ME 635: Modeling and Simulation
Homework 4

Data Modeling and Numerical Methods

10/3/2022

I pledge my honor that I have abided by the Stevens Honor System

Submitted by,

Viral Panchal

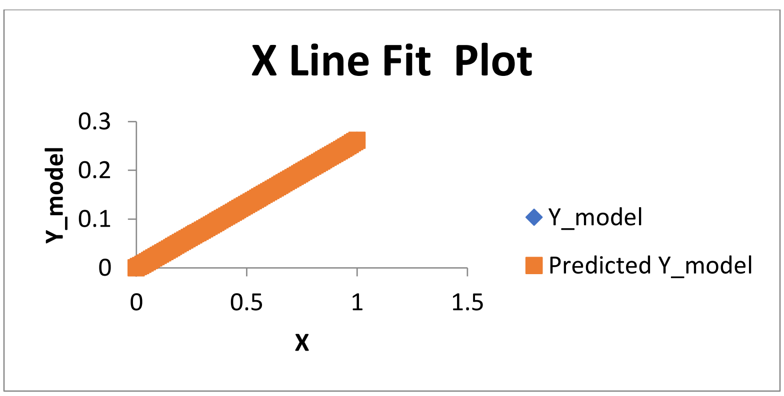
1. **(20 Pts):** For the data given in ***HW4_Problem1.txt*** (two comma separated values), find :
    a. Coefficients for the best-fit quadratic model and the RMS error.

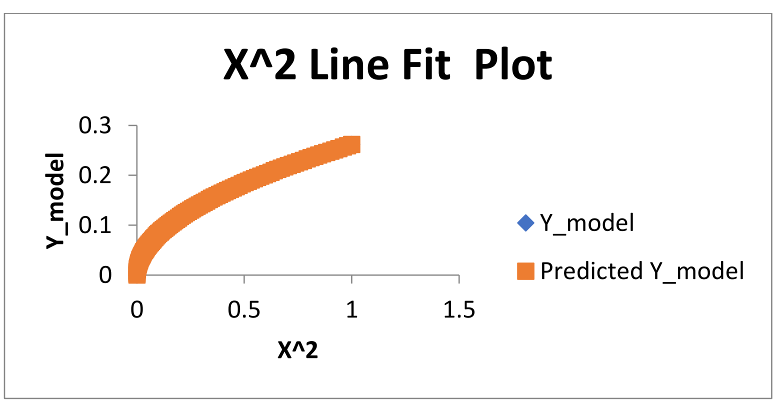
Solution:

$$y = a_0 + a_1(x) + a_2(x^2)$$

		Coefficients
A0	Intercept	8.98478E-05
A1	X	0.258913142
A2	X^2	0.002653255
	N	101
	sum(residual^2)	1.39054E-07
	RMSE	3.71048E-05

Plots:





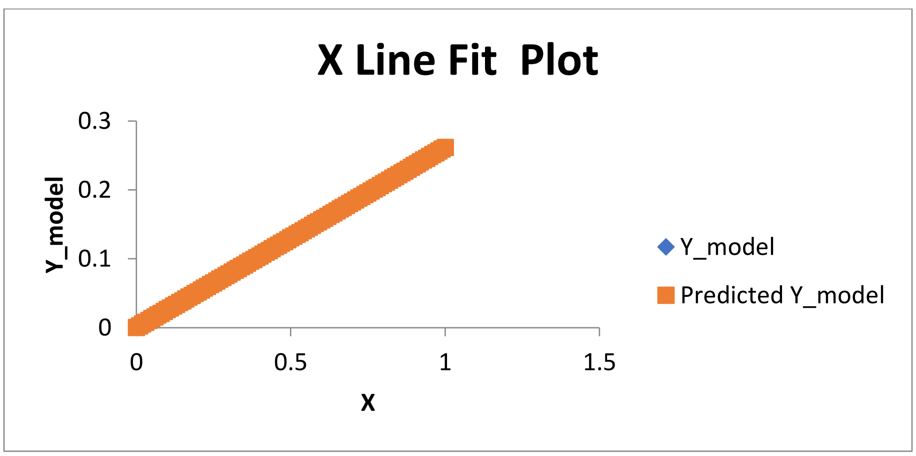
b. Find the coefficients a_0 and a_1 if $\boldsymbol{y_{model} = a_0 * x + a_1 * sinh(x)}$ and the RMS error

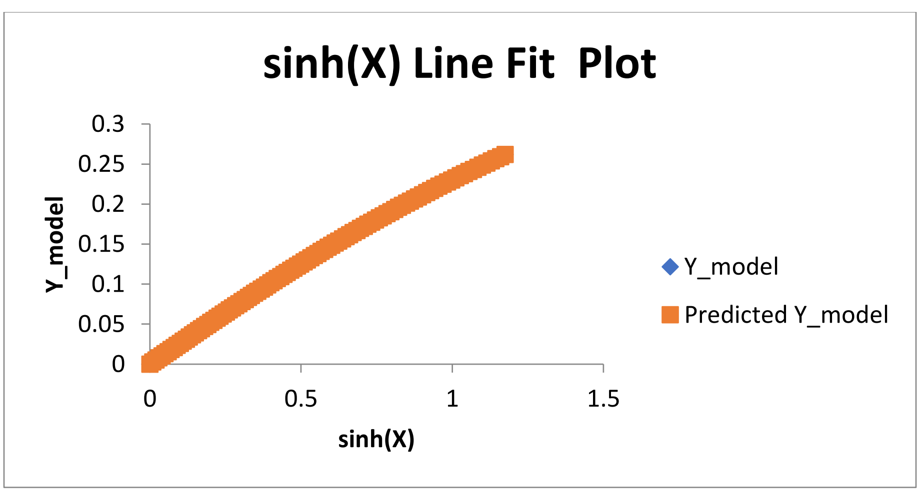
Solution:

$y_{model} = a_0 * x + a_1 * sinh(x)$

	Intercept	0
A0	X	0.25
A1	sinh(X)	0.01
	N	101
	Sum(resiudal^2)	7.35629E-18
	RMSE	2.69879E-10

Plots:





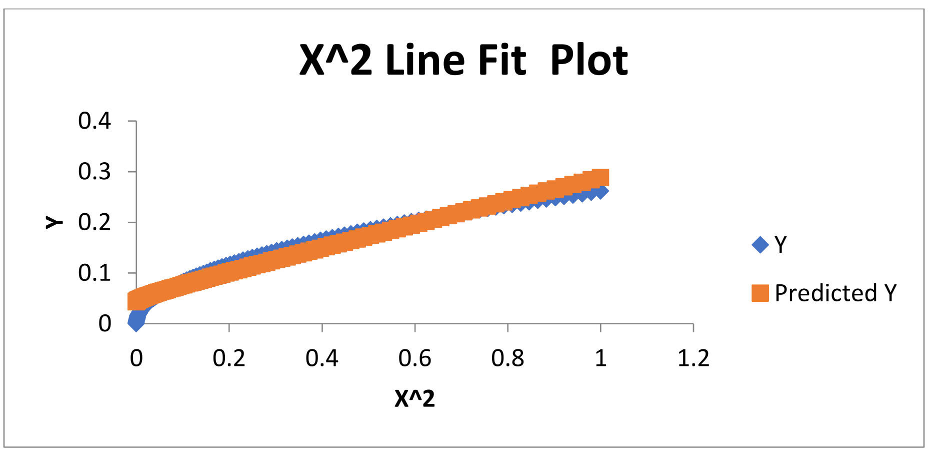
c. Find the coefficients b_0 and b_1 if $\boldsymbol{y_{model} = b_0 * x^2 + b_1 * exp(x)}$

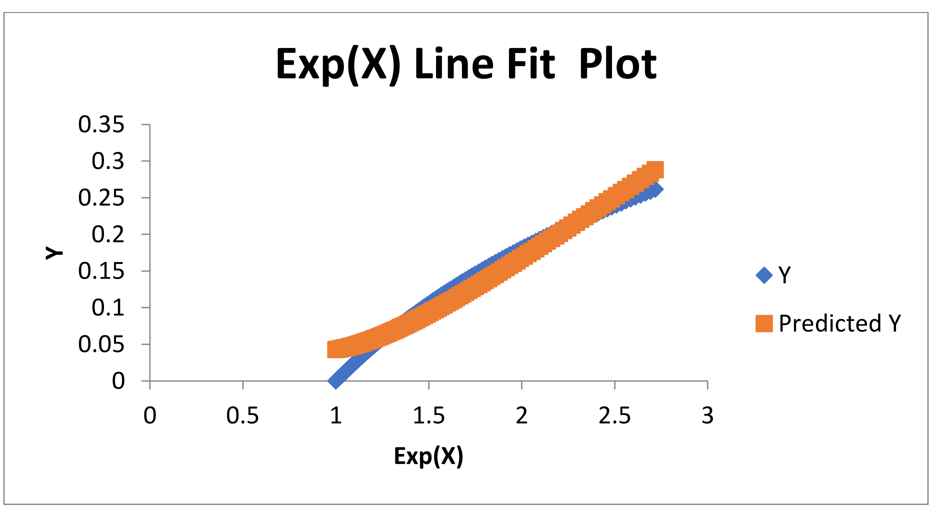
Solution:

$$y_{model} = b_0 * x^2 + b_1 * exp(x)$$

	Intercept	0
B0	X^2	0.171587
B1	Exp(X)	0.042931
	N	101
	Sum(residual^2)	0.027193
	RMSE	0.016409

Plots:



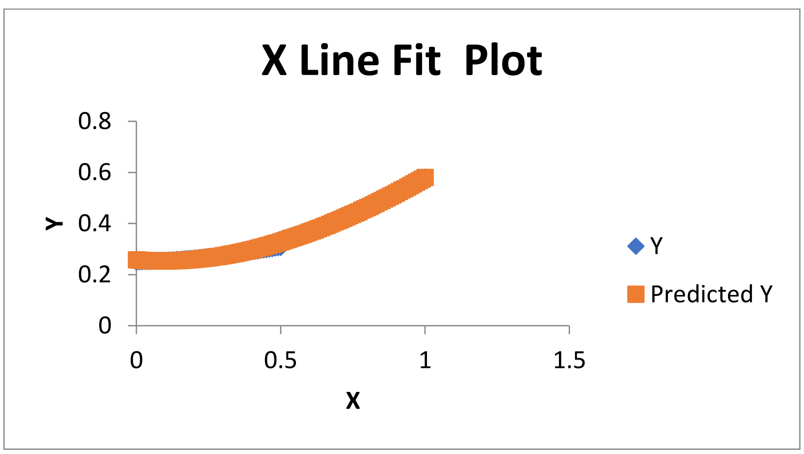


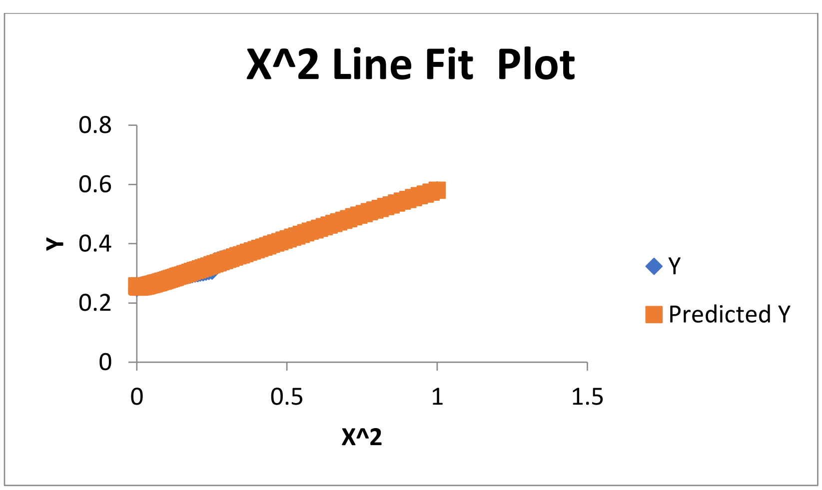
2. **(20 Pts):** For the data given in **HW4_problem2.txt** (two comma separated values: X,Y), find:
   a. Best-fit cubic model for the entire regime ($0 \le X \le 1$)

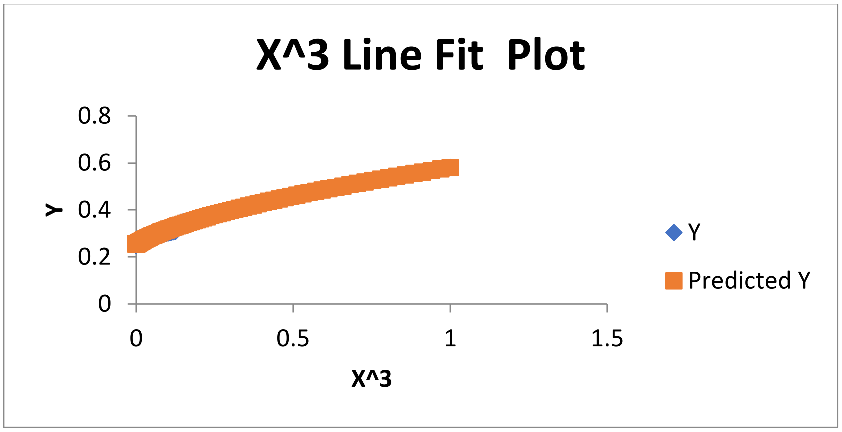
Solution:

		Coefficients
A0	Intercept	0.258027996
A1	X	-0.089978217
A2	X^2	0.512793483
A3	X^3	-0.100118874
	N	101
	Sum(Residual^2)	0.003662277
	RMSE	0.006021642

Plots:





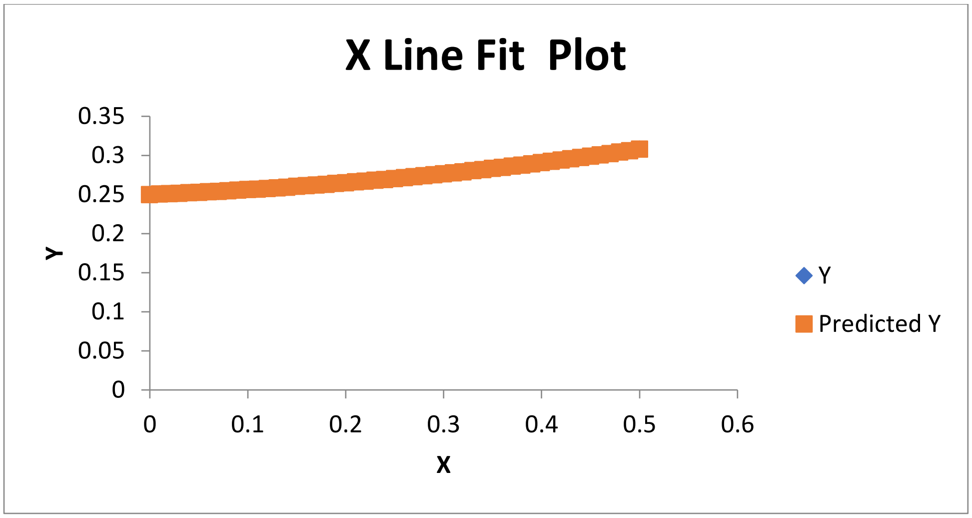


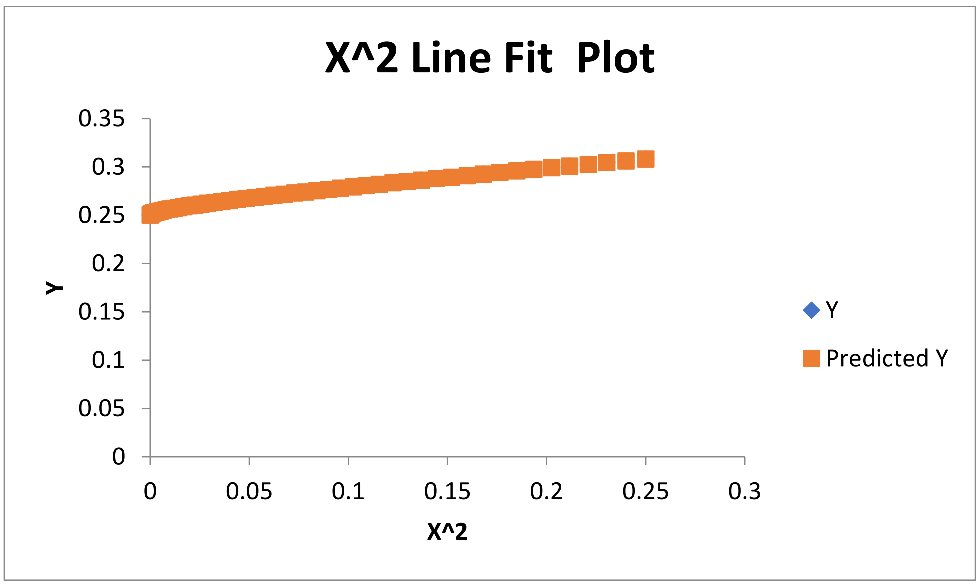
b. Two piece-wise cubic models, first fit, $f_1(x)$, is valid from $0 \leq X \leq 0.5$ and the second fit, $f_2(x)$ is valid from: $0.5 < X \leq 1$. (Note: the point X=0.5 belongs to the first fit).
c. For the fits determined in 3(b), Plot the two functions $f_1(X)$ and $f_2(X)$ and comment on the continuity (C_0 and C_1: data and slope continuity) of the two models at X = 0.5. If $f_2(X)$ is extrapolated to X = 0.5.

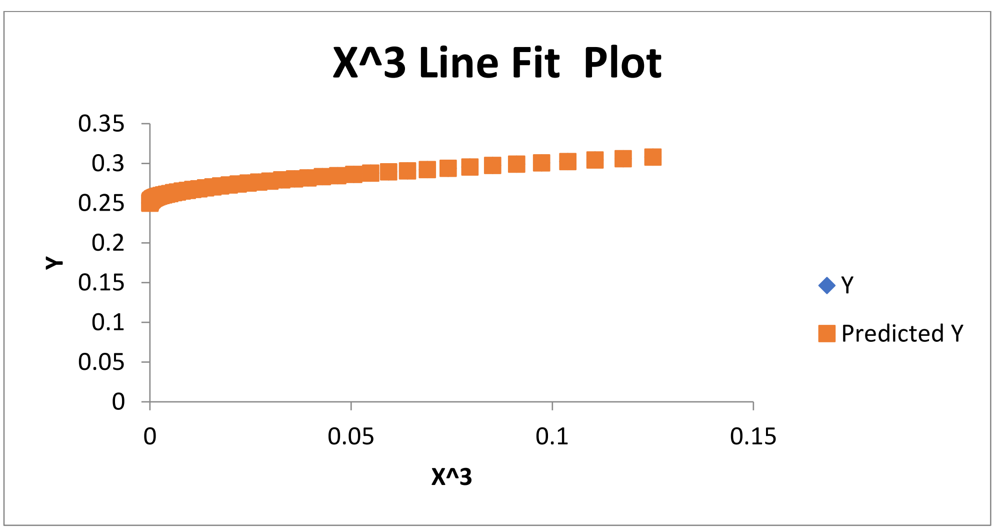
Solution:

First function from 0 – 0.5

		Coefficients
A0	Intercept	0.249991167
A1	X	0.050385743
A2	X^2	0.121477951
A3	X^3	0.019189636
	N	51
	Sum(Residual^2)	6.68572E-10
	RMSE	3.62067E-06

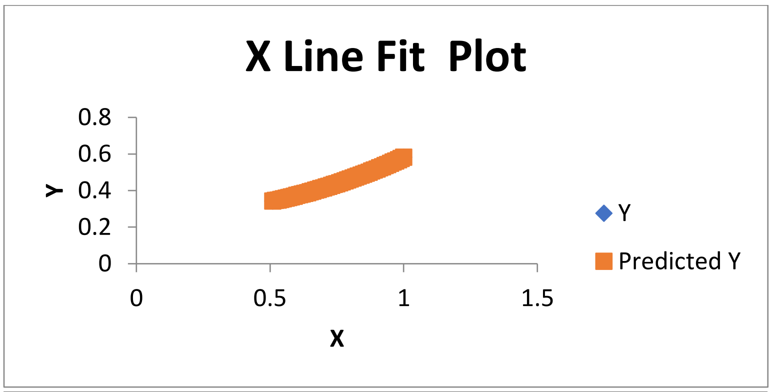

X Line Fit Plot
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Y
0
0.1
0.2
0.3
0.4
0.5
0.6
X
Y
Predicted Y

