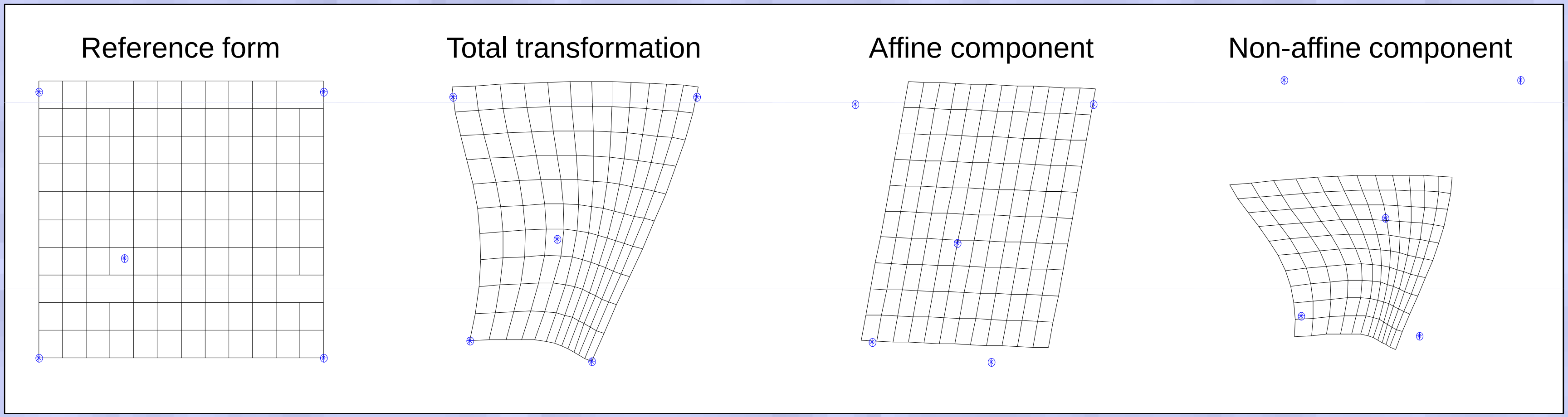
Multivariate statistics and geometric morphometrics

- Eigenanalysis used in several ways in geometric morphometrics:
  - Calculation of *partial warps*.
  - Use of *partial warp scores* in PCA, DFA, and CCA.
  - Direct use of landmark coordinates in PCA and DFA.

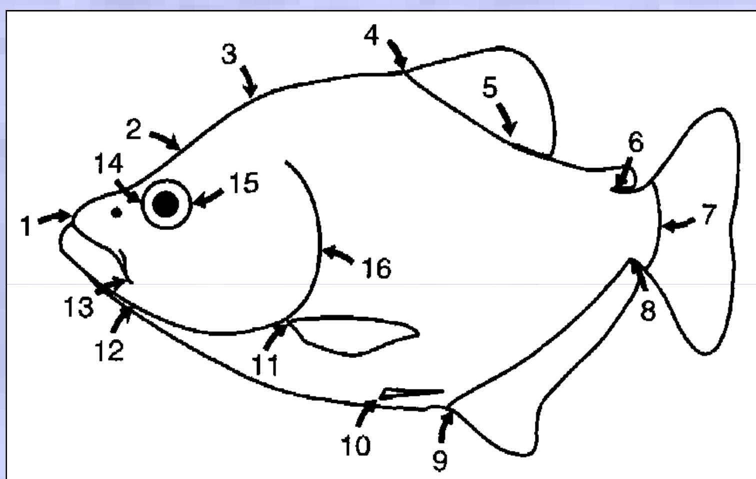
Partial warps

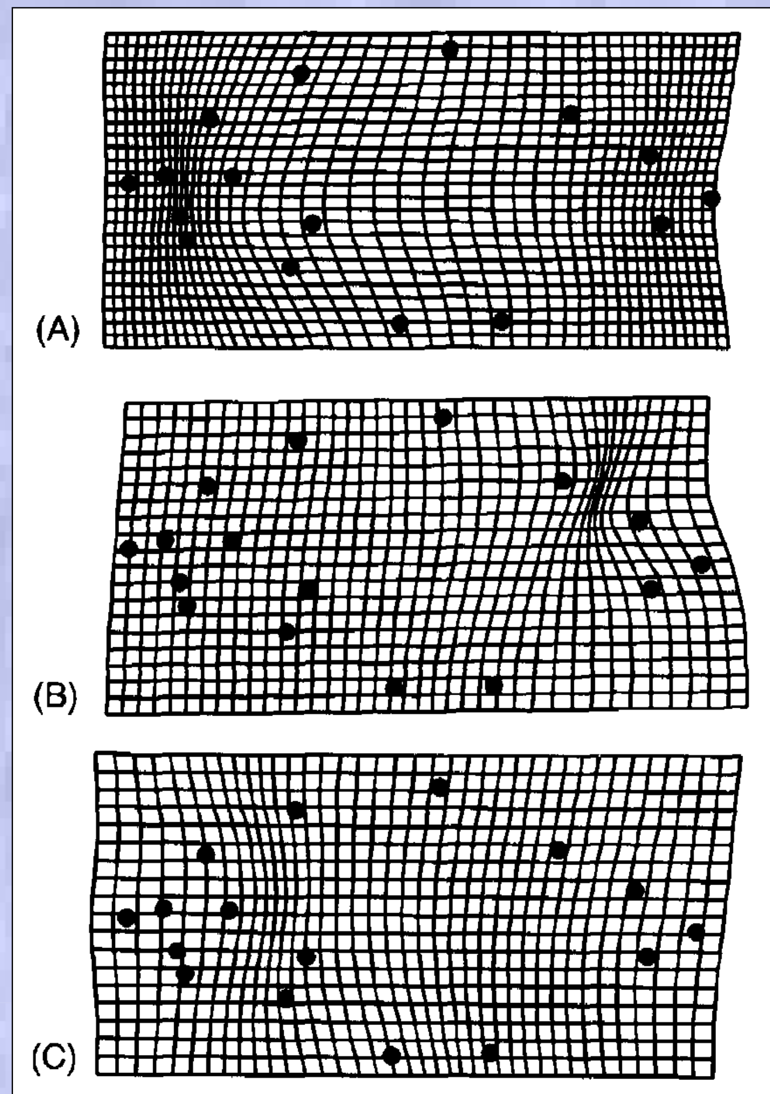
- Recall: thin-plate spline decomposes shape difference into *global* and *local* components:
  – *Uniform*, *affine component* is a tilted plane viewed in perspective.
  – *Non-uniform*, *non-affine component* characterizes regional deformations (warping of the thin plate).
    - Characterized by bending-energy matrix.
  – *Total deformation* is sum of the two components.



