Electric Charges & Fields

Electric charges
- Two kinds of charges + ve and – ve
- S.I unit is coulomb (C)
  1 C = charge flowing through a wire in 1 sec if current in it is 1A
  $1e = 1.602 \times 10^{-19}$ C

In 1985, Charles Augustin de Coulomb gave Coulomb's law

Quantization of charge
All charges must be a multiple of charge (e)
$Q = ne$
Where n is an integer

Conservation of charge
It is not possible to create or destroy net charge of an isolated system.

Coulomb's Law
- Force between two charged particles

$\vec{F} = \frac{k\,q_1 q_2 \vec{r}}{r^3} = \frac{k q_1 q_2 \hat{r}}{r^2}$, force is attractive when q_1 and q_2 are of opposite sign, with else repulsive

$|\vec{F}| = \frac{k\,q_1 q_2}{r^2}$

$k = \frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$

- Force between multiple charges

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \ldots\ldots = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{1i}^2} \hat{r}_{1i}$

where $\hat{r}_{1i}$ is the unit vector along charge particle under consideration
r_{1i} = distance between I and i^{th} particle

"The electric force with which two charges attract or repel one another are not affected by the presence of other charges". It is known as Superposition Principle.

Electric Dipole
moment, $\vec{p} = q\vec{d}$

Electric field due to dipole at axial position

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2p}{r^3}\right)\hat{r}$

Electric field due to dipole at equatorial position

$|E| = \frac{1}{4\pi\varepsilon_0}\left(\frac{p}{r^3}\right)$

Torque on an electric dipole placed in an electric field $(\vec{E})$

$\vec{\tau} = \vec{p} \times \vec{E} = p\,E \sin\theta$

Electric Field
Electric field due to point charge
$\vec{E} = \frac{\vec{F}}{q} = k\frac{Q}{r^2}$ (N/C)

$\because$ unit = N/C

- Due to discrete distribution of charges

$\vec{E} = \sum_{i}^{n} \vec{E}_i$

- Due to continuous distribution of charges

$\vec{E} = k \int \frac{dQ}{r^3}\vec{r}$

$|\vec{E}| = k \int \frac{dQ}{r^2}$

Electric field lines
- Always normal to conducting surface
- Lines originating from +ve charge
- Terminating at –ve charge
- Never intersect each other
- Do not form closed loop
- Are imaginary lines.

Electric flux
$\Delta\phi = \vec{E}\Delta\vec{S}$, $\Delta\vec{S}$ = area vector
$\vec{E}$ = electric field

$\phi = \int \vec{E}.\vec{ds}$

Gauss's Theorem
$\oint \vec{E}.\vec{ds} = q_{in}/\varepsilon_0 = \phi$

where, q_{in} = net charge enclosed
ϕ = flux through a closed surface

Applications of Gauss's Law
Electric field due to uniformly charged sphere at
- Outside point
  $E = Q/4\pi\varepsilon_0 r^2$
- Internal point
  $E = Q/4\pi\varepsilon_0 R^3$
- The surface
  $E = Q/4\pi\varepsilon_0 R^2$

- Field due to linear charge distribution
  $E = \lambda/2\pi\,\varepsilon_0 r$,
  λ = linear charge density
- Field due to a plane sheet of charge
  $E = \sigma/2\varepsilon_0$,
  σ = charge per unit area
- Field due to a charged conducting plate
  $E = \sigma/\varepsilon_0$,
  σ = charge per unit area
- Field inside a conductor = 0

UNIT – I

Electrostatics

CHAPTER

1 ELECTRIC CHARGES AND FIELDS

Syllabus

- *Electric charges; Conservation of charge; Coulomb's law—force between two point charges, forces between multiple charges; Superposition principle and continuous charge distribution.*
- *Electric field; Electric field due to a point charge; Electric field lines; Electric dipole, Electric field due to a dipole; Torque on a dipole in uniform electric field.*
- *Electric flux; Statement of Gauss's theorem and its applications to find the field due to infinitely long straight wire, uniformly charged infinite plane sheet.*

Chapter Analysis

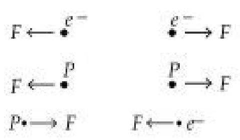
List of Concepts Name	2017	2018	2019
Electric Field and Dipole	2 Q (3 marks) 2 Q (5 marks)	1 Q (3 marks)	1 Q 1 mark) 1 Q (3 marks) OR 1 Q (3 marks)
Gauss's Theorem and Its Applications	1 Q (5 mark)	2 Q (3 marks) 1 Q (5 marks)	1 Q (5 marks)

TOPIC-1
Electric Field and Dipole

Revision Notes

Electric Charge

- Electric charge is the property of a matter due to which, it experiences a force when placed in an electromagnetic field.
- Point charge is an accumulation of the electric charges at a point, without spatial extent.
- Electrons are the smallest and lightest fundamental particles in an atom having negative charge as these are surrounded by invisible force known as electrostatic field.
- Protons are comparatively larger and heavier than electrons with positive electrical charge which is similar in strength as electrostatic field in an electron with opposite polarity.
- Two electrons or two protons will tend to repel each other as they carry like charges, negative and positive respectively.
- The electron and proton will get attracted towards each other due to their unlike charges.



- The charge present on the electron is equal and opposite to charge on the proton.

  Charge on a proton $= +1{\cdot}6 \times 10^{-19}$ C

  and charge on an electron $= -1{\cdot}6 \times 10^{-19}$ C

Scan to know more about this topic

Electric Charges & Field

Electrostatic Charge

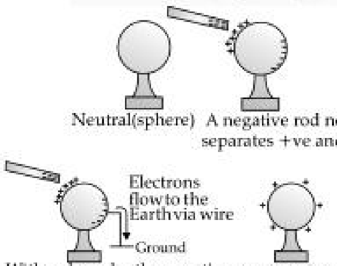
- Electrostatic charge means the charge is at rest.
- Electrostatic charge is a fundamental physical quantity like length, mass and time.
- Charge on a body is expressed as $q = \pm ne$
- The magnitude of charge is independent of the speed of the particle.
- Based on the flow of charge across them, materials are classified as :
  - Conductors - allow electric charge to flow freely, e.g. metals
  - Semi-conductors - behave as the conductor or insulator depending on the number of free electron and hole availability. e.g. silicon
  - Insulators - do not allow electric charge to flow, e.g. rubber, wood, plastic etc.
- **Net charge on a body is given by :**
  - Charging by friction - charging insulators
  - Charging by conduction - charging metals and other conductors
  - Charging by induction - wireless charging

Scan to know more about this topic

Electric Field

Charging by Induction

- Charging by induction means charging without the contact.
- On rubbing a glass rod and silk cloth piece together, glass rod gets positively charged whereas silk cloth gets negatively charged.
- If a plastic rod is rubbed with wool, it becomes negatively charged.
- If a negatively charged rod is brought near neutral metal with insulator mounting, it repels free electrons and attracts positive charges on metal.
- If far end is connected to Earth by a wire, electrons will flow towards ground while positive charges are kept captive by the rod.



Neutral(sphere) A negative rod near by separates +ve and –ve charges

With rod nearby the negative end may be discharged. With rod away, the positive charges distribute evenly

Properties of Electric Charge

Addition of charges

- If a system contains three point charges q_1, q_2 and q_3, then the total charge of the system will be the algebraic addition of q_1, q_2 and q_3, *i.e.*, charges will add up.

$$q = q_1 + q_2 + q_3$$

Conservation of charges

- Electric charge is always conserved. It is the sum of positive and negative charges present in an isolated system, which remains constant.
- Charge cannot be created or destroyed in a process, but only exists in positive-negative pairs.

Quantization of charges

- Electric charge is always quantized *i.e.*, electric charge is always an integral multiple of charge 'e'.
- Net charge q_{net} of an object having N_e electrons, N_p protons and N_n neutrons is :

  $q_{net} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = \pm ne$
- Neutron (n) : $m = 1.675 \times 10^{-27}$ kg; $q = 0$
- Proton (p) : $m = 1.673 \times 10^{-27}$ kg; $q = +1.6 \times 10^{-19}$ C
- Electron (e): $m = 9.11 \times 10^{-31}$ kg; $q = -1.6 \times 10^{-19}$ C

Scan to know more about this topic

Electric Dipole

Coulomb's Law

- The force of attraction or repulsion between two point charges q_1 and q_2 separated by a distance r is directly proportional to product of magnitude of charges and inversely proportional to square of distance between charges, written as :

$$F = k\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

where,
F = Force of attraction/repulsion between charges q_1 & q_2.
q_1, q_2= Magnitudes of charge 1 and charge 2 respectively
r = Distance between charges q_1, q_2
k = Constant whose value depends on medium where charges are kept. $k = \frac{1}{4\pi\varepsilon}$

As $\varepsilon = K'\varepsilon_0$, $k = \frac{1}{4\pi K'\varepsilon_0}$

ε_0 = permittivity of vacuum = 8.854×10^{-12} F/m
K' = relative permittivity of medium or dielectric constant.

- For vacuum, relative permittivity, $K' = 1$,
- As $\varepsilon = K\varepsilon_0$, force of attraction/repulsion among two electric charges q_1, q_2 placed in vacuum and medium is :

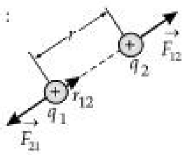
$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \text{ (vacuum)}$$

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\cdot\frac{q_1 q_2}{r^2} \text{ (medium)}$$

- The unit coulomb (C) is derived from the SI unit ampere (A) of the electric current.
- Current is the rate $\frac{dq}{dt}$ at which charge moves past a point or through a region, $i = \frac{dq}{dt}$, hence 1 C=(1 A) × (1 s).
- The vector form of Coulomb force with $\hat{r}_{12}$ = unit vector from q_1 to q_2 is given as :

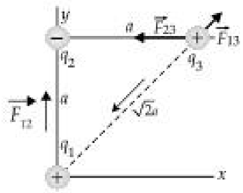
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} \text{ and } \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

$$\Rightarrow \quad \vec{F}_{21} = -\vec{F}_{12}$$



Principle of Superposition

- The force on any charge due to other charges at rest is the vector sum of all the forces on that charge due to the other charges, taken one at a time.
- The individual forces are unaffected due to presence of other charges.
- Force exerted by q_1 on $q_3 = \vec{F}_{13}$
- Force exerted by q_2 on $q_3 = \vec{F}_{23}$
- Net force exerted on q_3 is vector sum of $\vec{F}_{13}$ and $\vec{F}_{23}$

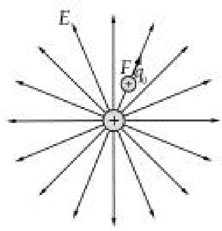


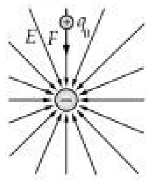
Electric field

- The space around a charge up to which its electric force can be experienced is called electric field.
- If a test charge q_0 is placed at a point where electric field is E, then force on the test charge is $F = q_0E$
- The electric field strength due to a point source charge 'q' at an observation point 'A' at a distance 'r' from the source charge is given by :

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^3}\vec{r}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$



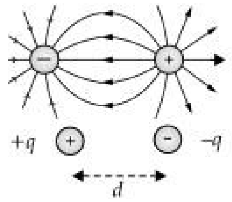


- The unit of electric field is N/C.
- Electric field inside the cavity of a charged conductor is zero.
- If a charged/uncharged conductor is placed in an external field, the field in conductor is zero.
- In case of charged conductor, electric field is independent of the shape of conductor.

