Modeling and Simulation, MC312

Lab-4 (2025)

Mukesh Tiwari

Diffusion of Innovation (Bass Model)

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Background

Many technologies/products exhibit S-shaped adoption: slow initial growth, rapid increase, and eventual saturation.

Introduction: Rogers and Diffusion of Innovations

When we think of population models, it is natural to begin with examples from biology. Yet, populations can also be defined in social terms. A well-known case is the spread of new ideas and technologies, described by Everett Rogers in his classic studies of the diffusion of innovations. Rogers observed that when a new technology is introduced into a community, very few people adopt it spontaneously at the outset. He called these early users *innovators* or *seed adopters*. Their decisions are often independent of what others in the community are doing. Instead, they adopt because of personal interest, curiosity, resources, or exposure to outside information.

What happens after this initial seeding is crucial. Rogers emphasized that the majority of adoption does not come from independent choices made in isolation, but from the *influence of peers*. Once a handful of innovators begin to use the new product or idea, their neighbors, friends, or colleagues are exposed to it. These interactions—through observation, conversation, or imitation—create a social feedback process. The willingness of one individual to adopt increases when others around them have already done so. As adoption spreads from one person to another, the population is no longer a collection of isolated decision makers. It becomes an *interacting population*, in which each individual's state (adopter or non-adopter) depends on the states of others.

Fig. 1 adapted from Rogers' work, illustrates exactly how innovations spread in an interacting population. In the earliest stage, the cumulative adoption curve rises only very slowly. This part of the curve is driven by a small set of *seed adopters*—the innovators—who take up the new idea or product independently of others. Their role is crucial: without this initial seeding, the curve would remain flat and the innovation would fail to take off. Once a visible group of innovators is present, interaction within the population begins to dominate. Peers observe, imitate, and exchange information. The result is an accelerating rise in adoption, which appears as the steep middle section of the S-shaped curve. Here we find the early majority and late majority, whose decisions are strongly shaped by the presence of adopters around them. Adoption is no longer an isolated decision but the outcome of social influence in an interacting population.

Finally, as the market becomes saturated and only a few non-adopters remain, the cumulative curve levels off. This tail end represents the laggards, whose adoption comes last and often reluctantly. The full S-shape is therefore a signature of both seed adoption at the beginning and peer-to-peer interaction throughout the process. In this way, the Rogers curve makes visible the transition from independent action by a few innovators to widespread adoption driven by social interaction.

From Rogers' Curve to a Modeling Framework

The S–shaped curve observed by Rogers invites us to ask: what mechanisms generate this pattern? A useful way to think about it is to separate two sources of influence on adoption:

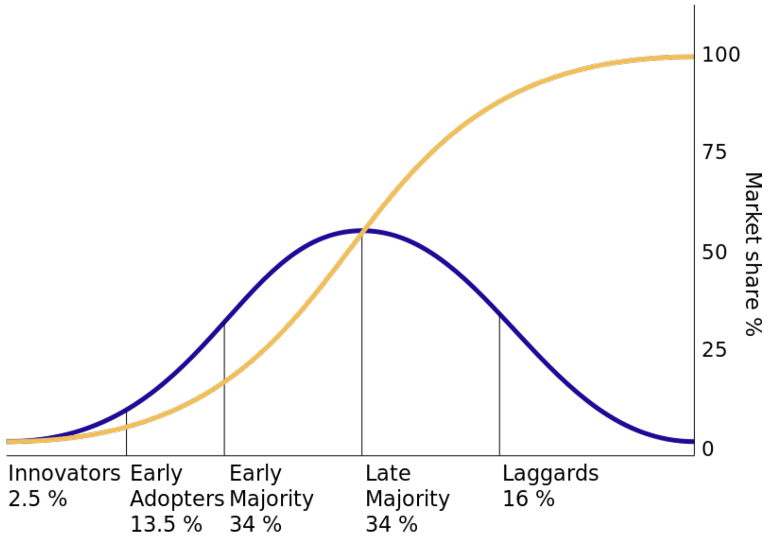

Figure 1: The diffusion of innovations according to Rogers. With successive groups of consumers adopting the new technology (shown in blue), its market share (yellow) will eventually reach the saturation level. The yellow curve is known as the logistic function. (figure and caption taken from wikipedia)

- **External influence (advertising, media, policy):** Individuals adopt independently of how many others have already done so. These are the "innovators" in Rogers' terminology, triggered by exposure to information campaigns, incentives, or direct promotion. The rate of adoption here depends on a constant push from outside the social system.
- **Internal influence (word of mouth, peer imitation):** Individuals adopt because they see others around them doing so. The likelihood of adoption increases with the fraction of adopters in the population. This captures the essence of interacting populations: adoption spreads through contact and communication between peers.

