Electrostatic Potential & Capacitance

Electric Potential

- Work done per unit test charge by an external agent in moving the test charge from reference point to the desired point. Its SI unit is J/C
  $V_0 = \frac{\text{Work}}{\text{charge}}$. Its SI Unit

Electric potential due to a point charge

$V_P = Q/4\pi\varepsilon_0 r$

Potential due to a system of charges

$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{Q_i}{r_i}$

Potential due to a dipole

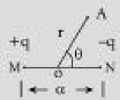
$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$

Where $p = qd$

$\theta = \angle AON$

At $\theta = 0$, $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$ [axial position]

At, $\theta = 90°$, $V = 0$ [equitorial position]



In 1774, Alessandro Volta wrote treatise"on the forces of attraction of electric fire".

Equipotential Surface

- Potential is same at all the points of the surface.
- Component of electric field parallel to an equipotential surface is zero.

Electric Potential Energy

- It is negative count of work done by the electric force as the configuration of the system changes.
- $Ur_2 - Ur_1 = -W = \frac{q_1 \cdot q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$
- If the separation between charges is 'r' then $U_{(r)} = \frac{q_1 \cdot q_2}{4\pi\varepsilon_0 r}$
- Potential Difference,
  $V_B - V_A = \frac{U_B - U_A}{q}$
  $U_B - U_A$ = change in Potential Energy
  q = test charge

Potential Energy of a dipole

$dU = pE\sin\theta d\theta$

If we choose P.E. of dipole to be zero when $\theta = 90°$ then

$U_\theta - U_{90°} = \int_{90°}^{\theta} pE\sin\theta\, d\theta$

$U_\theta = -pE\cos\theta = -\vec{p}\cdot\vec{E}$

If it is rotated through angle θ against the torque

Conductors & Insulators

Conductors	Insulators
Such a material which when placed in an electric field, the free electrons move in a direction opposite to the field.	Such a material in which electrons are tightly bound, & when exposed in an electric field, electrons do not move *i.e.* having no free electrons.

- Electric field inside a conductor is zero.
- Electric field is always perpendicular to the charged surface.
- In static state, there will be no additional charge in a conductor.

Dielectric

- $F = \frac{q_1 q_2}{4\pi K\varepsilon_0 r^2}$, K = dielectric constant of medium
- A dielectric is an electrical insulator that can be polarized by "application of" electric field.

Capacitance, $C = \frac{Q}{V}$

Capacitance of a parallel plate capacitor

$C = K\varepsilon_0 A/d$, K = dielectric constant

Capacitance when material slab is inserted between the plates

$C = K\varepsilon_0 A/[Kd - x(K-1)]$

where x = thickness of the slab inserted

Capacitance of a spherical capacitor

$C = 4\pi\varepsilon_0 / \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For isolated sphere

$C = 4\pi\varepsilon_0 R$

Parallel grouping of capacitors

$C = C_1 + C_2 + C_3 + \cdots + C_n$

for two, $C = C_1 + C_2$

Series grouping of capacitors

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$

For two, $C = \frac{C_1 C_2}{C_1 + C_2}$

Energy stored in a capacitor

$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = 1/2\, QV$

CHAPTER

2 ELECTROSTATIC POTENTIAL AND CAPACITANCE

Syllabus

- *Electric potential, potential difference, electric potential due to a point charge, electric dipole and system of charges; equi-potential surfaces, electrical potential energy of a system of two point charges and of electric dipole in an electrostatic field.*
- *Conductors and insulators, free charges and bound charges inside a conductor, Dielectrics and electric polarisation, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.*

Chapter Analysis

List of Concepts Name	2017	2018	2019
Electric Potential	1 Q (1 mark) 1 Q (3 marks)	1 Q (3 marks)	1 Q (1 mark) 1 Q (3 marks)
Capacitance	1 Q (2 marks) 1 Q (3 marks)		1 Q (5 marks)

TOPIC-1
Electric Potential

Revision Notes

Electric potential

- Electric potential is the amount of work done by an external force in moving a unit positive charge from infinity to a point in electrostatic field without producing an acceleration.
- It is written as $V = \frac{W}{q}$

  where, W = work done in moving charge q through the field, q = charge being moved through the field.
- The SI units of electric potential are $\frac{J}{C}$, Volt, $\frac{Nm}{C}$

Scan to know more about this topic

Electric Potential

Potential difference

- Electric potential difference is defined as the amount of work done to carry a unit charge from one point to another in an electric field.

  Electric potential difference = $\frac{\text{Work}}{\text{Charge}} = \frac{\Delta PE}{\text{Charge}} = \frac{W}{q}$

Between two points A and B, $W_{AB} = -V_{AB} \times q$
where, $V_{AB} = V_B - V_A$ is potential difference between A and B.

➢ In a region of space having an electric field, the work done by electric field dW, when positive point charge q, is displaced by a distance ds, then,

$$dW = q\vec{E}.\vec{ds}$$

$$\Delta V = V_{AB} = V_B - V_A = -\frac{W_{AB}}{q} = -\frac{\int_A^B q\vec{E}.\vec{ds}}{q} = -\int_A^B \vec{E}.\vec{ds}$$

Scan to know more about this topic

Electric Potential Due to Point Charge

Electric potential due to point charge

➢ The electric potential by point charge q, at a distance r from the charge, can be written as,

$$V_E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

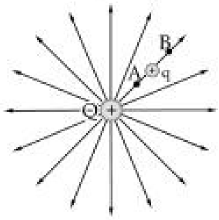
where, ε_0 is permittivity of vacuum (free space)

➢ Electric potential is a scalar quantity.

➢ Dimension of Electric potential is $[M L^2T^{-3}A^{-1}]$

➢ For a single point charge q the potential difference between A and B is given by,

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}.\vec{ds} = -\int_A^B Eds\cos 0^\circ = -E\int_A^B ds$$



where, E is the field due to a point charge, $ds = dr$, so that,

$$V_B - V_A = -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0}\cdot\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

➢ If $r_B = \infty$, then $V_B = 0$ so,

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_A} = \frac{kq}{r_A}$$

Dipole and system of charges

➢ Electric dipole consists of two equal and but opposite electric charges which are separated by a distance.

➢ The net potential due to a dipole at any point on its equatorial line is always zero. So, work done in moving a charge on equatorial line is always zero.

➢ Electric potential due to dipole at a point at distance r and making an angle θ with the dipole moment p is given by, $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}.\hat{r}}{r^2} (r >> a)$

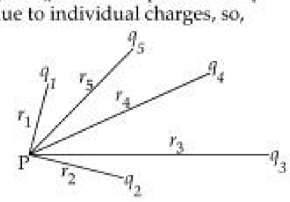
➢ Potential at a point due to system of charges is the sum of potentials due to individual charges.

➢ In a system of charges $q_1, q_2, q_3, \ldots q_n$ having positive vectors $r_1, r_2, r_3, \ldots r_n$ relative to point P, the potential at point P due to total charge configuration is algebraic sum of potentials due to individual charges, so,

$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_n}{r_n}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$



➢ It is known that in a uniformly charged spherical shell, here electric potential outside the shell is given as :

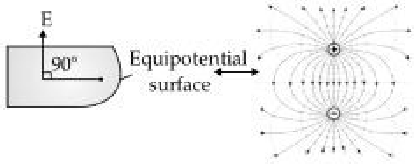
$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \qquad (r \geq R)$$

where, q is total charge on shell and R is shell radius.

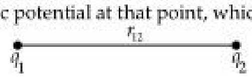
Equipotential surfaces

Equipotential Surface is a surface in space on which all points have same potential. It requires no work to move the charge on such surface, hence the surface will have no electric field, so E will be at right angle to the surface.

- Work done in moving a charge over equipotential surface is zero.
- Electric field is always perpendicular to equipotential surface.



- Spacing among equipotential surfaces allows to locate regions of strong and weak fields.
- Equipotential surfaces **never intersect** each other. If they intersect then the intersecting point of two equipotential surfaces, results in two values of electric potential at that point, which is impossible.

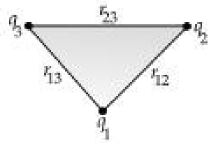


- Potential energy of a system of two charges

  $U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$

- Potential energy of a system of three charges

  $U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$

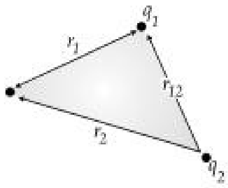


- Potential energy due to single charge in an external field :

  Potential energy of a charge q at a distance r in an external field

  $U = qV(\vec{r})$

  Here, $V(\vec{r})$ is the external potential at point r



- Potential energy due to two charges in an external field

  $U = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$

- Potential energy of a dipole in an external field :

  When a dipole of charge $q_1 = +q$ and $q_2 = -q$ having separation '$2a$' is placed in an external field ($\vec{E}$).

  $U(\theta) = -pE\cos\theta$

  Here, $p = 2aq$ and θ is the angle between electric field and dipole.

Key Formulae

- Electric Potential $V = \frac{W}{q}$, measured in volt; 1 volt = 1 Joule / coulomb.
- Electric potential difference or "voltage" $(\Delta V) = V_f - V_i = \frac{\Delta U}{q} = \frac{W}{q}$
- Electric potential due to a point charge q at a distance r away : $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
- Finding V from E : $V_f - V_i = -\int_i^f \vec{E} . d\vec{S}$
- Potential energy of two point charges in absence of external electric field : $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}}\right]$
- Potential energy of two point charges in presence of external electric field : $q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

  Note : All symbols have their usual meanings.

Objective Type Questions (1 mark each)

(I) MULTIPLE CHOICE QUESTIONS

Q. 1. The electrostatic potential on the surface of a charged conducting sphere is 100 V.

Two statements are made in this regard :

S_1 : At any point inside the sphere, electric field intensity is zero.

S_2 : At any point inside the sphere, the electrostatic potential is 100 V.

Which of the following is a correct statement?

(a) S_1 is true, but S_2 is false.

(b) Both S_1 and S_2 are false.

(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2.

(d) S_1 is true, S_2 is also true but the statements are independent. [NCERT Exemplar]

Ans. Correct option : (c)

Explanation : The relation between electric field intensity E and potential (V) is,

$$E = -\frac{dV}{dr}$$

where, Electric field intensity, $E = 0$ inside the sphere

So that, $\frac{dV}{dr} = 0$

This means that V = constant. So, if $E = 0$ inside charged sphere, the potential is constant or $V = 100$ everywhere inside the sphere and it verifies the shielding effect also. So, it justifies the option (c).

Q. 2. Equipotential surface at a great distance from a collection of charges whose total sum is not zero are approximately :

(a) spheres. (b) planes.

(c) paraboloids. (d) ellipsoids. [NCERT Exemplar]

Ans. Correct option : (a)

Explanation : The equipotential surface, are perpendicular to the field lines. So there must be electric field, which cannot be without charge.

So, the algebraic sum of all charges must not be zero. Equipotential surface at a great distance means that space of charge is negligible as compared to distance. So, the collection of charges is considered as a point charge.

