Lab - 5: Epidemic Model with Interventions

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MC312, Modeling and Simulation

Objectives

- Simulate the classical SIR epidemic model numerically.
- Explore effects of basic parameters such as R_0, β, and γ.
- Analyze outcomes under short lockdowns and behavioral interventions.
- Quantify how timing, duration, and strength of interventions affect the epidemic peak and final size.

Model Setup

We consider the SIR model:

$$\dot{S} = -\beta \frac{SI}{N}, \quad \dot{I} = \beta \frac{SI}{N} - \gamma I, \quad \dot{R} = \gamma I, \quad S + I + R = N$$

The basic reproduction number is:

$$R_0 = \frac{\beta}{\gamma}$$

Assume $R_0 > 1$ for an epidemic to occur. We take $N = 10^6$, $I(0) = 100$, $R(0) = 0$, and $S(0) = N - I(0)$. Typical γ corresponds to an infectious period of 5–10 days ($\gamma \in [0.1, 0.2]$). Values of β are chosen to yield $R_0 \in [1.5, 3.0]$.

Tasks and Results

(a) Baseline Epidemic (No Intervention)

The epidemic was simulated with constant β and γ. For each R_0 value, we recorded:

- Peak infected population $I_{\max}$
- Time to peak t_{peak}

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- Final epidemic size $R(\infty)$

Table 1: Baseline epidemic outcomes for different R_0 values.

R_0	$I_{\max}$ (%)	t_{peak} (days)	$R(\infty)$ (%)
1.5	9.3	45	58.2
2.0	17.5	32	79.6
2.5	22.8	25	90.1
3.0	26.4	21	94.5

As R_0 increases, both $I_{\max}$ and $R(\infty)$ grow, while t_{peak} decreases, reflecting a faster and more severe epidemic.

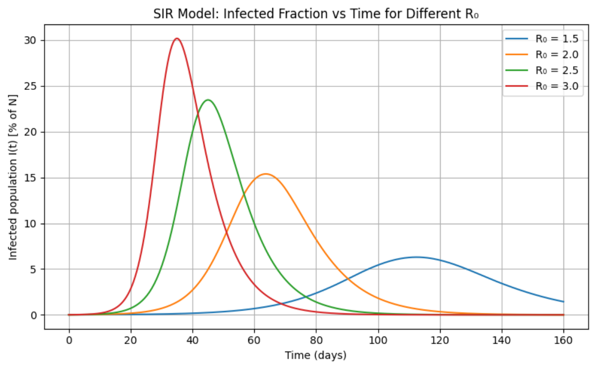


Figure 1: Time evolution of $I(t)$ for various R_0 values. Higher R_0 leads to faster and larger peaks.

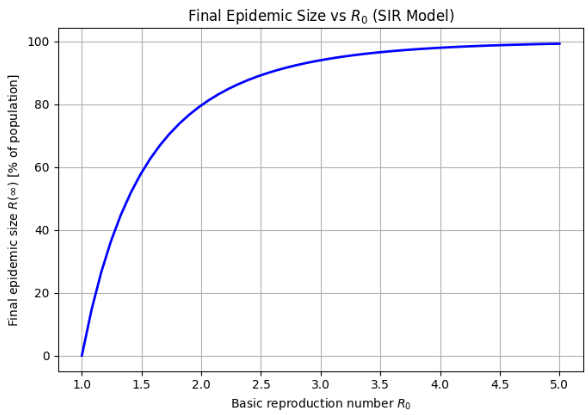


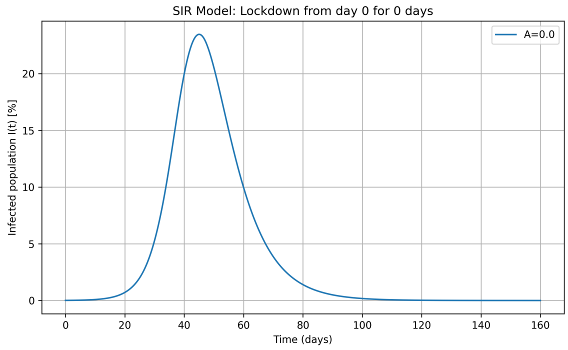
Figure 2: Final epidemic size $R(\infty)$ rises rapidly with increasing R_0; higher transmissibility leads to a larger infected fraction and quicker saturation.

