1. **Use PCA to compress images from the sklearn digits dataset. Reconstruct the images using fewer principal components and compare the results.**
   Goals:

   - Apply PCA to reduce the dimensionality of the digit images.
   - Reconstruct the compressed images using a selected number of components.
   - Compare the visual quality of the reconstructed images with the original ones using a side-by-side plot.

2. **Perform PCA on the iris dataset to understand how much variance is explained by each principal component. Additionally, you must determine how many principal components are required to capture at least 95% of the total variance in the data.**
   Goals:

   - Apply PCA to the standardized Iris dataset.
   - Calculate and plot the cumulative explained variance for each principal component.
   - Identify the minimum number of components needed to explain 95% of the variance.
   - Visualize this with a plot that shows the cumulative explained variance.

3. **Perform Singular Value Decomposition (SVD) on a randomly generated matrix and verify that the original matrix can be reconstructed using the product of the decomposed matrices.**
   Goals:

   - Generate a random matrix of size 5x5.
   - Perform SVD on this matrix to obtain the U, Σ (singular values), and V^T matrices.
   - Reconstruct the original matrix using the decomposed matrices.
   - Compare the original and reconstructed matrices and compute the difference.

4. **Apply both PCA and SVD for dimensionality reduction on the Breast Cancer dataset, and compare the results in terms of variance explained and reconstruction accuracy.**
   Goals:

   - Apply PCA to the standardized dataset and reduce it to 5 components.
   - Perform SVD on the same dataset, also reducing it to 5 components.
   - Compare the explained variance for PCA and the reconstruction accuracy for both methods.
   - Calculate and report the reconstruction error (mean squared error) for both methods.

5. **You are given a small collection of text documents. Use `TruncatedSVD` on a TF-IDF matrix derived from a small set of text documents to reduce the dimensionality of the matrix and reconstruct it.**
   Goals:

   - Convert a set of text documents into a TF-IDF matrix using `TfidfVectorizer`.
   - Apply TruncatedSVD to reduce the dimensionality of the TF-IDF matrix to 2 components.
   - Reconstruct the original TF-IDF matrix from the reduced representation.
   - Compare the reconstructed matrix with the original one to assess how much information was retained.