When only external influence is present, the adoption curve rises smoothly and gradually, but rarely produces the sharp acceleration seen in real diffusion. When only internal influence operates, adoption can accelerate dramatically once a few seed adopters are present, but without that initial seeding the process never starts. The observed S–shaped curve arises most naturally when *both forces act together*: advertising or promotion creates the first wave of innovators, and peer–to–peer influence then drives the rapid spread through the majority of the population. This perspective leads directly to mathematical models in which the rate of change of adopters is written as a combination of an *external term* and an *internal interaction term*. In the next part of the lab we will make these ideas precise, and explore how varying the relative strength of advertising versus word–of–mouth changes the shape of the adoption curve.

Modeling Framework: Advertisement vs Word-of-Mouth

Typically the mathematical model for such diffusion problems has the form

$$\dot{N} = \alpha(t)(C - N(t)),$$

where $N(t)$ is the cumulative number of adopters by time t, C is the total market potential, and $\alpha(t)$ is the effective rate of adoption. Three variants are common:

- **External influence (advertisement):** $\alpha(t) = p$, a constant.

- **Internal influence (word of mouth):** $\alpha(t) = \frac{q}{C} N(t)$.
- **Mixed influence (Bass model):** $\alpha(t) = p + \frac{q}{C} N(t)$.

Mixed Influence Model (Bass): Peak Timing and Width

We focus on the mixed (Bass) model for the fraction of adopters $x(t) = N(t)/C$:

$$\dot{x} = (p + q\,x)(1 - x), \qquad 0 \le x(t) \le 1, \quad p > 0,\ q > 0.$$

Let the *adoption rate* be $r(t) = \dot{x}(t)$ (the bell-shaped curve), and the *cumulative adoption* be $x(t)$ (the S-curve).

1. **Numerical Explorations.**
   (a) For a grid of parameter pairs (p, q) (e.g., $p \in [0.01, 0.1]$, $q \in [0.1, 1]$ with a few representative values), simulate $x(t)$ and compute the time t^* at which $r(t)$ attains its maximum.
   (b) Plot t^* against the ratio p/q. Propose and fit a simple functional form $t^* \approx f(p/q)$ (e.g., log-linear, power law, or rational form). Outside the lab hours try getting an analytical solution.
   (c) Width of the adoption curve: Define the *width* of $r(t)$ as the full width at half maximum (FWHM): the time difference between the two points where $r(t)$ equals one-half of its peak value. Compute this width w numerically for the same (p, q) grid.

Provide a discussion, based on your understanding of the introduction to the problem and the numerical simulations. If you can adjust only one lever of p or q, which one would you adjust. What are some limitations of thinking of adoption purely from a model like this.

EV Adoption: How far along the curve?

Let us consider the dataset `electric-car-stocks.csv`, which reports the cumulative number of electric cars in use across many countries and years. Using these data, compare EV adoption between developed nations (e.g., EU countries, the United States) and developing nations (e.g., India, Brazil, Chile). From the cumulative adoption numbers, estimate the parameters p, q, and C of the mixed (Bass) model. From the fitted curves, determine how far along the adoption curve each country is (refer to Fig. 1). That is, relate your findings to Rogers' adopter categories: based on the fitted curve, classify whether each country is still in the *innovator/early adopter* stage, has moved into the *early majority*, or is approaching the *late majority/laggards.* Also discuss whether the estimated C appears plausible or overly optimistic for each country. Comment on or discuss how well EVs are being adopted globally, supported by your fitted parameters and positions on the curve.

For fitting. To estimate the three parameters (p, q, C) from the EV adoption data, you may use:

- **MATLAB:** `lsqcurvefit` (Optimization Toolbox) or `fminsearch`.
- **Python:** `scipy.optimize.curve_fit`.

Important: Before fitting, rebase the years to start at zero (e.g. set $t = \text{Year} - 2010$) so that the estimated parameters p and q remain of reasonable magnitude. After fitting, compute and plot the residuals (errors) defined as

$$\text{error}(t) = N_{\text{data}}(t) - N_{\text{fit}}(t).$$

Make a plot of errors versus time for each country you analyze.