Electric potential due to point charge is,

$$V = k_e \frac{q}{r}$$

which explains that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, forms spherical surface.

Q. 3. A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge :

(a) remains constant because the electric field is uniform.

(b) increases because the charge moves along the electric field.

(c) decreases because the charge moves along the electric field.

(d) decreases because the charge moves opposite to the electric field. [NCERT Exemplar]

Ans. Correct option : (c)

Explanation : As we know that, an equipotential surface is always perpendicular to the direction of electric field. Positive charge experiences the force in the direction of electric field. When a positive charge is released from rest in uniform electric field, its velocity increases in the direction of electric field. So K.E. increases, and the P.E. decreases due to law of conservation of energy.

Q. 4. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.

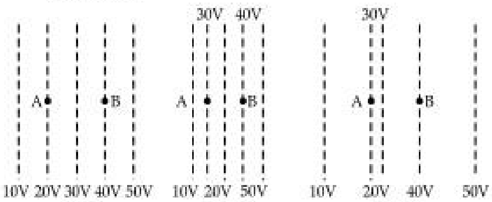


Fig I Fig II Fig III

(a) The work done in Figure (I) is the greatest.

(b) The work done in Figure (II) is least.

(c) The work done is the same in Figure (I), Figure (II) and Figure (III).

(d) The work done in Figure (III) is greater than Figure (II), but equal to that in Figure (I). [NCERT Exemplar]

Ans. Correct option : (c)

Explanation : The work done by the electrostatic force is given by $W_{12} = q(V_2 - V_1)$

As the potential difference between A and B in all three figures are equal *i.e.* 20 V, so work done by any charge in moving from A to B surface will be equal.

Q. 5. On moving a charge of 20 C by 2 cm, 2 J of work is done. Then, the potential difference between the points is : A

(a) 0.1V (b) 8 V

(c) 2 V (d) 0.5 V

Ans. Correct option : (a)

Detailed Answer :

The electric potential at any point due to a charge Q is given by,

$$V = \frac{W}{Q} = \frac{2J}{20C} = 0.1V$$

Q. 6. What is the angle between electric field and equipotential surface? U

(a) 90° always (b) 0° always

(c) 0° to 90° (d) 0° to 180°

Ans. Correct option : (a)

Detailed Answer :

Electric field is always perpendicular to the equipotential surface at any point. If the electric field lines are not considered as perpendicular, there would be a non zero component of electric field along the surface of conductor and charges could not be considered at rest.

(II) FILL IN THE BLANKS

Q. 1. The electric potential is constant in a given region. The electric field in that region is .... U

Ans. Zero

Q. 2. The electric potential .... along the lines of force. R

Ans. increases

Q3. The dielectric constant of an insulator cannot be.......... R

Ans. infinite

(III) VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. Why the electric potential at a point on the equatorial line of an electric dipole is zero? U

Ans. Since the electric dipole is a scalar quantity and the distance of the point from each charge is same. So, the net potential at the point will be zero because of the same magnitude and the opposite sign.

Q. 2. Can two equipotential surfaces intersect each other? Justify your answer. U

Ans. The normal drawn at any point on the equipotential surface will give the direction of electric field at that point. If the two surfaces intersect each other, there will be two directions of the electric field at that point which is not possible. So, the two equipotential surfaces cannot intersect each other.

Q. 3 Define the term potential energy for a charge 'q' at a distance 'r' in an external field.

Ans. The electric potential energy at a point which is at a distance 'r' from charge 'q' is defined as the amount of work done in bringing a unit positive charge from infinity to that point without any acceleration against the electrostatic force.

AI Q. 4. A point charge Q is placed at point 'O' as shown in figure. Is the potential at point A, *i.e.*, V_A, greater, smaller or equal to potential, V_B at point B, when Q is (i) positive, and (ii) negative charge?

O• A• B•

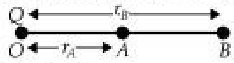
Ans. (i) $V_A > V_B$ ½

(ii) $V_A < V_B$ ½

[CBSE Marking Scheme 2017]

Detailed Answer :

Let r_A is the distance of point A from point charge Q and r_B is the distance of point B from point charge Q.



Potential at point A :

$$V(r_A) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r_A}$$

Potential at point B :

$$V(r_B) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r_B}$$

Since $r_A < r_B$, so when :

charge Q is positive; $V_A > V_B$ ½

charge Q is negative; $V_A < V_B$ ½

Q. 5. Does the charge given to a metallic sphere depend on whether it is hollow or solid?

U **[Delhi I, II, III 2017]**

Ans. No ½

Because the charge resides only on the surface of the conductor ½

[CBSE Marking Scheme 2017]

Q. 6. An electron is accelerated through a potential difference V. Write the expression for its final speed, if it was initially at rest.

A & E **[Delhi/O.D. Comptt. I, II, III 2018]**

Ans. $v = \sqrt{\frac{2eV}{m}}$ 1

[CBSE Marking Scheme, 2018]

Detailed Answer :

Kinetic energy of electron = Electric energy gain by the electron

$$\Rightarrow \quad \frac{1}{2}m(v^2 - u^2) = eV$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = eV \qquad [\text{as } u = 0]$$

$$\therefore \quad v = \sqrt{\frac{2eV}{m}} \qquad 1$$

Q. 7. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor ?

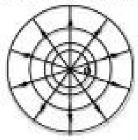
U **[O.D. I, II, III 2015]**

Ans. If it were not so, the presence of a component of the field along the surface would violate its equipotential nature. 1

[CBSE Marking Scheme 2015]

Detailed Answer :

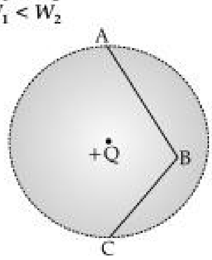
In an equipotential surface, the potential for a point charge is given by $V = \frac{kQ}{r}$. As electric field lines point radially away from the charge, they are perpendicular to equipotential surface. If the electric field lines are not perpendicular to the surface of conductor, there exists a non-zero component of electric field along the surface of conductor where charges could not be at rest. ½



½

Q. 8. In the given figure, charge $+Q$ is placed at the centre of a dotted circle. Work done in taking another charge $+q$ from A to B is W_1 and from B to C is W_2. Which one of the following is correct ?

(i) $W_1 > W_2$
(ii) $W_1 = W_2$
(iii) $W_1 < W_2$



U [CBSE SQP 2017-18]

Ans. (ii)

As $V_A - V_B = V_B - V_C$ magnitude of work done is same.

[CBSE Marking Scheme 2017] 1

Detailed Answer :

As we know $E = -\frac{dV}{dr}$ and inside a ring there is zero electric field which means constant potential

$\therefore \quad V_A - V_B = V_B - V_C$

$\therefore$ Magnitude of work done is same. 1

Commonly Made Error

- The students usually apply the formula for the potential due to a point charge instead of the relationship between electric field and potential.

Answering Tip

- Points A and B are at same potential lies on an equipotential surface.

Short Answer Type Questions (2 marks each)

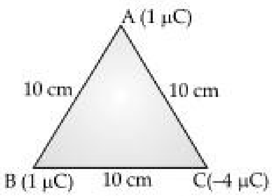
AI **Q. 1. Calculate the amount of work done to dissociate a system of three charges 1 μC, 1 μC and – 4 μC placed on the vertices of an equilateral triangle of side 10 cm.**

Ans. Given :

$q_A = 1 \times 10^{-6}\,C = q_B$

$q_C = -4 \times 10^{-6}\,C$

$r = 10\text{ cm} = 0.1\text{ m}$



$W = W_{AB} + W_{BC} + W_{AC}$

$W = \frac{1}{4\pi\varepsilon_0 r}[q_Aq_B + q_Bq_C + q_Cq_A]$

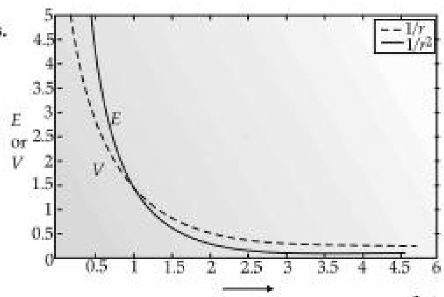
$W = 9 \times 10^9 \times \frac{1}{0.1}[1 \times 1 - 4 \times 1 - 4 \times 1] \times 10^{-12}\,J$

$W = \frac{-9 \times 10^9 \times 7 \times 10^{-12}}{0.1}\,J$

$W = -0.63\,J$ 2

AI **Q. 2. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q.**

Ans.



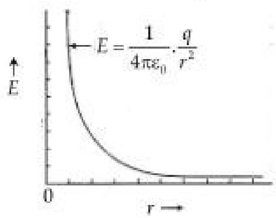
1 + 1

Detailed Answer :

Due to point charge :

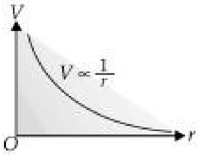
Electric field, $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$

$\because \quad E \propto \frac{1}{r^2}$ ½

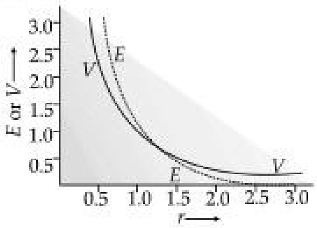


Electric potential, $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$

$\because \quad V \propto \frac{1}{r}$ ½



Now on plotting electric field and electric potential with distance r



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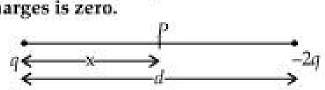
Q. 3. Two point charges q and $-2q$ are kept d distance apart. Find the location of the point relative to charge q at which potential due to this system of charges is zero.

Ans.



½

Let P be the required point at a distance x from charge q

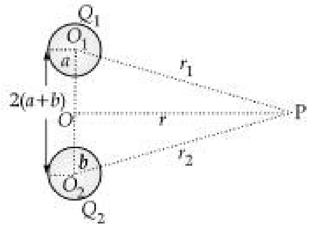
$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-2q)}{(d-x)} = 0 \qquad ½$$

$$\frac{1}{x} = \frac{2}{d-x} \qquad ½$$

$$\Rightarrow \quad x = \frac{d}{3}$$

$\therefore$ The required point is at a distance $\frac{d}{3}$ from the charge q. ½

AI Q. 4. Find the potential energy associated with a charge q if it were present at the point P with respect to the 'set-up' of two charged spheres, arranged as shown. Here O is the mid-point of the line O_1O_2.