Electric field lines

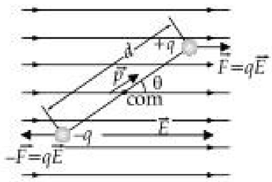
- Electric field lines are imaginary lines that extend from positive charge towards negative charge.
- Direction of electric field lines around positive charge is imagined by positive test charge q_0 located around source charge.
- Electric field has same direction as force on positive test charge.
- Electric field lines linked with negative charge are directed inward described by force on positive test charge q_0.
- The electric field lines never cross each other.
- Strength of electric field is encoded in density of field lines.

Electric Dipole



- The system formed by two equal and opposite charges separated by a small distance is called an electric dipole.
- The electric field exists due to a dipole.
- The force on a dipole in a uniform electric field is zero in both stable as well as unstable equilibrium.
- The potential energy of a dipole in an uniform electric field is minimum for a stable equilibrium and maximum in an unstable equilibrium.

Moment of Force



- In a dipole, when net force on dipole due to electric field is zero and center of mass of dipole remains fixed, then forces on charged ends produce net torque τ about its center of mass.

$$\tau = F\,d\sin\theta = qE(d)\sin\theta = pE\sin\theta$$

$$\vec{\tau} = \vec{p}\times\vec{E}$$

  - If $\theta = 0^\circ$ or 360°, dipole exists in stable equilibrium state.
  - If $\theta = 180^\circ$, dipole exists in unstable equilibrium state.

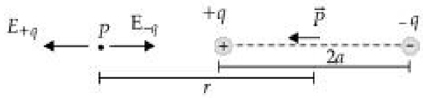
- In uniform electric field, dipole experiences torque, net force on dipole is zero.
- In uniform electric field, dipole experiences a rotatory motion.
- In non-uniform electric field, dipole experiences torque and net force.
- In non-uniform electric field, dipole experiences rotatory and translatory motion.
  - The torque aligns dipole with electric field and it becomes zero.
  - The direction of torque is normal to the plane going inward.

Electric Dipole Moment

- Dipole moment is a vector quantity whose unit is coulomb-meter (Cm).
- Dipole moment vector of electric dipole is $\vec{p} = \vec{q}\times 2a$ between pair of charges q, $-q$ along the line.
- For point P at distance r from centre of dipole on charge q, for $r>>a$, total field at point P is

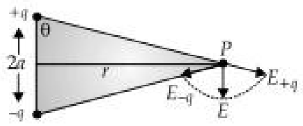
$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \text{ (if } a << r)$$



- For point P on the equatorial plane due to charges $+q$ and $-q$, electric field of dipole at a large distance

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$



Know the Terms

- **1 coulomb :** When two point charges placed at a distance of 1 m in vacuum, repel/attract each other with force of 9×10^9 N, the charge on each is known as 1 coulomb.
- **Electric line of force :** It is a curve drawn in such a way that the tangent at each point to curve gives the direction of the net field at that point.

Know the Formulae

- Coulomb's force : $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$;

  where all alphabets have their usual meanings.
- Electric field due to point charge q : $E = \frac{k|q|}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$
- Electric field due to a dipole at a point on the dipole axis : $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}$ $(r >>> a)$
- Electric field at a point on equatorial plane : $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$ $(r >>> a)$
- Torque on an electric dipole placed in an electric field, $\tau = pE\sin\theta$

Objective Type Questions

(1 mark each)

(I) MULTIPLE CHOICE QUESTIONS

Q. 1. Plastic rod rubbed with fur and glass rod rubbed with silk U

(a) repel each other. (b) mix up with each other.

(c) attract each other. (d) None of the above.

Ans. Correct option : (c)

Explanation : Rubbing a rod with certain materials will cause the rod to become charged. If a plastic rod rubbed with fur becomes negatively charged and a glass rod rubbed with silk becomes positively charged, both will attract each other.

Q. 2. Electric charge between two bodies can be produced by U

(a) sticking. (b) rubbing.

(c) oiling. (d) passing AC current.

Ans. Correct option : (b)

Explanation : The triboelectric effect is a type of contact electrification in which certain materials become electrically charged after they come into frictional contact with a different material.

Q. 3. Electric charges when remain at rest under the action of electric forces is called U

(a) electrostatic. (b) electric flux.

(c) electric field. (d) electric field lines.

Ans. Correct option : (a)

Explanation : Coulomb force, also called electrostatic force or Coulomb interaction, attraction or repulsion of charged particles or objects because of their electric charges.

Q. 4. Law stating that "force is directly proportional to product of charges and inversely proportional to square of separation between them" is called U

(a) Newton's law. (b) Coulomb's law.

(c) Gauss's law. (d) Ohm's law.

Ans. Correct option : (b)

Explanation : Coulomb's law states that : The magnitude of the electrostatic force of attraction or repulsion between two-point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distances between them.

Q. 5. Out of the following, which is not a property of field lines? R

(a) Field lines are continuous curves without any breaks

(b) Two field lines cannot cross each other

(c) Field lines start at positive charges and end at negative charges

(d) They always form closed loops.

Ans. Correct option : (d)

Explanation : The electric field lines may not always form the closed loops just like in case of a point charge. So, the statement is wrong.

Q. 6. If E_1 be the electric field strength of a short dipole at a point on its axial line and E_2 that on the equatorial line at the same distance, then U

(a) $E_1 = E_2$

(b) $E_1 = 2E_2$

(c) $E_2 = 2E_1$

(d) None of these.

Ans. Correct option : (b)

Explanation : The electric field at any axial point is twice the electric field at any equatorial point in case of a dipole, if the distance taken is same. The electric filed at an equatorial point is Kp/r^3 and at axial line is given by $2Kp/r^3$. Their ratio will be 1: 2 .

Commonly Made Error

- Some students do not remember the axial electric field is double than that of the equatorial field. They write that the equatorial field is double than that of axial field.

Answering Tip

- The students should carefully remember that ratio of axial electric field to that of equatorial electric field of a dipole is 2 : 1

(II) FILL IN THE BLANKS

Q. 1. The S.I. unit of permittivity of free space is .... R

Ans. $m^{-3}\,kg^{-1}\,s^4\,A^2$

Q. 2. The electric field lines for an electric dipole emerge from .... charge.

Ans. positive

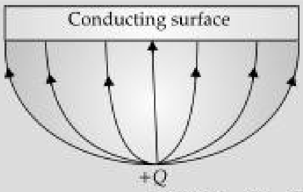
Q. 3 If an object attains +1 C of charge, then the number of electrons lost by that object is ....

Ans. 6.25×10^{18}.

(III) VERY SHORT ANSWER TYPE QUESTIONS

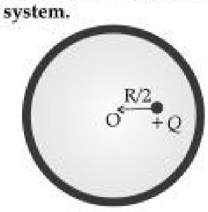
Q. 1. A point charge $+Q$ is placed in the vicinity of a conducting surface. Draw the electric field lines between the surface and the charge.
R **[O.D. Comptt. I, II, III 2017]**

Ans.



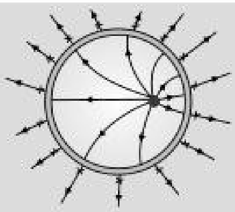
[CBSE Marking Scheme, 2017] 1

Q. 2. Figure shows a point charge $+Q$, located at a distance $R/2$ from the centre of a spherical metal shell. Draw the electric field lines for the given system.



A **[CBSE SQP 2016]**

Ans.



Inside ½
Outside ½
[CBSE Marking Scheme, 2016]

AI **Q. 3. Why should electrostatic field be zero inside a conductor ?** U

Ans. In the static equilibrium, there is no current inside, or on the surface of the conductor, hence the electric field is zero everywhere inside the conductor.

Alternatively,

Since the charge inside the conductor is zero, the electric field is also zero. **1**

OR

Alternatively,

Since the conductor is uncharged so the electric field inside it is zero.

Q. 4. What is the value of the angle between the vectors $\vec{p}$ and $\vec{E}$ for which the potential energy of an electric dipole of dipole moment $\vec{p}$, kept in an external electric field $\vec{E}$, has the maximum value.
A **[CBSE SQP 2014]**

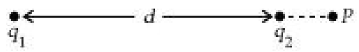
Ans. P.E. $= -\vec{p}\cdot\vec{E} = -pE\cos\theta$ ½

$\therefore$ P.E. is the maximum when $\cos\theta = -1$,

i.e., $\theta = \pi\ (= 180^\circ)$. ½

[CBSE Marking Scheme, 2014]

AI **Q. 5. Two point charges 'q_1' and 'q_2' are placed at a distance 'd' apart as shown in the figure. The electric field intensity is zero at a point 'P' on the line joining them as shown. Write two conclusions that you can draw from this.**



Ans. (i) The two point charges (q_1 and q_2) should be of opposite nature. ½

(ii) Magnitude of charge q_1 must be greater than that of charge q_2. ½

AI **Q. 6. Two equal balls having equal positive charge 'q' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two ?**

Ans. The force decreases. **1**

Detailed Answer :

The force given as $F = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q^2}{\left(d - t + t\sqrt{K}\right)^2}$ will decrease when a plastic is introduced, as the dielectric constant K for plastic is more than 1.

Q. 7. When is the torque on a dipole in an electric field maximum? U

Ans. The torque on an electric dipole is maximum when it is held perpendicular to the field. **1**

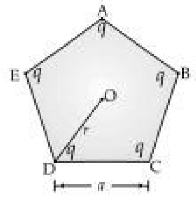
Short Answer Type Questions (2 marks each)

AI Q. 1. Five charges, q each are placed at the corners of a regular pentagon of side 'a'.

(i) What will be the electric field at O if the charge from one of the corners (say A) is removed ?

(ii) What will be the electric field at O if the charge q at A is replaced by $-q$?