- *Non-uniform* portion of deformation:
  - Describes changes that vary in nature and extent across the organism.
  - Can be further decomposed into set of *orthogonal* components: *partial warps*.
    - Eigenvectors of bending-energy matrix.
  - Characterize changes at progressively smaller, more localized spatial scales:
    - 1st partial warp describes change at largest scale (lowest bending energy).
    - 2nd partial warp describes change at smaller scale.
    - Etc.
  - For *k* landmarks, can calculate *k*-3 partial warps.

- Partial warps 1–3 for Fink’s ontogenetic piranha data:





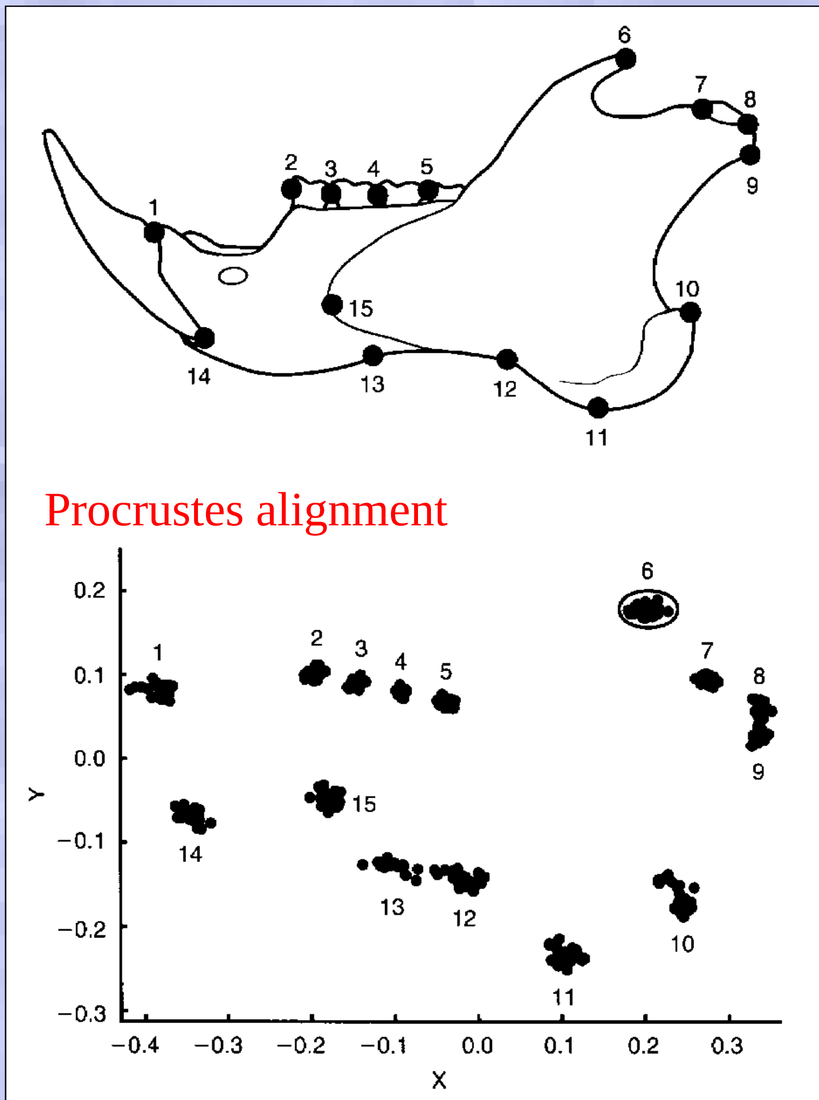
Large scale

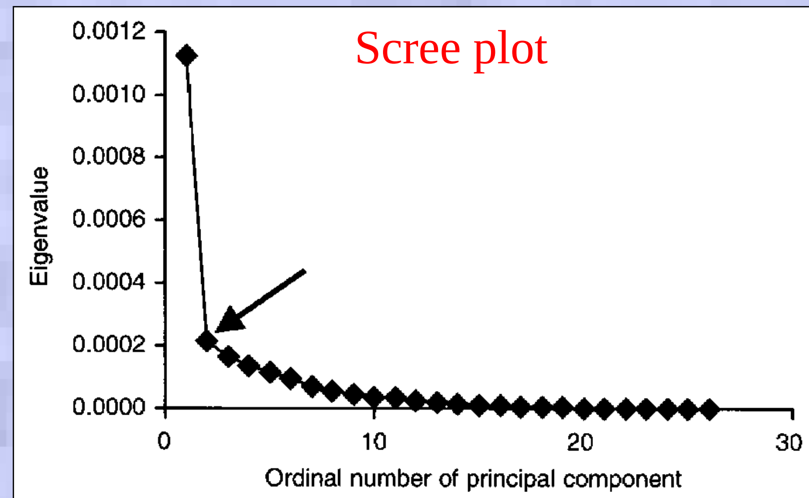
Moderate scale

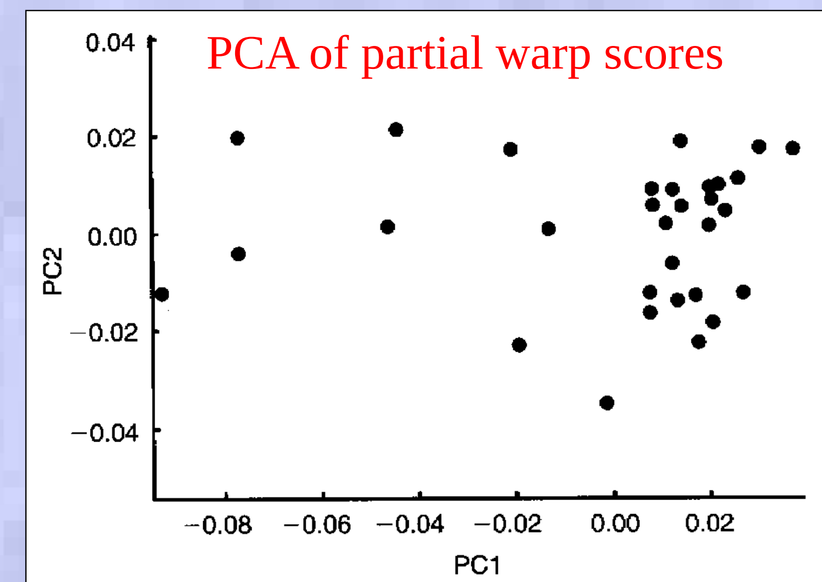
Small scale

- Corresponding to each partial warp is a vector of *partial warp scores.*
- Calculated by projecting differences between aligned specimens and reference form onto partial warps.