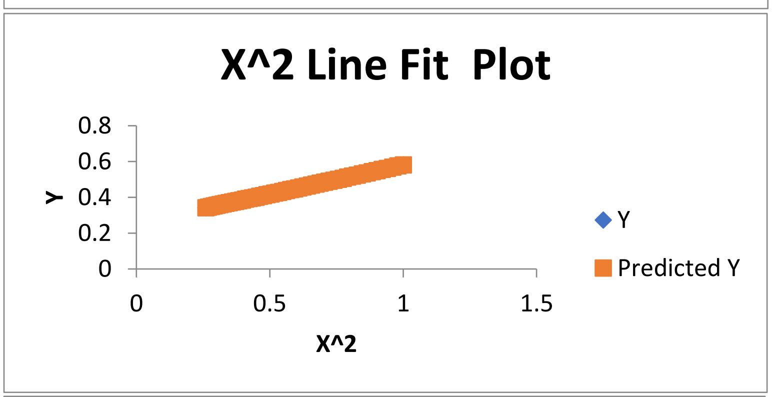

X^2 Line Fit Plot
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Y
0
0.05
0.1
0.15
0.2
0.25
0.3
X^2
Y
Predicted Y

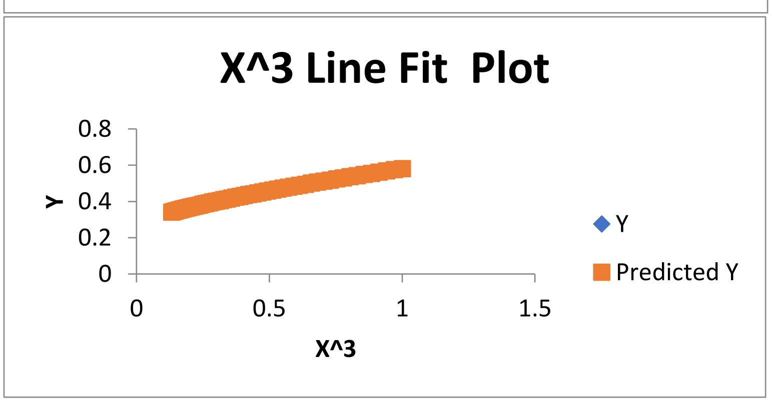

X^3 Line Fit Plot
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Y
0
0.05
0.1
0.15
X^3
Y
Predicted Y

Second function from 0.5 – 1 (not including 0.5)

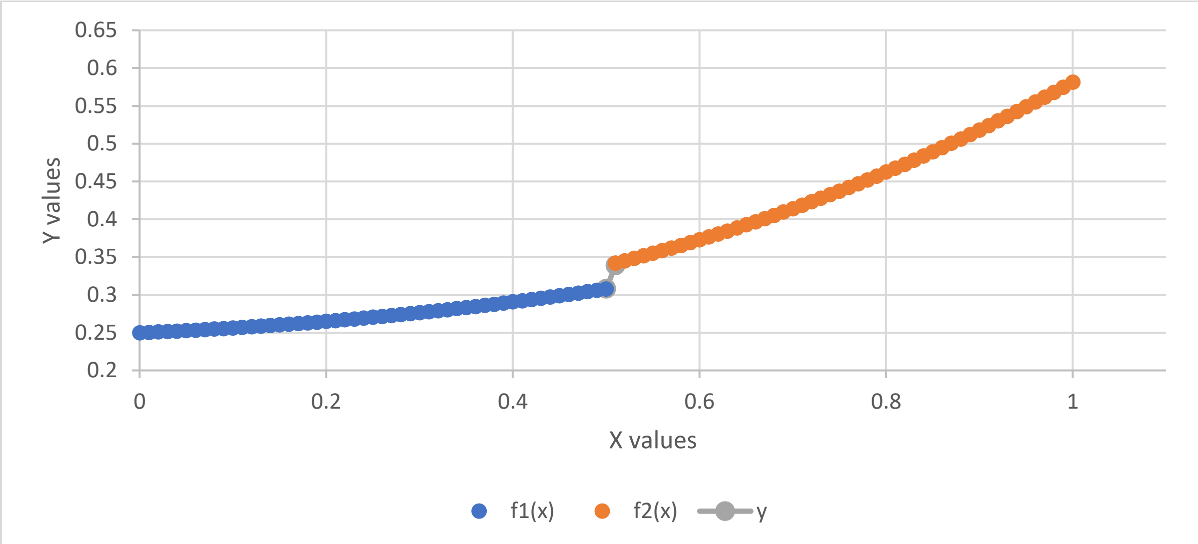
		Coefficients
A0	Intercept	0.266252339
A1	X	-0.009421796
A2	X^2	0.293626389
A3	X^3	0.030908769
	N	50
	Sum(residual^2)	8.11183E-10
	RMSE	4.02786E-06







Plotting both the functions:



When x = 0.5;

f1(x)	0.30795223
f2(x)	0.338811634
df1(x)	0.186255921
df2(x)	0.30738617

From the above plot and numerical computation, we can see that the functions do not coincide at x = 0.5. Also, they don't have the same slope at x = 0.5. Hence, they cannot be termed as continuous.