Ans.

$$r_1 = O_1P = \sqrt{r^2 + (2a+b)^2} \qquad ½$$

$$r_2 = O_2P = \sqrt{r^2 + (a+2b)^2} \qquad ½$$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right] \qquad ½$$

$\therefore$ The potential energy of charge q, at $P = qV$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{Q_1}{[r^2+(2a+b)^2]^{½}} + \frac{Q_2}{[r^2+(a+2b)^2]^{½}}\right] \qquad ½$$

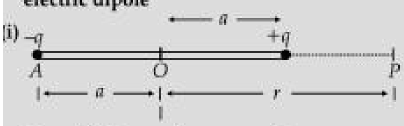
Long Answer Type Questions-I

(3 marks each)

Q. 1. (i) Derive the expression for the electric potential due to an electric dipole at a point on its axial line.
(ii) Depict the equipotential surface due to electric dipole. R **[Delhi II 2017]**

Ans. (i) Derivation of expression for electric potential due to electric dipole on axial line 2
(ii) Depiction of equipotential surfaces due to an electric dipole 1

(i)



Potential due to charge $-q$ at A :

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{(r+a)} \qquad ½$$

Potential due to charge $+q$ at B :

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r-a)} \qquad ½$$

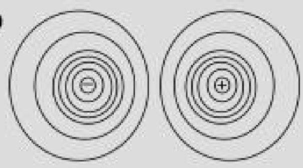
Potential at point P, $V = V_A + V_B$

Net potential at point P :

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{(r+a)} + \frac{1}{(r-a)}\right] \qquad ½$$

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times 2a}{(r^2-a^2)} \qquad ½$$

(ii)



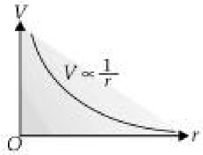
1

[CBSE Marking Scheme 2017]

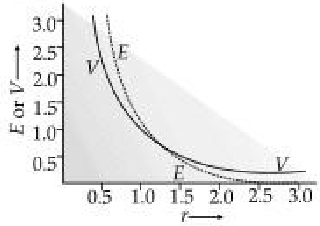
AI Q. 2. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.

Electric potential, $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$

$\because \qquad V \propto \frac{1}{r}$ ½



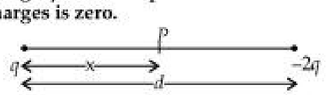
Now on plotting electric field and electric potential with distance r



1

Q. 3. Two point charges q and $-2q$ are kept d distance apart. Find the location of the point relative to charge q at which potential due to this system of charges is zero.

Ans.



½

Let P be the required point at a distance x from charge q

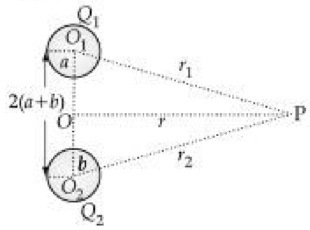
$\therefore \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-2q)}{(d-x)} = 0$ ½

$\frac{1}{x} = \frac{2}{d-x}$ ½

$\Rightarrow \qquad x = \frac{d}{3}$

$\therefore$ The required point is at a distance $\frac{d}{3}$ from the charge q. ½

Q. 4. Find the potential energy associated with a charge q if it were present at the point P with respect to the 'set-up' of two charged spheres, arranged as shown. Here O is the mid-point of the line O_1O_2.



Ans. $r_1 = O_1P = \sqrt{r^2+(2a+b)^2}$ ½

$r_2 = O_2P = \sqrt{r^2+(a+2b)^2}$ ½

$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r_1}+\frac{Q_2}{r_2}\right]$ ½

$\therefore$ The potential energy of charge q, at $P = qV$

$= \frac{q}{4\pi\varepsilon_0}\left[\frac{Q_1}{[r^2+(2a+b)^2]^{1/2}}+\frac{Q_2}{[r^2+(a+2b)^2]^{1/2}}\right]$ ½

Long Answer Type Questions-I

(3 marks each)

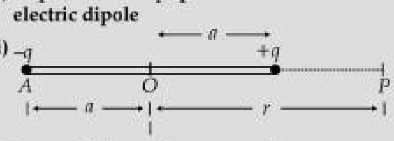
Q. 1. (i) Derive the expression for the electric potential due to an electric dipole at a point on its axial line.

(ii) Depict the equipotential surface due to electric dipole. **R [Delhi II 2017]**

Ans. (i) Derivation of expression for electric potential due to electric dipole on axial line **2**

(ii) Depiction of equipotential surfaces due to an electric dipole **1**

(i)



Potential due to charge $-q$ at A :

$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{(r+a)}$ ½

Potential due to charge $+q$ at B :

$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r-a)}$ ½

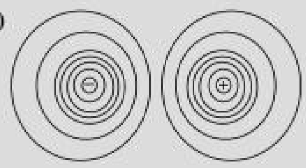
Potential at point P, $V = V_A + V_B$

Net potential at point P :

$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{(r+a)}+\frac{1}{(r-a)}\right]$ ½

$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times 2a}{(r^2-a^2)}$ ½

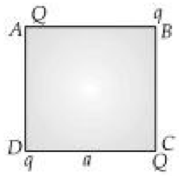
(ii)



1

[CBSE Marking Scheme 2017]

Q. 2. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.



Find the

(i) resultant electric force on a charge Q, and

(ii) potential energy of this system.

[Delhi/O.D. CBSE 2018]

Ans. (i) Finding the resultant force on a charge Q 2

(ii) Potential Energy of the system 1

(i)Let us find the force on the charge Q at the point C

Force due to the other charge Q

$$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{(a\sqrt{2})^2}$$

$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q^2}{2a^2}\right)\text{(along } AC\text{)} \qquad ½$$

Force due to the charge q (at B),

$$\vec{F_2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{a^2} \text{ along } BC$$

Force due to the charge q (at D),

$$\vec{F_3} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{a^2} \text{ along } DC \qquad ½$$

Resultant of these two equal forces $\vec{F_2}$ & $\vec{F_3}$

$$\vec{F_{23}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ(\sqrt{2})}{a^2} \text{ (along } AC\text{)} \qquad ½$$

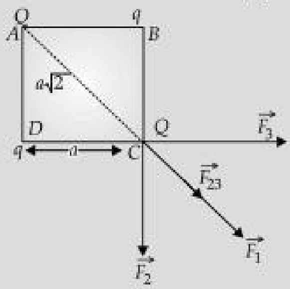
$\therefore$ Net force on charge Q (at point C)

$$\vec{F} = \vec{F_1}+\vec{F_{23}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{a^2}\left[\frac{Q}{2}+\sqrt{2}q\right] \qquad ½$$

This force is directed along AC.

(For the charge Q, at the point A, the force will have the same magnitude but will be directed along CA)

[**Note :** Don't deduct marks if the student does not write the direction of the net force , F]



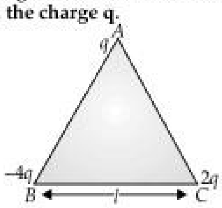
(ii) Potential energy of the system

$$= \frac{1}{4\pi\varepsilon_0}\left[4\frac{qQ}{a}+\frac{q^2}{a\sqrt{2}}+\frac{Q^2}{a\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left[4qQ+\frac{q^2}{\sqrt{2}}+\frac{Q^2}{\sqrt{2}}\right] \qquad 1$$

[CBSE Marking Scheme 2018]

Q. 3. (i) Three point charges q, – $4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q.



(ii) Find out the amount of the work done to separate the charges at infinite distance.

A [Delhi/OD CBSE 2018]

Ans. (i) Finding the magnitude of the resultant force on charge q 2

(ii) Finding the work done 1

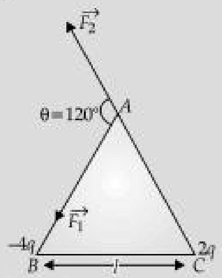
(i) Force on charge q due to the charge – $4q$ ½

$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{l^2}\right)\text{, along } AB$$

Force on the charge q, due to the charge $2q$

$$\vec{F_2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q^2}{l^2}\right)\text{, along } CA$$

The forces F_1 and F_2 are inclined to each other at an angle of 120°



Hence, resultant electric force on charge q

$$F = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta} \qquad ½$$

$$= \sqrt{F_1^2+F_2^2+2F_1F_2\cos 120°}$$

$$= \sqrt{F_1^2+F_2^2-F_1F_2} \qquad ½$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l^2}\right)\sqrt{16+4-8}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{3}q^2}{l^2}\right) \qquad ½$$

(ii) Net *P.E.* of the system

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l}[-4+2-8]$$

$$= \frac{(-10)}{4\pi\varepsilon_0}\cdot\frac{q^2}{l} \qquad ½$$

$$\therefore \quad \text{Work done} = \frac{10\,q^2}{4\pi\varepsilon_0 l} = \frac{5q^2}{2\pi\varepsilon_0 l} \qquad ½$$

[CBSE Marking Scheme 2018]

Detailed Answer :

(ii) The amount of work done to separate the charges at infinite distance is equal to the (-ve) potential energy of the given system. Now we know that the potential energy of three charges at the corners of an equilateral triangle *ABC* of side *l* is given by,

$$U_{PE} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

given $q_1 = q$, $q_2 = -4q$, $q_3 = 2q$, $r_{12} = r_{13} = r_{23} = l$

$$\Rightarrow \quad U_{PE} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q(-4q)}{l} + \frac{q.2q}{l} + \frac{2q(-4q)}{l}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l}[-4+2-8]$$

$$U_{PE} = -\frac{10q^2}{4\pi\varepsilon_0 l}$$

Therefore work done, $W = -U_{PE} = -\left(\frac{-10q^2}{4\pi\varepsilon_0 l}\right)$

$$\Rightarrow \quad \boxed{W = +\frac{5q^2}{2\pi\varepsilon_0 l}} \qquad 1$$

Q. 4. A particle, having a charge +5 μC, is initially at rest at the point x = 30 cm on the x-axis. The particle begins to move due to the presence of a charge Q that is kept fixed at the origin. Find the kinetic energy of the particle at the instant it has moved 15 cm from its initial position if (a) Q = +15 μC and (b) Q = – 15 μC

U **[SQP 2018-19]**

Ans. From energy conservation,

$$U_i + K_i = U_f + K_f$$

$$kQq/r_i + 0 = kQq/r_f + K_f \qquad ½$$

$$K_f = kQq(1/r_i - 1/r_f) \qquad ½$$

When Q is +15 μC, q will move 15 cm away from it. Hence r_f = 45 cm

$K_f = 9 \times 10^9 \times 15 \times 10^{-6} \times 5 \times 10^{-6}\,[1/(30 \times 10^{-2}) - 1/(45\times10^{-2})]$ ½

$= 0.75$ J ½

When Q is – 15 μC, q will move 15 cm towards it. Hence r_f = 15 cm

$K_f = 9 \times 10^9 \times (-15 \times 10^{-6}) \times 5 \times 10^{-6}\,[(1/30 \times 10^{-2}) - 1/(15 \times 10^{-2})]$ ½

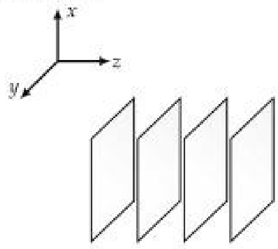
$= 2.25$ J ½

[CBSE Marking Scheme 2018-19]

Q. 5. (a) Draw the equipotential surfaces corresponding to a uniform electric field in the Z-direction.

(b) Derive an expression for the electric potential at any point along the axial line of an electric dipole. **3**

Ans. (a) Note that the electric field is always perpendicular to the equipotential surface. Hence, the equipotential surfaces are going to be infinite plane sheets parallel to the x-y plane. Also, the electric field strength is proportional to the distance between the surfaces. Hence, the surfaces will be evenly spaced.



