

# Cryptographic primitives

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# Hash functions

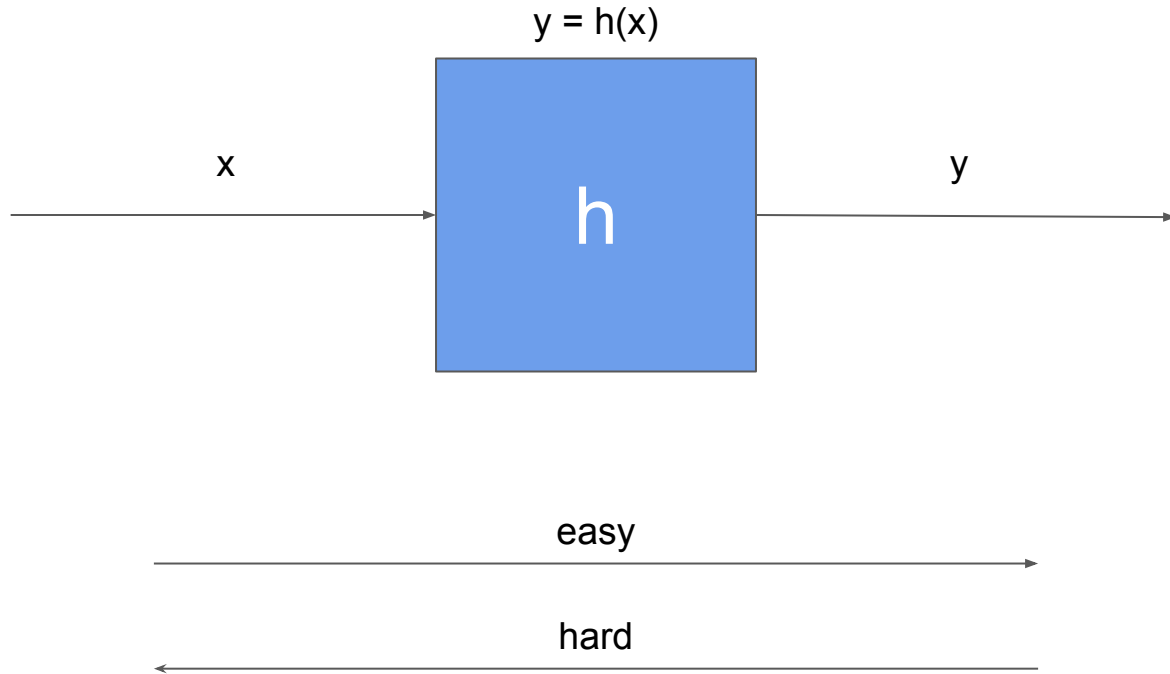
# Hash functions

- Map data of arbitrary size to a string of a fixed length
- $h(x) = y$ , where  $h: \{0, 1\}^* \rightarrow \{0, 1\}^n$
- Various applications: hash tables, proof-of-work, digital fingerprint, file integrity, cache mechanisms, identifiers, time-stamping

# Cryptographic Hash Functions

- $h(x) = y$
- Easy to calculate  $y$ , given  $x$
- Practically infeasible to find  $x$  from  $y$
- Practically infeasible to alter  $x$  without altering  $y$
- Practically infeasible to find different  $x, x'$  that produce the same  $y$

# Cryptographic Hash Functions



# sha2

- Descendant of sha1
- It is considered safe for cryptographic applications
- SHA224:  $\{0,1\}^* \rightarrow \{0,1\}^{224}$
- SHA256:  $\{0,1\}^* \rightarrow \{0,1\}^{256}$
- SHA384:  $\{0,1\}^* \rightarrow \{0,1\}^{384}$
- SHA512:  $\{0,1\}^* \rightarrow \{0,1\}^{512}$

# Example of sha256

sha256('Hello World!')

=

7F83B1657FF1FC53B92DC18148A1D65DFC2D4B1FA3D6

77284ADDD200126D9069

# 'Reversing' a hash

- Hashes are designed to be one-way, irreversible
- So, the reversing can be done only by **brute-force**
  - Try all possible strings of length 0, 1, 2, 3 etc...
  - Try words from a dictionary



# Hash dictionary attack

```
foreach (word in dictionary){  
    if(SHA256(word) == c) {  
        return word;  
    }  
}
```

# Hash dictionary attack

```
 $\Sigma$  = {'a', 'b', ..., 'z'};  
numdigits = 0;  
m = "";  
while (SHA256(m) != c) {  
    try {  
        m = increment(m,  $\Sigma$ , numdigits);  
    }  
    catch (OutOfBoundsException e) {  
        ++numdigits;  
        m = repeat( $\Sigma$ [0], numdigits);  
    }  
}  
return m;
```