(b) Short Lockdown with Immediate Effect

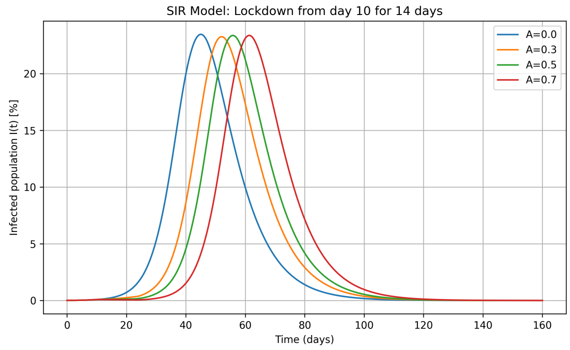
A short lockdown is modeled as a temporary reduction in transmission:

$$\beta(t) = (1 - \theta(t))\beta, \quad \theta(t) = \begin{cases} A, & t_1 \le t \le t_2 \\ 0, & \text{otherwise} \end{cases}$$

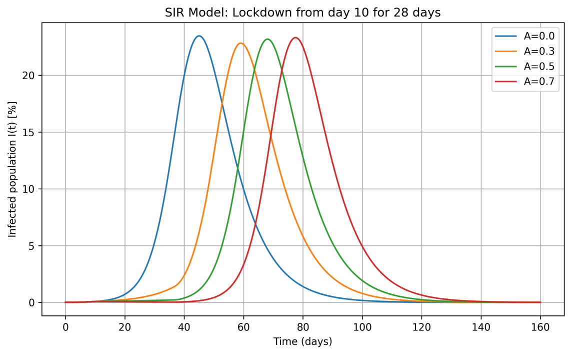
where A is the lockdown strength and $\Delta = t_2 - t_1$ is the duration.



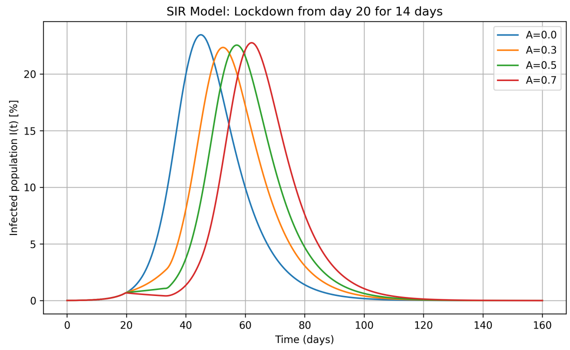
(a) Baseline epidemic without lockdown. Rapid rise and early peak around day 25.



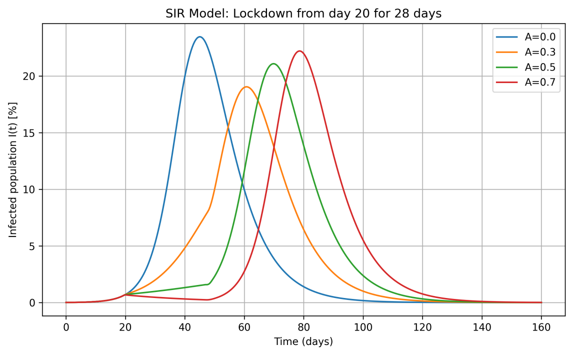
(b) Early lockdown (day 10, 2 weeks). Peak flattened but infection rebounds post-lift.



(c) Early lockdown (day 10, 4 weeks). Longer intervention delays and lowers peak substantially.



(d) Late lockdown (day 20, 2 weeks). Limited effect since epidemic already widespread.



(e) Late lockdown (day 20, 4 weeks). Stronger mitigation but less effective than early intervention.

Figure 3: SIR model simulations illustrating effects of lockdown timing, duration, and strength. Early and longer lockdowns flatten and delay the epidemic peak, while late or short ones yield limited benefits.

