Grade: 10

Chapter No. 6: Triangles

TID: v22cb10ma0607

Research Doc: Link

Video Overview:

Idea 1: Fact + Activity.

KT and MT	Content Approach	Reference Links	Merits/ Demerits
KT:	Hook and Content Flow		
MT:	Activity: Presenter is playing with the magnetic rods and with a smile on his face, looks up at the viewers.		
	Presenter: Oh, hey everyone! Well I found these in my old box of toys. Not just that I discovered something which could a quite interesting fact if I was as interested in shapes as I am now,		
	Now have a look at this."		
	Presenter attaches four magnetic sticks to small spherical shaped balls and makes a square,		

Presenter: "As you can see, this shape seems like a square, but it's not very stable."

Presenter changes its shape while pushing its corners.

Presenter: Like, I can change it to something like parallelogram or rhombus.

And it's the same with any other polygon, but,

If I connect three magnetic rods and construct a triangle, it's not that easy to change its shape. And that's why triangles are considered as the most sturdy and strong shapes out of others.

But if I make a bigger magnetic triangle, something this big,

Will that be similar to a triangle as small as this, Well we've studied several criteria of similarities in triangle.

Like, in previous videos we proved several of its criteria like AAA, AA, and SAS criteria of similarity.

But for this time, let's try something exclusive,

FSA: SAS criteria of similarity.

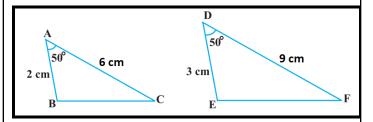
SAS criterion for similarity

Let's perform an activity to understand the criterion.

Activity

Step-1:

Draw two triangles ABC and DEF such that AB = 2 cm, $\angle A = 50^{\circ}$, AC = 5 cm, DE = 3 cm, $\angle D = 50^{\circ}$ and DF = 10 cm as shown in the figure.



Step-2: Obtain the value of $\frac{AB}{DE}$ and $\frac{AC}{DF}$, we get

$$\frac{AB}{DE} = \frac{2}{3}$$
 and $\frac{AC}{DF} = \frac{6}{9} = \frac{2}{3}$

That is.

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{2}{3}$$

And also, $\angle A = \angle D = 50^{\circ}$

Step-3: Measure \angle B, \angle C, \angle E and \angle F. You will find that \angle B = \angle E and \angle C = \angle F. That is, \angle A = \angle D, \angle B = \angle E and \angle C = \angle F. Therefore, by AAA criterion for similarity, $\triangle ABC \sim \triangle DEF$

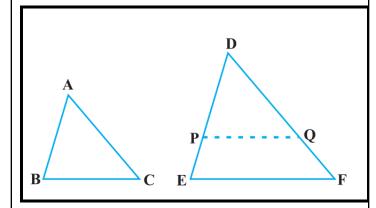
You can repeat this activity by creating multiple pairs of these triangles with one angle of a triangle equal to one angle of another triangle and the sides including these angles are proportional. Everytime, you will find that the triangles are similar.

It is due to the following criterion of similarity of triangles:

Theorem:

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This criterion is referred to as the SAS (Side–Angle–Side) similarity criterion for two triangles.



Consider two triangles ABC and DEF such that $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF} (< 1)$

To prove: $\triangle ABC \sim \triangle DEF$

Construction: Mark points P and Q on DE and DF respectively such that DP = AB and DQ = AC. Join PQ.

Proof:

In $\triangle ABC$ and $\triangle DPQ$, we have AB = DP [by construction] $\angle A = \angle D$ AC = DQ [by construction]

Therefore, by SAS criterion of congruence, we get $\triangle ABC \cong \triangle DPQ$ (1)

Now, $\frac{AB}{DE} = \frac{AC}{DF}$ [given] $\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ [:: AB = DP and AC = DQ] $\Rightarrow PQ \mid\mid EF$ [by converse of the Thale's theorem] $\Rightarrow \angle DPQ = \angle DEF$ and $\angle DQP = \angle DFE$ [corresponding angles]

Thus, in $\triangle DPQ$ and $\triangle DEF$ $\angle DPQ = \angle DEF$ $\angle DQP = \angle DFE$

Therefore, by AA criterion for similarity

$\Delta DPQ \sim \Delta DEF$ (2)	
From equation (1) and (2), we have $\triangle ABC \cong \triangle DPQ$ and $\triangle DPQ \sim \triangle DEF$ $\Rightarrow \triangle ABC \sim \triangle DPQ$ and $\triangle DPQ \sim \triangle DEF$ [all congruent triangles are similar] $\Rightarrow \triangle ABC \sim \triangle DEF$	
Remark: If two triangles ABC and DEF are similar, then: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \qquad \text{[ratio of corresponding sides are equal]}$ $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB + BC + AC}{DE + EF + DF} \qquad \text{[using ratio and proportion]}$	
$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta DEF}$	

Idea 2: Anthony james: Sculptures

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KT:	Hook and Content Flow		

MT:

Presenter is deeply engaged in his laptop and attentively looking at it. Instantly, breaking the fourth wall he turns to the viewers.

Presenter: Oh, hey everyone! I was browsing just randomly on my laptop and I just came across this artist named Anthony james.

He has made a few sculptures and they're just awesome.

Have a look at this tesseract like a sculpture filled with mirrors.

And all of them are just triangles.

Isn't this fascinating, I feel like there's no end to all these triangles.

But are these all triangles similar, well they look similar.

So how do we find similarities between triangles, Well we have learned a few concepts like AAA, AA and SSS criteria of similarities but let's try something new this time.

FSA: SAS criteria of similarity.

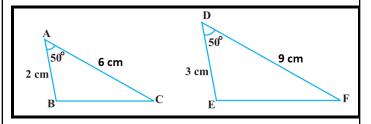
SAS criterion for similarity

Let's perform an activity to understand the criterion.

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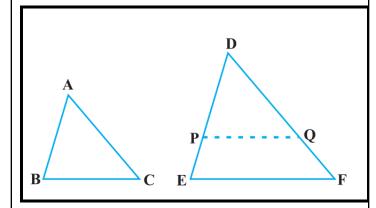
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Proof:

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Therefore, by SAS criterion of congruence, we get $\triangle ABC \cong \triangle DPQ$ (1)

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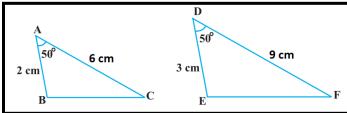
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Idea 3: Explicit along with the Activity

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	Activity		
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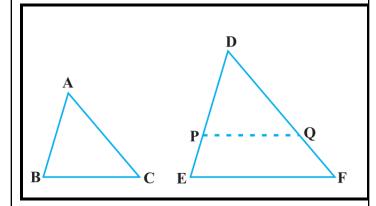
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Remark:

If two triangles ABC and DEF are similar, then: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \qquad \text{[ratio of corresponding sides are equal]}$ $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB + BC + AC}{DE + EF + DF} \qquad \text{[using ratio]}$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta DEF}$$