R [SQP 2017] [NCERT Exemplar]



Ans. (i) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$ along OA **1**

(ii) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{r^2}$ along OA **1**

[CBSE Marking Scheme, 2017]

Detailed Answer :

(i) If a charge q is removed from point A, a negative charge is developed at A where electric field will be

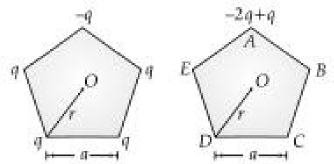
$$\vec{E}_A + \vec{E}_{four\,charges} = 0$$

Hence $\vec{E}_{four\,charges} = -\vec{E}_A$ or $\left|\vec{E}_{four\,charges}\right| = \left|\vec{E}_A\right|$

When charge q is removed from A, net electric field at the centre due to remaining charges

$$\left|\vec{E}_{four\,charges}\right| = \left|\vec{E}_A\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ along } OA \quad \textbf{1}$$

(ii) If a charge q is replaced by charge $-q$ at point A, this generates a net electric field at point O as a result of $-2q$ charge, so



$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ along } OA.$$

Hence net electric field at the centre

$$\vec{E}_{net} = \vec{E}_{-q} + \vec{E}_{four\,charges} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\vec{E}_{net} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2} \text{ along } OA. \quad \textbf{1}$$

Commonly Made Error

- Some candidates find resultant force F as a scalar sum of all forces, but resultant force is equal to vector sum of all forces.

Answering Tips

- Students should apply vector sum at the place of scalar sum for vector quantities.

AI Q. 2. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ Nm. Calculate the potential energy of the dipole, if it has charge ± 8 nC.

Ans. Torque $\quad \tau = pE\sin\theta$ ½

$$4\sqrt{3} = pE\sin 60° = pE\frac{\sqrt{3}}{2}$$

$$4\sqrt{3} = pE\frac{\sqrt{3}}{2}$$

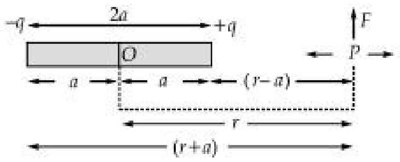
Hence, $\quad pE = 8$ ½

Potential energy, $\quad U = -pE\cos\theta$ ½

or $\quad U = -8 \times \cos 60° = -4$ J ½

AI Q. 3. Find the expression for electric field intensity in an axial position due to electric dipole. **R**

Ans. Consider an electric dipole whose length is $2a$ and centre at O. From the mid-point O, consider a point P at a distance r, where the electric field intensity is to be determined.



We have $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

Case I, $\quad E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r-a)^2}$ (due to positive charge)

Case II, $\quad E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r+a)^2}$ (due to negative charge)

Then, the net electric field $E = E_1 - E_2$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2\cdot(r+a)^2}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0} \cdot \frac{4ar}{(r^2-a^2)^2}$$

for, $r^2 >>> a^2$

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0}\left|\frac{4ar}{r^4}\right|$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\times q\times 2a\times r}{r^4}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\vec{p}}{r^3}\ \text{N/C}\ [\because p = 2qa]\ 2$$

Commonly Made Error

- Several candidates derive an expression for intensity of electric field E at a point in the broadside position *i.e.*, coaxial position, instead of that in the end-on position as required. Some candidates do not understand which derivation to write. Hence they write both the derivations. Many candidates are not able to draw the correctly labelled diagram.

Answering Tips

- Students should derive the expression by using proper vector diagram and find the resultant by vectorial method.

Long Answer Type Questions-I

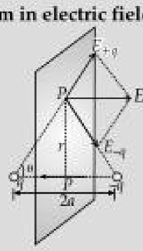
(3 marks each)

AI Q. 1. (i) Derive the expression for electric field at a point on the equatorial line of an electric dipole.

(ii) Depict the orientation of the dipole in (i) stable, (ii) unstable equilibrium in a uniform electric field. A **[Delhi I 2017]**

Ans. Derivation of expression of electric field on equatorial line of electric dipole **2**

Depiction of orientation for stable and unstable equilibrium in electric field **½+½**



(i) Let the point 'P' be at a distance 'r' from the mid point of the dipole.

$$|E_{+q}| = \frac{q}{4\pi\varepsilon_0\left(r^2+a^2\right)} \qquad ½$$

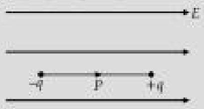
$$|E_{-q}| = \frac{q}{4\pi\varepsilon_0\left(r^2+a^2\right)} \qquad ½$$

Both are equal and their directions are as shown in the figure, hence net electric field

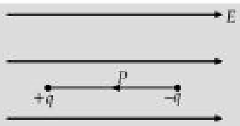
$$\vec{E} = \left[-\left(E_{+q}+E_{-q}\right)\cos\theta\right]\hat{p} \qquad ½$$

$$= -\frac{2qa}{4\pi\varepsilon_0\left(r^2+a^2\right)^{3/2}}\hat{p} \qquad ½$$

(ii) Stable equilibrium $\theta = 0°$ ½



Unstable equilibrium $\theta = 180°$ ½



[CBSE Marking Scheme, 2017]

AI Q. 2. (i) Obtain the expression for the torque $\vec{\tau}$ experienced by an electric dipole of dipole moment $\vec{p}$ in a uniform electric field, $\vec{E}$.

(ii) What will happen if the field were not uniform? R **[Delhi III 2017]**

Ans. (i) Obtaining expression for torque $\vec{\tau}$ experienced by electric dipole in uniform electric field **2**

(ii) Effect of non-uniform electric field **1**

(i) Force on $+q$, $\vec{F} = q\vec{E}$ ½

Force on $-q$, $\vec{F} = -q\vec{E}$ ½

Magnitude of torque $\vec{\tau} = qE \times 2a\sin\theta$ ½

$= 2qaE\sin\theta$

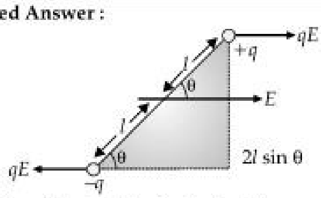
$\vec{\tau} = \vec{p}\times\vec{E}$ ½

(ii) If the electric field is non-uniform, the dipole experiences a translatory force as well as a torque. 1

[CBSE Marking Scheme, 2017]

Detailed Answer :

(i)



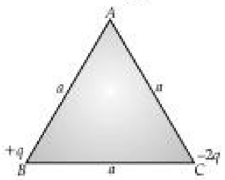
½

Consider electric dipole kept in an uniform electric field at an angle θ where a dipole experience a torque, so, torque generated by parallel forces qE will act as couple as ½

$$|\vec{\tau}| = qE2l\sin\theta \quad ½$$
$$= pE\sin\theta \quad [\text{as } p = 2ql] \ ½$$
$$|\vec{\tau}| = |\vec{p}\times\vec{E}|$$

(ii) When the field is non-uniform, force acting on both ends will not be equal, hence they result in formation of couple and net force. With this, dipole experiences rotational as well as linear force. **1**

AI Q. 3. Two point charges + q and $-2q$ are placed at the vertices 'B' and 'C' of an equilateral triangle ABC of side 'a' as given in the figure. Obtain the expression for (i) the magnitude and (ii) the direction of the resultant electric field at the vertex A due to these two charges.

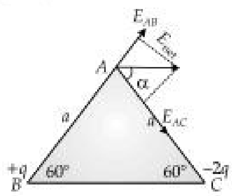


A

Ans. (i) The magnitude

$$\left|\vec{E}_{AB}\right| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2} = E \quad ½$$

$$\left|\vec{E}_{AC}\right| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{a^2} = 2E \quad ½$$



½

$$E_{net} = \sqrt{(2E)^2+(E^2)+2\times 2E\times E\cos 120^\circ}$$

$$E_{net} = \sqrt{(2E)^2+E^2+2\times 2E\times E\times\left(-\frac{1}{2}\right)}$$

$$E_{net} = \sqrt{4E^2+E^2-2E^2}$$

$$E_{net} = E\sqrt{3} = \frac{1}{4\pi\varepsilon_0}\frac{q\sqrt{3}}{a^2} \quad ½$$

(ii) The direction of resultant electric field at vertex A

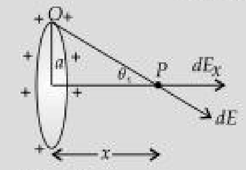
$$\tan\alpha = \frac{E_{AB}\sin 120^\circ}{E_{AC}+E_{AB}\cos 120^\circ} \quad ½$$

$$\tan\alpha = \frac{E\times\frac{\sqrt{3}}{2}}{2E+E\times\left(\frac{-1}{2}\right)} = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^\circ \text{ (with side } AC) \quad ½$$

AI Q. 4. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence, show that for points at large distances from the ring, it behaves like a point charge.
A [Delhi I, II, III, 2016]

Ans.



$OP = \sqrt{a^2+x^2}$, by Pythagoras theorem

where, a is radius of ring

x is the distance from centre to point 'P'

$$dE = \frac{kdQ}{a^2+x^2}$$

$$dE_x = \frac{kdQ}{(a^2+x^2)}\cos\theta \quad 1$$

and dE_y will be zero because all components will cancel out each other.

$$\because \quad \cos\theta = \frac{x}{(a^2+x^2)^{1/2}}$$

$$dE_x = \frac{kdQ}{a^2+x^2}\cdot\frac{x}{(a^2+x^2)^{1/2}}$$

$$E_x = \int\frac{kxdQ}{(a^2+x^2)^{3/2}}$$

$$E_x = \frac{kx}{(a^2+x^2)^{3/2}}\int dQ$$

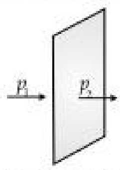
$$E = \frac{kQx}{(a^2+x^2)^{3/2}} \quad 1$$

Now, if $x >> a$

$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{x^2}$, which is equal to field produced by a point change.

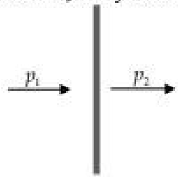
$\therefore$ For large distance, ring behaves as a point charge. **1 [CBSE Marking Scheme, 2016]**

Q. 5. (i) An electric dipole is kept first to the left and then to the right of a negatively charged infinite plane sheet having a uniform surface charge density. The arrows p_1 and p_2 show the directions of its electric dipole moments in the two cases.



Identify for each case, whether the dipole is in stable or unstable equilibrium. Justify each answer.

(ii) Next, the dipole is kept in a similar way (as shown), near an infinitely long straight wire having uniform negative linear charge density.



