Wow, such a masterpiece!

Oh, hey everyone! Are you wondering what this book is?
Well,

This book is filled with the achievements and the life-journeys of various mathematicians of the ancient world.

And, this chapter is one of my favourite chapters. It’s about Thales.

Do you know who Thales was?

Let me tell you.

Thales was one of the most famous Greek mathematicians who lived from around six hundred twenty-four B.C to five hundred forty-five B.C.

He was so knowledgeable that he was named the first of Greece's seven sages.

He gave one of the most important theorems about triangles known as the Basic Proportionality Theorem.

But what is the basic proportionality theorem that Thales proposed?

Let’s understand it.

PRESENTER
The basic proportionality theorem

states that, 'If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.'

Let's consider a triangle 'ABC'.

And we are given that a line 'DE' is parallel to the side 'BC' intersecting the other two sides 'AB' and 'AC' at points 'D' and 'E' respectively.

We need to prove that 'AD' divided by 'DB' equals 'AE' divided by 'EC'.

To prove this theorem, we will do a construction. Let's join the points 'B' and 'E' and the points 'C' and 'D'.

Now, draw a perpendicular from point 'E' to meet side 'AB' at point 'N'.

So, line segment 'EN' is perpendicular to side 'AB'.

Then draw another perpendicular from point 'D' to meet the side 'AC' at the point 'M'.

Thus, the line segment 'DM' is perpendicular to side 'AC'.

Let's now begin with the proof.

Since the LHS of the statement to be proved is AD by DB, let us consider two triangles, ADE and BDE, whose bases are AD and DB respectively.

We have learnt that the area of a triangle is half times the base multiplied by the corresponding height.

So if we consider triangle 'ADE' where the base is 'AD' and the corresponding height is 'EN', what will be the area of triangle ADE?

Correct! The area of the triangle 'ADE' will be equal to half multiplied by 'AD' multiplied by 'EN'.

Let's consider this as our equation one.

Similarly, consider the triangle 'BDE'.

If we consider side BD to be the base of this triangle, what is the corresponding height?

For the base 'BD', the corresponding height is 'EN'.

But how can the height of triangle BDE be outside the triangle?

That is because angle BDE is an obtuse-angle.

And the height corresponding to the base of a triangle, whose one of the base angles is obtuse, will lie in the exterior of the triangle.

Now, can you tell me the area of the triangle ‘BDE’?

Correct! Area of the triangle ‘BDE’ equals half times ‘BD’ multiplied by ‘EN’.

Let this be our equation two.

To get the LHS of the statement to be proved, let us divide equation one by equation two, and we will get—

Area of triangle ‘ADE’ divided by area of triangle ‘BDE’ equals to half multiplied by ‘AD’ multiplied by ‘EN’, whole divided by,

half multiplied by BD multiplied by ‘EN’.

Thus, the area of triangle ‘ADE’ divided by the area of triangle ‘BDE’ equals ‘AD’ divided by ‘BD’.
And this becomes our equation three.

Moving on, since the RHS of the statement to be proved is AE by EC,

let us consider two triangles, ADE and CDE, whose bases are AE and EC respectively.

What is the area of triangle ‘ADE’?

Correct! The area of triangle ‘ADE’ is equal to half multiplied by ‘AE’ multiplied by ‘DM’.

Let this be equation four.

In the triangle 'CDE', if we consider the side 'EC' as the base, what is the corresponding height?

You're right! 'DM' will be the height of the triangle 'CDE' corresponding to the base 'EC'.

Again, why is the height of the triangle CDE outside the triangle?

Correct! That is because base angle CED is an obtuse-angle.

And, what would be the area of triangle 'CDE'?

Area of the triangle 'CDE' will be equal to half multiplied by 'EC' multiplied by 'DM'.
And that's going to be equation five.

To get the RHS of the statement to be proved, let us divide equation four by equation five, and we will get—

Area of triangle 'ADE' divided by area of triangle 'CDE' equals half multiplied by 'AE' multiplied by 'DM', whole divided by,

half multiplied by EC multiplied by DM.

And so, the area of triangle 'ADE' divided by area of triangle 'CDE' is equal to 'AE' divided by 'EC'.
Let us mark this as equation six.

Now here, it gets interesting.

Look at the triangles 'BDE' and 'CDE'.

These two triangles are between the same parallel lines 'BC' and 'DE'.

And what about their bases?

Yes! Both the triangles 'BDE' and 'CDE' have the same base 'DE'.

