VIRAL PANCHAL

HW 04

Q.1) Gauss - Seidel iterative method.

Given:

$$\begin{bmatrix} 4 & 0 & 1 & 0 & 1 \\ 2 & 5 & -1 & 1 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 0 & 1 & 0 & 4 & -2 \\ 1 & 0 & -1 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 32 \\ 19 \\ 14 \\ -2 \\ 41 \end{bmatrix}$$

$4x_1 + x_3 + x_5 = 32$; $x_1 = \frac{32 - x_3 - x_5}{4}$

$2x_1 + 5x_2 - x_3 + x_4 = 19$; $x_2 = \frac{19 - 2x_1 + x_3 - x_4}{5}$

$x_1 + 3x_3 - x_4 = 14$; $x_3 = \frac{14 - x_1 + x_4}{3}$

$x_2 + 4x_4 - 2x_5 = -2$; $x_4 = \frac{-2 - x_2 + 2x_5}{4}$

$x_1 - x_3 + 5x_5 = 41$; $x_5 = \frac{41 - x_1 + x_3}{5}$

(I) $x_1 = \frac{32 - 0 - 0}{4} = 8$; $x_2 = \frac{19 - 2(8) + 0 - 0}{5} = 0.6$

$x_3 = \frac{14 - 8 + 0}{3} = 2$; $x_4 = \frac{-2 - (0.6) + 2(0)}{4} = -0.65$

$x_5 = \frac{41 - 8 + 2}{5} = 7$

(II) $x_1 = \dfrac{32 - 2 - 7}{4} = 5.75$, $x_2 = \dfrac{19 - 2(5.75) + 2 + 0.65}{5}$

$x_2 = 2.03$

$x_3 = \dfrac{14 - 8 + (-0.65)}{3} = 2.533$

$x_4 = \dfrac{-2 - 2.03 + 14}{4} = 2.4925$

$x_5 = \dfrac{41 - 5.75 + 2.533}{5} = 7.5567$

(III) $x_1 = \dfrac{32 - 2.533 - 7.5567}{4} = 5.4775$

$x_2 = \dfrac{19 - 2(5.4775) + 2.5333 - 2.4925}{5} = 1.6172$

$x_3 = \dfrac{14 - (5.4775) + (2.4425)}{3} = 3.6717$

$x_4 = \dfrac{-2 - (1.6172) + 2(7.5567)}{4} = 2.8740$

$x_5 = \dfrac{41 - (5.4775) + 3.6717}{5} = 7.8388$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5.4775 \\ 1.6172 \\ 3.6717 \\ 2.8740 \\ 7.8388 \end{bmatrix}$$

Ans..

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Q.2) Condition number:

Given:

$$A = \begin{bmatrix} 10 & 12 & 0 \\ 0 & 2 & 8 \\ 2 & 4 & 8 \end{bmatrix}$$

$$\text{cond}[A] = \|A\|_1 \, \|A^{-1}\|_1$$

$$\|A\|_1 = \max_{1 \le i \le N} \sum_{i=1}^{N} |a_{ij}|$$

$$= \max \begin{bmatrix} |10| + |0| + |2| \\ |12| + |2| + |4| \\ |0| + |8| + |8| \end{bmatrix}^T = \max \begin{bmatrix} 12 \\ 18 \\ 16 \end{bmatrix}^T$$

$$\therefore \|A\|_1 = 18$$

$$A^{-1} = \begin{bmatrix} -0.5 & -3 & 3 \\ 0.5 & 2.5 & -2.5 \\ -0.125 & -0.5 & 0.625 \end{bmatrix}$$

$$\|A^{-1}\| = \max \begin{bmatrix} |-0.5| + |0.5| + |-0.125| \\ |-3| + |2.5| + |-0.5| \\ |3| + |-2.5| + |0.625| \end{bmatrix}^T$$

$$= \max \begin{bmatrix} 1.125 \\ 6 \\ 6.125 \end{bmatrix}^T = 6.125$$

$$\therefore \text{cond}[A] = 18 \times 6.125 = 110.25 \quad // \text{ Ans}$$

Q.3) Fixed point iteration.

$$\left.\begin{aligned} f_1 &= y - x^3 + 3x^2 - 4x \\ f_2 &= y^2 - x - 2 = 0 \end{aligned}\right\} \text{Given}$$

$f_1 = y - x^3 + 3x^2 - 4x$

$\therefore x = \dfrac{y - x^3 + 3x^2}{4}$

$f_2 = y^2 - x - 2$

$\therefore y^2 = x + 2$

$y = \pm\sqrt{x+2}$