ME 635: Modeling and Simulation
Homework 5

Finite Element method
10/11/2022

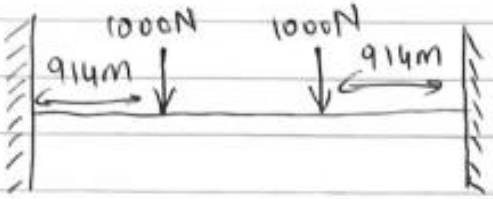
I pledge my honor that I have abided by the Stevens Honor System

Submitted by,
Viral Panchal

HW 05

Beam deflection curve.

Q1)



$E = 200 \times 10^9 \ N/m^2$

$b = 50.8mm$

$h = 100mm$

$$K = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^3 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^3 \end{bmatrix}$$

$K = K_1 + K_2 + K_3$

$K_i = K(l_i, E, I)$ ← Matlab

$K = \Sigma K_i$ //

↓

8x8 matrix ← Matlab

Q1 MATLAB program:

```
%ME635 - HW05 - Q1
%Viral Panchal

close all
clear all
clc

l = 3048*(10^-3);
x = 0:0.001:l;
E = 200*(10^9);
I = (50.8*(10^-3))*((100*(10^-3))^3)/12;
l1 = 914*(10^-3);
l3 = 914*(10^-3);
l2 = l-l1-l3;

p2 = -1000;
m2 = 0;
p3 = -1000;
m3 = 0;

k1 = global_stiff(l1,E,I);
fprintf('k1 = \n');
disp(k1)
k2 = global_stiff(l2,E,I);
fprintf('k2 = \n');
disp(k2)
k3 = global_stiff(l3,E,I);
fprintf('k3 = \n');
disp(k3)

fprintf('K = k1 + k2 + k3 \n\n');
k_final = zeros(8);
k_final(1:4,1:4) = k1;
k_final(3:6,3:6) = k_final(3:6,3:6) + k2;
k_final(5:8,5:8) = k_final(5:8,5:8) + k3;
fprintf('K =\n');
disp(k_final)

% Calculation forces and displacements
force_vec = [p2 m2 p3 m3]';
displacement_vec = [0; 0; k_final(3:6,3:6)\force_vec; 0;0];
force_vec = k_final*displacement_vec;
fprintf('Force = \n');
disp(force_vec)

% Caculating the points on the equation
eq_1 =
displacement(displacement_vec(1),displacement_vec(2),displacement_vec(3),displacement_vec(4),l1,0
:0.01:1);
eq_2 =
displacement(displacement_vec(3),displacement_vec(4),displacement_vec(5),displacement_vec(6),l2,0
```

```
:0.01:1);
eq_3 =
displacement(displacement_vec(5),displacement_vec(6),displacement_vec(7),displacement_vec(8),l3,0
:0.01:1);
plot(linspace(0,l1,101),eq_1)
hold on
plot(linspace(l1,l1+l2,101),eq_2)
hold on
plot(linspace(l1+l2,l1+l2+l3,101),eq_3)
legend('Eq_1','Eq_2','Eq_3')
grid on
```

Global_stiff – function

```
function K = global_stiff(l,E,I)

K = ((E*I)/(l^3))* [12      6*l     -12     6*l
                    6*l     4*l^2   -6*l    2*l^2
                    -12     -6*l     12      -6*l
                    6*l     2*l^2   -6*l    4*l^2];

end
```