  - Represent *k*-3 points within the tangent space spanned by the partial warps.
  - *X* and *Y* components, indicating directions of partial warp.
  - Express contribution that each partial warp makes to the total deformation.
  - Can be used as set of 2(*k*-3) *shape variables* to represent individuals in multivariate analyses.
- *Principal components* of partial warp scores are sometimes called *relative warps*.

- Example from Zelditch et al. (2004):
  - Sample of 31 lower jaws of fox squirrels:







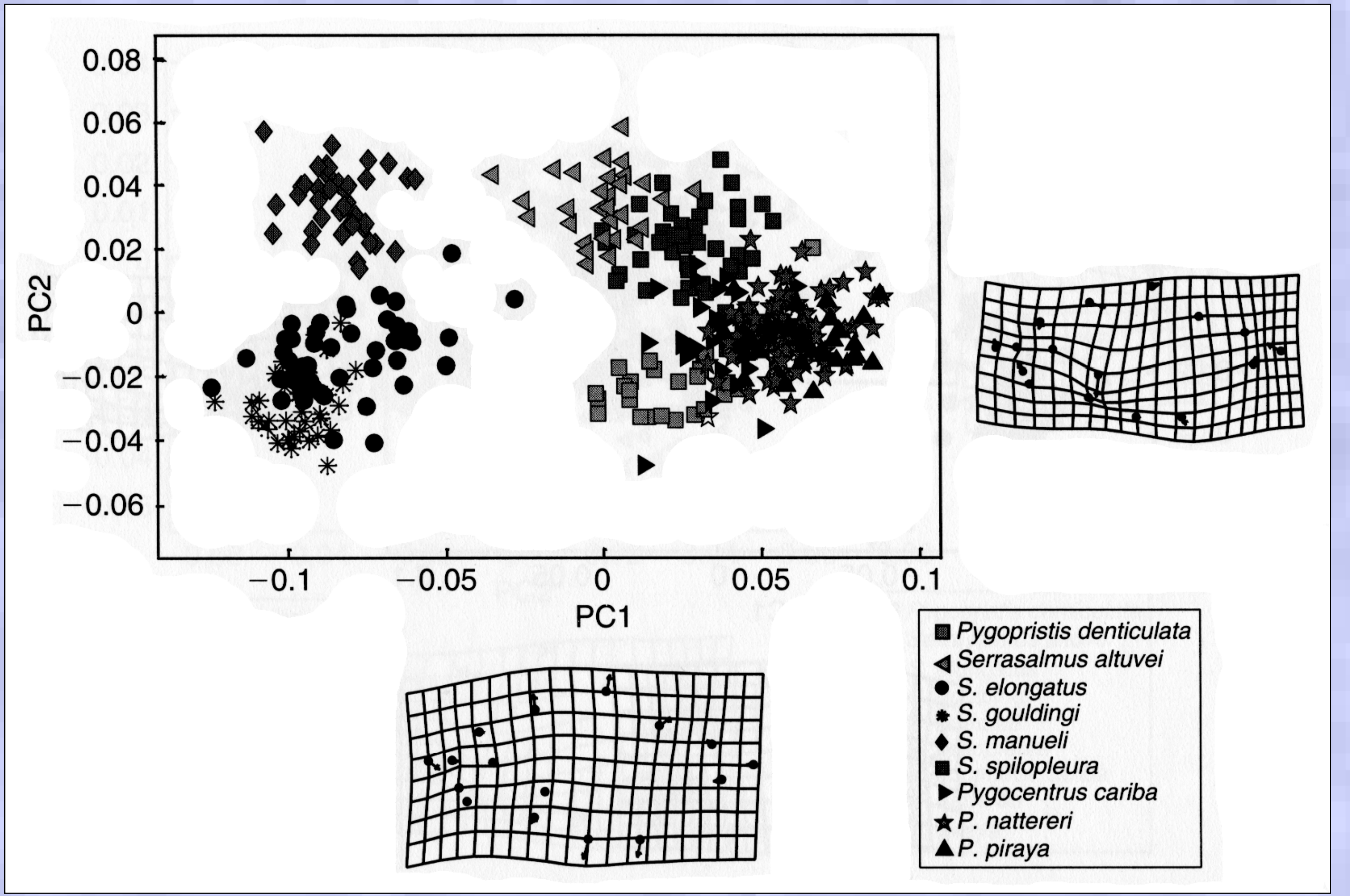
- *Principal warp* = partial warp interpreted as a bent surface of the thin-plate spline.
- *Uniform component* of TPS is sometimes called the *0^{th} principal warp*.
  - Orthogonal to all partial warps.
- The *uniform component* of the TPS can also be decomposed into orthogonal components, characterizing:

  (1) compression/dilatation

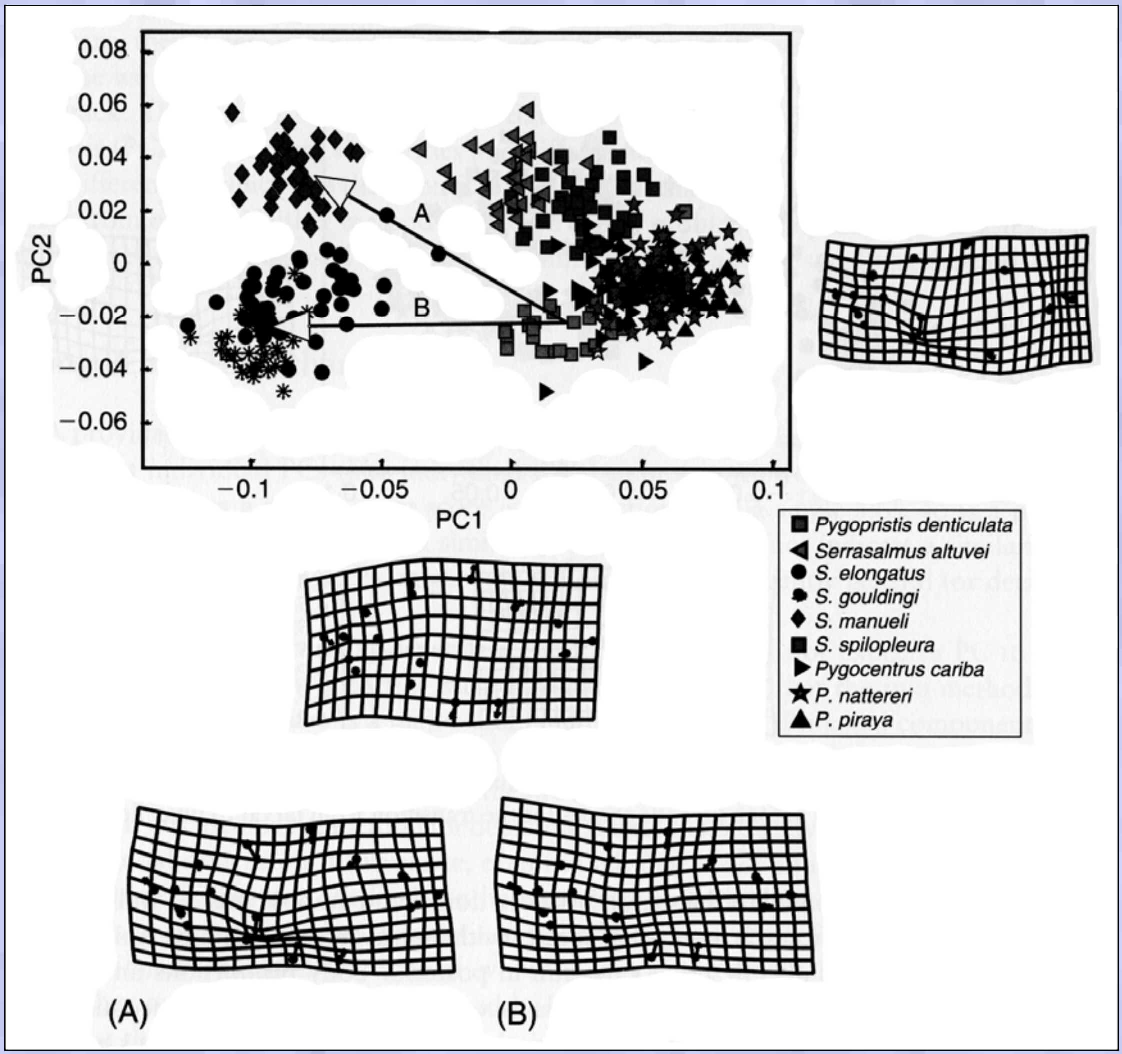
  (2) shear
  - Not an eigenanalysis.

- Important ironical note:
  - Bending energy matrix is a function of *only* the reference (concensus) form of the TPS deformation.
  - *Partial warps are not features of the shape change*.
    - Warps do not describe variation among individual forms, or deformations between forms.
    - Cannot be interpreted as components of the deformation.
    - Provide a useful coordinate system within the space in which shape change is analyzed and visualized.
      - Tangent space.
      - Projections of forms are partial-warp scores.

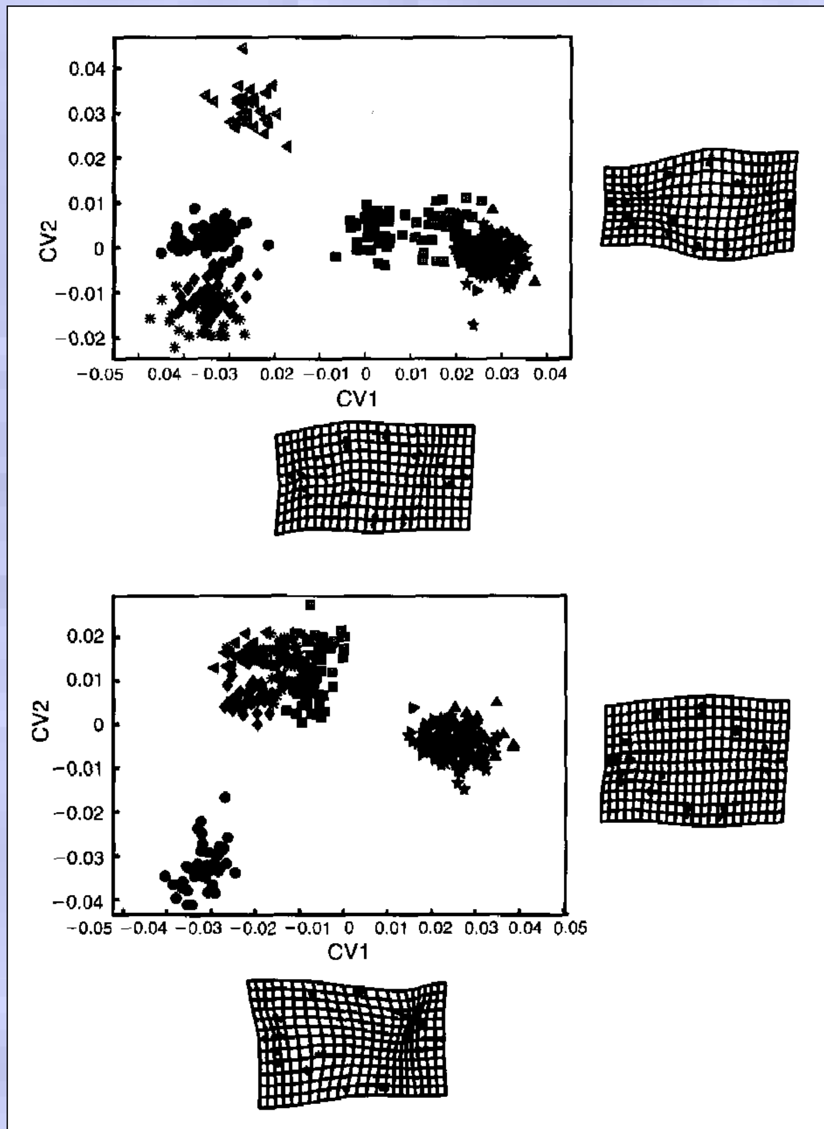
- Examples from Zelditch et al. (2004):
  - PCA (*relative warps*) of juveniles of 9 species of piranhas.
  - Data: matrix of *partial warp scores* on *1st partial warp.*