Will the dipole be in equilibrium at these two positions? Justify your answer. A **[SQP 2018-19]**

Ans. (i) p_1 : stable equilibrium ½

p_2 : unstable equilibrium

The electric field, on either side, is directed towards the negatively charged sheet and its magnitude is independent of the distance of the field point from the sheet. For position p_1, dipole moment and electric field are parallel. For position p_2, they are anti parallel. ½+½

(ii) The dipole will not be in equilibrium in any of the two positions. ½

The electric field due to an infinite straight charged wire is non-uniform ($E \propto 1/r$). ½

Hence, there will be a net non-zero force on the dipole in each case. ½

[CBSE Marking Scheme, 2018-19]

Q. 6. An electric dipole is placed in a uniform electric field.

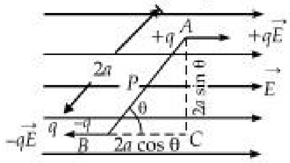
(i) Show that no translatory force acts on it.

(ii) Derive an expression for the torque acting on it.

(iii) Find work done in rotating the dipole through 180°.

A **[Delhi Comptt. I, II, III, 2014, O.D. I, II, III, 2013, 2012]**

Ans. Electric dipole of charges $+q$ and $-q$ separated by distance $2a$ is shown in the figure.



It is placed in a uniform electric field at an angle θ with it.

(i) Force on charge $+q$, $\vec{F_1} = q\vec{E}$, in the direction of $\vec{E}$. ½

Force on charge $-q$, $\vec{F_2} = -q\vec{E}$, in the opposite direction of $\vec{E}$.

$\therefore$ Net translatory force on dipole $= \vec{F_1} + \vec{F_2}$

$$= +q\vec{E} - q\vec{E} = \vec{0}$$

Hence, no translatory force acts on it. ½

(ii) But the two equal, parallel and unlike forces form a couple in which a torque is given by

τ = Force × perpendicular distance between the two forces

$$\tau = qE \times 2a\sin\theta$$

$$\tau = pE\sin\theta$$

where, $p = q \times 2a$ = dipole moment 1

(iii) Work done in rotating the dipole through 180° is

$$W = \int dW$$

$$W = \int_{0^\circ}^{180^\circ} \tau d\theta = pE\int_{0^\circ}^{180^\circ} \sin\theta d\theta$$

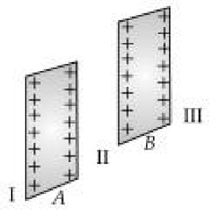
$$W = pE[-\cos\theta]_{0^\circ}^{180^\circ}$$

$$W = -pE[\cos 180^\circ - \cos 0^\circ]$$

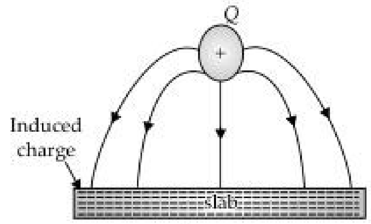
$$W = pE[1 + 1] = 2pE$$

AI **Q. 7. (i) A point charge (+Q) is kept in the vicinity of an uncharged conducting plate. Sketch electric field lines between the charge and the plate.**

(ii) Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the fields in the regions marked II and III.



Ans. (i)



1

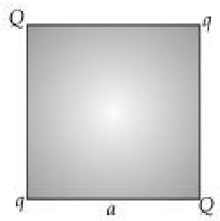
(ii) (a) For region II, $E_{II} = \frac{1}{2\varepsilon_0}(\sigma_1 - \sigma_2)$ ½

towards right side from sheet A to sheet B. ½

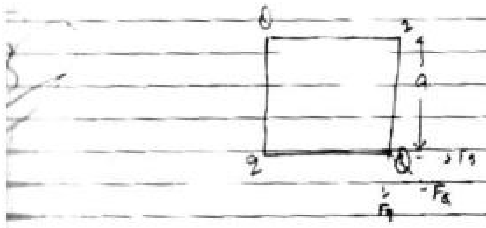
(b) For region III, $E_{III} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_2)$ ½

towards right side away from the two sheets. ½

Q. 8. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure. Find the resultant electric force on a charge Q A **[CBSE Board Paper 2018]**



Ans.



Force on Q acts due to q, q and Q

as the force due to ~~both~~ q are equal with angle of 90°

⇒ ~~Resulta~~ Resultant force F is given by

$$\sqrt{F^2+F^2} = \sqrt{2}F \quad \text{(acts along diameter)}$$

where $F = \frac{KQq}{a^2}$

Now, force due to Q acts along diameter

$$\Rightarrow F_Q = \frac{KQ^2}{(\sqrt{2}a)^2} = \frac{KQ^2}{2a^2}$$

⇒ Net force = $\sqrt{2}F + F_Q$ (as they act along diameter)

$$\Rightarrow \frac{\sqrt{2}QqK}{a^2} + \frac{KQ^2}{2a^2}$$

$$= \frac{K(2\sqrt{2}Qq + Q^2)}{2a^2}$$

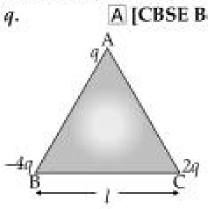
$$= \frac{KQ}{2a^2}\left[2\sqrt{2}q + Q\right] \text{ N}$$

along the diameter away from charge Q.

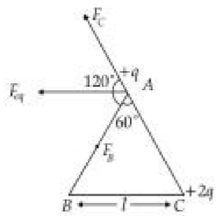
where $K = \frac{1}{4\pi\epsilon_0}$

[Topper's Answer, 2018]

Q. 9. Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side 'l' as shown in the figure. Obtain the expression for the magnitude of the resultant electric force acting on the charge q. A **[CBSE Board Paper 2018]**



Ans. The resultant force on the charge q at A is



$$F_q = \sqrt{F_C^2 + F_B^2 + 2F_BF_C\cos\theta}$$

Now,

$$|\vec{F}_C| = \frac{k.q.2q}{l^2} = \frac{2kq^2}{l^2} = F_c \qquad ...(1)$$

$$\overline{F_B} = \frac{kq(-4q)}{l^2} = -\frac{4kq^2}{l^2} \Rightarrow F_B = \frac{4kq^2}{l^2} \qquad ...(2)$$

Now from equation (1) and (2)

$F_B = 2F_c$

and angle between F_B and F_C is 120°, so force on q is

$$F_q = \sqrt{F_c^2 + (2F_c)^2 + 2.(2F_c).F_c.\cos 120°}$$

$$= \sqrt{5F_c^2 + 4F_c^2\left(-\frac{1}{2}\right)} = \sqrt{5F_c^2 - 2F_c^2} = F_c\sqrt{3}$$

$$\boxed{F_q = 2\sqrt{3}\frac{kq^2}{l^2}} \qquad \left[\because \text{from eqn(1)}, F_c = \frac{2kq^2}{l^2}\right] \qquad \textbf{3}$$

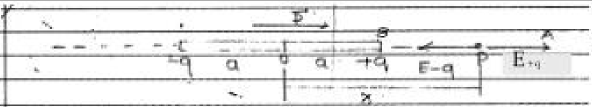
Long Answer Type Questions - II **(5 marks each)**

AI Q. 1. (a) Derive an expression for the electric field E due to a dipole of length '$2a$' at a point distant r from the centre of the dipole on the axial line.

(b) Draw a graph of E versus r for $r >> a$.

(c) If this dipole were kept in a uniform external electric field E_0, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases. R

Ans. (a)



Consider a dipole having dipole moment p. Electric field due to −q at the point P is along PB

$$E_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2} \text{ along PB}$$

Electric field due to +q at the point P is along PA

$$E_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} \text{ along PA}$$

The net electric field

$$= E_{+q} - E_{-q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} q\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2}$$

$$= \frac{2q\,r \times 2a}{4\pi\varepsilon_0(r^2-a^2)^2}$$

$$= \frac{2pr}{4\pi\varepsilon_0(r^2-a^2)^2} \text{ along PA.}$$

(b) Electric field at point P is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2} \text{ in the direction of } \vec{p}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}r}{(r^2-a^2)^2}$$

For $r \gg a$

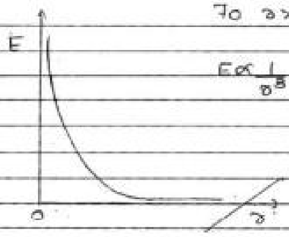
$$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{r^4}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$E \propto \frac{1}{r^3}$$

For $r \gg a$

$E \propto \frac{1}{r^3}$



(c) When $\theta = 0$

energy $= -PE$

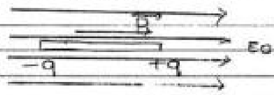
Torque $= 0$.

It is stable equilibrium

When dipole is in stable equilibrium

potential energy $P.E = -PE$

Torque $= 0$



$\vec{p}$ is parallel to $\vec{E_0}$

The dipole is placed parallel to the electric field.

In unstable equilibrium

$\theta = 180°$

$P.E = -PE\cos 180$

$= -PE \times -1$

$= PE$

potential energy is maximum

The dipole is placed antiparallel to the field

Torque

$\tau = P \times E$

$PE\sin\theta$

In Ist case $\tau = PE\sin 0$

$= 0$

In IInd case $\tau = PE\sin 180$

$= 0$

[Topper's Answer 2017]

AI Q. 2. (i) Define electric dipole moment. Is it a scalar or a vector quantity? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

(ii) Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero. R

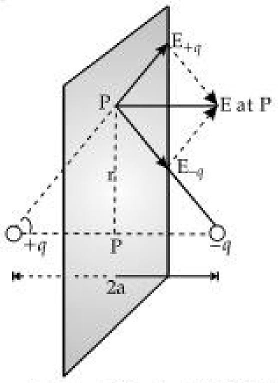
Ans. (i) Electric dipole moment : The strength of an electric dipole is measured by the quantity of electric dipole moment. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges.

Electric dipole moment, $p = q \times 2a$

It is a vector quantity.

In vector form, it is written as $\vec{p} = q \times 2\vec{a}\,\hat{p}$, where the direction of $2\vec{a}\,\hat{p}$ is from negative charge to positive charge.

Electric field of dipole at points on the equatorial plane :



The magnitudes of the electric field due to the two charges $+q$ and $-q$ are given by,

$$|\vec{E}_{+q}| = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{(r^2+a^2)}$$

$$|\vec{E}_{-q}| = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{(r^2+a^2)}$$

$$|\vec{E}_{+q}| = |\vec{E}_{-q}|$$

The direction of $|\vec{E}_{+q}|$ and $|\vec{E}_{-q}|$ are shown in the figure. The components normal to the dipole axis cancel out. The components along the dipole axis add up.