(b) Try yourself, Similar to Q.1 (i), Long Answer Type Questions I **[CBSE 2019 Paper Delhi Set I]**

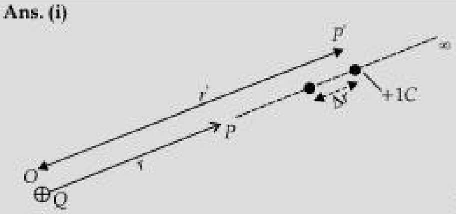
Long Answer Type Questions-II

(5 marks each)

AI **Q. 1. (i) Obtain an expression for the potential due to a point charge.**

(ii) Prove that the potential due to an electric dipole (length 2a) varies as the 'inverse square' of the distance of the 'field point' from the centre of the dipole for $r > a$. U **[Comptt. Delhi. I, II, III 2016]**

Ans. (i)



½

Consider a point charge 'Q' kept at point O.

Let P be the field point at distance r.

At some intermediate point P' the electrostatic force on the unit positive charge is : ½

$$= \frac{Q\times 1}{4\pi\varepsilon_0 r'^2}$$

Work done against this force from r' to $r' + \Delta r'$ is

$$\Delta W = \frac{Q}{4\pi\varepsilon_0 r'^2}\Delta r'. \quad ½$$

Total work done 'W' by the external Force from ∞ to r

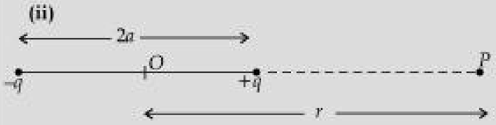
$$W = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2}\Delta r' \quad ½$$

$$W = \frac{Q}{4\pi\varepsilon_0 r}$$

Hence potential at this point

$$V = W = \frac{Q}{4\pi\varepsilon_0 r} \quad ½$$

(ii)



Potential at point P due to charge $(-q)$ ½

$$V_1 = \frac{-1}{4\pi\varepsilon_0}\cdot\frac{q}{(r+a)}$$

Potential due to charge $+q$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r-a)} \quad ½$$

Hence total potential at point P

$$V = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{(r+a)} + \frac{1}{(r-a)}\right]$$

$$= \frac{q\times 2a}{4\pi\varepsilon_0(r^2-a^2)} \quad ½$$

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(r^2-a^2)}$$

where, $\vec{p} = q \times 2a$ = dipole moment for $r >> a$ ½

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$$

$$\Rightarrow \quad V \propto \frac{1}{r^2}. \quad ½$$

[CBSE Marking Scheme 2016]

Q. 2. Two point charges q and $-q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively.

(i) Find the electrostatic potential at $(0, 0, z)$ and $(x, y, 0)$.

(ii) How much work is done in moving a small test charge from the point $(5,0,0)$ to $(-7, 0, 0)$ along the x-axis ?

(iii) How would your answer change if the path of the test charge between the same points is not along the x-axis but along any other random path ?

(iv) If the above point charges are now placed in the same positions in the uniform external electric field $\vec{E}$, what would be the potential energy of the charge system in its orientation of unstable equilibrium ?

Justify your answer in each case.

A [Comptt. Delhi/OD I, II, III 2018]

Ans. (i) Finding the electrostatic potential **2**

(ii) Finding the work done **1**

(iii) Effect of change of path **1**

(iv) Potential energy of the system **1**

(with justification in each case)

(i) We have, for a point charge,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \quad ½$$

(a) At point $(0, 0, z)$:

Potential due to the charge $(+q)$

$$V_+ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(z+a)} \quad ½$$

Potential due to the charge $(-q)$

$$V_- = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{(z-a)}$$

Total Potential at $(0, 0, z)$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{z+a} - \frac{1}{z-a}\right] \quad ½$$

$$= \frac{-2qa}{4\pi\varepsilon_0(z^2-a^2)}$$

(b) At point $(x, y, 0)$

Potential due to the charge $+q$

$$V_+ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{x^2+y^2+a^2}}$$

Potential due to the charge $(-q)$

$$V_- = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{\sqrt{x^2+y^2+a^2}}$$

Total potential at $(x, y, 0)$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2+a^2}} - \frac{1}{\sqrt{x^2+y^2+a^2}}\right) = 0 \quad ½$$

Note : Give full credit of part (b) if a student writes that the point $(x, y, 0)$ is equidistant from charges $+q$ and $-q$. Hence, total potential due to them at the given point will be zero.

(ii) Work done $= q[V_1 - V_2]$ ½

$V_1 = 0$ and $V_2 = 0$

$\therefore$ Work done $= 0$

Where V_1 and V_2 are the total potential due to dipole at point (5, 0, 0) and (– 7, 0, 0) ½

(iii) There would be no change.
This is because the electrostatic field is a conservative field. **1**

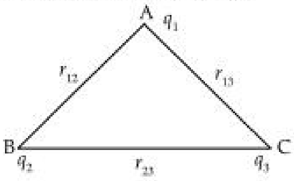
(**Alternatively :** The work done, in moving a test charge between two given points is independent of the path taken.) **1**

(iv) The two given charges make an electric dipole of dipole moment $\vec{p} = q.2\vec{a}$.

P.E. in position of unstable equilibrium (where $\vec{p}$ and $\vec{E}$ are antiparallel to each other) ½

$= + pE = 2aqE$ **[CBSE Marking Scheme, 2018]** ½

Q. 3. (a) Define electrostatic potential at a point. Write its SI unit. Three charges q_1, q_2 and q_3 are kept respectively at points A, B and C as shown in figure. Write the expression for electrostatic potential energy of the system.



(b) Depict the equipotential surfaces due to :

(i) an electric dipole

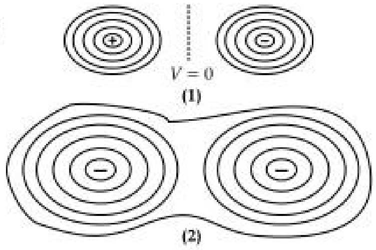
(ii) two identical negative charges separated by a small distance.

Ans. (a) Definition of electrostatic potential – SI unit J/c or Volt. 1

Deduction of expression of electrostatic potential energy of given system of charges : **2**

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

(b)



1 + 1

[CBSE Sample Question Paper 2019-20]

TOPIC-2
Capacitance

Revision Notes

Conductors and insulators

- Conductors are the materials through which charge can move freely. **Examples :** metals, semi-metals as carbon, graphite, antimony and arsenic.
- Insulators are materials in which the electrical current will not flow easily. Such materials cannot be grounded and do not easily transfer electrons. **Examples :** plastics and glass.

Dielectrics

- These are the materials in which induced dipole moment is linearly proportional to applied electric field.
- Electrical displacement or electrical flux density $D = \varepsilon_r\varepsilon_0 E$.
  where, ε_r = relative permittivity, ε_0 = permittivity of free space and E is electric field.
- If a dielectric is kept in between the plates of capacitor, capacitance increases by factor 'κ' (kappa) known as dielectric constant, so $C = \kappa\varepsilon_0\frac{A}{d}$

  where, A = area of plates

  κ = dielectric constant of material also called relative permittivity $\kappa = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

Material	Dielectric Constant (κ)	Dielectric strength (10^6 V/m)
Air	1.00059	3
Paper	3.7	16
Pyrex Glass	5.6	14
Water	80	-

- In dielectric, polarisation and production of induced charge takes place when dielectric is kept in an external electric field.

Electric polarization

- Electric polarization P is the difference between electric fields D (induced) and E (imposed) in dielectric due to bound and free charges written as $P = \frac{D - E}{4\pi}$

Scan to know more about this topic

Capacitance

- In term of electric susceptibility : $P = \chi_e E$
- In MKS : $P = \varepsilon_0 \chi_e E$,
- The dielectric constant κ is always greater than 1 as $\chi_e > 0$

Capacitor

- A capacitor is a device which is used to store charge.
- Amount of charge 'Q' stored by the capacitor depends on voltage applied and size of capacitor.
- Capacitor consists of two similar conducting plates placed in front of each other where one plate is connected to positive terminal while other plate is connected to negative terminal.
- Electric charge stored between plates of capacitor is directly proportional to potential difference between its plates, *i.e.*,

$$Q = CV$$

where, C = Capacitance of capacitor, V= potential difference between the plates

- In capacitor, energy is stored in the form of electrical energy, in the space between the plates.

Capacitance

- Capacitance of a capacitor is ratio of magnitude of charge stored on the plate to potential difference between the plates, written as $C = \frac{Q}{\Delta V}$

where, C = capacitance in farads (F), Q = charge in Coulombs (C), ΔV = electric potential difference in Volts (V),

- SI unit of capacitance is farad (F)
- $1\,F = \frac{1\,C}{1\,V} = 9 \times 10^{11}$ stat farad,

Where, stat-farad is electrostatic unit of capacitance in C.G.S. system

- Capacitance of a conductor depends on size, shape, medium and other conductors in surrounding.
- Parallel plate capacitor with dielectric among its plates has capacitance which is given as :

$$C = \kappa\varepsilon_0 \frac{A}{d},$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

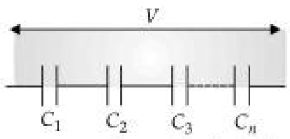
- Capacitor having capacitance of 1 Farad is too large for electronics applications, so components with lesser values of capacitance such as μ (micro), n (nano) and p (pico) are applied such as :

PREFIX	MULTIPLIER	
μ	10^{-6} (millionth)	$1\,\mu F = 10^{-6}\,F$
n	10^{-9} (thousand-millionth)	$1\,nF = 10^{-9}\,F$
p	10^{-12} (million-millionth)	$1\,pF = 10^{-12}\,F$

Combination of capacitors in series and parallel

Capacitors in series

- (i) If a number of capacitors of capacitances C_1, C_2, C_3, ... C_n are connected in series, then their equivalent capacitance is given by :



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots \frac{1}{C_n}$$

- In series combination, the charge on each capacitor is same, but the potential difference on each capacitor depends on their respective capacitance, *i.e.*,

$$q_1 = q_2 = q_3 \ldots\ldots\ldots\ldots\ldots q_n = q$$

- If V_1, V_2, V_3, ..., V_n be the potential differences across the capacitors and V be the emf of the charging battery, then

$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots + V_n$$

- As charge on each capacitor is same, therefore

$$q = V_1C_1 = V_2C_2 = V_3C_3 \ldots\ldots\ldots\ldots$$

the potential difference is inversely proportional to the capacitance, *i.e.*,

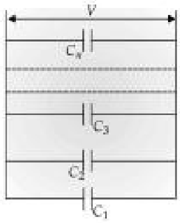
$$V \propto \frac{1}{C}$$

- In series, potential difference across largest capacitance is minimum.
- The equivalent capacitance in series combination is less than the smallest capacitance in combination.