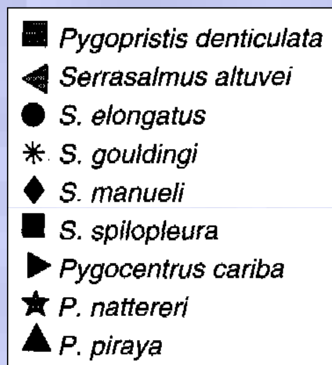


- Analysis of direction in which species differ from *Pygopristis denticulata* in juvenile shape:

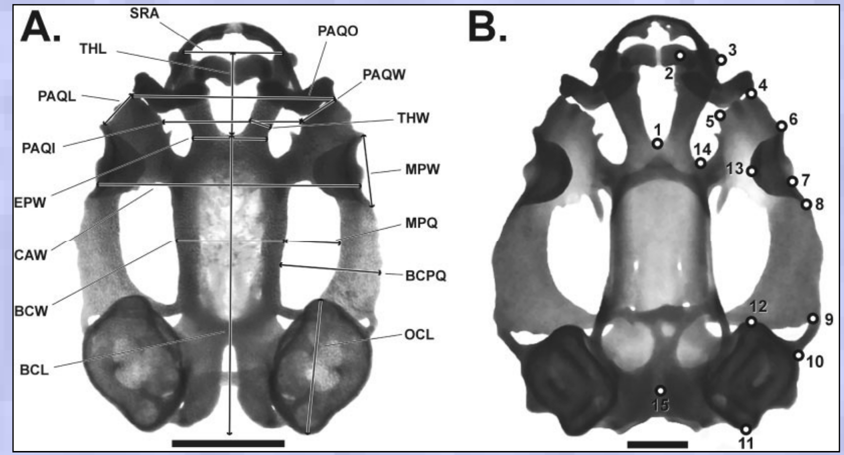


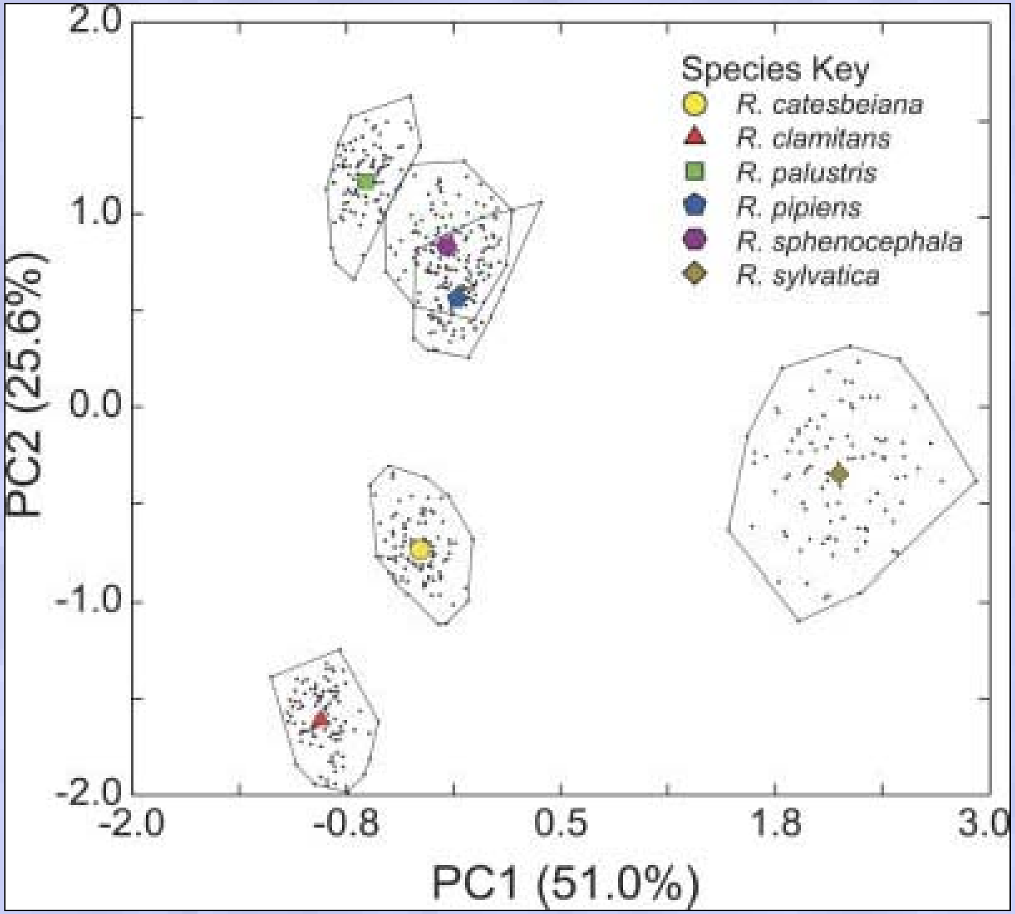
- Discriminant analysis (DFA, =CVA) of juvenile and adult piranhas:

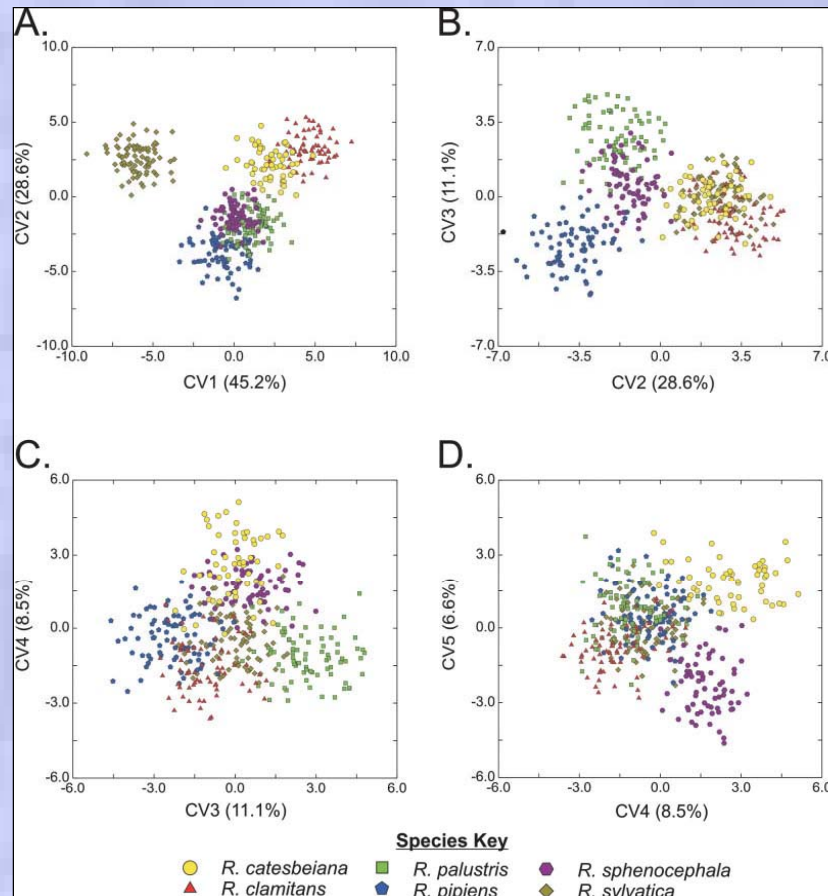




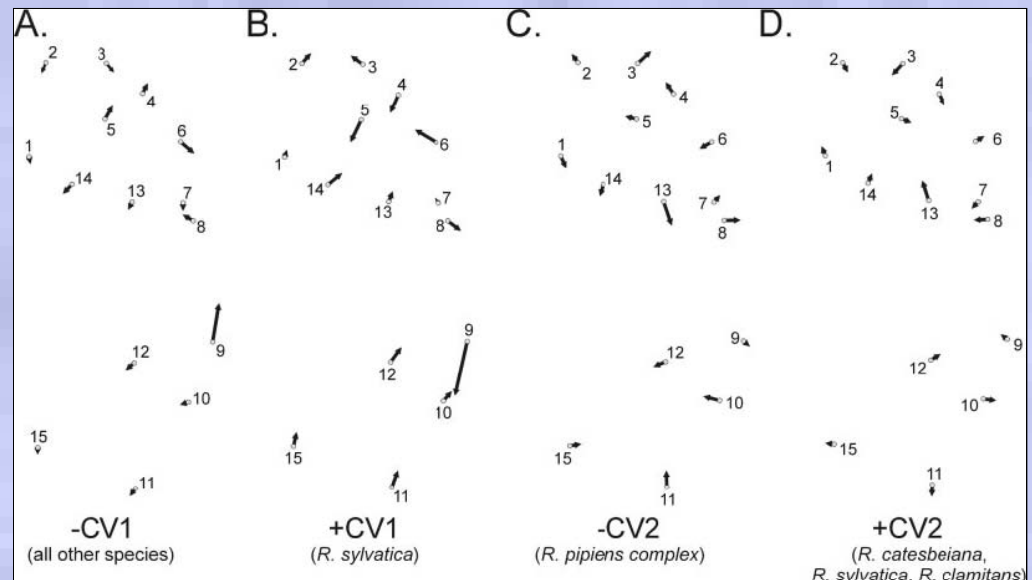
- Larson (2005): ontogenetic development in *Rana* tadpoles.
  - Used landmarks for PCA and CVA of partial warp scores, and distances for allometric analyses:







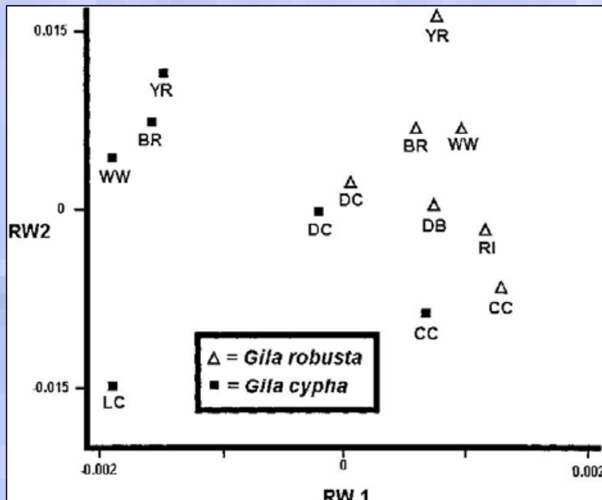
A.
B.
C.
D.
CV1 (45.2%)
CV2 (28.6%)
CV3 (11.1%)
CV4 (8.5%)
CV5 (6.6%)
Species Key
R. catesbeiana
R. clamitans
R. palustris
R. pipiens
R. sphenocephala
R. sylvatica