∴ Total electric field

$$\vec{E} = -(E_{+q} + E_{-q})\cos\theta\,\hat{p}$$

[Negative sign shows that field is opposite to $\hat{p}$]

$$\vec{E} = \frac{-2qa}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}\hat{p} \quad \text{...(i)}$$

At large distances ($r >> a$), this reduces to

$$\vec{E} = \frac{-2qa}{4\pi\varepsilon_0 r^3}\hat{p} \quad \text{...(ii)}$$

$$|\vec{E}_{+q}| = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{(r^2+a^2)} \quad \text{...(iii)}$$

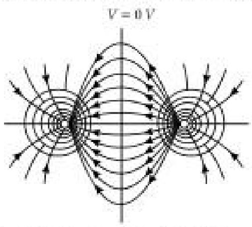
$$|\vec{E}_{-q}| = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{(r^2+a^2)} \quad \text{...(iv)}$$

$$\therefore \quad |\vec{E}_{+q}| = |\vec{E}_{-q}|$$

$$\because \quad \vec{p} = 2qa\hat{p}$$

$$\therefore \quad \vec{E} = \frac{\vec{p}}{4\pi\varepsilon_0 r^3}$$ **3**

(ii) Equipotential surface due to electric dipole :

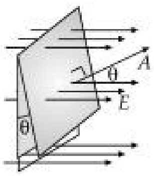


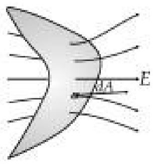
The potential due to the dipole is zero at the line bisecting the dipole length. **2**

TOPIC-2
Gauss's Theorem and its Applications

Revision Notes

Electric Flux





- Electric flux is proportional to algebraic number of electric field lines passing through the surface, outgoing lines with positive sign, incoming lines with negative sign.
- Due to arbitrary arrangement of electric field lines, electric flux can be quantify as $\phi_E = EA$
- If vector A is perpendicular to surface, magnitude of vector A parallel to electric field is $A\cos\theta$

  $A_{||} = A\cos\theta$

  $\phi_E = EA_{||} = EA\cos\theta$
- In non-uniform electric field, the flux will be $\phi_E = \int E dA$

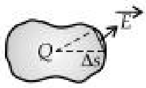
Continuous Charge Distribution

- It is a system in which the charge is uniformly distributed over the material. In this system, infinite number of charges are closely packed and have minor space among them. Unlike the discrete charge system, the continuous charge distribution is uninterrupted and continuous in the material. There are three types of continuous charge distribution system.
- For linear charge distribution (λ), $\vec{F} = \dfrac{q_0}{4\pi\varepsilon_0}\int_x \dfrac{\lambda}{r^2}dx\hat{r}$ (Where, λ = linear charge density)
- For surface charge distribution (σ), $\vec{F} = \dfrac{q_0}{4\pi\varepsilon_0}\int_s \dfrac{\sigma}{r^2}dS\hat{r}$ (Where, σ = surface charge density)
- For volume charge distribution (ρ), $\vec{F} = \dfrac{q_0}{4\pi\varepsilon_0}\int_v \dfrac{\rho}{r^2}dV\hat{r}$ (Where, ρ = volume charge density)

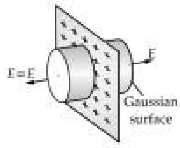
Scan to know more about this topic

Gauss Theorem

Gauss theorem

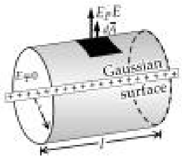


- The net outward normal electric flux through any closed surface of any shape is equal to $1/\varepsilon_0$ times to net charge enclosed by the surface
- The electric field flux at all points on Gaussian surface is $\phi = E\oint dA = \dfrac{q}{\varepsilon_0}$



- If there is a positive flux, net positive charge is enclosed.
- If there is a negative flux, net negative charge is enclosed.
- If there is zero flux, no net charge is enclosed.
- The expression for electric field due to a point charge on Gaussian surface is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$



- In an insulating sheet, charge remains in the sheet, so electric field, $E = \dfrac{\sigma}{2\varepsilon_0}$
- Gauss theorem works in cases of cylindrical, spherical and rectangular symmetries.
- The field outside the wire points radially outward which depends on distance from wire, $\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$, where, λ is linear density of charge.
- **Closed surface :** It is a surface which divides the space inside and outside region, where one can't move from one region to another without crossing the surface.
- **Gaussian surface :** It is a hypothetical closed surface having similar symmetry as problem on which we are working.
- **Electrostatic Shielding :** It is the phenomenon of protecting a certain region of space from external electric field.
- **Dielectric :** The non-conducting material in which charges are easily produced on the application of electric field is called dielectric. *e.g.* Air, H_2 gas, glass, mica, paraffin wax, transformer oil etc.

Key Formulae

- Electric flux through an area A : $\phi = E.A = EA\cos\theta$
- Electric flux through a Gaussian surface : $\phi = \oint E.dS$
- Gauss's Law : $\phi = \dfrac{q_{enc}}{\varepsilon_0}$
- Electric Field due to an infinite line of charge : $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{2k\lambda}{r}$

  where, E = electric field [N/C], λ = charge per unit length [C/m]

  ε_0 = permittivity of free space = 8.85×10^{-12} [$C^2/N\ m^2$], r = distance (m), $k = 9 \times 10^9$ N m^2 C^{-2}
- Electric field due to a ring at a distance x is : $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{qx}{(r^2+x^2)^{3/2}}$
- When, $x >>> r$: $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{x^2}$
- When $x <<< r$: $E = 0$

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Scan to know more about this topic

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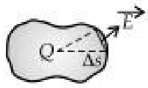
Gauss theorem



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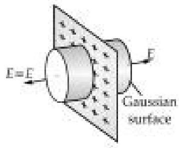

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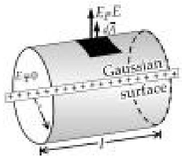

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- When, $x >>> r$: $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}$
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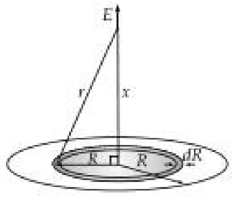
➢ Electric field due to a charged disc : $$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2 + x^2}}\right]$$



where ,

E = electric field [N/C]

σ = charge per unit area [C/m^2]

$\varepsilon_0 = 8.85 \times 10^{-12}$ [C^2/Nm^2]

x = distance from charge [m]

R = radius of the disc [m]

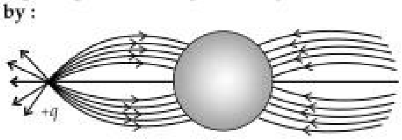
➢ Electric field due to a thin infinite sheet : $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

Objective Type Questions

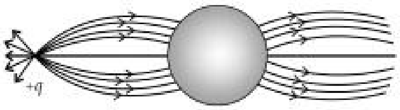
(1 mark each)

(I) MULTIPLE CHOICE QUESTIONS

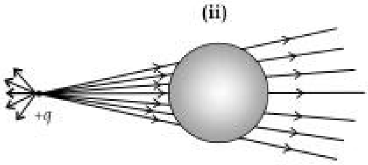
Q. 1. A point positive charge is brought near an isolated conducting sphere in Figure. The electric field is best given by :



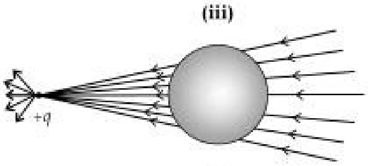
(i)



(ii)



(iii)



(iv)

(a) Fig (i) **(b) Fig (iii)**
(c) Fig (ii) **(d) Fig (iv)**

[NCERT Exemplar]

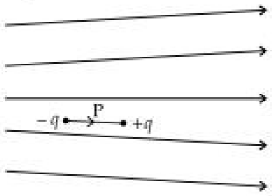
Ans. Correct option : (b)

Explanation : As given charge is $+q$ therefore lines of forces in positive charge must be outwards from positive charge q. Now, as the positive charge is kept near an isolated conducting sphere, due to induction left part of sphere gets accumulated negative charge and right part positive, and lines of force from right part of sphere must emerge outwards normally.

So, verifies the answer **(b)**.

As lines of force are not perpendicular to the surface of sphere, so options (b) and (d) are not true again.

Q. 2. Figure shows electric field lines in which an electric dipole *P* is placed as shown. Which of the following statements is correct?

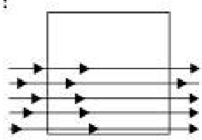


(a) The dipole will not experience any force.
(b) The dipole will experience a force towards right.
(c) The dipole will experience a force towards left.
(d) The dipole will experience a force upwards.
[NCERT Exemplar]

Ans. Correct option : (c)

Explanation : We know electric field emerges radially outward from positive point charge.
In the figure given above, space between field lines is increasing (or density of electric field line is decreasing). In other words, the electric force is decreasing while moving from left to right.
Thus, the force on charge – q is greater than the force on charge + q in turn dipole will experience a force towards left direction.

Q. 3. A square surface of side L metres is in the plane of the paper. A uniform electric E(volt m^{-1}), also in the plane of the paper, is limited only to the lower half of the square surface as shown in the figure. The electric flux (in SI units) associated with the surface is : U



(a) EL^2 **(b) EL^2 / 2ε_0**
(c) EL^2 / 2 **(d) Zero**

Ans. Correct option : (d)

Explanation : As the electric field lines are parallel to the surface of the square, so there will not be any field lines crossing the surface. Hence, the electric flux through the surface will be zero.

Commonly Made Error

- Most of the candidates are unable to Gauss's law correctly. They cannot use the expression to know the value of angle for the formula.

Answering Tip

- The given figure should be carefully observed to know that the electric field lines are parallel to the surface and no lines will cross the surface.

Q. 4. A charge Q is enclosed by a Gaussian spherical surface of radius R. If the radius is doubled, then the outward electric flux will : A
(a) increase four times **(b) be reduced to half**
(c) remain the same **(d) be doubled.**

Ans. Correct option : (c)

Explanation : The electric flux passing through a surface depends upon the charge enclosed inside a surface. Since the enclosed charge remains same and the radius of the Gaussian surface is doubled, there will not be any change in the electric flux and it will remain same.

Q. 5. Gauss's law is valid for : U
(a) any closed surface
(b) only regular closed surfaces
(c) any open surface
(d) only irregular open surfaces.

Ans. Correct option : (A)

Explanation : The Gauss's law can only be used if we have a Gaussian surface enclosing the charge and the Gaussian surface is always closed.

(II) FILL IN THE BLANKS

Q. 1. The electric field due to a line charge is .... proportional to the distance from the point of observation R

Ans. inversely

Q. 2. The electric flux through a Gaussian surface is .... of the shape of the surface. R

Ans. independent

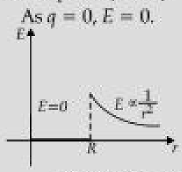
Q. 3. When a charge Q is placed at the corner of the cube, the factor by which the flux of the cube changes as compared to it being placed at the centre of the cube is......... A

Ans. 1/8

(III) VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. Draw a plot showing variation of electric field with distance from centre of a solid conducting sphere of radius *R* having a charge of +*Q* on its surface. U **[Delhi (Comptt.) I, II, III 2017]**

Ans. Field at an inside point (r < R)
As q = 0, E = 0.