# Commitment schemes

# Commitment schemes

- Alice wants to commit to a value “b”
- Alice and Bob do not trust each other
- **Binding:** Bob wants to know that Alice will not change her mind about b
- **Hiding:** Alice does not want to reveal her value beforehand

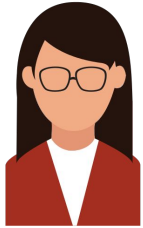
# Commitment schemes: phases

- Commitment phase:
  - Alice chooses and commits to a value
  - Sends a secret to Bob
  - **Hiding:** Bob cannot use the secret to find the value of Alice
- Reveal phase:
  - Bob learns the initial value Alice chose
  - **Binding:** Bob confirms that Alice didn't change her value using the secret

# Commitment schemes: using a hash

- Commitment phase:
  - Alice computes  $\mathbf{c} = \mathbf{H}(\mathbf{b})$
  - Send  $\mathbf{c}$  to Bob
  - Bob cannot reverse  $\mathbf{c}$  to get  $\mathbf{b}$

# Commitment schemes: commitment phase



Computes  $c = h(b)$

$c$



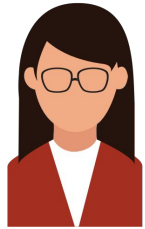
$c$

# Commitment schemes: using a hash

- Reveal phase:
  - Alice sent **b** to Bob
  - Bob checks that  $H(b) = c$



# Reveal phase: reveal phase



$b$

$b$



Checks that  
 $c = h(b)$

# Proof of work

- Cryptographic primitive that describes the proof of executed CPU cycles
- Alice wants to prove that Bob will dedicate his CPU power only for her
- How can she do that ?

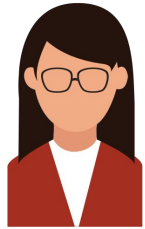
# Proof of work

- Alice chooses a random *salt*  $K$ , big enough (i.e. 128 bits)
- Alice sends to Bob the salt  $K$ , and a target parameter  $T$ .
- The target parameter defines the amount of work to be done

# Proof of work

- Bob calculates  $x$  such that
  - $H(K || x) < T$
- $x$  is the *nonce*
- Bob sends  $x$  to Alice
- Alice validates that  $H(K || x) < T$

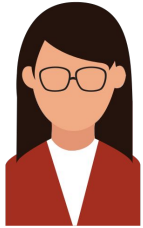
# Proof of work: Request



K, T

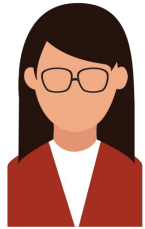


# Proof of work: Work



$H(K \parallel x) < T$  ?

# Proof of work: Proof



$H(K \parallel x) < T ?$

x



# Proof of work: Algorithm

```
do {  
    x = rand();  
} while (H(K || x) >= T);  
return x;
```



# Proof of work

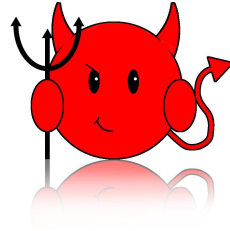
- Very important to blockchain consensus mechanisms
- More details in the following lectures...

# Digital signatures

# Digital signatures



“See you at 9pm”



“See you at 10pm”



Did Alice send the message ?  
Is this the message Alice wrote ?

# Digital Signatures

- **Integrity:** The signed message was not *altered* in transit
- **Authentication:** The signature was created by a known sender
- **Non-repudiation:** The sender cannot deny having signed the message

# Digital Signatures

- The sender produce a key pair ( $vk$ ,  $sk$ )
- $sk$ : signing key
- $vk$ : verification key

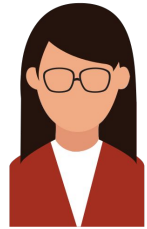
# Digital Signatures

- Verification key is **public**
  - publicly verifiable
  - transferable signatures
- Signing key is private

# Digital Signatures

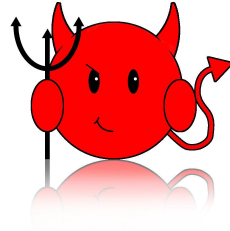
- $\sigma = \text{sign}(\text{sk}, m)$
- $\text{verify}(\text{vk}, \sigma, m) = \{0, 1\}$

# Digital signatures



$\sigma = \text{sign}(\text{sk}, \text{'See you at 9pm'})$

See you at 9pm



$\sigma$

See you at 10pm



$\text{verify}(\text{vk}, \sigma, \text{'See you at 10pm'})$   
= false

Not Alice!!



# Digital Signatures

- Digital signatures are very important to blockchain systems
- A valid signature implies ownership
- More details in the following lectures...