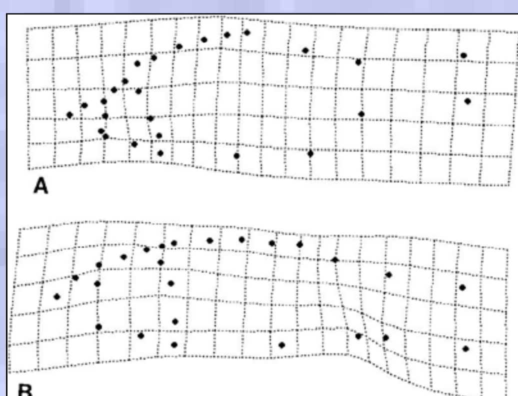


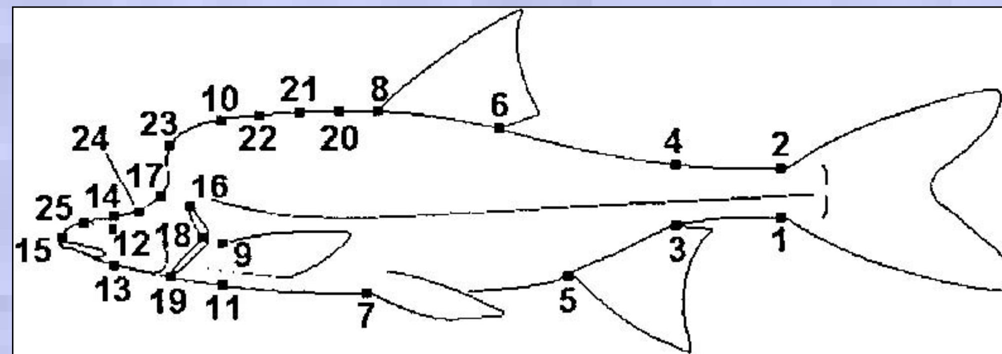
A.
B.
C.
D.
-CV1
(all other species)
+CV1
(R. sylvatica)
-CV2
(R. pipiens complex)
+CV2
(R. catesbeiana,
R. sylvatica, R. clamitans)

- Douglas et al. (2001): Differentiation of *Gila* populations in upper Colorado drainage based on partial warp scores:

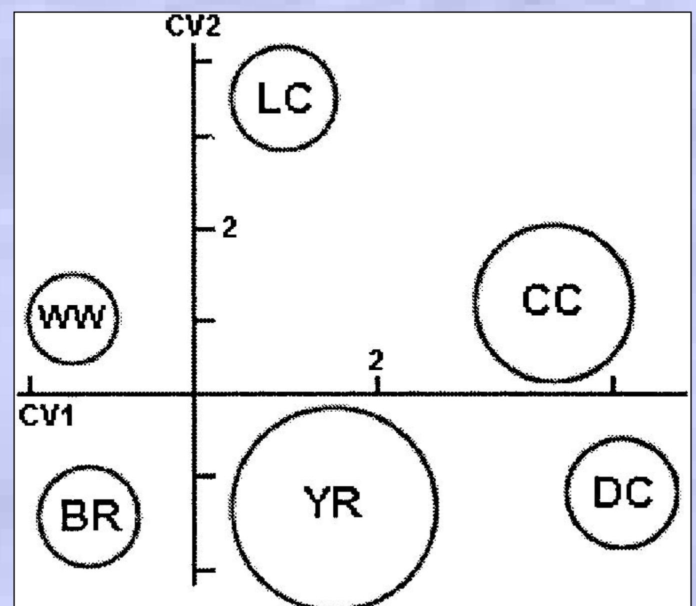
Relative warps:



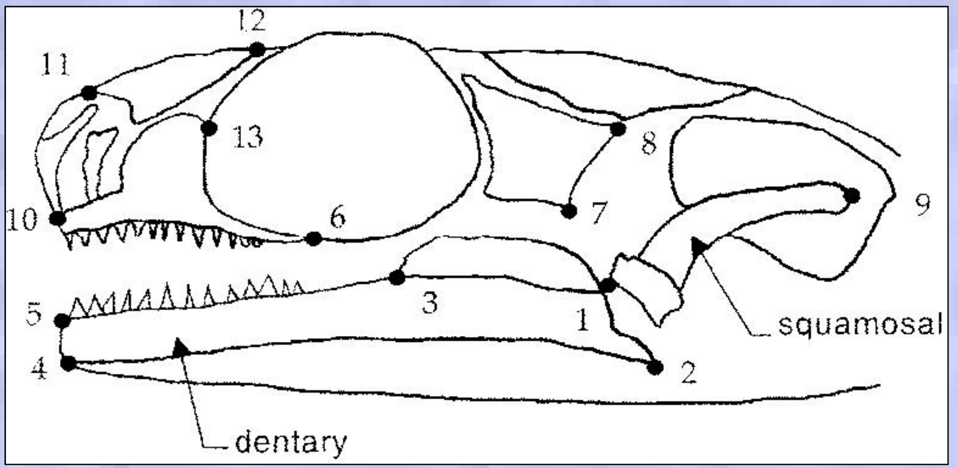




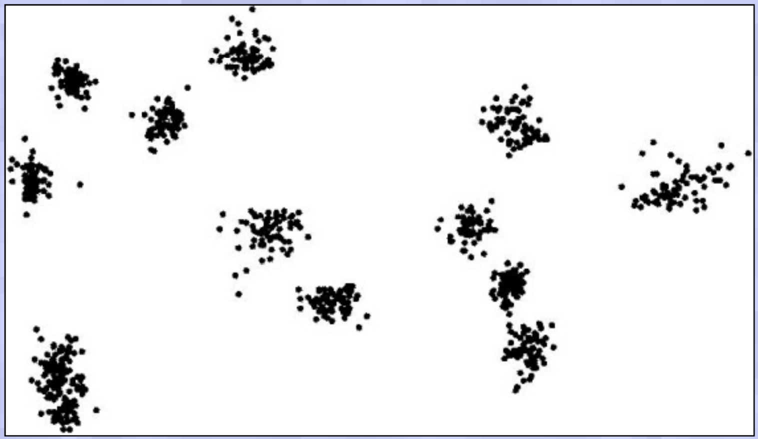
Discriminant analysis:



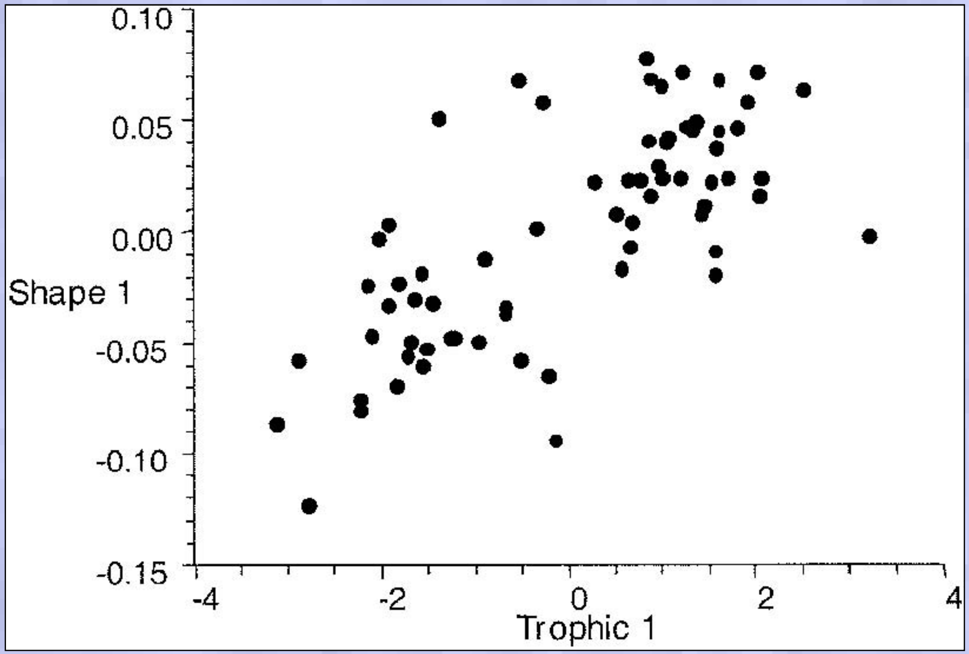
- Rohlf & Corti (2000): use of partial warp scores in canonical correlation analysis (actually, PLS) of *Plethodon* skulls:
  - Two species from single locality: 38 *P. hoffmani* + 31 *P. cinereus*.
  - *Skull landmarks* characterizing trophic morphology.
  - *Stomach contents*: importance ranks of 16 prey items.



Procrustes GLS:



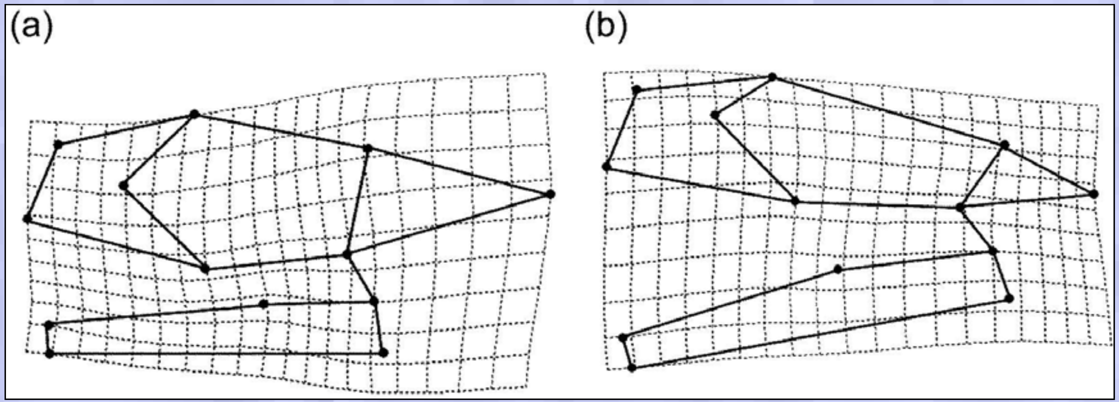
First canonical axes:



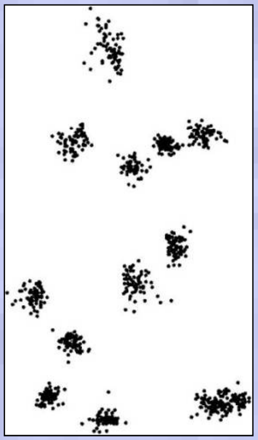
Trophic axes:

	Dimensions		
Variable	1	2	3
Acarina	0.028	0.039	0.037
Eggs	−0.004	0.139	−0.171
Isoptera	0.020	−0.190	0.071
Collembola	0.015	−0.118	0.141
Chelonethida	0.026	0.004	0.042
Hymenoptera	−0.578	0.005	0.656
Gastropoda	0.106	0.134	0.278
Larvae	−0.073	0.409	0.226
Coleoptera	0.679	−0.342	0.472
Diptera	−0.291	−0.185	0.169
Araneida	0.078	0.130	0.078
Isopoda	0.260	0.754	0.152
Orthoptera	−0.051	0.061	−0.221
Diplopoda	0.042	0.009	−0.054
Oligochaeta	0.153	−0.001	0.224
Chilopoda	−0.017	0.057	0.025
Singular values	0.0547	0.0096	0.0076
Correlations	0.725	0.557	0.442

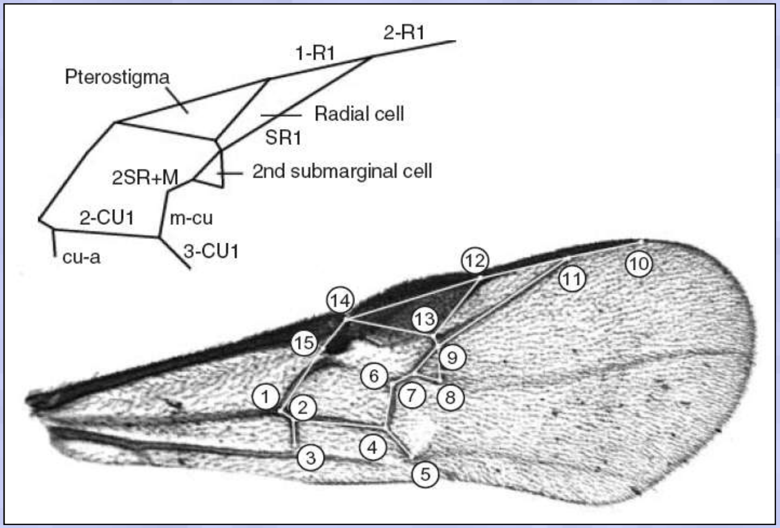
Extent of variation on 1st shape axis:



- Use of *landmark coordinates* as data:
  - Avoids necessity of TPS and partial warp decompositions.
  - Sets of digitized landmarks superimposed by Procrustes GLS transformation.
  - *X* and *Y residuals* for each landmark from consensus configuration used as 2*k* variables.



- *Ex.*: Baylac et al. (2003): wing venation in parasitoid wasps:



- PCA and DFA of two species of *Bassus*: