

HW.

Q.4) Polynomial interpolation

$$T(x) = \sum T_i N_i(x)$$

$$x = \{0, \dots, 1\}$$
$$i = 1, \dots, 5$$

$$T(0) = T_1 = 100$$

$$T(1) = T_2 = 100$$

$$T(0.5) = T_3 = 150$$

$$T(0.25) = T_4 = 120$$

$$T(0.75) = T_5 = 130$$

$$x_1 = 0; x_2 = 1;$$

$$x_3 = 0.5; x_4 = 0.25$$

$$x_5 = 0.75$$

Interpolation functions,

$$N_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}$$
$$= \frac{(x-1)(x-0.5)(x-0.25)(x-0.75)}{(-1)(-0.5)(-0.25)(-0.75)}$$

$$N_1(x) = \frac{(x-1)(x-0.5)(x-0.25)(x-0.75)}{0.09375}$$

$$N_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}$$
$$= \frac{x(x-0.5)(x-0.25)(x-0.75)}{(1)(1-0.5)(1-0.25)(1-0.75)}$$
$$= \frac{x(x-0.5)(x-0.25)(x-0.75)}{0.09375}$$

$$\begin{aligned}
 N_3(x) &= \frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)} \\
 &= \frac{(x-0)(x-1)(x-0.25)(x-0.75)}{(0.5-0)(0.5-1)(0.5-0.25)(0.5-0.75)} \\
 N_3(x) &= \frac{x(x-1)(x-0.25)(x-0.75)}{0.015625}
 \end{aligned}$$

$$\begin{aligned}
 N_4(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)} \\
 &= \frac{(x-0)(x-1)(x-0.5)(x-0.75)}{(0.25-0)(0.25-1)(0.25-0.5)(0.25-0.75)} \\
 N_4(x) &= \frac{x(x-1)(x-0.5)(x-0.75)}{-0.02344}
 \end{aligned}$$

$$\begin{aligned}
 N_5(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)} \\
 &= \frac{(x-0)(x-1)(x-0.5)(x-0.25)}{(0.75)(0.75-1)(0.75-0.5)(0.75-0.25)} \\
 &= \frac{x(x-1)(x-0.5)(x-0.25)}{-0.02344}
 \end{aligned}$$

$$T(x) = \underbrace{T_1}_{\text{constants}} N_1(x) + \underbrace{T_2}_{\text{constants}} N_2(x) + \underbrace{T_3}_{\text{constants}} N_3(x) + \underbrace{T_4}_{\text{constants}} N_4(x) + \underbrace{T_5}_{\text{constants}} N_5(x)$$

$$\therefore T(x) =$$

Plotted in Matlab. for $x \in [0, 1]$