Modeling and Simulation, MC312

Lab-5 (2025)

(Epidemic Model with interventions)

Due: September 29, 2025

Objective

In this lab we will numerically explore the SIR model discussed in the class. The first step is for you to identify a set of relevant parameter values (population size, initial conditions, recovery rate, and transmission rate corresponding to a chosen R_0) that will serve as the baseline for your study. After fixing these, the objective is to explore the role of different interventions, such as short lockdowns, gradual behavioral changes, and vaccination. The goal is to understand how timing, duration, and strength of interventions affect peak infection, time to peak, and final epidemic size.

Model Setup

We consider the classic SIR model:

$$\dot{S} = -\beta \frac{SI}{N}, \quad \dot{I} = \beta \frac{SI}{N} - \gamma I, \quad \dot{R} = \gamma I, \quad S + I + R = N.$$

The basic reproduction number is $R_0 = \beta/\gamma$. Assume $R_0 > 1$, so that an epidemic is possible.

Directions for exploration:

- Choose N to be of order 10^5–10^6 for convenience.
- Start with a small number of infecteds $I(0)$ (e.g. 10–100) and set $R(0) = 0$, $S(0) = N - I(0)$.
- Select γ corresponding to an infectious period between 5–10 days.
- Explore different R_0 values (e.g. between 1.5 and 3.0) by adjusting β.

Tasks

1. Baseline Epidemic (no intervention)

- Simulate the epidemic with constant β and γ of your choice.
- Record the peak infected population I_{max}, the time to peak t_{peak}, and the final epidemic size $R(\infty)$.
- Compare outcomes for different R_0 values using a single figure of $I(t)$.
- Make a plot of the dependence of the epidemic size $R(\infty)$ on R_0. Discuss how the final size changes as R_0 increases.

2. Short Lockdown with Immediate Effect

Lockdown reduces contact rate. One way to capture this effect in the model is to reduce the transmission coefficient β by a factor $(1-\theta(t))$, where $\theta(t)$ represents the effectiveness (or severity) of the lockdown. Thus,

$$\beta(t) = (1-\theta(t))\beta, \quad \theta(t) = \begin{cases} A, & t_1 \le t \le t_2, \\ 0, & \text{otherwise.} \end{cases}$$

Here $A \in [0,1]$ is the strength of the lockdown: $A = 0$ means no effect, while larger values of A correspond to stronger reductions in contacts (for example, $A = 0.5$ means the transmission rate is cut in half during the lockdown period).

- Choose values for A (strength of lockdown), t_1 (start time), and $\Delta = t_2 - t_1$ (duration).
- Explore systematically how these parameters affect I_{max}, t_{peak}, and $R(\infty)$.
- Comment on whether an earlier lockdown always reduces the peak.
- Investigate the conditions under which the epidemic *rebounds* after the lockdown. Hint: a rebound can occur if, at the end of lockdown, a large fraction of the population is still susceptible, so that the effective reproduction number

  $$R_{\text{eff}}(t) = R_0 \frac{S(t)}{N}$$

  is still greater than 1 when restrictions are lifted.
- Do not take very large values of Δ: in reality, long lockdowns may not be feasible due to economic and social costs. For guidance, think in terms of days: a lockdown of 14 days (2 weeks) to 28 days (4 weeks) is realistic, while values much longer than this should be avoided. Study how the choice of timing t_1 is often as important as the duration. Since γ is usually expressed per day (e.g. $\gamma = 1/7$ for a 7-day infectious period), one time unit in your simulation corresponds to one day. Thus, a lockdown of $\Delta = 14$ means about two weeks, and $\Delta = 28$ means about four weeks.

Context. During the COVID-19 pandemic, lockdowns were implemented around the world as a way to reduce transmission when vaccines or treatments were not yet available. Numerical studies of the SIR model show why this was important: introducing a lockdown reduces the effective transmission rate $\beta(t)$ and can delay or lower the peak of infections. Even if the final epidemic size $R(\infty)$ is not greatly reduced, the *shift in the peak* buys valuable time for health systems to prepare, for hospitals to avoid being overwhelmed, and for policymakers to roll out additional measures (such as vaccination). Your simulations allow you to investigate how such interventions lead to the "flattening" and "delaying" of the epidemic curve that was widely discussed during COVID-19.

3. Extra Credit: Behavior-Aware Lockdown Ramp

In reality, a lockdown ("lock") and its lifting ("unlock") rarely produce an instantaneous change in contact behavior. When a lockdown is announced, people take time to adapt: at first compliance is limited, but over several days the effectiveness builds up toward a maximum level. When restrictions are lifted, people do not immediately return to normal. Instead, they remain cautious for some time, and only gradually relax and increase their contact levels.

For this extra credit task, try to construct your own way of modeling this kind of gradual change. You might let the lockdown effectiveness increase smoothly during the lock period, and then decrease gradually after it ends. The key idea is to capture the *delay in compliance* during the lock and the *slow relaxation* after the unlock.

- Propose a simple model that represents these “lock” and “unlock” phases.
- Run simulations comparing your behavior-aware lockdown to an “instant” lockdown of the same nominal strength and duration.
- Discuss how gradual compliance and gradual relaxation change the trajectory of $I(t)$. For example, does a slower ramp-up reduce effectiveness? Does a slower ramp-down help prevent a sharp rebound?