Capacitors in parallel

- (i) If a number of capacitors of capacitances C_1, C_2, C_3................C_n are connected in parallel, then their equivalent capacitance is given by,

$$C_p = C_1 + C_2 + C_3 + \ldots\ldots\ldots\ldots + C_n$$



- In parallel combination, the potential difference across each capacitor is same and equal to the emf of the charging battery, *i.e.*,

$$V_1 = V_2 = V_3 = \ldots\ldots\ldots\ldots = V_n = V$$

while the charge on different capacitors may be different.

- If $q_1, q_2, q_3, \ldots\ldots\ldots, q_n$ be the charges on the different capacitors, then

$$q_1 + q_2 + q_3 + \ldots\ldots\ldots\ldots + q_n = VC_p$$

- As potential drop across each capacitor is same, so

$$\Rightarrow \qquad V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} = \ldots\ldots = \frac{q_n}{C_n}$$

- The charges on capacitors are directly proportional to capacitances, *i.e.*, $q \propto C$
- Parallel combination is useful when large capacitance with large charge gets accumulated on combination.
- Force of attraction between parallel plate capacitor will be $F = \frac{1}{2}\left[\frac{QV}{d}\right] = \frac{1}{2}QE$ where Q is charge on capacitor.

Capacitance of parallel plate capacitor with and without dielectric medium between the plates

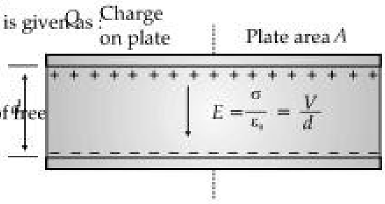
- Parallel plate capacitor is a capacitor with two identical plane parallel plates separated by a small distance where space between them is filled by dielectric medium.
- The electric field between two large parallel plates is given as :

$$E = \frac{\sigma}{\varepsilon_0},$$

Where, σ = charge density and ε_0 = permittivity of free

Surface charge density,

$$\sigma = \frac{Q}{A},$$



where, Q = charge on plate and A = plate area

- Capacitance of parallel-plate capacitor with area A separated by a distance d is written as

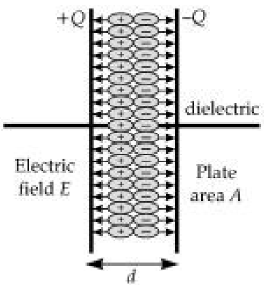
$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

Scan to know more about this topic

Capacitance in Series & Parallel

- If a dielectric slab is placed in between the plates of a capacitor, then its capacitance will increase by certain amount.
- Capacitance of parallel plate capacitor depends on plate area A, distance d between the plates, medium between the plates (κ) and not on charge on the plates or potential difference between the plates.
- If we have number of dielectric slabs of same area as the plates of the capacitor and thicknesses t_1, t_2, t_3..... and dielectric constant $\kappa_1, \kappa_2, \kappa_3$...... between the plates, then the capacitance of the capacitor is given by

$$C = \frac{\varepsilon_0 A}{\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + \frac{t_3}{\kappa_3} + \ldots},$$



Where, $d = t_1 + t_2 + t_3 + \ldots\ldots$

- If slab of conductor of thickness t is introduced between the plates, then

$$C = \frac{\varepsilon_0 A}{\frac{t}{\kappa} + \frac{(d-t)}{1}} = \frac{\varepsilon_0 A}{\frac{t}{\infty} + \frac{(d-t)}{1}}$$

$$C = \frac{\varepsilon_0 A}{d-t} \qquad (\because \kappa = \infty \text{ for a conductor})$$

- When the medium between the plates consists of slabs of same thickness but areas A_1, A_2, A_3,... and dielectric constants κ_1, κ_2, κ_3..., then capacitance is given by

$$C = \frac{\varepsilon_0(\kappa_1 A_1 + \kappa_2 A_2 + \kappa_3 A_3 \ldots)}{d}$$

$$\therefore \quad \kappa = \frac{C_m}{C_0} = \frac{\text{capacitance in medium}}{\text{capacitance in vacuum}}$$

- When space between the plates is partly filled with medium of thickness t and dielectric constant κ, then capacitance will be :

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)}$$

When there is no medium between the plates, then $\kappa = 1$, so $C_{vacuum} = \frac{\varepsilon_0 A}{d}$

- Capacitance of spherical conductor of radius R in a medium of dielectric constant κ is given by,

$$C = 4\pi\varepsilon_0\kappa R$$

Energy stored in capacitor

- In capacitor, energy gets stored when a work is done on moving a positive charge from negative conductor to positive conductor against the repulsive forces.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

- **Polar atom :** Atom in which positive and negative charges possess asymmetric charge distribution about its centre.
- **Polarisation :** The stretching of atoms of a dielectric slab under an applied electric field.
- **Dielectric strength :** The maximum value of electric field that can be applied to dielectric without its electric breakdown.
- **Dielectric :** It is an electrically insulated or non-conducting material considered for its electric susceptibility.
- **Permittivity :** It is a property of a dielectric medium that shows the forces which electric charges placed in medium exerts on each other.

OR

It is the measure of resistance that is encountered when forming an electric field in a particular medium. More specifically, permittivity describes the amount of charge needed to generate one unit of electric flux in a particular medium.

Key Formulae

- Capacitance, $C = \frac{Q}{V}$, measured in Farad; 1 F = 1 coulomb/volt
- Parallel plate capacitor :

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

- Cylindrical capacitor :

$$C = 2\pi\kappa\varepsilon_0 \frac{L}{\ln(b/a)}$$

where, L= length [m], b= radius of the outer conductor [m], a = radius of the inner conductor [m]

- Spherical capacitor :

$$C = 4\pi\kappa\,\varepsilon_0\left(\frac{ab}{b-a}\right)$$

where, b = radius of the outer conductor [m], a = radius of the inner conductor [m]

- Maximum charge on a capacitor :

$$Q = VC$$

- For capacitors connected in series, the charge Q is equal for each capacitor as well as for the total equivalent. If the **dielectric constant** κ is changed, the capacitance is multiplied by κ, the voltage is divided by κ and Q is unchanged. In vacuum $\kappa = 1$ and when dielectrics are used, replace ε_0 with $\kappa\,\varepsilon_0$.
- Electrical energy stored in a capacitor : [Joules (J)]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

➢ Surface charge density or Charge per unit area : [C/m^2]

$$\sigma = \frac{q}{A}$$

➢ **Energy density :**

- Electric energy density is also called Electrostatic pressure.
- Electric force between plates of capacitor,

$$F = \frac{1}{2}\varepsilon_0 E^2 A$$

- Energy stored in terms of Energy density,

$$\frac{E}{A \times d} = \frac{1}{2}\varepsilon_0 E^2$$

$$U = \frac{1}{2}\varepsilon_0 E^2$$

where, U = energy per unit volume [J/m^3], ε_0= permittivity of free space, = 8.85×10^{-12} C^2/Nm^2, E = energy [J]

- **Capacitors in series :**

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \cdots$$

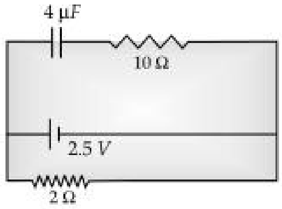
- **Capacitors in parallel :**

$$C_{eff} = C_1 + C_2 \ldots$$

Objective Type Questions **(1 mark each)**

(I) MULTIPLE CHOICE QUESTIONS

Q. 1. A capacitor of 4 μF is connected as shown in the figure. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be :



(a) 0 μC **(b) 4 μC**
(c) 16 μC **(d) 8 μC**

[NCERT Exemplar]

Ans. Correct option : (d)

Explanation : As capacitor offer infinite resistance for DC circuit. So current from cell will not flow across branch of 4 μF and 10 Ω. So current will flow across 2 ohm branch.

So current flows through across 2 Ω resistance from left to right is,

$$I = \frac{V}{(R+r)} = \frac{2.5\text{ V}}{(2+0.5)} = 1\text{ A}$$

So Potential Difference (PD) across 2 Ω resistance $V = RI = 2 \times 1 = 2$ Volt

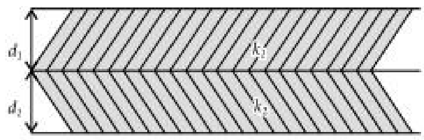
As battery, capacitor and 2 branches are in parallel. So PD will remain same across all three branches.

As current does not flow through capacitor branch so no potential drop will be across 10 Ω.

So PD across 4 μF capacitor = 2 Volt

$Q = CV = 4\,\mu\text{F} \times 2\text{ V} = 8\,\mu\text{C}$

Q. 2. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant κ_1 and the other has thickness d_2 and dielectric constant κ_2 as shown in Figure. This arrangement can be thought as a dielectric slab of thickness d (= $d_1 + d_2$) and effective dielectric constant κ. The κ is :



(a) $\frac{k_1 d_1 + k_2 d_2}{d_1 + d_2}$ **(b)** $\frac{k_1 d_1 + k_2 d_2}{k_1 + k_2}$

(c) $\frac{k_1 k_2 (d_1 + d_2)}{(k_1 d_2 + k_2 d_1)}$ **(d)** $\frac{2k_1 k_2}{k_1 + k_2}$

[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : Capacitance of a parallel plate capacitor filled with dielectric of constant k_1 and thickness d_1 is,

$$C_1 = \frac{k_1\varepsilon_0 A}{d_1}$$

Similarly, for other capacitance of a parallel plate capacitor filled with dielectric of constant k_2 and thickness d_2 is,

$$C_2 = \frac{k_2\varepsilon_0 A}{d_2}$$

Both capacitors are in series so equivalent capacitance is given by

Use this solution in the question

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{\dfrac{K_1\varepsilon_0 A}{d_1}\ \dfrac{K_2\varepsilon_0 A}{d_2}}{\dfrac{K_1\varepsilon_0 A}{d_1} + \dfrac{K_2\varepsilon_0 A}{d_2}} = \frac{K_1K_2\varepsilon_0 A}{K_1D_2 + K_2D_1} \quad \text{...(i)}$$

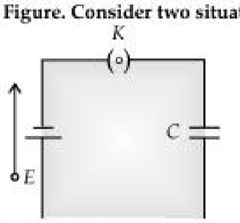
We can write the equivalent capacitance as

$$C = \frac{K\varepsilon_0 A}{d_1 + d_2} \quad \text{...(ii)}$$

On comparing (i) and (ii) we have

$$K = \frac{K_1K_2(d_1 + d_2)}{K_1d_2 + K_2d_1}$$

Q. 3. A parallel plate capacitor is connected to a battery as shown in Figure. Consider two situations :



A : Key *K* is kept closed and plates of capacitors are moved apart using insulating handle.

B : Key *K* is opened and plates of capacitors are moved apart using insulating handle.

Choose the correct option(s).