2

[CBSE Marking Scheme, 2017]

Q. 2. State Gauss's law in electrostatics.
R **[Delhi I, II, III 2016]**

Ans. Gauss's law in electrostatics : "The surface integral of electrostatic field $\vec{E}$ produced by any source over any closed surface S enclosing a volume V in vacuum, *i.e.*, total electric flux over the closed surface S in vacuum, is $\frac{1}{\varepsilon_0}$ times the total charge (Q) contained inside S, *i.e.*,

$$\phi_E = \oint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$ 1

[CBSE Marking Scheme, 2016]

Commonly Made Error

- Most of the candidates are unable in stating Gauss's law correctly. Key words like net charges, closed surface etc. are missed by students. Some candidates write magnetic flux instead of electric flux.

Answering Tip

- Student should keep the precaution about the keywords while doing the practice of Gauss law.

AI Q. 3. What is the electric flux through a cube of side 1 cm which encloses an electric dipole ? U

Ans. Zero (as net charge enclosed by the surface is zero.) 1[CBSE Marking Scheme, 2015]

Detailed Answer :

In a cubic surface, the net electric charge will be zero since dipole carries equal and opposite charges. It is observed that the net electric flux through closed cubic surface will be

$$= \frac{\text{Charge enclosed}}{\varepsilon_0}$$

and because the charge enclosed is zero,

$\therefore$ electric flux is also zero. 1

Q. 4. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased ? U [Delhi I, II, III 2016]

Ans. Electric flux remains unaffected. 1 **[CBSE Marking Scheme, 2016]**

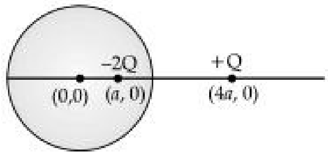
Detailed Answer :

There will be no effect on electric flux, when the radius of Gaussian surface is increased because the charge enclosed by the gaussian surface remains the same.

The flux through Gaussian surface is given by $\phi = \frac{q_{enc}}{\varepsilon_0}$. As the flux is independent of radius, so there will be no effect on electric flux when radius of Gaussian surface increases.

AI Q. 5. Two charges of magnitudes –2Q and +Q are located at points (*a*, 0) and (4*a*, 0) respectively. What is the electric flux due to these charges through a sphere of radius '3*a*' with its centre at the origin ? U

Ans. Gauss's theorem states that the electric flux through a closed surface enclosing a charge is equal to $\left(\frac{1}{\varepsilon_0}\right)$ times the magnitude of the charge enclosed.



The sphere encloses a charge of – 2Q, so

$$\phi = \frac{2Q}{\varepsilon_0} \qquad 1$$

Q. 6. A charge Q is kept at the centre of the cube. What is the electric flux through the two opposite faces of the cube? A

Ans. Electric flux $(\Phi) = \frac{1}{\varepsilon_0} \times \frac{2}{6} = \frac{1}{3\varepsilon_0}$

Q. 7. Is electric flux a scalar or a vector quantity? Give reason. U

Ans The electric flux is the number of electric field lines crossing an imaginary area. So, it is a scalar quantity.

Short Answer Type Questions (2 marks each)

AI Q. 1. Given a uniform electric field $\vec{E} = 5 \times 10^3\ \hat{i}$ N/C. Find the flux of this field through a square of side 10 cm on a side whose plane is parallel to the *y-z* plane. What would be the flux through the same square if the plane makes a 30° angle with the *x*-axis ? A

Ans. Flux of this field through a square of 10 cm 1

Flux of the square with normal making 30° angle 1

$\phi = EA\cos\theta$ ½

$\phi = 5 \times 10^3 \times 10^{-2} \cos 0^\circ\ \text{NC}^{-1}\,\text{m}^2$

$\phi = 50\ \text{NC}^{-1}\,\text{m}^2$ ½

$\phi = 5 \times 10^3 \times 10^{-2} \cos 60^\circ\ \text{NC}^{-1}\,\text{m}^2$ ½

$= 25\ \text{NC}^{-1}\,\text{m}^2$ ½

Detailed Answer :

Given :

$E = 5 \times 10^3 \hat{i}$ N/C along (+) positive direction of *x*-axis.

Surface area, $A = 10\text{ cm} \times 10\text{ cm}$

$= 0.10\text{ m} \times 0.10\text{ m}$

$= 10^{-2}\text{ m}^2$

(i) In case of plane parallel to *y-z* plane, normal to plane will be along *x*-axis, so $\phi = 0^\circ$

Electric flux will be calculated using $\phi = |\vec{E}|A\cos\theta$ ½

$= 5 \times 10^3 \times 10^{-2} \times \cos 0^\circ$

$= 50\ \text{Nm}^2/\text{C}$ ½

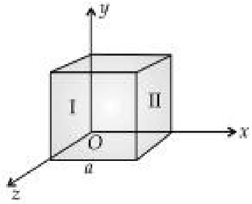
(ii) Since plane is making an angle of 30° with *x*-axis, so normal to its plane will make 60° with *x*-axis, so $\theta = 60^\circ$

Now finding Electric flux again with $\phi = |\vec{E}|A\cos\theta$

$= 5 \times 10^3 \times 10^{-2} \times \cos 60^\circ$ ½

$= 25\ \text{Nm}^2/\text{C}$ ½

AI Q. 2. Given the electric field in the region $\vec{E} = 2x\hat{i}$, find the net electric flux through the cube and the charge enclosed by it. R&U **[Delhi I, II, III 2015]**



Ans. (ii) From the given diagram,

Only the face perpendicular to the direction of x-axis, contribute to the electric flux. The remaining faces of the cube gives zero contribution. **1**

Total flux $\phi = \phi_{\mathrm{I}} + \phi_{\mathrm{II}}$

$= \oint_{\mathrm{I}} \vec{E}.\vec{ds} + \oint_{\mathrm{II}} \vec{E}.\vec{ds}$ ½

$= 0 + 2(a).\, a^2$ ½

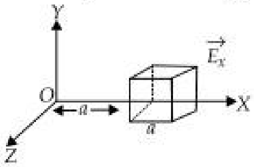
$= 2a^3$ ½

Charge enclosed, $q = \varepsilon_0\phi = 2a^3\varepsilon_0$

Long Answer Type Questions-I

(3 marks each)

Q. 1. Define electric flux and write its SI unit. The electric field components in the figure shown are :



$E_x = \alpha x$, $E_y = 0$, $E_z = 0$ where $\alpha = \frac{100\,\mathrm{N}}{\mathrm{C\,m}}$. Calculate the charge within the cube, assuming $a = 0.1$ m. A & E **[OD Comptt. I, II, III 2018]**

Ans. Definition of Electric flux **1**

SI unit ½

Formula (Gauss's Law) ½

Calculation of Charge within the cube **1**

Electric Flux is the dot product of electric field and area vector. **1**

Also accept

$$\phi = \oint \vec{E}\cdot\vec{ds}$$

SI Unit : Nm^2/C or volt-meter ½

For a given case

$$\phi = \phi_1 + \phi_2 = [E_x(at\ x = 2a) - E_x(at\ x = a)]a^2$$

$$= [\alpha(2a) - \alpha(a)]a^2$$

$$= \alpha a^3$$

$$= 100 \times (0{\cdot}1)^3 = 0{\cdot}1\ \mathrm{Nm^2/C}$$ ½

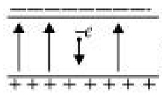
But $\phi = \dfrac{q}{\varepsilon_0}$

$\therefore$ $q = \varepsilon_0\phi = 8{\cdot}854 \times 10^{-12} \times 10^{-1}$ C

$= 0.8854$ pC

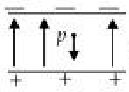
[CBSE Marking Scheme, 2018]

AI Q. 2. An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 N/C



(a)

Calculate the time it takes to fall through this distance starting from rest.



(b)

If the direction of the field is reversed (Fig. b) keeping its magnitude unchanged, calculate the time taken by a proton to fall through this distance starting from rest. A & E **[OD Comptt. I, II, III 2018]**

Ans. Relevant formulae **1**

Calculation of time taken by the electron **1**

Calculation of time taken by the proton **1**

We have

Force $= qE$

Acceleration

$$a = \frac{qE}{m}$$

Also

$$s = \frac{1}{2}at^2 \quad \text{as } u = 0$$

$\therefore$ $t = \sqrt{\dfrac{2s}{a}}$ ½

(i) For the electron

$a = \dfrac{eE}{m}$ ½

$\therefore$ $t = \sqrt{\dfrac{3\times10^{-2}\times9.1\times10^{-31}}{1.6\times10^{-19}\times2.0\times10^{4}}}$

$= 2.92$ ns ½

(ii) For proton

$t = \sqrt{\dfrac{2\times1.5\times10^{-2}\times1.67\times10^{-27}}{1.6\times10^{-19}\times2\times10^{4}}}$

$= 0.125\ \mu$s ½

[CBSE Marking Scheme, 2018]

Q. 3. State Gauss's law in electrostatics. Derive an expression for the electric field due to an infinitely long straight uniformly charged wire.

A & E **[Delhi Comptt. I, II, III 2017]**

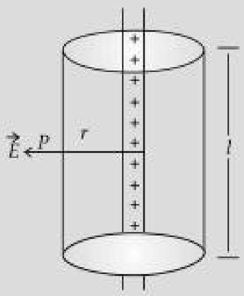
Ans. Statement of Gauss Law 1

Derivation of electric field due to infinitely long straight uniformly charged wire 2

The surface integral of electric field over a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the charge enclosed by the surface.

Alternatively,

$$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$ 1



½

Flux through the Gaussian surface

= flux through the curved cylindrical part of the surface

= E × 2πrl ½

Charge enclosed by the surface = $\frac{\lambda}{2\pi\varepsilon_0 r}$

$$= E \times (2\pi rl)$$

$$= \frac{\lambda l}{\varepsilon_0}$$ ½

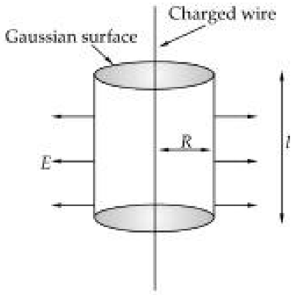
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ ½

[CBSE Marking Scheme, 2017]

Detailed Answer :

Gauss's Law states that total electric flux over the closed surface S will be $1/\varepsilon_0$ times the total charge which is $\phi_E = \oint E ds = \frac{q}{\varepsilon_0}$ 1

In a long straight wire with uniform charge per unit length λ, there should be electric field generated by charge distribution for cylindrical symmetry. Also, field to point will radially be away from the wire.



