05241(5.4-1)(5.4-2.2)(5.4-3.4) + 0.000603(5.4-1)(5.4-2.2)(5.4-3.4)(5.4-4.8) \\ &\quad + (-0.00251)(5.4-1)(5.4-2.2)(5.4-3.4)(5.4-4.8)(5.4-6) \end{aligned}$$

$$= 2 + 2.9335 + (-2.93329) + 1.4759 + 0.01019 + 0.00545$$

$$f(5.4) = 3.51175 \quad \dots \quad \{ \text{Used calculator} \}$$