(a) In A : *Q* remains same but *C* changes.

(b) In B : *V* remains same but *C* changes.

(c) In A : *V* remains same and hence *Q* changes.

(d) In B : *Q* remains same and hence *V* changes.

[NCERT Exemp. Q. 2.13, Page 13]

Ans. Correct options : (c) and (d)

Explanation :

(i) In case A : When the space between the plates of capacitor increases, the capacitance decreases by relation,

$$C = \frac{K\varepsilon_0 A}{d}$$

But battery remains same, *i.e.*, potential difference across plate remains '*V*' same. So by $Q = CV$ relation, *Q* also decreases verifies answer (c) and discards answer (a).

(ii) In case B : *K* is open, and capacitance decreases by moving apart plates of capacitor, so by relation $Q = CV$, here *K* is open, so charge *Q* remains same in turn *V* will increase on decreasing *C*. Hence answer (d) is verified.

Q. 4. Two capacitors of capacitance C are connected in series. If one of them is filled with dielectric substance K, what is the effective capacitance? A

(a) $\frac{KC}{(1+K)}$ **(b)** C(1 + K)

(c) $\frac{2KC}{1+k}$ **(d)** None of the above

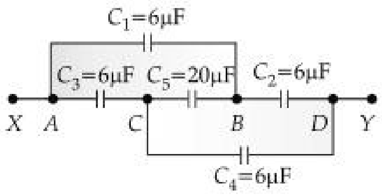
Ans. Correct option : (a)

***Explanation* :**

We have now two capacitors of capacitances C and KC connected in series.

$$C_{eq} = \frac{C \times KC}{C + KC} = \frac{KC}{1 + K}$$

Q. 5. What is the effective capacitance between the points X and Y ? A



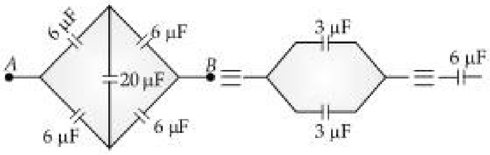
(a) 24 μF **(b)** 18 μF

(c) 12 μF **(d)** 6 μF

Ans. Correct option : (d)

***Explanation* :**

Reconstruction of the circuit results



The equivalent capacitance is equal to 3 μF + 3μF = 6μF

Commonly Made Error

- The students are unable to draw the circuit for the application of the formula for the series and parallel combination.

Answering Tip

- The concept of Wheatstone bridge should be carefully understood in this case.

(II) FILL IN THE BLANKS

Q. 1. The capacitance of a spherical capacitor is directly proportional to its ....

Ans. radius

Q. 2. A 1-farad capacitor can store .... charge at 1 volt.

Ans. 1 coulomb

Q. 3. The ratio of electric flux density to electric field intensity is known as ....

Ans. dielectric constant

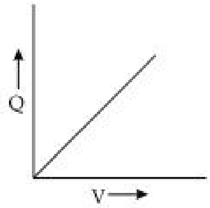
(III) VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. Can we give any desired charge to the capacitor?

Ans. No, we cannot give any desired charge to the capacitor. As we increase the charge after a certain amount, the air near the capacitor gets ionized and the leakage of the charge starts from it. The electric field between the two plates attains the breakdown value of the air.

Q. 2. Sketch the graph to show how a charge Q acquired by a capacitor of capacitance C varies with the increase in potential difference.

Ans. As the charge Q is directly proportional to the potential difference, the graph between Q and V will be a straight line having slope $\frac{Q}{V} = C$.



Q. 3. By what factor does the capacitance of a metal sphere increase if its volume is tripled?

Ans. Let the initial radius of the sphere be r

Capacitance of the sphere $C_1 = 4\pi\varepsilon_0 r$

Now volume of the sphere $V = \frac{4}{3}\pi r^3$

Let R be the radius of the sphere having volume 3V

So $3V = \frac{4}{3}\pi R^3$

$$3 \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R^3$$

$$R = (3r^3)^{\frac{1}{3}} r$$

Capacitance of the sphere having radius R will be

$$C_2 = 4\pi\varepsilon_0 R$$

$$C_2 = 4\pi\varepsilon_0 r(3)^{\frac{1}{3}}$$

$$C_2 = 3^{\frac{1}{3}} C_1$$

$$\frac{C_2}{C_1} = 3^{\frac{1}{3}} = 1.44$$

Short Answer Type Questions (2 marks each)

AI Q. 1. The battery remains connected to a parallel plate capacitor and a dielectric slab is inserted between the plates. What will be effect on its (i) potential difference (ii) capacity (iii) electric field and (iv) energy stored ?

A [CBSE SQP I 2017]

Ans. When a battery remains connected,

(i) potential difference V remains constant ½

(ii) capacity C increases ½

(iii) electric field will remain same ½

(iv) energy stored $\frac{1}{2}CV^2$ increases as C increases ½

[CBSE Marking Scheme 2017]

Detailed Answer :

When a battery remains connected to a parallel plate capacitor and if a dielectric slab is inserted between the plates of capacitor, then

(i) there will be no change in the potential difference as the capacitor remained connected with the battery. ½

(ii) capacity or capacitance will increase since with the introduction of dielectric slab, capacitance of capacitor will result $C = \frac{\kappa\varepsilon_0 A}{d}$ where $\kappa > 1$ resulting an increase in C. ½

(iii) Electric field will remain same as there will be no change in potential difference and distance between the plates. ½

(iv) Energy stored will be increased since from the expression $U = \frac{1}{2}CV^2$, potential difference V remains same while C increases which finally increases the energy of capacitor. ½

AI Q. 2. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that of stored initially in the single capacitor. A

Ans. Energy stored in a capacitor

$$U = \frac{1}{2}CV^2 \text{ or } \frac{1}{2}\frac{q^2}{C} \quad ½$$

Capacitance of the (parallel) combination

$$= C + C = 2C \quad ½$$

Here, the total charge, Q, remains the same :

$\therefore$ Initial energy $= \frac{1}{2} \cdot \frac{q^2}{C}$

and Final energy $= \frac{1}{2}\frac{q^2}{2C}$ ½

$\therefore$ Ratio of energies $= \frac{\text{final energy}}{\text{initial energy}}$

$$= \frac{\frac{1}{4}\frac{q^2}{C}}{\frac{1}{2}\frac{q^2}{C}} \quad ½$$

$$= \frac{2}{4} = \frac{1}{2}$$

AI Q. 3. A slab of material of dielectric constant κ has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. U

Ans. Initially when there is vacuum between the two plates, the capacitance of the two parallel plates is,

$$C_0 = \frac{\varepsilon_0 A}{d} \quad ½$$

where, A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced.

Now if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E.

Now the gap between the plates is divided in two parts, for distance t there is electric field E and for the remaining distance $(d - t)$, the electric field is E_0. ½

If V be the potential difference between the plates of the capacitor, then

$$V = Et + E_0(d - t)$$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} \quad \left(\because t = \frac{d}{2}\right)$$

$$= \frac{d}{2}(E + E_0) \quad \left(\because \frac{E_0}{E} = \kappa\right)$$

$$\Rightarrow \quad V = \frac{d}{2}\left(\frac{E_0}{\kappa} + E_0\right)$$

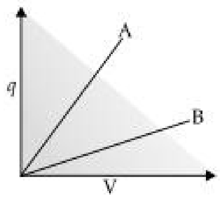
$$= \frac{dE_0}{2\kappa}(\kappa + 1)$$

Now, $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$

$$\Rightarrow \quad V = \frac{d}{2\kappa} \cdot \frac{q}{\varepsilon_0 A}(\kappa + 1)$$

We know, $C = \frac{q}{V} = \frac{2\kappa\varepsilon_0 A}{d(\kappa + 1)}$ 1

AI Q.4. The given graph shows variation of charge q versus potential difference V for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that of C_1. Which line (A or B) corresponds to C_1 and why ?

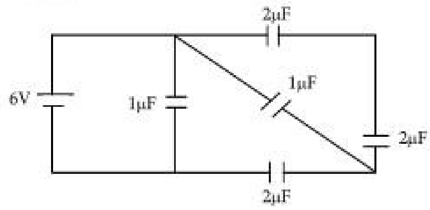


Ans. For C_1

Line B 1

Since slope (q/V) of Line B is lesser than that of Line A. 1

Q. 6. Find total energy stored in capacitors given in the circuit



[CBSE Sample Question Paper 2019-20]

Ans. C_2 and C_3 are in series

$$\frac{1}{C'} = \frac{1}{2} + \frac{1}{2} = 1$$

$$C' = 1\mu f \quad 1$$

C' and C_4 are in ||

$$C'' = 1 + 1 = 2\mu f$$

C'' and C_5 are in parallel

$$\frac{1}{C'''} = \frac{1}{2} + \frac{1}{2} \Rightarrow C''' = 1\mu f$$

C''' and C_1 are in parallel

$$C_{eq} = 1 + 1 = 2\mu f \quad 1$$

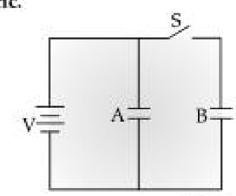
Energy Stored

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 6^2 = 36 \times 10^{-6} J$$

Long Answer Type Questions-I

(3 marks each)

AI Q. 1. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant κ. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



U **[O.D. I, II, III 2017]**

Ans. Formula for energy stored ½
Energy stored before 1
Energy stored after 1
Ratio ½

$$\text{Energy stored} = \frac{1}{2}CV^2 = \left(\frac{1}{2}\frac{Q^2}{C}\right) \quad ½$$

Net capacitance with switch S closed

$$= C + C = 2C \quad ½$$

$$\text{Energy stored} = \frac{1}{2}\times 2C\times V^2 = CV^2 \quad ½$$

After the switch S is opened, capacitance of each capacitor = κ × C

∴ Energy stored in capacitor A

$$A = \frac{1}{2}\kappa CV^2$$

For capacitor B,

$$\text{Energy stored} = \frac{1}{2}\frac{Q^2}{\kappa C} = \frac{1}{2}\frac{C^2V^2}{\kappa C}$$

$$= \frac{1}{2}\frac{CV^2}{\kappa} \quad ½$$

$$\therefore \quad \text{Total Energy stored} = \frac{1}{2}\kappa CV^2 + \frac{1}{2}\frac{CV^2}{\kappa}$$

$$= \frac{1}{2}CV^2\left(\kappa + \frac{1}{\kappa}\right) \quad ½$$

$$= \frac{1}{2}CV^2\left(\frac{\kappa^2+1}{\kappa}\right)$$

$$\therefore \quad \text{Required ratio} = \frac{2CV^2\kappa}{CV^2(\kappa^2+1)}$$

$$= \frac{2\kappa}{(\kappa^2+1)} \quad ½$$

[CBSE Marking Scheme 2017]

Q. 2. A capacitor of unknown capacitance is connected across a battery of V volt. A charge of 360 μC is stored in it. When the potential across the capacitor is reduced by 120 V, the charge stored in the capacitor becomes 120 μC. Calculate V and the unknown capacitance. What would have been the charge on the capacitor if the voltage were increased by 120 V? U **[Delhi (Comptt.) I 2017]**