½

In this, cylindrical gaussian surface is co-axial with the wire of radius R and length l where symmetry implies to electric field generated by wire that will be perpendicular to curved surface of cylinder, so as per Gauss's law,

$$E(R) \times 2\pi R l = \frac{\lambda l}{\varepsilon_0}$$ ½

where, $E(R)$ is electric field strength which acts at perpendicular distance R from the wire.

In figure, left part shows electric flux through Gaussian surface while right part shows total charge enclosed by cylinder which is divided by ε_0. ½

Further, $$E(R) = \frac{\lambda}{2\pi\varepsilon_0 R}$$ ½

Here, the field points radially away from the wire when $\lambda > 0$, and radially towards the wire when $\lambda < 0$.

Commonly Made Error

- Some candidates do not know the correct expression. Few candidates are not sure about $\frac{1}{R}$ or $\frac{1}{R^2}$ (dependence of E on R).

Answering Tips

- Students should practice for the application of Gaussian Surface then find the required electric field.

AI **Q. 4. A charge Q is distributed uniformly over a metallic sphere of radius R. Obtain the expression for the electric field (E) and electric potential (V) at a point $0 < x < R$.**

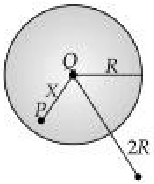
Show or plot the variation of E and V with x for $0 < x < 2R$. A & E

Ans. Expression for electric field 1½

Expression for potential ½

Plot of graph (E vs r) ½

Plot of graph (V vs r) ½



½

By Gauss's law

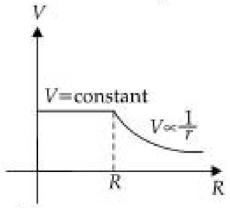
$$\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0} \qquad ½$$

$$q = 0 \text{ in interval } 0<x<R$$

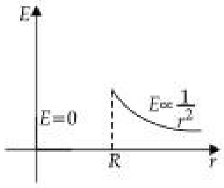
$$E = 0 \qquad ½$$

or $$E = -\frac{dV}{dr} \qquad ½$$

Hence, V = constant $= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$



½



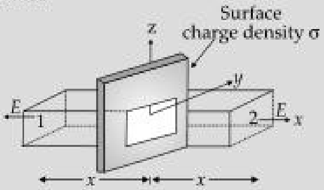
½

[Even if a student draws E and V for $0<r<R$ award ½ + ½ mark]

Q. 5. Using Gauss's law in electrostatics, deduce an expression for electric field intensity due to a uniformly charged infinite plane sheet. If another identical sheet is placed parallel to it, show that there is no electric field in the region between the two sheets. A & E **[Foreign III 2017]**

Ans. Derivation of expression for electric field **2**

Proving that there is no electric field between plates **1**



½

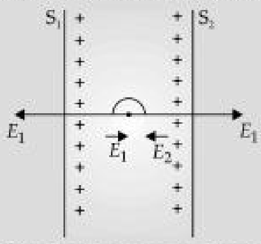
By Gauss's law $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$ ½

$$\therefore \quad 2EA = \frac{\sigma A}{\varepsilon_0} \qquad ½$$

$$\therefore \quad E = \frac{\sigma}{2\varepsilon_0}$$

or $$\vec{E} = \frac{\sigma}{2\varepsilon_0}A \qquad ½$$

Electric field between two identical charged sheets



$\because$ Both the sheets have same charge density, their electric fields will be equal and opposite in the region between the two sheets. ½

Hence the net field is zero. ½

[CBSE Marking Scheme, 2017]

Detailed Answer :

In the figure, if σ is uniform surface charge density of an infinite plane sheet with x-axis to be normal to the plane, then by symmetry, electric field will not depend on y and z coordinates and their directions. Considering Gaussian rectangular parallelpiped surface with cross-sectional area A, two faces 1 and 2 will contribute to flux where electric field lines result as parallel to other faces which do not contribute to total flux.

It is analysed that unit vector which is normal to first surface will be in negative x direction while unit vector which is normal to second surface will be in positive $+x$ direction, so flux $E.\Delta S$ by the surfaces will be equal and add up.

Hence,

net flux through Gaussian surface = $2EA$

charge enclosed by closed surface = σA

As per Gauss's law,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0} \qquad ½$$

$$\therefore \quad 2EA = \frac{\sigma A}{\varepsilon_0} \qquad ½$$

Or, $$E = \frac{\sigma}{2\varepsilon_0} \qquad ½$$

This shows that the electric field around an infinite plane of charge does not vary with distance from the plane.

In terms of vector,

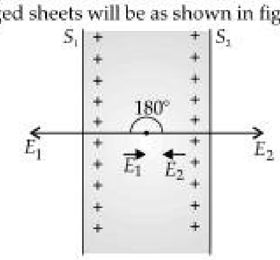
$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

where,

$\hat{n}$ = unit vector normal to plane and going away from it

$\vec{E}$ = directed away from plate if σ is (+) positive and toward the plate if σ is (–) negative.

Since the two charged infinite plates have identical charges, so electric field between two identical charged sheets will be as shown in figure :



Now electric field due to surface 1

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}$$

Now electric field due to surface 2

$$\vec{E}_2 = -\frac{\sigma}{2\varepsilon_0} \qquad ½$$

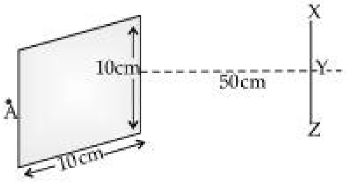
Now the resultant electric field between the uniformly charged infinite plates = $\vec{E}_1 + \vec{E}_2$, θ = 180°

So, $$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$
$$= E_1 - E_2$$
$$= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

Q. 6. Given a uniformly charged plane/sheet of surface charge density σ = 2 × 10^{17} C/cm^2.

(i) Find the electric field intensity at a point *A*, 5 mm away from the sheet on the left side.

(ii) Given a straight line with three points *X*, *Y* and *Z* placed 50 cm away from the charged sheet on the right side. At which of these points, the field due to the sheet remains the same as that of point *A* and why ? A & E **[CBSE SQP 2016]**



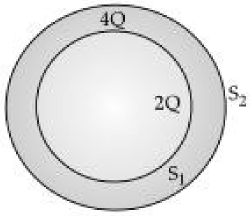
Ans. (i) At point *A*, $E = \frac{\sigma}{2\varepsilon_0} = \frac{2\times10^{17}}{2\times8.85\times10^{-12}}$

$= 1.12 \times 10^{28}$ N/C 1½

(ii) At point *Y*, Because at 50 cm, the charge sheet acts as a finite sheet and thus the magnitude remains same towards the middle region of the plane sheet.

[CBSE Marking Scheme, 2016] 1½

AI **Q. 7. Consider two hollow concentric spheres, S_1 and S_2, enclosing charges 2*Q* and 4*Q* respectively as shown in the figure. (i) Find out the ratio of the electric flux through them. (ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air ? Deduce the necessary expression.**



Ans. According to the Gauss's law, ½

$$\phi = \oint \vec{E} \cdot d\vec{s} = \frac{q_{enclosed}}{\varepsilon_0}$$

∴ For sphere S_1, flux enclosed = $\phi_1 = \frac{2Q}{\varepsilon_0}$ ½

and for sphere S_2, flux enclosed = $\phi_2 = \frac{2Q+4Q}{\varepsilon_0}$ ½

$$\phi_2 = \frac{6Q}{\varepsilon_0}$$

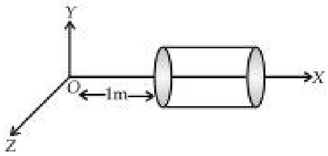
$$\therefore \qquad \frac{\phi_1}{\phi_2} = \frac{1}{3} \qquad ½$$

When a medium of dielectric constant ε_r is introduced in the space inside sphere S_1, the flux through S_1 would be $\phi_1' = \frac{2Q}{\varepsilon_r}$. 1

AI **Q. 8. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional co-ordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50\,x\,\hat{i}$, where *E* is in NC^{-1} and *x* is in metres. Find**

(i) Net flux through the cylinder.

(ii) Charge enclosed by the cylinder.



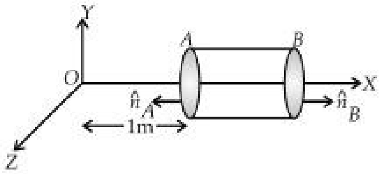
A & E

Ans. (i) Given :

$$\vec{E} = 50x\,\hat{i}$$

and $\Delta S = 25$ cm^2 or 25×10^{-4} m^2

As the electric field is only along the *x*-axis, hence, flux will pass only through the cross-section of cylinder.



½

Magnitude of electric field at cross-section A,

$$E_A = 50 \times 1 = 50 \text{ N/C.}$$

Magnitude of electric field at cross-section B,

$$E_B = 50 \times 2 = 100 \text{ N/C}$$

The corresponding electric fluxes are

$$\oint \phi_A = \vec{E} . \Delta\vec{S}$$
$$= 50 \times 25 \times 10^{-4} \times \cos 180^\circ$$
$$= -0.125 \text{ Nm}^2/\text{C}^2 \quad ½$$

$$\oint \phi_B = \vec{E} . \Delta\vec{S}$$
$$= 100 \times 25 \times 10^{-4} \times \cos 0^\circ$$
$$= 0.25 \text{ Nm}^2/\text{C}^2 \quad ½$$

So, the net flux through the cylinder,

$$\oint \phi = \oint \phi_A + \oint \phi_B$$
$$= -0.125 + 0.25$$
$$= 0.125 \text{ Nm}^2/\text{C}^2 \quad ½$$

(ii) Using the Gauss's law,

$$\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$$

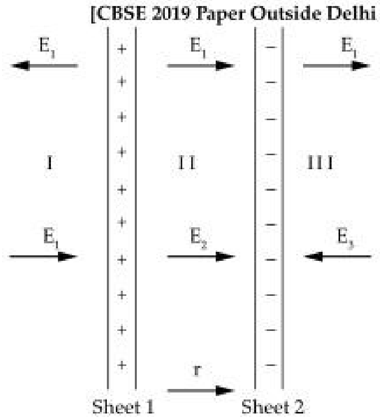
$$\Rightarrow \quad 0.125 = \frac{q}{8.85 \times 10^{-12}}$$

$$\Rightarrow \quad q = 8.85 \times 10^{-12} \times 0.125$$
$$= 1.107 \times 10^{-12} \text{ C.} \quad 1$$

Q. 9. Two large charged plane sheets of charge densities s and –2s C/m^2 are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet, and (iii) between the two sheets.

[CBSE 2019 Paper Outside Delhi Set-I]

Ans.



(a) E.F. left of the plate 1 (region I)

$$\vec{E}_I = \vec{E}_1 + \vec{E}_2 = -\frac{s}{2\varepsilon_0}\hat{r} + \frac{2s}{2\varepsilon_0}\hat{r}$$

(b) II (between the plates)

$$\vec{E}_{II} = \frac{s}{2\varepsilon_0}\hat{r} + \frac{2s}{2\varepsilon_0}\hat{r}$$

(C) Region III (Right of plate 2)

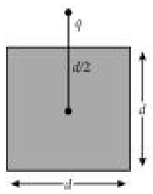
$$\vec{E}_{III} = \frac{s}{2\varepsilon_0}\hat{r} - \frac{2s}{2\varepsilon_0}\hat{r}$$

Long Answer Type Questions - II

(5 marks each)

AI **Q. 1. (i) Define electric flux. Is it a scalar or a vector quantity ?**

A point charge q is at a distance of $d/2$ directly above the centre of a square of side d, as shown in the figure. Use Gauss's law to obtain the expression for the electric flux through the square.



(ii) If the point charge is now moved to a distance 'd' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.

R&A [CBSE Delhi OD 2018]

Ans. (i) Definition of electric flux **1**

Stating scalar/ vector **½**

Gauss's law **½**

Derivation of the expression for electric flux **1**

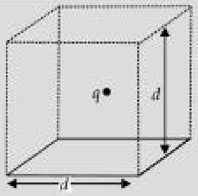
(ii) Explanation of change in electric flux **2**

(i) Electric flux through a given surface is defined as the dot product of electric field and area vector over that surface. **1**

Alternatively $\phi = \oint_s \vec{E}.d\vec{s}$

Electric flux, through a surface equals the surface integral of the electric field over that surface. ½

It is a scalar quantity. ½



Constructing a cube of side 'd' so that charge 'q' gets placed within of this cube (Gaussian surface)

According to Gauss's law the Electric flux

$$\phi = \frac{\text{Charge enclosed}}{\varepsilon_0}$$

$$= \frac{q}{\varepsilon_0} \qquad ½$$

This is the total flux through all the six faces of the cube. Hence electric flux through the square

$$\frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0} \qquad ½$$

(ii) If the charge is moved to a distance d and the side of the square is doubled the cube will be constructed to have a side $2d$ but the total charge enclosed in it will remain the same. Hence the total flux through the cube and therefore the flux through the square will remain the same as before.

[Deduct 1 mark if the student just writes No change/ not affected without giving any explanation.] **1+1**

[CBSE Marking Scheme 2018]

Q. 2. (i) Use Gauss's law to derive the expression for the electric field ($\vec{E}$) due to a straight uniformly charged infinite line of charge density λ C/m.

(ii) Draw a graph to show the variation of E with perpendicular distance r from line of charge.

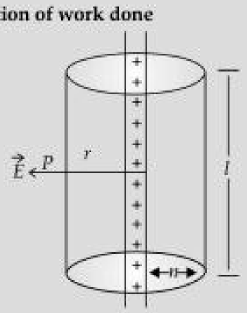
(iii) Find the work done in bringing a charge q from perpendicular distance r_1 to r_2 ($r_2 > r_1$). A

Ans. (i) Derivation of the expression for electric field $\vec{E}$ **3**

(ii) Graph to show the required variation of the electric field **1**

(iii) Calculation of work done **1**

(i)



½

To calculate the electric field, imagine a cylindrical Gaussian surface, since the field is everywhere radial, flux through two ends of the cylindrical Gaussian surface is zero. ½

At cylindrical part of the surface electric field $\vec{E}$ is normal to the surface at every point and its magnitude is constant.

Therefore flux through the Gaussian surface = Flux through the curved cylindrical part of the surface,

$$= E \times 2\pi r l \qquad \text{...(i) ½}$$

Applying Gauss's Law

Flux, $$\phi = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Total charge enclosed = Linear charge density $\times l$

$$= \lambda l$$

$$\phi = \frac{\lambda l}{\varepsilon_0} \qquad \text{....(ii) ½}$$

Using Equations (i) & (ii)

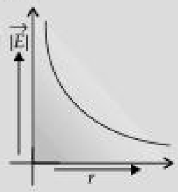
$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad ½$$

In vector notation, $$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n} \qquad ½$$

(where $\hat{n}$ is a unit vector normal to the line charge)

(ii) The required graph is as shown :



1

(iii) Work done in moving the charge 'q' through a small displacement 'dr'

$$dW = \vec{F}.d\vec{r}$$

$$dW = q\vec{E}.d\vec{r}$$

$$= qEdr\cos 0°$$

$$dW = q \times \frac{\lambda}{2\pi\varepsilon_0 r}dr \qquad ½$$

Work done in moving the given charge from r_1 to r_2 ($r_2 > r_1$)

$$W = \int_{r_1}^{r_2} dW = \int_{r_1}^{r_2} \frac{\lambda q dr}{2\pi\varepsilon_0 r}$$

$$W = \frac{\lambda q}{2\pi\varepsilon_0}[\log_e r_2 - \log_e r_1] \qquad ½$$

$$W = \frac{\lambda q}{2\pi\varepsilon_0}\left[\log_e \frac{r_2}{r_1}\right]$$

[CBSE Marking Scheme, 2018]

Q. 3. (i) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ.

(ii) An infinitely large thin plane sheet has a uniform surface charge density +σ. Obtain an expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet. R **[OD II, 2017]**

Ans. (i) Similar to Q. 7, Short Answer Type II **3**

(ii) Amount of work done :

$$W = q\int_{\infty}^{r} \vec{E}.d\vec{r} \qquad \tfrac{1}{2}$$

$$W = q\int_{\infty}^{r} (-E.dr) \qquad \tfrac{1}{2}$$

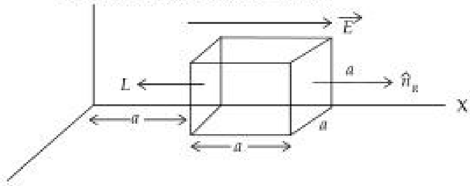
$$W = -q\int_{\infty}^{r}\left(\frac{\sigma}{2\varepsilon_0}\right)dr \qquad \tfrac{1}{2}$$

$$W = \frac{q\sigma}{2\varepsilon_0}|\infty - r| = \infty \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2017]

Q. 4. (a) Using Gauss's law, derive expression for intensity of electric field at any point near the infinitely long straight uniformly charged wire.

(b) The electric field components in the following figure are $E_x = ax$, $E_y = 0$, $E_z = 0$; in which $a = 400$ N/C m. Calculate (i) the electric flux through the cube, and (ii) the charge within the cube assume that $a = 0.1$ m.



[CBSE SQP – 2020]

Ans. (a) Try yourself, Similar to Q. 2(i), Long Answer Type Questions **3**

(b) Since the electric field has only an x component, for faces perpendicular to x direction, the angle between E and ΔS is $\pm \pi/2$. Therefore, the flux $\phi = E.\Delta S$ is separately zero for each such face of the cube. Now the magnitude of the electric field at the left face is :

Net flux through the cube

$$= -(400a)a^2 + a^2(400 \times 2a) \qquad 1$$

$$= -400a^3 + 800a^3$$

$$= 400a^3$$

$$= 400 \times (0.1)^3$$

$$= 0.4 \text{ Nm}^2\text{C}^{-1}$$

(b) We can use Gauss's law to find the total charge q inside the cube.

We have $\phi = q/\varepsilon_0$ or $q = \phi\varepsilon_0$. Therefore, **1**

$$q = 8.85 \times 10^{-12} \times 0.4$$

$$= 3.540 \times 10^{-12} \text{ C}$$

Electrostatic Potential & Capacitance

Electric Potential

- Work done per unit test charge by an external agent in moving the test charge from reference point to the desired point. Its SI unit is J/C
  V_0 = Work. Its SI Unit charge

Electric potential due to a point charge

$V_P = Q/4\pi\varepsilon_0 r$

Potential due to a system of charges

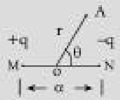
$$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{Q_i}{r_i}$$

Potential due to a dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Where $p = qd$

$\theta = \angle AON$



At $\theta = 0$, $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$ [axial position]

At, $\theta = 90°$, $V = 0$ [equitorial position]

In 1774, Alessandro Volta wrote treatise"on the forces of attraction of electric fire".

Equipotential Surface

- Potential is same at all the points of the surface.
- Component of electric field parallel to an equipotential surface is zero.

Electric Potential Energy

- It is negative count of work done by the electric force as the configuration of the system changes.
- $U_{r_2} - U_{r_1} = -W = \frac{q_1 \cdot q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$
- If the separation between charges is 'r' then $U_{(r)} = \frac{q_1 \cdot q_2}{4\pi\varepsilon_0 r}$
- Potential Difference,
  $V_B - V_A = \frac{U_B - U_A}{q}$
  $U_B - U_A$ = change in Potential Energy
  q = test charge

Potential Energy of a dipole

$dU = pE\sin\theta d\theta$

If we choose P.E. of dipole to be zero when $\theta = 90°$ then

$$U_\theta - U_{90°} = \int_{90°}^{\theta} pE\sin\theta\, d\theta$$

$U_\theta = -pE\cos\theta = -\vec{p}\cdot\vec{E}$

If it is rotated through angle θ against the torque

Conductors & Insulators

Conductors	Insulators
Such a material which when placed in an electric field, the free electrons move in a direction opposite to the field.	Such a material in which electrons are tightly bound, & when exposed in an electric field, electrons do not move *i.e.* having no free electrons.

- Electric field inside a conductor is zero.
- Electric field is always perpendicular to the charged surface.
- In static state, there will be no additional charge in a conductor.

Dielectric

- $F = \frac{q_1 q_2}{4\pi K\varepsilon_0 r^2}$, K = dielectric constant of medium
- A dielectric is an electrical insulator that can be polarized by "application of" electric field.

Capacitance, $C = \frac{Q}{V}$

Capacitance of a parallel plate capacitor
$C = K\varepsilon_0 A/d$, K = dielectric constant

Capacitance when material slab is inserted between the plates
$C = K\varepsilon_0 A/[Kd - x(K-1)]$
where x = thickness of the slab inserted

Capacitance of a spherical capacitor
$$C = 4\pi\varepsilon_0 \Big/ \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
For isolated sphere
$C = 4\pi\varepsilon_0 R$

Parallel grouping of capacitors
$C = C_1 + C_2 + C_3 + \cdots + C_n$
for two, $C = C_1 + C_2$

Series grouping of capacitors
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$
For two, $C = \frac{C_1 C_2}{C_1 + C_2}$

Energy stored in a capacitor
$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = 1/2\, QV$$