What can you say about the areas of two triangles on the same base and between the same parallels?

Correct! They are equal in area.

So, we can say that the area of triangle 'BDE' equals the area of triangle 'CDE'.

This means, their reciprocals will also be the same. That is, 1 by area of triangle BDE equals 1 by area of triangle CDE.

Now if we multiply both sides of this equation by the area of triangle ADE, we will get—

Area of triangle 'ADE' divided by area of triangle 'BDE' is equal to area of triangle 'ADE' divided by area of triangle 'CDE'.

Let this be equation seven.

From equations three, six, and seven, we get, 'AD' by 'DB' is equal to 'AE' by 'EC'.

Hence, we have proved that 'AD' by 'DB' is equal to 'AE' by 'EC'.

This is the basic proportionality theorem.

Or the Thales theorem.

Thus, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Let us now look at the converse statement of this theorem.

But what do you mean by the converse of a theorem?

Let us consider a given theorem in the form…'if p then q'.

Here 'p' is the given part of the theorem and 'q' is the part required to be proved.

Then the converse of this theorem will be obtained by interchanging the given part and the part to be proved of the theorem.

That is, the converse of a theorem will be in the form of 'if q then p'.

So, can you tell me the converse of the basic proportionality theorem?

Right! The converse of this theorem will be—

if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. But is this converse statement true?

Let's see.

Consider a triangle 'ABC'.

Here, we are given that AD by DB is equal to AE by EC, and

we have to prove that segment 'DE' is parallel to side 'BC'.

So for construction,
let a line intersect sides 'AB' and 'AC' at points 'D' and 'E' respectively such that 'AD' divided by 'DB' equals 'AE' divided by 'EC'.

Let's prove the converse of the basic proportionality theorem using the indirect method of proof.

But do you know what the indirect method of proof is?

You can watch our special video on 'indirect method of proof' to know about it.

Let us get back to our proof. So, to prove the converse of the basic proportionality theorem by indirect method, let's assume that segment 'DE' is not parallel to the side 'BC'.

If DE is not parallel to side BC, then there must exist another line that is parallel to side BC.

So, let us consider another point 'E' dash on side 'AC' such that segment 'DE' dash is parallel to the side 'BC'.

Since 'DE' dash is parallel to 'BC', by the basic proportionality theorem,

we can say that—
'AD' by 'DB' is equal to 'AE' dash by 'E' dash C.

Let this be equation one.

And we are given that 'AD' by 'DB' is equal to 'AE' by 'EC'.

Let's consider this as equation two.

In equations one and two, what do you observe about their left hand side?

Correct! The left hand side of both the equations are the same.

So, what can you conclude about the right hand side of these equations?

Correct! Their right hand side must also be the same.

So, we can say that 'AE' dash by 'E' dash C equals 'AE' by 'EC'.

Let us add one on both sides of this equation.

So, we will get,

'AE' dash by 'E' dash C, plus one,
equals 'AE' by 'EC', plus one.

Simplifying this, we get—
'AE' dash plus 'E' dash C, whole divided by 'E' dash C, equals 'AE' plus 'EC', whole divided by 'EC'.

But what is 'AE' dash plus 'E' dash C?

Right! It's AC.

And what is AE plus EC?

Yes, AE plus EC is AC.

So, the equation can now be written as 'AC' by 'E' dash C equals 'AC' by 'EC'.

Simplifying this, we can say, 'E' dash 'C' is equal to 'EC'.

As points 'E' and 'E' dash are on the same line segment 'AC', 'E' dash 'C' will be equal to 'EC' only when points 'E' and 'E' dash coincide.

In other words, 'E' and 'E' dash must be the same point.

Hence, we can say that 'DE' and 'DE' dash are the same.

Earlier, we began by assuming that 'DE' dash is parallel to the side 'BC'.

So, we can say that segment 'DE' is parallel to side 'BC'.

Thus, we can conclude that the converse of the basic proportionality theorem is true.

In general, if a line divides any two sides of a triangle in the same ratio, then the line will be parallel to the third side.

And that's all about the basic proportionality theorem and its converse.

Let's now quickly summarise all that we learnt.

The basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

It is also known as the Thales theorem.

The converse of basic proportionality theorem states that if a line divides any two sides of a triangle in the same ratio, then the line will be parallel to the third side.

We have studied the basic proportionality theorem and its converse. But, what are its applications?

Well, watch our next video to know more about it.