Ans. Calculation of V and unknown capacitance 2
Calculation of charge when voltage is increased by 120 V 1

$$\text{Capacitance} \qquad C = \frac{Q_1}{V_1} \quad ½$$

$$\text{Also,} \qquad C = \frac{Q_2}{V_2} \text{ and } C = \frac{Q_3}{V_3} \quad ½$$

$$\frac{360\mu C}{V} = \frac{120\mu C}{(V-120)} \quad ½$$

So, $3V - 360V = V$

$$V = 180\ V \quad ½$$

$$C = \frac{360\ \mu V}{180\ V} = 2\ \mu F \quad ½$$

$$2\mu F = \frac{Q_3}{300} \quad ½$$

$$Q_3 = 600\ \mu\ C \quad ½$$

[CBSE Marking Scheme 2017]

Detailed Answer :

(i) Let the initial voltage = V volts

Charge stored, Q_1 = 360 μC

$$Q_1 = CV_1 = CV \qquad ...(i)\ ½$$

Now the changed potential

$$V_2 = (V-120) \text{ volts}$$

Charge stored, Q_2 = 120 μC

$$Q_2 = CV_2 \qquad ...(ii)\ ½$$

Now divide equation (i) by (ii), we have:

$$\frac{Q_1}{Q_2} = \frac{CV_1}{CV_2}$$

$$\frac{360\ \mu C}{120\ \mu C} = \frac{V}{V-120} \quad ½$$

On solving, we get V = 180 volts

Now value of unknown capacitance C can be calculated as.

$$C = \frac{Q_1}{V} = \frac{360\times 10^{-6}}{180}\text{F}$$

$$= 2\times 10^{-6} F = 2\ \mu F \quad ½$$

(ii) When the voltage applied increases to 120 V, then

$$V_3 = 180 + 120 = 300\ V \quad ½$$

Finally the charge stored in the capacitor will be :

$$Q_3 = CV_3$$

$$= 2 \times 10^{-6} \times 300\ C$$
$$= 600\ \mu C \qquad ½$$

Q. 3. (i) How many electrons must be added to one plate and removed from other so as to store 25.0 J of energy in a 5.0 *n*F parallel plate capacitor ?

(ii) How would you modify this capacitor so that it can store 50.0 J of energy without changing the charge on its plates ? U **[SQP 2017-18]**

Ans. (i) $C = 5 \times 10^{-9}\ F,\ U = 25\ J$

$$U = \frac{Q^2}{2C}$$
$$Q^2 = 2UC \qquad ½$$
$$= 2 \times 25 \times 5 \times 10^{-9}$$
$$Q = 5{\cdot}0 \times 10^{-4}\ C \qquad ½$$
$$Q = ne \qquad ½$$
$$n = \frac{Q}{e} = \frac{5{\cdot}0 \times 10^{-4}C}{1{\cdot}60 \times 10^{-19}}$$
$$= 3{\cdot}125 \times 10^{15} \text{ electrons} \ ½$$

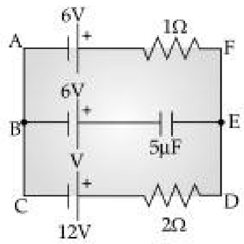
(ii) Without changing charge on the plates, we can make C half.

$$C = \frac{\varepsilon_0 A}{d}$$

Double the plate separation or inserting dielectric of dielectric constant of a value such that C will become half. **1**

[CBSE Marking Scheme 2017]

Q. 4. In the given circuit, with steady current, calculate the potential difference across the capacitor and the charge stored in it. U **[Foreign I 2017]**



Ans. Value of current **1**

Value of voltage **1**

Value of charge **1**

In the loop *ACDFA*

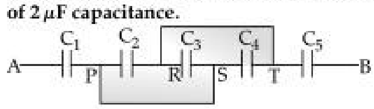
$$I = \frac{12-6}{(1+2)} = 2A \qquad 1$$
$$V_{AF} = V_{BE} \qquad ½$$
$$= 6 + 2 = 6 + V_C$$
$$V_C = 2\ V \qquad ½$$

Charge $Q = CV_C = 5\ \mu F \times 2\ V$

$$= 10\ \mu C \qquad 1$$

[CBSE Marking Scheme 2017]

AI **Q. 5. (i) Find equivalent capacitance between *A* and *B* in the combination given below. Each capacitor is of 2 μF capacitance.**



(ii) If a DC source of 7 V is connected across *AB*, how much charge is drawn from the source and what is the energy stored in the network ? U **[Delhi I 2017]**

Ans. (i) Calculation of equivalent capacitance **1**

(ii) Calculation of charge and energy stored **1+1**

(i) Capacitors C_2, C_3, and C_4 are in parallel

$$\therefore \quad C_{234} = C_2 + C_3 + C_4$$
$$\therefore \quad C_{234} = 6\ \mu F \qquad ½$$

Capacitors C_1, C_{234}, and C_5 are in series

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{234}} + \frac{1}{C_5}$$
$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{2} = \frac{7}{6}(\mu F)^{-1}$$
$$C_{eq} = \frac{6}{7}\mu F \qquad ½$$

(ii) Charge drawn from the source

$$Q = C_{eq}V \qquad ½$$
$$= \frac{6}{7} \times 7\ \mu C = 6\ \mu C \qquad ½$$

Energy stored, $U = \dfrac{Q^2}{2C_{eq}}$

$$= \frac{6 \times 6 \times 10^{-12} \times 7}{2 \times 6 \times 10^{-6}} J \qquad ½$$
$$= 21\ \mu J \qquad ½$$

[CBSE Marking Scheme 2017]

Q. 6. A 12 *pF* capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor ? If another capacitor of 6 *pF* is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor. U **[Delhi II 2017]**

Ans. Calculation of electrostatic energy in 12 *pF* capacitor **1**

Total charge stored in combination **1**

Potential difference across each capacitor ½ + ½

Energy stored, in the capacitor of capacitance 12 *pF*,

$$U = \frac{1}{2}CV^2 \qquad ½$$
$$= \frac{1}{2} \times 12 \times 10^{-12} \times 50 \times 50 J$$
$$= 1.5 \times 10^{-8}\ J \qquad ½$$

C = Equivalent capacitance of 12 *pF* and 6 *pF*, in series, is given by

$$\frac{1}{C} = \frac{1}{12}+\frac{1}{6}=\frac{1+2}{12}$$

$\therefore$ $C = 4\,pF$

Charge stored across each capacitor ½

$q = CV$

$= 4 \times 10^{-12} \times 50\,C$

$= 2 \times 10^{-10}\,C$

Charge on each capacitor 12 pF as well as 6 pF

$\therefore$ Potential difference across capacitor C_1 ½

$$\therefore \quad V_1 = \frac{2\times10^{-10}}{12\times10^{-12}}$$

$$= \frac{50}{3}V \qquad ½$$

$\therefore$ Potential difference across capacitor C_2

$$\therefore \quad V_2 = \frac{2\times10^{-10}}{6\times10^{-12}}$$

$$= \frac{100}{3}V \qquad ½$$

[CBSE Marking Scheme 2017]

Q. 7. Two identical capacitors of 12 pF each are connected in series across a battery of 50 V. How much electrostatic energy is stored in the combination ? If these were connected in parallel across the same battery, how much energy will be stored in the combination now ?
Also find the charge drawn from the battery in each case. [U] **[Delhi III 2017]**

Ans. Equivalent capacitance in series ½
Energy in series combination ½
Charge in series combination ½
Equivalent capacitance in parallel combination ½
Energy in parallel combination ½
Charge in parallel combination ½

In series combination :

$$\frac{1}{C_s} = \left(\frac{1}{12}+\frac{1}{12}\right)(pF)^{-1} \qquad ½$$

$\therefore$ $C_s = 6 \times 10^{-12}$ F

$$U_s = \frac{1}{2}CV^2$$

$$U_s = \frac{1}{2} \times 6 \times 10^{-12} \times 50 \times 50\,J$$

$U_s = 75 \times 10^{-10}\,J$ ½

$q_s = C_sV$

$= 6 \times 10^{-12} \times 50\,C$

$= 300 \times 10^{-12}\,C$

$= 3 \times 10^{-10}\,C$ ½

In parallel combination :

$C_p = (12 + 12)\,pF$ ½

$\therefore$ $C_p = 24 \times 10^{-12}$ F

$$U_p = \frac{1}{2}\times24\times10^{-12}\times2500\,J$$

$U_p = 3 \times 10^{-8}$ J

$q_p = C_pV$ ½

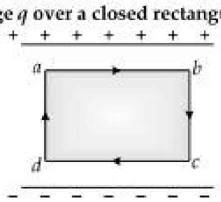
$= 24 \times 10^{-12} \times 50\,C$

$= 1{\cdot}2 \times 10^{-9}\,C$ ½

[CBSE Marking Scheme 2017]

[AI] **Q. 8. (i) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.**

(ii) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge *q* over a closed rectangular loop *a b c d a*.



[A]

Ans. (i) Work done by the source of potential, in storing an additional charge (dq), is

$$dW = V.dq$$

But $$V = \frac{q}{C}$$

$$\Rightarrow \quad dW = \frac{q}{C}dq \qquad ½$$

Total work done in storing the charge q,

$$\int dW = \int_0^q \frac{q}{C}dq$$

$$W = \frac{1}{C}\left(\frac{q^2}{2}\right)_0^q = \frac{q^2}{2C} \qquad ½$$

This work is stored as electrostatic energy in the capacitor.

$$\therefore \quad U = \frac{1}{2}CV^2 \qquad (\because q = CV)$$

$$\text{Energy stored per unit volume} = \frac{\frac{1}{2}CV^2}{Ad}$$

$$U = \frac{\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2}{Ad}$$

$$U = \frac{1}{2}\varepsilon_0 E^2 \qquad 1$$

(ii) Work done in moving the charge q from a to b, and from c to d is zero because electric field is perpendicular to the displacement. ½

Work done from b to c = – Work done from d to a ½

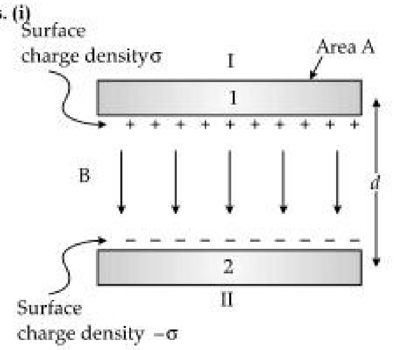
$\therefore$ Total work done in moving a charge q over the closed loop = 0

[AI] **Q. 9. (i) Derive the expression for the capacitance of a parallel plate capacitor having plate area *A* and plate separation *d*.**

(ii) Two charged spherical conductors of radii R_1 and R_2 when connected by a conducting wire acquire

charges q_1 and q_2 respectively. Find the ratio of their surface charge densities in terms of their radii. [A]