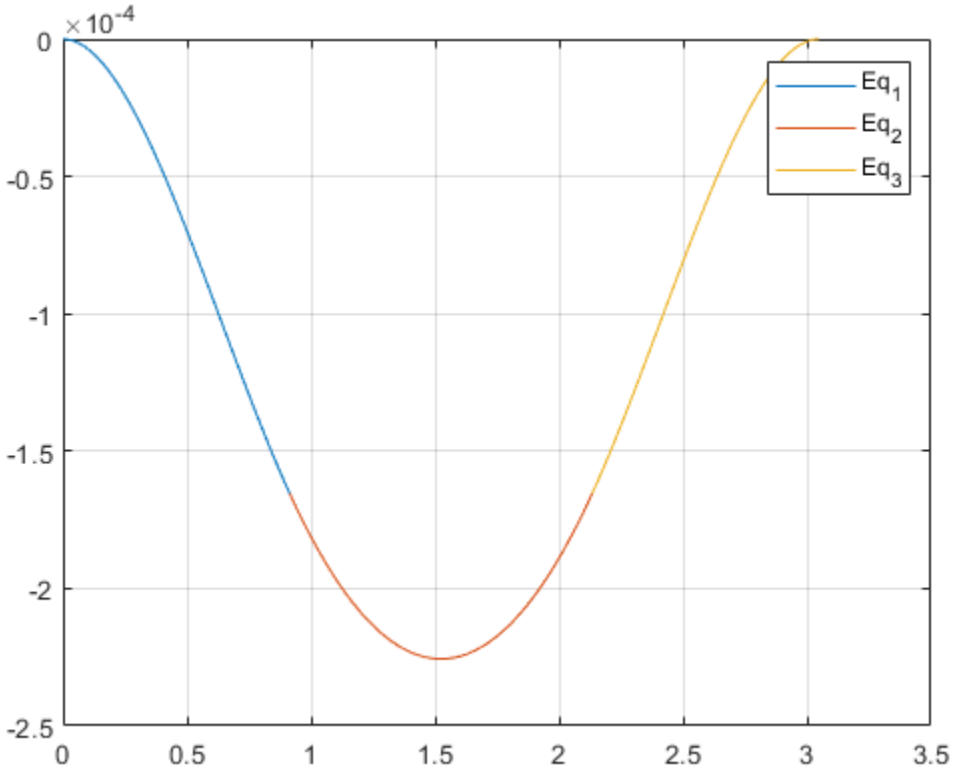
Displacement – function

```
function V = displacement(V1,tet1,V2,tet2,Lin,zet)

V = (1-3*zet.*zet+2*zet.*zet.*zet)*V1 + Lin*(zet-
2*zet.*zet+zet.*zet.*zet)*tet1+(3*zet.*zet-
2*zet.*zet.*zet)*V2+Lin*(zet.*zet.*zet-zet.*zet)*tet2;

end
```

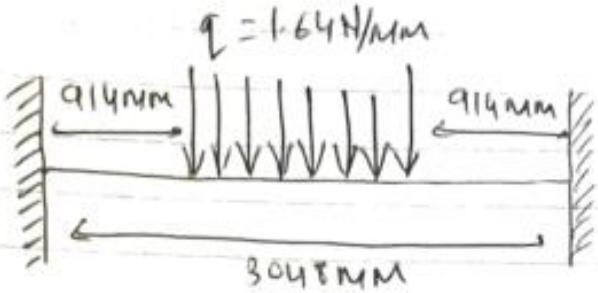
Matlab Output:

```
k1 =
  1.0e+07 *

   1.3306   0.6081  -1.3306   0.6081
   0.6081   0.3705  -0.6081   0.1853
  -1.3306  -0.6081   1.3306  -0.6081
   0.6081   0.1853  -0.6081   0.3705

k2 =
  1.0e+06 *

   5.5952   3.4131  -5.5952   3.4131
   3.4131   2.7760  -3.4131   1.3880
  -5.5952  -3.4131   5.5952  -3.4131
   3.4131   1.3880  -3.4131   2.7760

k3 =
  1.0e+07 *

   1.3306   0.6081  -1.3306   0.6081
   0.6081   0.3705  -0.6081   0.1853
  -1.3306  -0.6081   1.3306  -0.6081
   0.6081   0.1853  -0.6081   0.3705

K = k1 + k2 + k3

K =
  1.0e+07 *

 Columns 1 through 7

   1.3306   0.6081  -1.3306   0.6081        0        0        0
   0.6081   0.3705  -0.6081   0.1853        0        0        0
  -1.3306  -0.6081   1.8901  -0.2668   -0.5595   0.3413        0
   0.6081   0.1853  -0.2668   0.6481   -0.3413   0.1388        0
        0        0  -0.5595  -0.3413    1.8901   0.2668   -1.3306
        0        0   0.3413   0.1388    0.2668   0.6481   -0.6081
        0        0        0        0   -1.3306  -0.6081    1.3306
        0        0        0        0    0.6081   0.1853   -0.6081

 Column 8

        0
        0
        0
        0
   0.6081
```

```
    0.1853
   -0.6081
    0.3705

Force =
   1.0e+03 *

    1.0000
    0.6399
   -1.0000
    0.0000
   -1.0000
         0
    1.0000

-0.6399
```



Q.2)



$q = -1.64$ N/MM

$E = 200 \times 10^9$

$I = \frac{bh}{12}$

$$K = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^3 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^3 & -6l & 4l^3 \end{bmatrix}$$

$K = K_1 + K_2 + K_3$ ← Similar to Q1. solved in matlab and plotted the curve.

Q2 MATLAB Program:

```
% ME635 - HW05 - Q2
% ViralPanchal

close all
clear all
clc

l = 3048*(10^-3);
x =  0:0.001:l;
E = 200*(10^9);
I = (50.8*(10^-3))*((100*(10^-3))^3)/12;
l1 = 914*(10^-3);
l3 = 914*(10^-3);
l2 = l - l1 - l3;

q = -1640;
p2 = q*l2/2;
m2 = q*l2^2/12;
p3 = q*l2/2;
m3 = -q*l2^2/12;

k1 = global_stiff(l1,E,I);
fprintf('k1 = \n');
disp(k1)
k2 = global_stiff(l2,E,I);
fprintf('k2 = \n');
disp(k2)
k3 = global_stiff(l3,E,I);
fprintf('k3 = \n');
disp(k3)

fprintf('K = k1 + k2 + k3 \n\n');
k_final = zeros(8);
k_final(1:4,1:4) = k1;
k_final(3:6,3:6) = k_final(3:6,3:6) + k2;
k_final(5:8,5:8) = k_final(5:8,5:8) + k3

% Calculation forces and displacements
force_vec = [p2 m2 p3 m3]';
displacement_vec = [0; 0; k_final(3:6,3:6)\force_vec; 0;0];
force_vec = k_final*displacement_vec;
fprintf('Force = \n');
disp(force_vec)

% Caculating the points on the equation
eq_1 =
displacement(displacement_vec(1),displacement_vec(2),displacement_vec(3),displacement_vec(4),l1,0
:0.01:1);
eq_2 =
displacement(displacement_vec(3),displacement_vec(4),displacement_vec(5),displacement_vec(6),l2,0
:0.01:1);
```

```
eq_3 =
displacement(displacement_vec(5),displacement_vec(6),displacement_vec(7),displacement_vec(8),l3,0
:0.01:1);
plot(linspace(0,l1,101),eq_1)
hold on
plot(linspace(l1,l1+l2,101),eq_2)
hold on
plot(linspace(l1+l2,l1+l2+l3,101),eq_3)
legend('Eq_1','Eq_2','Eq_3')
grid on

% Slope
slope_eq = zeros();
for r = 1:(length(eq_1)-1)
    slope_eq(r,1) = ((eq_1(1,r+1)-eq_1(1,r))/l1)*100;
    slope_eq(r,2) = ((eq_2(1,r+1)-eq_2(1,r))/l2)*100;
    slope_eq(r,3) = ((eq_3(1,r+1)-eq_3(1,r))/l3)*100;
end

figure
plot(linspace(0,l1,100),slope_eq(:,1))
hold on
plot(linspace(l1,l1+l2,100),slope_eq(:,2))
hold on
plot(linspace(l1+l2,l1+l2+l3,100),slope_eq(:,3))
legend('eq_1','eq_2','eq_3')
grid on
title('Slope plot');
```

Global_stiff – function

```
function K = global_stiff(l,E,I)

K = ((E*I)/(l^3))* [12      6*l     -12     6*l
                    6*l     4*l^2   -6*l    2*l^2
                    -12     -6*l     12      -6*l
                    6*l     2*l^2   -6*l    4*l^2];

end
```

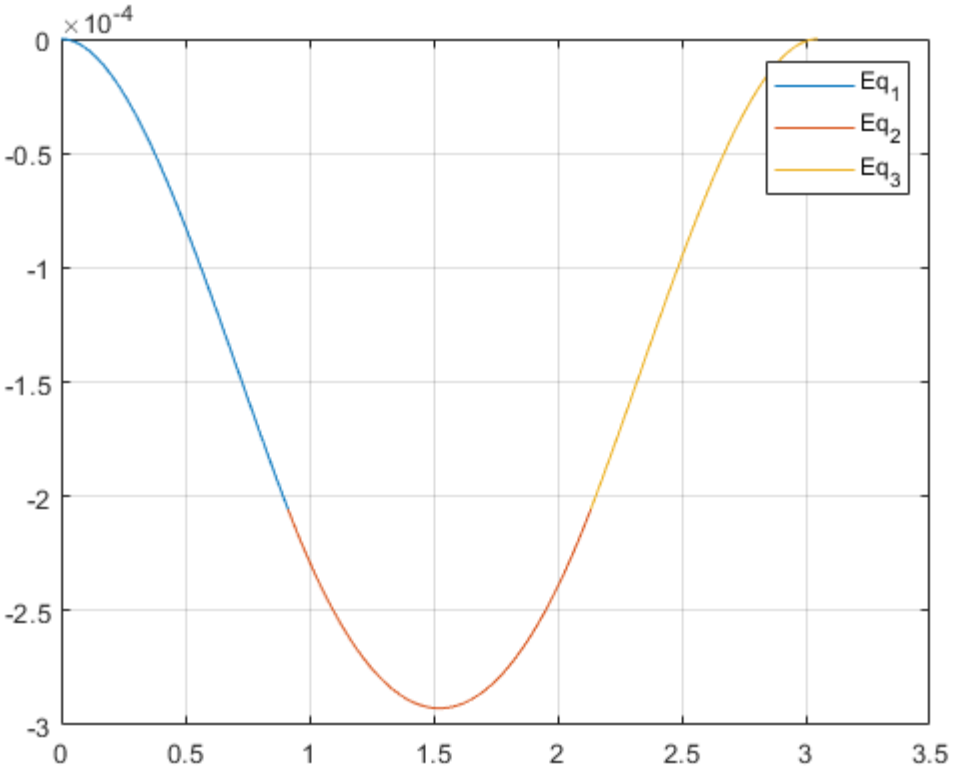
Displacement – function

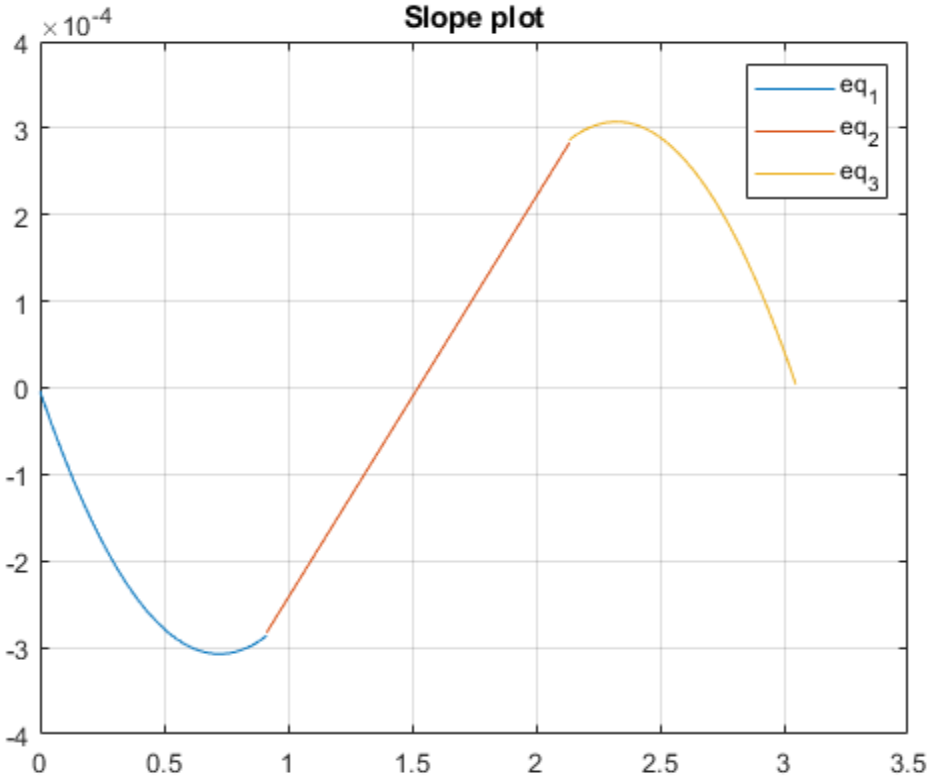
```
function V = displacement(V1,tet1,V2,tet2,Lin,zet)

V = (1-3*zet.*zet+2*zet.*zet.*zet)*V1 + Lin*(zet-
2*zet.*zet+zet.*zet.*zet)*tet1+(3*zet.*zet-
2*zet.*zet.*zet)*V2+Lin*(zet.*zet.*zet-zet.*zet)*tet2;

end
```

Matlab Output:

```
k1 =
   1.0e+07 *

    1.3306    0.6081   -1.3306    0.6081
    0.6081    0.3705   -0.6081    0.1853
   -1.3306   -0.6081    1.3306   -0.6081
    0.6081    0.1853   -0.6081    0.3705

k2 =
   1.0e+06 *

    5.5952    3.4131   -5.5952    3.4131
    3.4131    2.7760   -3.4131    1.3880
   -5.5952   -3.4131    5.5952   -3.4131
    3.4131    1.3880   -3.4131    2.7760

k3 =
   1.0e+07 *

    1.3306    0.6081   -1.3306    0.6081
    0.6081    0.3705   -0.6081    0.1853
   -1.3306   -0.6081    1.3306   -0.6081
    0.6081    0.1853   -0.6081    0.3705

K = k1 + k2 + k3


k_final =

   1.0e+07 *

  Columns 1 through 7

    1.3306    0.6081   -1.3306    0.6081         0         0         0
    0.6081    0.3705   -0.6081    0.1853         0         0         0
   -1.3306   -0.6081    1.8901   -0.2668   -0.5595    0.3413         0
    0.6081    0.1853   -0.2668    0.6481   -0.3413    0.1388         0
         0         0   -0.5595   -0.3413    1.8901    0.2668   -1.3306
         0         0    0.3413    0.1388    0.2668    0.6481   -0.6081
         0         0         0         0   -1.3306   -0.6081    1.3306
         0         0         0         0    0.6081    0.1853   -0.6081

  Column 8

         0
         0
         0
         0
    0.6081
    0.1853
   -0.6081
    0.3705
```

```
Force =
   1.0e+03 *

    1.0004
    0.7216
   -1.0004
   -0.2034
   -1.0004
    0.2034
    1.0004
   -0.7216
```





Published with MATLAB® R2021a