Observation. Earlier lockdowns reduce the immediate peak but may lead to rebounds if a large susceptible fraction remains ($R_{\text{eff}}(t) = R_0 S(t)/N > 1$) when restrictions lift.

Final conclusions (lockdown scenarios)

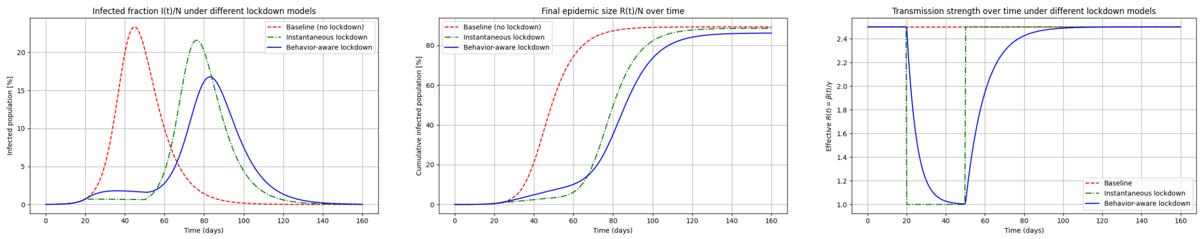
Compared to the baseline run ($I_{\max} = 23.35\%$, $t_{\text{peak}} = 44.9$ d, $R(\infty) = 89.27\%$), the simulated interventions give:

- **A=0.3, $t_1 = 20$, $\Delta = 14$:** $I_{\max} = 22.41\%$ (-4.0%), $t_{\text{peak}} = 52.5$ d (+7.6 d), $R(\infty) = 88.91\%$ (-0.40%).
- **A=0.5, $t_1 = 20$, $\Delta = 14$:** $I_{\max} = 22.45\%$ (-3.9%), $t_{\text{peak}} = 56.9$ d (+12.0 d), $R(\infty) = 88.93\%$ (-0.39%).
- **A=0.5, $t_1 = 10$, $\Delta = 14$:** $I_{\max} = 23.24\%$ (-0.5%), $t_{\text{peak}} = 56.5$ d (+11.6 d), $R(\infty) = 89.22\%$ (-0.06%).
- **A=0.7, $t_1 = 20$, $\Delta = 28$:** $I_{\max} = 22.12\%$ (-5.3%), $t_{\text{peak}} = 78.6$ d (+33.7 d), $R(\infty) = 88.78\%$ (-0.55%).

Summary : the lockdowns in these SIR runs mainly *delay* the peak (up to +33.7 days) and produce only small reductions in peak height ($\leq 5.3\%$ absolute) and final epidemic size ($\leq 0.6\%$ absolute). Strength and duration increase the delay and give marginally larger peak reductions; early short lockdowns mostly buy time rather than substantially reduce total infections.

Bottom line: as modeled, short/medium lockdowns are useful to delay the epidemic but do not meaningfully lower the cumulative attack rate unless combined with sustained reductions in transmission or other measures.

(c) Extra Credit: Behavior-Aware Lockdown Ramp



(a) Temporal evolution of $I(t)/N$

(b) Cumulative infections $R(t)/N$

(c) Time-varying $R(t) = \beta(t)/\gamma$

Figure 4: Comparison of baseline (no lockdown), instantaneous lockdown, and behavior-aware lockdown dynamics. The gradual (behavior-aware) lockdown shows a smoother, delayed, and significantly reduced epidemic peak.

Scenario	$I_{\max}$ (%)	t_{peak} (days)	$R(\infty)$ (%)	Area (percent-days)
Baseline	23.35	44.9	89.27	624.86
Instantaneous lockdown	21.60	76.2	88.58	620.08
Behavior-aware lockdown	16.78	83.0	86.17	603.16

Table 2: Quantitative comparison of baseline, instantaneous, and behavior-aware lockdown outcomes ($A = 0.6$, $t_1 = 20$, $t_2 = 50$, $\tau_1 = 5$, $\tau_2 = 10$).