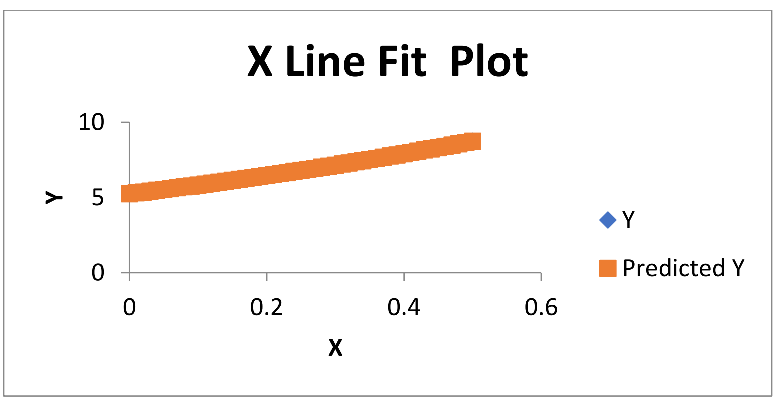
3. **(30 Pts): For the data given in HW4_Problem3.txt (two comma separated values: X, Y), model with two piece-wise polynomial models, a cubic function ($f_1(X)$) to fit from $0 \leq X \leq 0.5$ and a quartic (4^{th} order - $f_2(X)$) to fit from $0.5 \leq X \leq 1$, such that $f_2(X)$ maintains both C_0 and C_1 continuity at $X = 0.5$.**

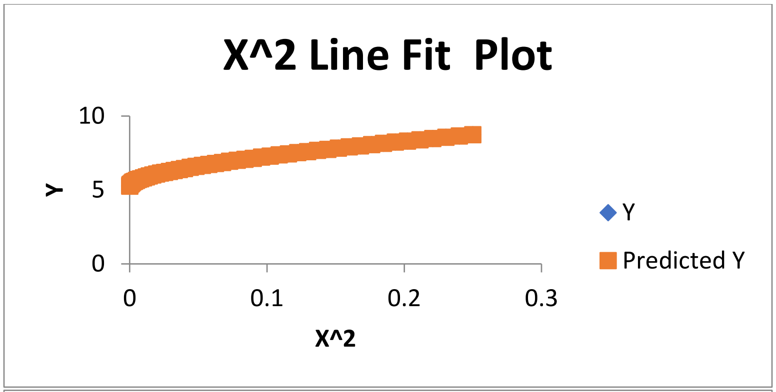
Solution:

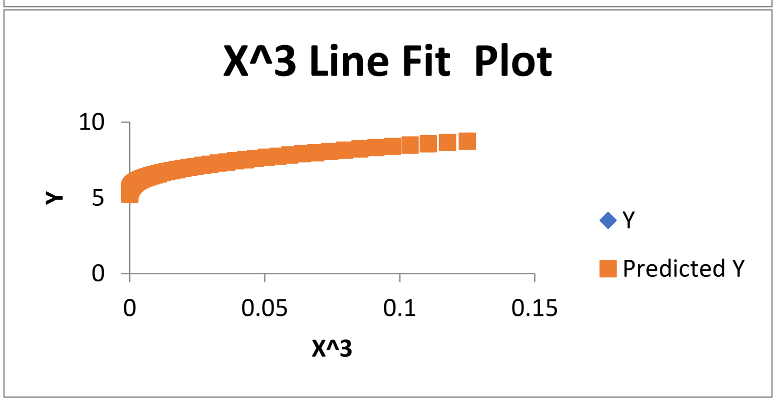
Function 1: Cubic function coefficients

		Coefficients
A0	Intercept	5.249793458
A1	X	5.508938721
A2	X^2	2.4195637
A3	X^3	1.073881937
	N	51
	sum(residual^2)	3.76857E-07
	RMSE	8.59613E-05

Plots:

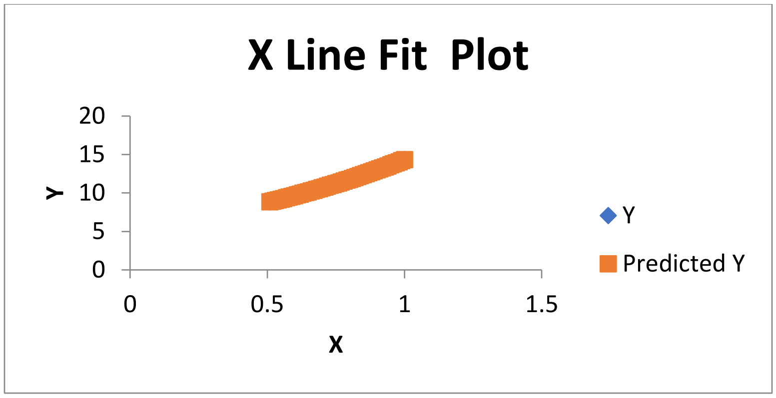


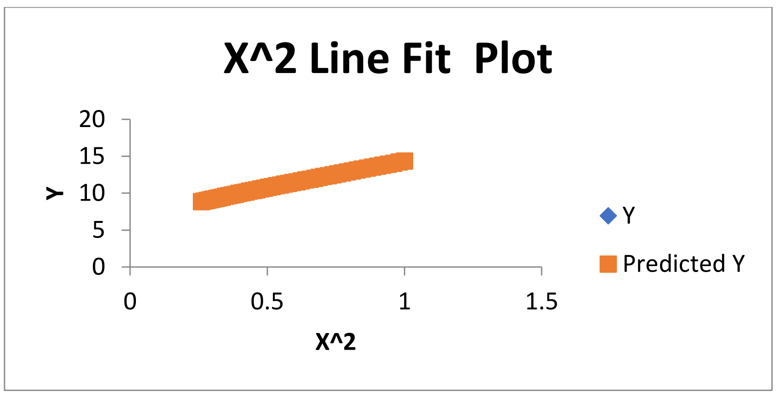


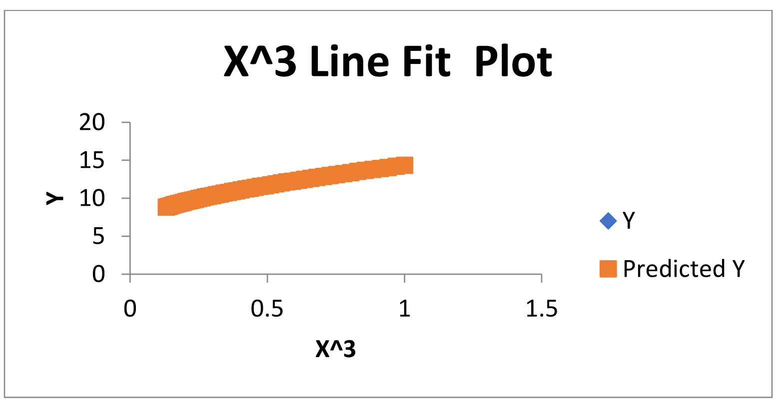


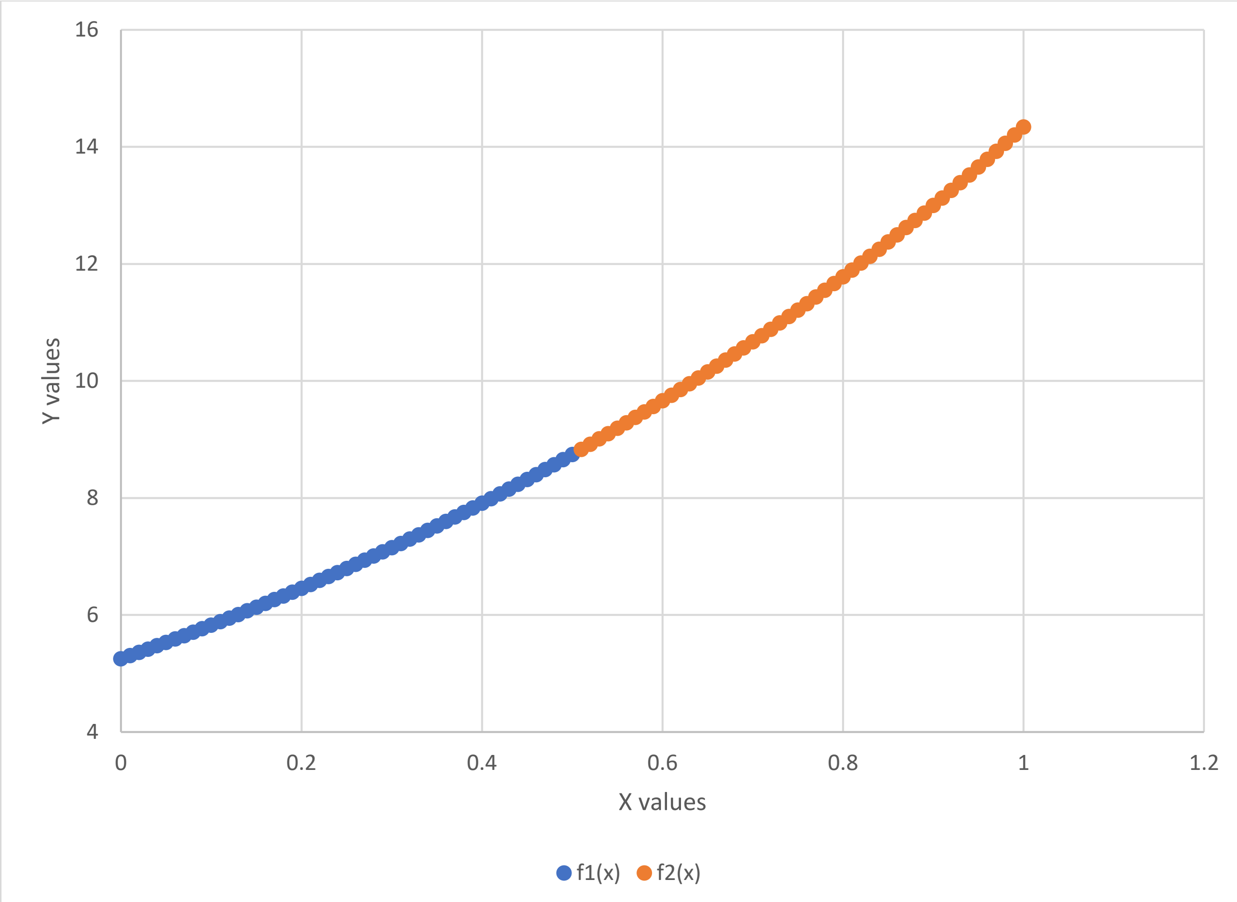
Second Function: Quartic function coefficients

		Coefficients
A0	Intercept	5.267089306
A1	X	5.383568678
A2	X^2	2.809486163
A3	X^3	0.436751548
A4	X^4	0.444505214
	N	50
	sum(residual^2)	5.39952E-10
	RMSE	3.28619E-06


X Line Fit Plot
20
15
10
5
0
Y
0
0.5
1
1.5
X
Y
Predicted Y


X^2 Line Fit Plot
20
15
10
5
0
Y
0
0.5
1
1.5
X^2
Y
Predicted Y


X^3 Line Fit Plot
20
15
10
5
0
Y
0
0.5
1
1.5
X^3
Y
Predicted Y



f1(x)	8.743388985
f2(x)	8.743620705
df1(x)	8.733913873
df2(x)	8.742871108

In this, the values for df1(x) and df2(x) at x = 0.5 is almost similar and hence the slope is said to be same for both the functions. Also, the values for both the function at x=0.5 are almost same numerically and hence, the two functions can be said to be continuous which can be even verified by looking at the plot provided above.

4. **(30 Pts):** Determine polynomial interpolation, $T(x) = \Sigma\, T_i N_i(x)$; where $x = \{0,..,1\}$ and $i=1...5$; such that $T(x=0) = T_1$, $T(x=1) = T_2$, $T(x=0.5) = T_3$, $T(X=0.25)= T_4$ and $T(x=0.75) = T_5$. Plot the five interpolation functions, $N_i(x)$.
Given $T_1 = 100$, $T_2 = 100$, $T_3=160$, $T_4=120$, $T_5=130$; Plot the temperature field, $T(x)$ and the five interpolation functions ($N_i(x)$, $i=1.......5$) Interpolation functions.

Q.4) Polynomial interpolation

$$T(x) = \Sigma T_i N_i(x) \qquad x = \{0, \dots 1\}$$
$$i = 1, \dots 5$$

$T(0) = T_1 = 100$

$T(1) = T_2 = 100$

$T(0.5) = T_3 = 150$

$T(0.25) = T_4 = 120$

$T(0.75) = T_5 = 130$

$x_1 = 0$; $x_2 = 1$; $x_3 = 0.5$; $x_4 = 0.25$

$x_5 = 0.75$

Interpolation functions,

$$N_1(x) = \frac{[(x-x_2)(x-x_3)(x-x_4)(x-x_5)]}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}$$
$$= \frac{(x-1)(x-0.5)(x-0.25)(x-0.75)}{(-1)(-0.5)(-0.25)(-0.75)}$$
$$N_1(x) = \frac{(x-1)(x-0.5)(x-0.25)(x-0.75)}{0.09375}$$

$$N_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}$$
$$= \frac{x(x-0.5)(x-0.25)(x-0.75)}{(1)(1-0.5)(1-0.25)(1-0.75)}$$
$$= \frac{x(x-0.5)(x-0.25)(x-0.75)}{0.09375}$$

$$N_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)}$$

$$= \frac{(x-0)(x-1)(x-0.25)(x-0.75)}{(0.5-0)(0.5-1)(0.5-0.25)(0.5-0.75)}$$

$$N_3(x) = \frac{x(x-1)(x-0.25)(x-0.75)}{0.015625}$$

$$N_4(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)}$$

$$= \frac{(x-0)(x-1)(x-0.5)(x-0.75)}{(0.25-0)(0.25-1)(0.25-0.5)(0.25-0.75)}$$

$$N_4(x) = \frac{x(x-1)(x-0.5)(x-0.75)}{\cancel{0.25}\ -0.02344}$$

$$N_5(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)}$$

$$= \frac{(x-0)(x-1)(x-0.5)(x-0.25)}{(0.75)(0.75-1)(0.75-0.5)(0.75-0.25)}$$

$$= \frac{x(x-1)(x-0.5)(x-0.25)}{-0.02344}$$

$$T(x) = \underset{\text{constants}}{T_1} N_1(x) + T_2 N_2(x) + T_3 N_3(x) + T_4 N_4(x) + T_5 N_5(x$$

$\therefore \cancel{T(x)} =$

Plotted in Matlab for $x \in [0,1]$

Matlab Program:

```
% ME635 HW04 - Q4
% Viral Panchal

close all
clear all
clc

i = 1;
N = zeros();
for x = 0:0.01:1
N_1 = ((x-1)*(x-0.5)*(x-0.25)*(x-0.75))/0.09375;
N(i,1) = N_1;
```

```
N_2 = ((x-0.5)*(x-0.25)*(x-0.75)*(x))/0.09375;
N(i,2) = N_2;
N_3 = ((x - 1)*(x-0.25)*(x-0.75)*(x))/0.015625;
N(i,3) = N_3;
N_4 = -((x - 1)*(x - 0.5)*(x-0.75)*(x))/0.02344;
N(i,4) = N_4;
N_5 = -((x - 1)*(x-0.5)*(x-0.25)*(x))/0.02344;
N(i,5) = N_5;
N(i,6) = x;
i = i+1;
end

plot(N(:,6),N(:,1))
hold on
plot(N(:,6),N(:,2))
hold on
plot(N(:,6),N(:,3))
hold on
plot(N(:,6),N(:,4))
hold on
plot(N(:,6),N(:,5))
legend('N_1(x)','N_2(x)','N_3(x)','N_4(x)','N_5(x)')
grid on
ylim([-0.7 1.4])

T1 = 100;
T2 = 100;
T3 = 150;
T4 = 120;
T5 = 130;

j = 1;
T = zeros();
% p = size(N,6)
for j = 1:1:size(N,1)
    T(j,1) = T1*N(j,1) + T2*N(j,2) + T3*N(j,3) + T4*N(j,4) + T5*N(j,5);
    j = j+1;
end

figure
plot(N(:,6),T(:,1))
legend('T(x) = Sum(T(x)*N_i(x))')
grid on
```

Output plots: