ME 635 – Modeling and Simulation

Fall 2022

Final Exam

Due Date: 12/16/2022

Report by,

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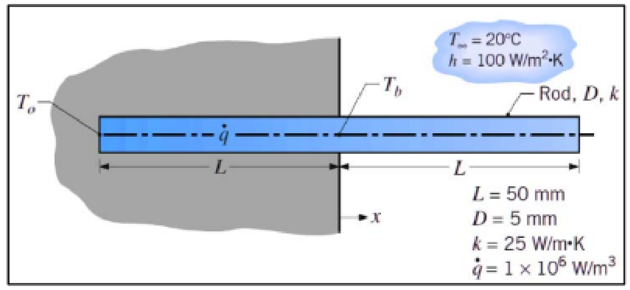
"I pledge my honor that I have abided by the Stevens Honor system"

Problem 1 [30 Points]

A thermal engineer wants to design a metal rod of length 2L, diameter D, and thermal conductivity k is inserted into a perfectly insulating wall, exposing one-half of its length to an airstream that is of temperature T_∞ and provides a convection coefficient h at the surface of the rod. An electromagnetic field induces volumetric energy generation at a uniform rate q within the embedded portion of the rod, as shown in Figure 1.



a) Derive an expression for the steady-state temperature T_b at the base of the exposed half of the rod. The exposed region may be approximated as a very long fin.

b) Derive an expression for the steady-state temperature T_o at the end of the embedded half of the rod.

c) Using numerical values provided in the figure to plot the temperature distribution in the rod from T_o at the end of the embedded half of the rod to the right end of the rod and describe key features of the temperature distribution. Does the rod behave as a very long fin?

Solution:

Given:

L = 50 mm = 0.050m

Total length of rod = 2L = 100 mm = 0.1m

D = 5 mm = 0.005m

K = 25 W/mk

$\dot{q}$ = 1e+6 W/m^3

A) Expression for the steady state temperature Tb

When the embedded portion of the rob is an infinite pin, then $0 \le x \le L$

Expression for heat transfer rate is given as:

$$\boldsymbol{q_f = \sqrt{h_\rho K A_c}\ (T_b - T_\infty)} \qquad \textbf{....Eq.1}$$

h → heat transfer coefficient

ρ → perimeter

k → thermal conductivity

A_c → Cross-sectional Area

T_b → Base Temperature

T_∞ → Airstream temperature

Now, applying energy balance for the embedded portion of the rod.

$\dot{q} = \frac{q_f}{A_c\, L}$ … Eq.2

From 1 & 2, we can get

$\frac{\dot{q}\, A_c\, L}{\sqrt{h\rho K A_c}} = (T_b - T_\infty)$

$$\boxed{\boldsymbol{T_b = \frac{\dot{q}\, A_c\, L}{\sqrt{h\rho K A_c}} + T_\infty} \quad \textbf{…… ANS 1}}$$

B) Expression for steady state temperature To

In the embedded region, the rod gives one dimensional heat transfer with uniform heat generation rate $\dot{q}$

The temperature distribution is given as –

$T(x) = \frac{\dot{q}\, L^2}{2k}\left(1 - \frac{X^2}{L^2}\right) + T_b$

$T(0) = T_o$

Hence, $\boxed{\boldsymbol{T_o = \frac{\dot{q}\, L^2}{2k} + T_b} \quad \textbf{…ANS 2}}$

This is the steady state temperature equation

C) Making a plot using numerical values given

Computing Tb,

$T_b = \frac{\dot{q}\, A_c\, L}{\sqrt{h\rho K A_c}} + T_\infty$

Substituting the values provided, we get **Tb = 328.53 k**

Now Solving for To,

$T_o = \frac{\dot{q}\, L^2}{2k} + T_b$

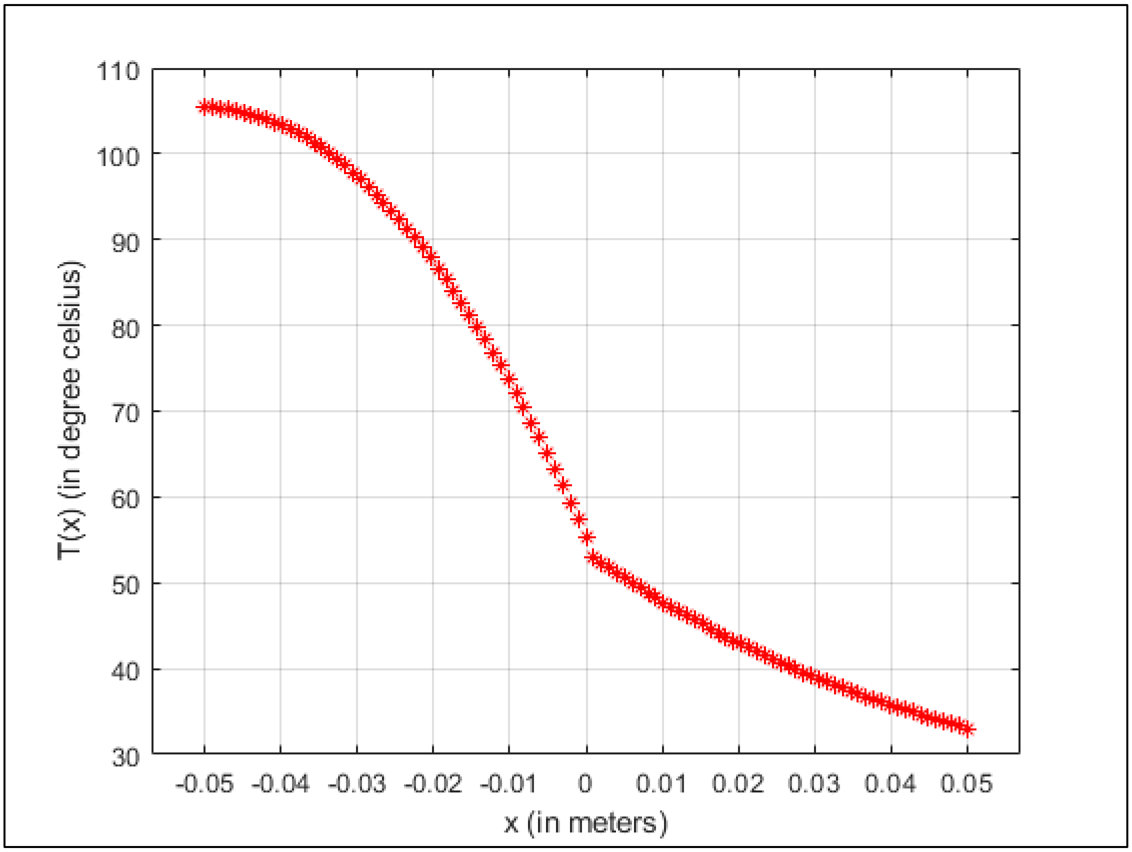
Substituting values in the above equation, we get **To = 105.38 K**

Now, plotting using **MATLAB:**

Program:

```
% Final Exam - Q1
% Viral Panchal

close all
clear all
clc

% Global variables
q_dot = 1e+6;
L = 0.050;  % in meter
d = 0.005;  % in meter
k = 25;
T_b = 328.53; % in kelvin
T_inf = 293.15; % in kelvin

% Variables for -L to 0
x1 = linspace(0,0.05,50);
x_1 = linspace(-0.05,0,50);
T1 = zeros();

for i = 1:length(x1)
    T1(i,1)= ((q_dot * (L^2 - abs(x1(i))^2))/(2*k))+T_b;
end

% Variables for 0 to L
x2 = linspace(0.05,1,50);
x_2 = linspace(0,0.05,50);
T2 = zeros();

for i = 1:length(x2)
    T2(i,1) = exp(-x2(i)) * (T_b-T_inf) + T_inf;
end

% Plotting both outputs
plot(x_1,T1-273.15,'r*')
hold on
plot(x_2(2:length(x_2)),T2(2:length(T2),1)-273.15,'r*')
ylabel('T(x) (in degree celsius)')
xlabel('x (in meters)')
axis padded
grid on
```

MATLAB output:



The temperature is converted from Kelvin to degree Celsius before plotting for easier understanding.

Problem 2 [30 Points]

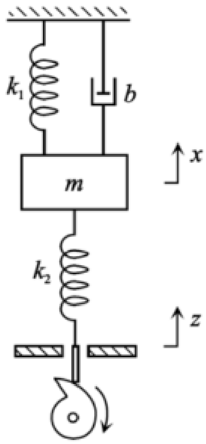


Figure 2

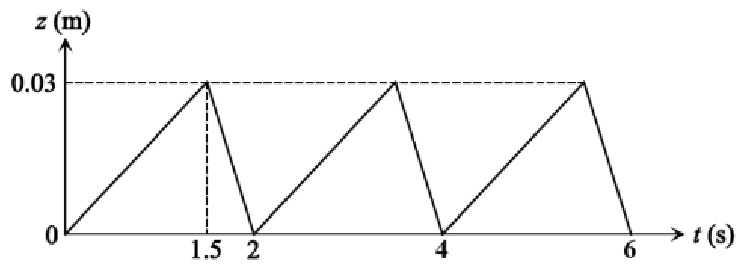


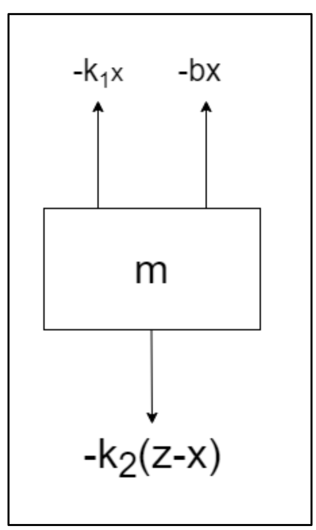
Figure 3

Consider the mass–spring–damper system shown in Figure 2, where the cam and follower impart a displacement z(t) in the form of a periodic sawtooth function (see Figure 3) to the lower end of the system. z(t) is the displacement input due to the surface of the cam. The values of the system parameters are m = 12 kg, b = 200 N·s/m, k_1 = 4000 N/m, and k_2 = 2000 N/m.

a) Draw the free-body diagram of the system.
b) Write the differential equations describing the motion of the mass shown in Figure.
c) Build a Simulink model based on the differential equation of motion of the system.
d) Use the Simulink to plot the displacement and velocity outputs as functions of t for 6 second.

Solution:

A) Free Body Diagram

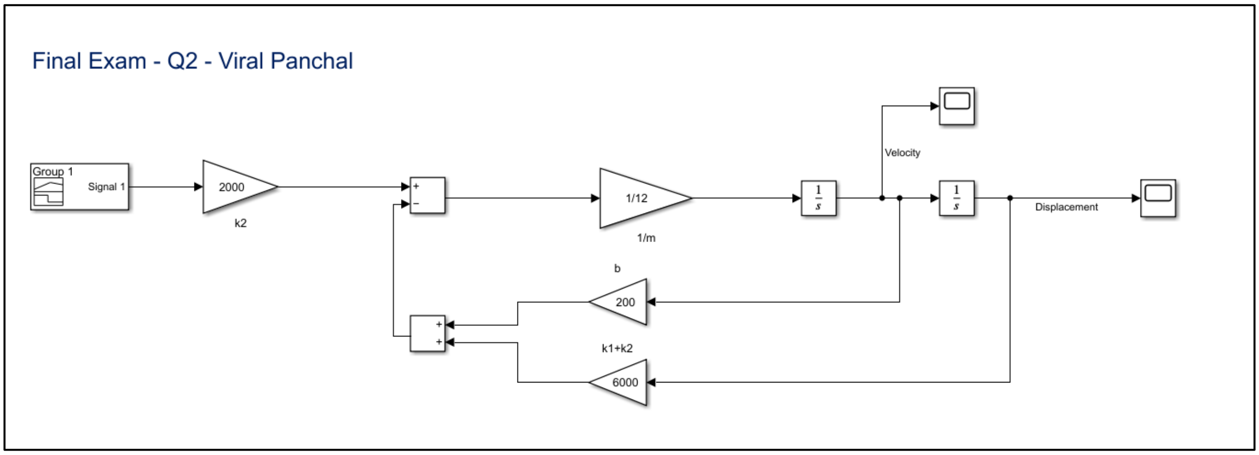


"K1" and "k2" are the spring constants. "b" is the damper as shown in the original figure.

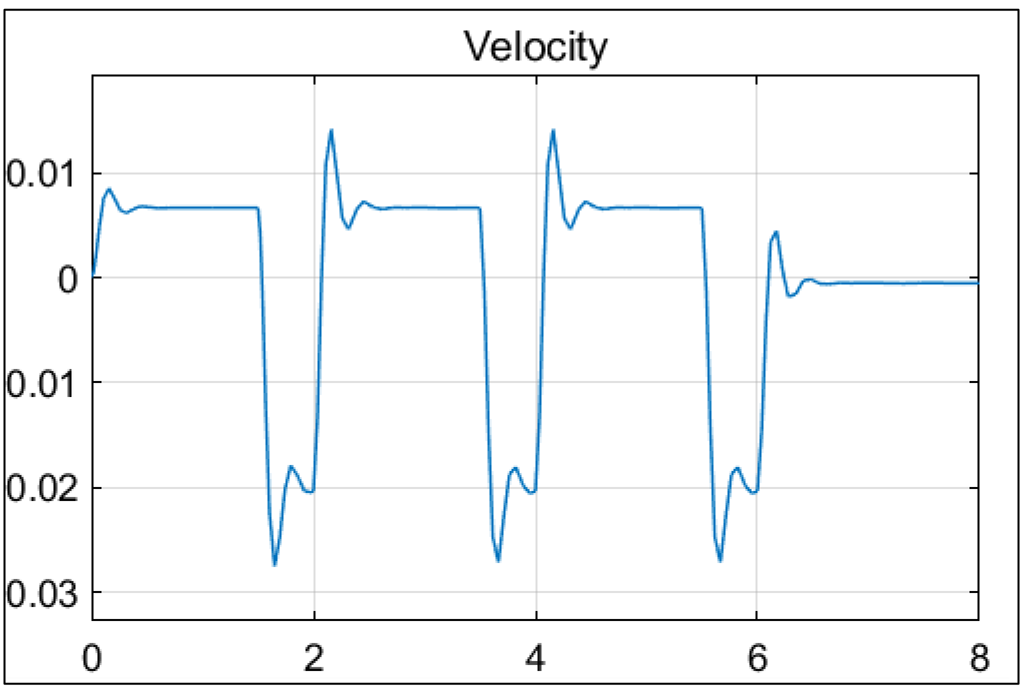
B) Differential equations:

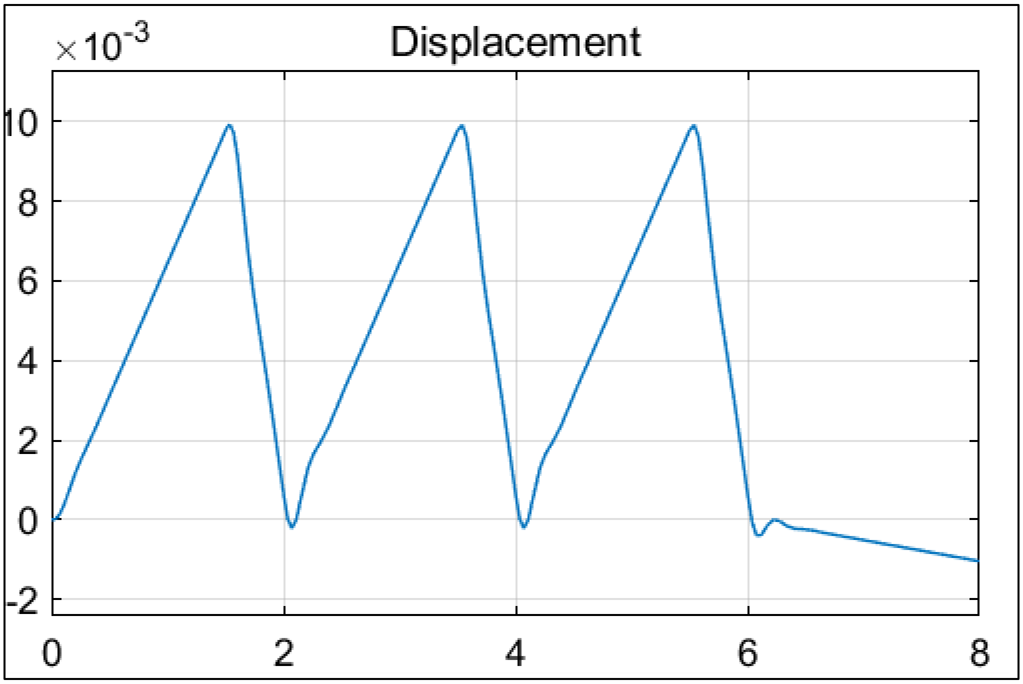
$$m\ddot{x} = -k_1x - b\dot{x} + k(z-x)$$
$$m\ddot{x} + k_1x + b\dot{x} + k_2x = k_2z$$
$$\ddot{x} = \frac{-k_1 + k_2}{m}x - \frac{b}{m}\dot{x} + \frac{k_2}{n}z$$

C) Simulink Model:



Simulink Output:





The signal input to the system is of 6 seconds, but the simulation time was set to 8 seconds to visualize the behavior after the input signal becomes constant.

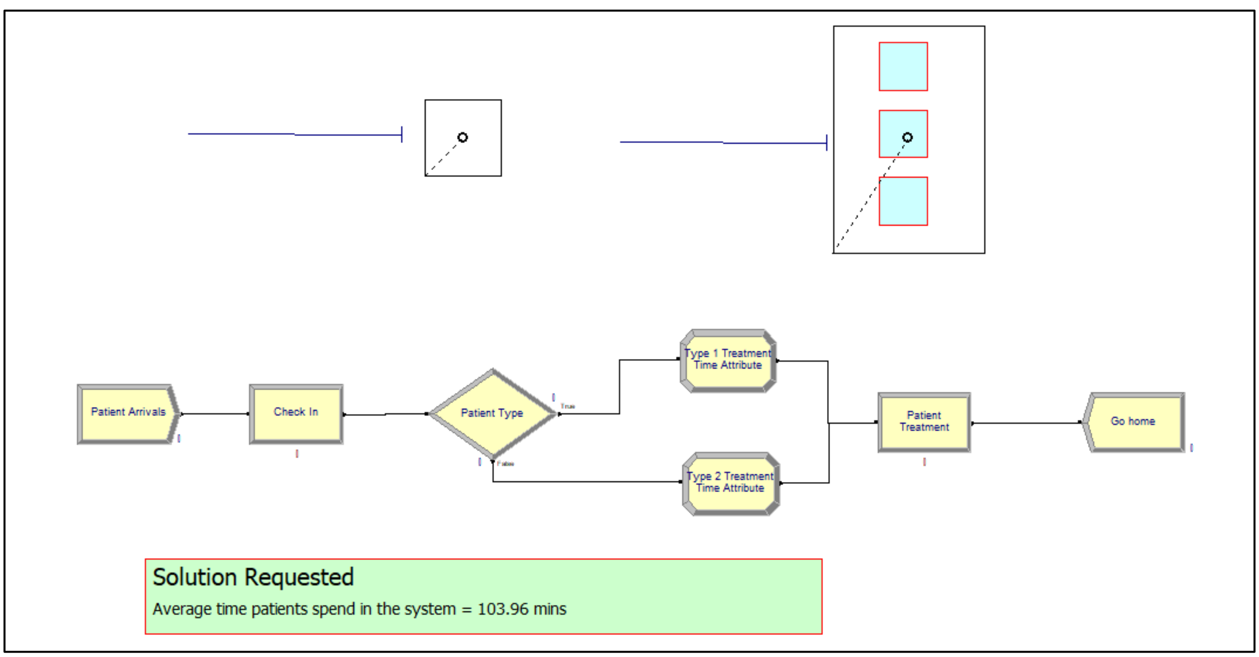
Problem 3 [40 Points]

a) An acute-care facility treats non-emergency patients (cuts, colds, etc.). Patients arrive according to an exponential interarrival time distribution with a mean of 11 (all times are in minutes). Upon arrival, they check in at a registration desk staffed by a single nurse. Registration times follow a triangular distribution with parameters 6, 10, and 19. After completing registration, they wait for an available examination room; there are three identical rooms. Data shows that the patients can be divided into two groups with regard to different examination times. The first group (55% of patients) has service times that follow a triangular distribution with parameters 14, 22, and 39. The second group (45%) has triangular service times with parameters 24, 36, and 59. Upon completion, patients are sent home. The facility is open 16 hours each day.

   1) Develop a model of this system and run it for 200 replications of one day each and observe the average total time patients spend in the system.
   2) Put text box in your Arena file with the numerical results requested.

Solution:

A) Arena Model:



Output:

Replications: 200 Time Units: Minutes

Entity

Time

VA Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	43.1741	< 0.15	40.5494	46.6825	21.9607	75.9379

NVA Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	0.00	< 0.00	0.00	0.00	0.00	0.00

Wait Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	60.7819	< 4.46	11.0164	155.30	0.00	315.31

Transfer Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	0.00	< 0.00	0.00	0.00	0.00	0.00

Other Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	0.00	< 0.00	0.00	0.00	0.00	0.00

Total Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	103.96	< 4.46	54.1955	200.54	22.6272	363.26

Other

Text box added in arena file as requested.

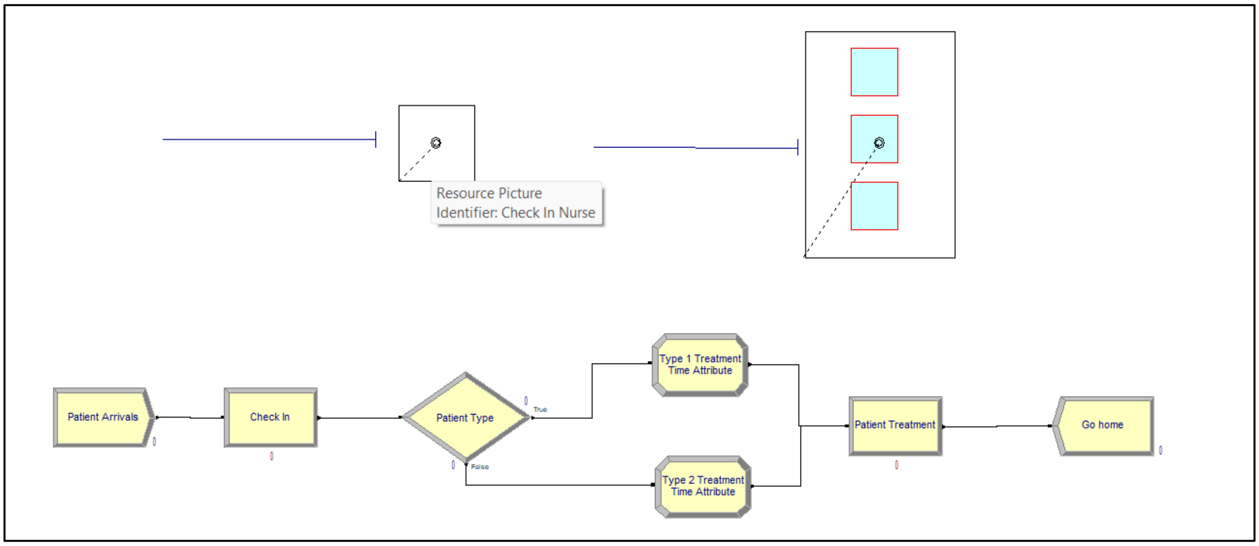
b) Modify your solution from Problem 1 to include lunch breaks for the doctors who staff the examination rooms. There are three doctors on duty for the first 3.5 hours of each 8-hour shift. For the next 90 minutes, the doctors take rotating 30-minute lunch breaks, resulting in only two doctors being available at any point during these 90 minutes. If all three doctors are busy with patients when the 90-minute lunch period begins, they wait until the first patient among the three is done, and that doctor takes the first 30-minute lunch break; each doctor gets a full 30-minute lunch, so if the 90-minute lunch period starts a little late, it also ends that much late. After all the lunch breaks end, all three doctors are available until the end of their 8-hour shift, at which point the second 8-hour shift begins with the same staffing and lunch (maybe call it dinner) rules. Note that the 8 hours in a doctor's shift include the 30-minute lunch break.

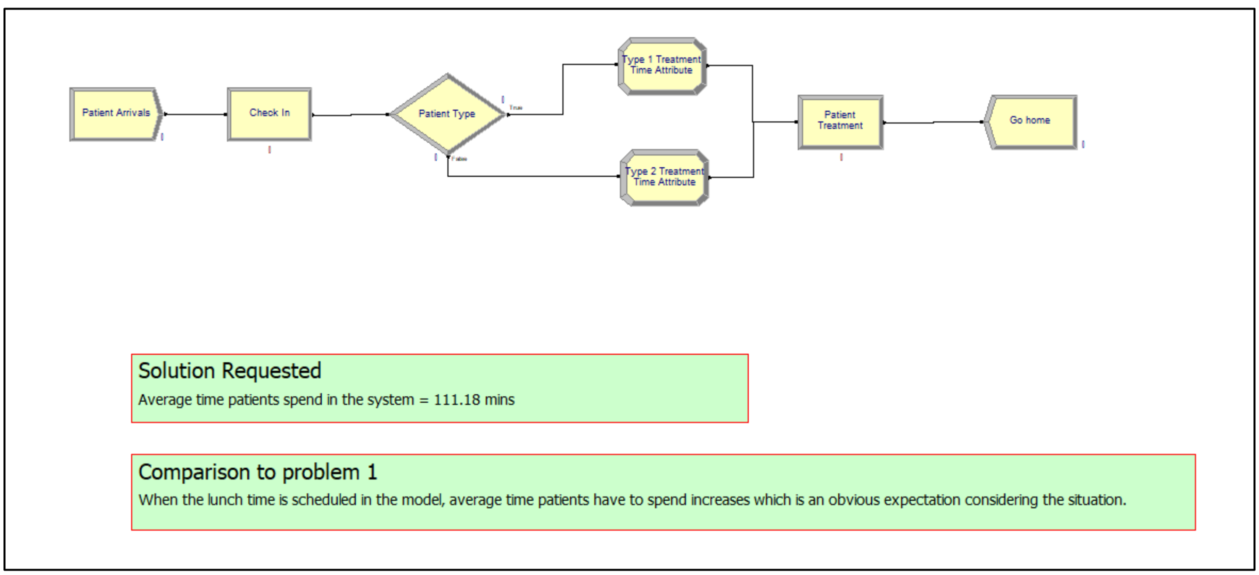
1) Develop a new model of this system, run it for 200 replications of one day each and observe the average total time patients spend in the system.
2) Put text box in your Arena file with the numerical results requested.
3) Compare your results to those from Problem 1 in a text box in your Arena file.

Solution:

Making a copy of the first model and making edits.

Arena model for 3B





Output:

Replications: 200 Time Units: Minutes

Entity

Time

VA Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	43.1635	< 0.15	40.5494	46.7229	21.9607	75.9379

NVA Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	0.00	< 0.00	0.00	0.00	0.00	0.00

Wait Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	68.0192	< 4.56	14.9247	182.43	0.00	326.51

Transfer Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	0.00	< 0.00	0.00	0.00	0.00	0.00

Other Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	0.00	< 0.00	0.00	0.00	0.00	0.00

Total Time	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Arrival Patient	111.18	< 4.57	58.3969	227.68	22.6272	374.46

Other

Text box added in the solution as requested.

Arena models for each question is attached individually in the submission.