Ans. (i)



Electric field between the plates of capacitor

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \qquad ½$$

$$\therefore \qquad V = Ed = \frac{Qd}{A\varepsilon_0} \qquad ½$$

Capacitance, $$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d} \qquad 1$$

(ii) When the two charged spherical conductors are connected by a conducting wire, they acquire the same potential. ½

i.e., $$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

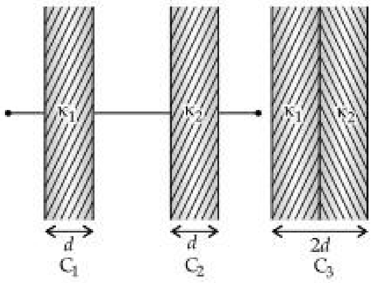
Hence, ratio of surface charge densities,

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1 R_2^2}{q_2 R_1^2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2} \times \frac{R_2^2}{R_1^2} = \frac{R_2}{R_1} \qquad ½$$

[AI] **Q. 10. The capacitors C_1 and C_2 having plates of area A each, are connected in series, as shown. Compare the capacitance of this combination with the capacitor C_3, again having plates of area A each, but 'made up' as shown in the figure.**



[U]

Ans. We have $$C_1 = \frac{A\varepsilon_0\kappa_1}{d} \qquad ½$$

and $$C_2 = \frac{A\varepsilon_0\kappa_2}{d}$$

$$\therefore \qquad C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{A\varepsilon_0}{d}\left(\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}\right) \qquad ½$$

Now, capacitor C_3 can be considered as made up of two capacitors C_1 and C_2, each of plate area A and separation d, connected in series. ½

We have $$C_1' = \frac{A\varepsilon_0\kappa_1}{d}$$

and $$C_2' = \frac{A\varepsilon_0 K_2}{d} \qquad ½$$

$$\Rightarrow \qquad C_3 = \frac{C_1'C_2'}{C_1'+C_2'} = \frac{A\varepsilon_0}{d}\left(\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}\right)$$

$$\therefore \qquad \frac{C_3}{C_{eq}} = 1 \qquad ½$$

Hence, the net capacitance of the combination is equal to that of C_3. ½

Q. 11. A 200μF parallel plate capacitor having plate separation of 5 mm is charged by a 100 V dc source. It remains connected to the source. Using an insulated handle, the distance between the plates is doubled and a dielectric slab of thickness 5 mm and dielectric constant 10 is introduced between the plates. Explain with reason, how the (i) capacitance, (ii) electric field between the plates, (iii) energy density of the capacitor will change ?

[O.D. I 2019] [A]

Ans. (i) Change in capacitance 1

(ii) Change in electric field 1

(iii) Change in electric density 1

Dielectric slab of thickness 5 mm is equivalent to an air capacitor of thickness $= \frac{5}{10}$ mm ½

Effective separation between the plates with air in between is = (5 + 0.50) mm = 5.5 mm

(i) Effective new capacitance

$$= 200\mu F \times \frac{5\text{mm}}{5.5\text{mm}} = \frac{2000}{11}\mu F$$

$$\approx 182\mu F \qquad 1$$

(ii) Effective new electric field

$$= \frac{100\text{V}}{5.5\times10^{-3}\text{m}} = \frac{20000}{11}$$

$$= 18182 \text{ V/m} \qquad ½$$

(iii) $$\frac{\text{New energy stored}}{\text{Original energy stored}} = \frac{\frac{1}{2}C'V^2}{\frac{1}{2}CV^2} = \frac{C'}{C} = \frac{10}{11} \qquad 1$$

New Energy density will be $\left(\frac{10}{11}\right)^2$ of the original energy density $= \frac{100}{121}$ of the original energy density.

[**Note :** If the student writes $C = \frac{A\varepsilon_0}{d}$

$$C_m = \frac{KA\varepsilon_0}{d}$$

$$E' = \frac{V}{d}$$

$$U = \frac{1}{2}\varepsilon_0 E^2$$

Award full marks.

[CBSE Marking Scheme 2019]

Detailed Answer :

Given : Capacitance of the capacitor, C = 200 μF

Potential of *dc* source, V = 100 V

Let 'A' be the area of the plate and '*d*' be the separation between the plates,

The capacitance of the capacitor is given as,

$$C = \frac{\varepsilon_0 A}{d}$$

or, $\varepsilon_0 A = Cd$ ...(i)

When the capacitor remains connected with the *dc* source than there will be no change in potential difference.

(i) Now, according the problem

Separation between the plates = $2d$

Thickness of dielectric slab, t = 5 mm $= 5.0 \times 10^{-3}$ m

Dielectric constant, K = 10

New capacitance of the capacitor

$$C' = \frac{A\varepsilon_0}{(d'-t)+\frac{t}{K}}$$

Here, $d' = 2d$ and $t = d$

$$C' = \frac{A\varepsilon_0}{(2d-d)+\frac{d}{K}}$$

$$= \frac{A\varepsilon_0}{d\left(1+\frac{1}{K}\right)} = \frac{Cd}{d\left(1+\frac{1}{K}\right)}$$

$$= \frac{CK}{K+1}$$

$$= \frac{10 \times 200\ \mu F}{(10+1)} = 182\ \mu F.$$

Hence, new capacitance of the capacitor will decrease.

(ii) Since, there in no change in the potential difference Hence, there would not be any change in electric field. It will be $\frac{100}{5.5\times 10^{-3}} = 18182$ V/m .

(iii) The Energy will decrease because

$$E = \frac{1}{2}CV^2$$

or, $E \propto C$ [V is constant]

Hence, the energy density will also decrease.

Long Answer Type Questions-II

(5 marks each)

Q. 1. (i) Define the capacitance of a capacitor. Obtain the expression for the capacitance of a parallel plate capacitor in vacuum in terms of plate area *A* and separation *d* between the plates.

(ii) A slab of material of dielectric constant κ has the same area as the plates of a parallel plate capacitor but has a thickness $\frac{3d}{4}$. Find the ratio of the capacitance with dielectric inside it to its capacitance without the dielectric.

U **[Foreign I, II, III 2017]**

Ans. (i) Definition of capacitance 1

Obtaining capacitance 2

(ii) Ratio of capacitances 2

(i) Capacitance equals the magnitude of the charge on each plate needed to raise the potential difference between the plates by unity. 1

OR

(i) Try yourself, Similar Q.9 (i), Long Answer Type Questions-I

(ii) Capacitance without dielectric,

$$C = \frac{A\varepsilon_0}{d}$$

Capacitance when filled with dielectric having thickness $\frac{3d}{4}$

$$C = \frac{A\varepsilon_0}{\left(d-t+\frac{t}{\kappa}\right)} \quad \text{½}$$

$$= \frac{A\varepsilon_0}{\left(d-\frac{3d}{4}+\frac{3d}{4\kappa}\right)} \quad \left[\text{As } t = \frac{3d}{4}\right]$$

$$= \frac{4\varepsilon_0 \kappa A}{d(\kappa+3)}$$

Ratio $$\frac{C'}{C} = \frac{A\varepsilon_0 4\kappa}{d(\kappa+3)} \times \frac{d}{A\varepsilon_0} \quad 1$$

$$= \frac{4\kappa}{(\kappa+3)} \quad \text{½}$$

[CBSE Marking Scheme 2017]

Q. 2. (i) Compare the individual dipole moment and the specimen dipole moment for H_2O molecule and O_2 molecule when placed in

(a) Absence of external electric field.

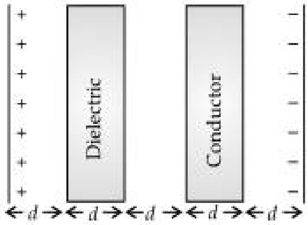
(b) Presence of external electric field.

Justify your answer.

(ii) Given two parallel conducting plates of area A and charge densities $+\sigma$ and $-\sigma$. A dielectric slab of constant κ and a conducting slab of thickness d each are inserted in between them as shown.

(a) Find the potential difference between the plates.

(b) Plot E versus x graph, taking $x = 0$ at positive plate and $x = 5d$ at negative plate.



U [CBSE SQP 2015-16]

Ans. (i)

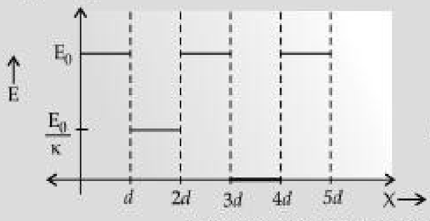
	Non-polar (O_2)	Polar (H_2O)
In absence of electric field		
Individual	No dipole moment exists.	Dipole moment exists.
Specimen	No dipole moment exists.	Dipoles are randomly oriented. Net P=0
In presence of electric field		
Individual	Dipole moment exists (molecules become polarised.)	Torque acts on the molecules to align them parallel to E.
Specimen	Dipole moment exists.	Net dipole moment exists parallel to E.

3

(ii) (a) $V = E_0 d + \dfrac{E_0}{\kappa} d + E_0 d + 0 + E_0 d$ ½

$V = 3\,E_0 d + \dfrac{E_0}{\kappa} d$ ½

(b) Graph :



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[CBSE Marking Scheme 2015]

AI **Q. 3.** A capacitor of capacitance C_1 is charged to a potential V_1 while another capacitor of capacitance C_2 is charged to a potential difference V_2. The capacitors are now disconnected from their respective charging batteries and connected in parallel to each other.

(i) Find the total energy stored in the two capacitors before they are connected.

(ii) Find the total energy stored in the parallel combination of the two capacitors.

(iii) Explain the reason for the difference of energy in parallel combination in comparison to the total energy before they are connected.

R [Comptt. Delhi/O.D. I, II, III 2018]

Ans. (i) Finding the total energy before the capacitors are connected 1

(ii) Finding the total energy in the parallel combination 3

(iii) Reason for difference 1

(i) We have

Energy stored in a capacitor $= \dfrac{1}{2}CV^2$ ½

$\therefore$ Energy stored in the charged capacitors

$$E_1 = \frac{1}{2}C_1V_1^2$$

And $$E_2 = \frac{1}{2}C_2V_2^2$$

$\therefore$ Total energy stored $= \dfrac{1}{2}C_1V_1^2 + \dfrac{1}{2}C_2V_2^2$ ½

(ii) Let V be the potential difference across the parallel combination.

Equivalent capacitance $= (C_1 + C_2)$ ½

Since charge is a conserved quantity, we have

$$(C_1 + C_2)V = C_1V_1 + C_2V_2$$ ½

$$\Rightarrow \quad V = \left[\frac{C_1V_1 + C_2V_2}{(C_1 + C_2)}\right] \qquad 1$$

$\therefore$ Total energy stored in the parallel combination

$$= \frac{1}{2}(C_1 + C_2)V^2 \qquad ½$$

$$= \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)} \qquad ½$$

(iii) The total energy of the parallel combination is different (less) from the total energy before the capacitors are connected. This is because some energy gets used up due to the movement of charges. **1**

[CBSE Marking Scheme, 2018]