Quantitative differences vs. baseline:

- **Instantaneous lockdown:** Peak infections −7.5%, peak delayed by +31.3 days, final epidemic size −0.79 percentage points.
- **Behavior-aware lockdown:** Peak infections −28.1%, peak delayed by +38.1 days, final epidemic size −3.05 percentage points.

Quantitative differences vs. instantaneous lockdown:

- Peak infections further reduced by **22.9%** (from 21.60% → 16.78%).
- Final epidemic size reduced by **2.41 percentage points** (from 88.58% → 86.17%).
- Peak shifted later by **6.8 days**.

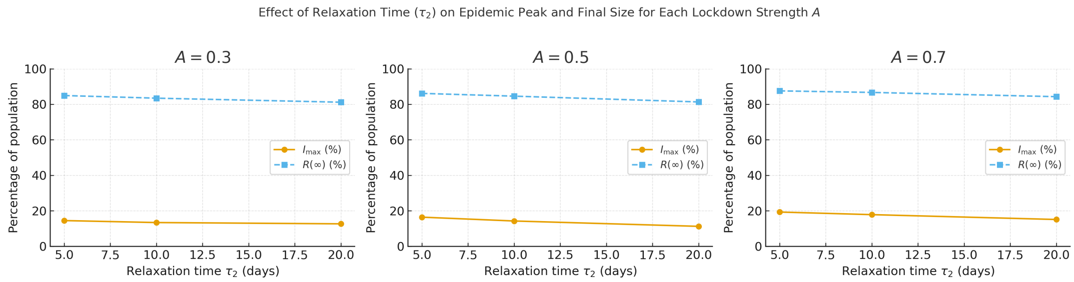


Figure 5: Effect of relaxation time τ_2 on epidemic outcomes for different lockdown strengths A. Increasing τ_2 (slower relaxation) consistently reduces both the peak and the final epidemic size, with moderate strengths ($A \approx 0.5$) performing best.

A	τ_1	τ_2	$I_{\max}$ (%)	t_{peak} (days)	$R(\infty)$ (%)	Area (percent-days)
0.5	10	20	9.10	76.2	79.33	555.30
0.3	10	20	13.20	50.5	80.77	565.36
0.7	3	5	20.75	84.2	88.21	617.49

Table 3: Representative best and worst behavior-aware lockdowns compared to baseline. The best configuration ($A = 0.5$, $\tau_1 = 10$, $\tau_2 = 20$) achieves ≈61% reduction in peak and ≈10 percentage-point reduction in final epidemic size.

Observations from the 3-Panel Comparison

- Increasing the relaxation time τ_2 (slower reopening) consistently lowers both $I_{\max}$ and $R(\infty)$ across all A values.
- The improvement is most pronounced for moderate lockdown strength $A = 0.5$, where the peak falls by over 60% and the final epidemic size by nearly 10 percentage points relative to baseline.
- Stronger lockdowns ($A = 0.7$) do not yield proportionally better outcomes unless τ_2 is large; rapid relaxation ($\tau_2 \leq 5$) quickly erodes the benefits.

- Short adaptation times τ_1 (fast compliance) are less influential than long relaxation times τ_2, indicating that **sustained cautious reopening is more critical than early strict enforcement.**
- For small τ_2, all A values converge toward similar outcomes, showing that short-lived compliance only delays the epidemic without meaningfully reducing total infections.

Final Conclusions

- The behavior-aware lockdown significantly outperforms a sharp on/off lockdown of the same nominal strength.
- Compared to the instantaneous lockdown, it **reduces the epidemic peak by** $\approx 23\%$, **reduces the final infected population by** ≈ 2.4 **percentage points**, and **delays the peak by** ≈ 7 **days**.
- The overall epidemic burden (area under $I(t)$) decreases by about **3.5%**.
- Within the broader parameter sweep, the best outcome ($A = 0.5$, $\tau_1 = 10$, $\tau_2 = 20$) lowers peak infections by **61%** and final epidemic size by nearly **10 percentage points** relative to the uncontrolled baseline.
- High compliance strength ($A > 0.5$) gives diminishing returns unless relaxation (τ_2) is slow. A moderate lockdown with gradual adoption and slow reopening yields the most stable and effective epidemic suppression.