Structural Macroeconometrics - Final Assignment

Federico Bertoia
Virginia Pagliero

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Introduction

The adjustment of the deficit has been one of the main tool for past and current fiscal policies, which implies that understanding the consequence of such measures become necessary to implement the most appropriate. Since the government is subject to an intertemporal budget constraint, the dynamics of the deficit in the long run is restricted: if taxes are increased today we can expect a future adjustment to counterbalance the debt.
However, several problems arise when looking at empirical results: *first*, response of economic variables have been often divergent from the traditional literature, such as the Keynesian or the Neoclassical theory[1]; *second*, as suggested by Bohn (1998) [5] and then resumed by Favero and Giavazzi (2007) [8], due to the nature of the analysis of fiscal shocks, i.e. Structural VAR, taxes and government spending seem to respond to economic variables but not to the level of debt.
As a consequence of this, the outcome of fiscal shocks may vary across time since the response of consumers and fiscal measures changes according to the intent of the government to adjust the deficit in the short run or not.

We will focus on the analysis of Blanchard & Perotti (2002) [4], that successfully addressed the dynamic effects on the economic activity due to fiscal policy shocks in the United States in the postwar period.
Their results proved that a positive shock of government spending increased both GDP and private consumption but it decreased private investment; on the other hand, a positive shock of tax had negative effect on both GDP and private investment[2].
However, when handling macroeconomics variables, nonstationarity is very common and therefore to solve the related econometric problems, we aim to replicate Blanchard & Perotti's analysis by accounting for possible *cointegration* among the variables: in order to do so, we follow the authors' method and use a Structural VAR properly adjusted. We think this is justified because of the properties of unit roots variables in cointegrated system studied by Engle and Granger (1987) [7], Stock (1987) [14] and from the impressing result regarding convergence of the Impulse Response Function from Phillips (1998) [12].
Since a cointegrated system suggests a long-run relation among the variables, we will mainly focus on two cases coming from different theoretical framework: first, the aforementioned budget constraint advised by Bohn (1998) should ensure long-run stationarity for taxes and spending; second the long-run tax reaction function from Angelini et Al. (2022) [2] which implies a unit elasticity of taxes to output.

Overall, for the no cointegration case our results are aligned with the current literature and the empirical results from Blanchard & Perotti: on the other hand, when we address cointegration, dynamics effects seem to be more persistent and do not stabilise within the horizon considered.

[1] according to Keynesian's view, a restrictive fiscal policy that affects government expenditure should consequently reduce private consumption and GDP through the multiplier which is in contrast with some of the empirical evidence that showed how a reduction of the government spending led to a significant increase in GDP and private consumption

[2] however, these results may be controversial since, following Favero and Giavazzi (2007), we can't see any evidence of stabilising fiscal policies until the 70s

We could explain this by recalling that the main unit roots' property is related to their shock's persistence and therefore we can anticipate that GDP will require more time to get stable again.
Additionally, even if we account for a cointegrated system, i.e. we allow for a leading common trend, that should solve the consistency issues regarding unit roots, we may still have biased Impulse Response Functions due to the omitted feedback from the debt level suggested by Favero and Giavazzi (2007).

The paper is organised as follow: in section 1 we will briefly discuss the problem regarding non stationary variables and how cointegration manage to solve it, then in section 2 we describe the methodology used and the identification solutions, whereas the empirical results are presented in section 3.

1 Problem

The analysis of Blanchard & Perotti focuses on a sample from 1960:Q1 to 1997:Q4 where there is no strong evidence of cointegration and therefore first differenced variables are used to ensure stationary.
This has become common practice in the current literature however, as pointed out by Christ[3] (1975) [6] and then proved by Phillips (1998), non stationary VAR with unit roots yields inconsistent estimated impulse response functions, $\hat{\phi}_h$. In presence of random walks, $\hat{\phi}_h$ converges asymptotically to a random object due to the persistent and cumulative nature of the shock: on the contrary, cointegrated processes specified through the error correction model ensures consistent estimates.
Since non stationarity is quite recurrent when handling macroeconomic variables, the presence of cointegration can be easily addressed and support the analysis of the long run equilibrium, which implies no endogenous tendency to deviate [1].
We define a set of $n \times 1$ non stationary variables w_t to be cointegrated if there exists a full rank $n \times r$ matrix β such that $\beta' w_t \sim I(0)$ and we can identify $n - r$ common trends driving the variables in the system.
As a result of this, we can estimate β through maximum likelihood and then the remaining parameters through OLS[4]: indeed, from the estimated error correction form we can resume the VAR parameters and therefore compute the consistent impulse response function.
However, we know that β is affected by lack of identification, which oblige us to establish a strategy and impose restrictions based on data: despite this, we know that $\alpha\beta'$ gives a unique estimate and therefore, this is true also for $\hat{\phi}_h$.

2 Methodology

Before presenting empirical results we will discuss our approach, in particular the specification of the VAR, the Structural VAR and the cointegration relations.

- Following Blanchard and Perotti (2002), we define a VAR model where $w_t = [T_t, G_t, GDP_t]'$, a linear trend and four lags are included: indeed, this is due to seasonal patterns in the response of taxes[5].

  $$w_t = \mu_t{}^{6} + A_1 w_{t-1} + A_2 w_{t-2} + A_3 w_{t-3} + A_4 w_{t-4} + u_t$$

  A covariance stationary process, i.e. $E(w_t) = const$ and $\Gamma(h)$ independent from time, would imply that the complex solutions of the characteristic polynomial $\Pi(L)$ are outside the unit circle, i.e. $|z_j| > 1$.

[3] in considering the multiplier effects of monetary and fiscal policy shocks he found great uncertainty across models, in particular regarding the asymptotic convergence of the impulse response functions
[4] from Stock(1987) and Phillips(1991) we understand that the least squares estimator of β is *super-consistent* but may be biased in finite sample: however, if we compute the dynamic OLS suggested by Stock and Watson (1993) we get consistent, asymptotically normal and efficient estimator
[5] however, this is not further confirmed by the BIC and the AIC Information Criteria, that suggest to include less lags
[6] deterministic part of the form $\mu_0 + \mu_1 t$ includes both the constant and the linear trend

This assumption is necessary to ensure $C(L) = \Pi(L)^{-1}$ and therefore compute the Vector Moving Average form of the process.

$$w_t = \sum_{h=0}^{\infty} C_h \mu + \sum_{h=0}^{\infty} C_h u_{t-h} \qquad \text{where} \quad \sum_{h=0}^{\infty} C_h < \infty$$

Hence, the impact multiplier C_h can identify the shock of innovations that will help us to create the Impulse Response Functions, i.e. capturing the impact and the dynamics of the structural shocks ε_t.
However, when there are some unit roots in the VAR system, the limit theory of the estimated coefficients changes and has some non normal components [13]: recall the VMA representation of a VAR(1).

$$C_h = (\Pi_1)^h \ , \ h = 0, 1, ...$$

$$C_h \to 0 \text{ as } h \to \infty$$

but instead now:

$$C_h = (\Pi_1)^h = \Pi_1 \ \text{ for } h \geq 1$$

This means that, in a SVAR framework, the impact of ε_t may never vanish: nevertheless, consistent coefficients estimates can be achieved by taking into considerations cointegration relationships.
Finally, we stress that a reduced form model, i.e. VAR, is always identified unlike the Structural VAR.

- As already mentioned, the simple VAR model is not appropriate to identify the structural shocks: since the innovations $u_t = [u_{T,t}, u_{G,t}, u_{GDP,t}]'$ are correlated, i.e. Σ_u is not diagonal, it would be impossible to recognise the impact of the shock from a variable to the other.
  In order to solve this problem, we can map the innovations into structural shocks ε_t that by construction have Σ_ε = diagonal and $E(\varepsilon_t) = 0$ and therefore allow to capture the impact of the unique structural shock on w_t.
  As the authors did, we will follow the AB model approach, which implies that the mapping procedure is implemented through two matrices.

$$Au_t = B\varepsilon_t \Rightarrow u_t = C\varepsilon_t \qquad \text{where} \quad C = A^{-1}B$$

  Where C captures the instantaneous impact of the structural shocks on the variables: however, this model implies several assumptions since we need identification on both A and B.
  The necessary order condition requires that we include at least 12 restrictions on the parameters of the two matrices, whereas the necessary and sufficient rank condition for local identification comes from the full rank of the Information matrix and it will be proved empirically.
  The identification procedure relies on results from Blanchard and Perotti (2002) that retrieve instantaneous parameters through institutional information: for istance, they manage to estimate the elasticity of taxes to GDP, i.e. a_{13} which equals on average 2.08, whereas there is no evidence of an immediate reaction of GDP from spending, hence $a_{23} = 0$[7].

$$A = \begin{bmatrix} 1 & 0 & -2.08 \\ 0 & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{22} & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}$$

  Afterwards, given the covariance matrix $E(uu') = \Sigma_u = A^{-1}BB'(A')^{-1}$ we can compute the concentrated likelihood that estimates the free parameters α and β of the matrices.

$$lnL_T(\Sigma_u) = const + \frac{T}{2}\log(det\Sigma_u^{-1}) - \frac{T}{2}tr[\Sigma_u^{-1}\hat{\Sigma}_u]$$

[7]intuitively the elasticity of taxes on spending and the other way around can be considered null

Where $\hat{\Sigma}_u$ is the covariance matrix which is known from the data whereas $\Sigma_u = A^{-1}(\alpha)B(\beta)B(\beta)'(A(\alpha)')^{-1}$. Once A and B are estimated and the conditions for local identification are satisfied, we can compute the Impulse Response Functions h steps ahead.

$$IR\hat{F}(h) = \hat{\phi}_h = \hat{C}_h\hat{C} \qquad \text{where} \quad \hat{C} = \hat{A}^{-1}\hat{B}$$

In particular, we will focus on the estimate of a shock j to the variable i after h periods which is obtained through the *selection matrix* and a stable *companion matrix*[8] and is necessary to capture the dynamics of a fiscal shock on the relevant variable, i.e. GDP.

$$\hat{\phi}_{i,j,h} = e_i'\hat{C}_{h,T}\hat{C}_T e_j \qquad \text{where} \quad \hat{C}_{h,T} = R(C\hat{o}m)^h R' \quad \& \quad \hat{C}_T = \hat{A}_T^{-1}\hat{B}_T$$

- The cointegration relation among the variables can be addressed through the VECM form, which is defined with restricted constant and linear trend.

  $$\Delta w_t = \mu_t + \Pi w_{t-1} + \Gamma_1\Delta w_{t-1} + \Gamma_2\Delta w_{t-2} + \Gamma_3\Delta w_{t-3} + u_t$$

  Where μ_t gathers the deterministic part, $\Gamma_i = -\sum_{i=1}^{p} A_{i+1}$ for $i = 1, 2, 3$ and $\Pi = \alpha\beta' = A_1 + A_2 + A_3 + A_4 - I_3$ and both α and β are $n \times r$.
  However, since we can't uniquely identify the two matrices, restrictions on β are necessary to correctly estimate the relations: indeed, this can be drawn empirically and we will select the constraints following our data and the already mentioned economic theories.
  Overall, from the common trend representation we can decompose our cointegrated system and write:

  $$w_t = Bf_t + I(0) + w_o$$

  where $f_t = \sum_{i=1}^{T} \varepsilon_{ij}$ is the common trend leading the relation, $I(0)$ are the innovations assumed as white noise[9] and w_0 are the initial conditions.
  Finally, it is important to stress that under cointegration assumption and $\Pi = \alpha\beta'$, the estimates are consistent and the impulse response function, computed from the VAR parameters, converges.

To summarise, in the next section we will present our empirical results that analyse the dynamic effects of a fiscal shock in the sample 1950:Q1 - 1979:Q3 for the cointegrated series under three specifications: (i) no constraints on β; (ii) β constrained according to Angelini et Al. (2022); (iii) β constrained according to Blanchard & Perotti (2002).

3 Empirical results

The relation among our variables has been studied several times in the literature and the main peculiarities are well-known, for instance the ratio of tax and spending with GDP (Figure 2b, 3b and 3a) suggests a constant and long lasting connection that we aim to analyse further.
Differently from Blanchard & Perotti (2002), we focus on the sample from 1950:Q1 to 1979:Q3: indeed, only in this period we can find strong evidence of common trends and therefore analyse the cointegration relation.
In order to do this, we use non detrended log data of the nominal variables divided by the GDP deflator and the population: this is necessary to highlight the evident linear trend (Figure 1) and treat the unit roots features.

[8] a stable matrix: (i) $com^j \to \mathbf{0}_{M\times M}$ as $j \to \infty$; (ii) $\sum_{j=0}^{\infty} com^j = (I_M - com)^{-1}$; (iii) $(I_M - comL)^{-1} = \sum_{h=j}^{\infty} com^j L^j$

[9] given ε_t a white noise process, it satisfies: (i) $E(\varepsilon_t) = 0$; (ii) $E(\varepsilon_t^2) < \infty$; (iii) $E(\varepsilon_t\varepsilon_{t-k}) = 0$

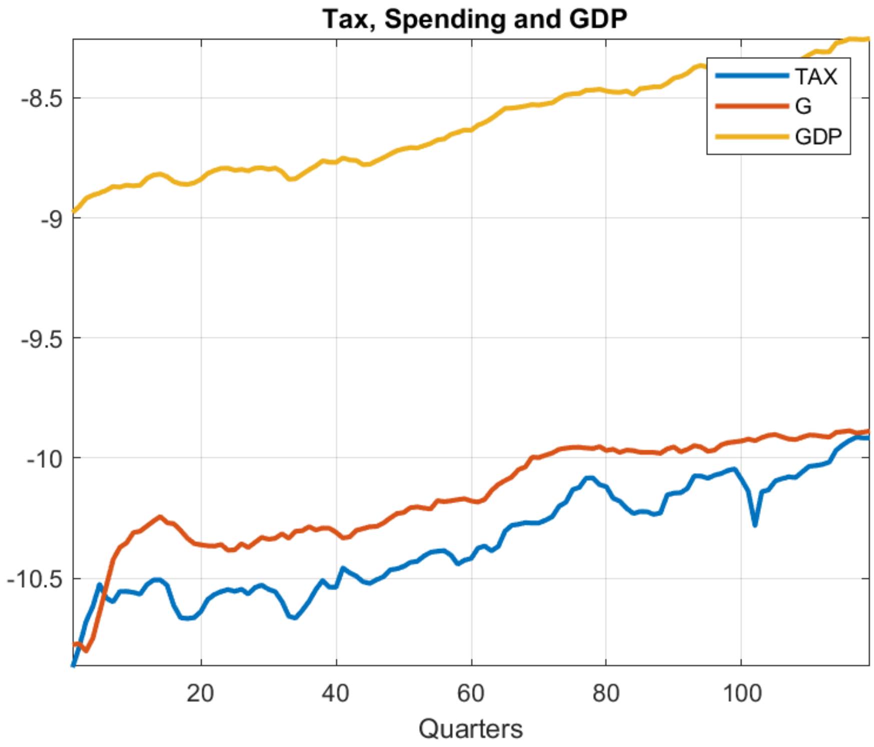

Figure 1: Plot of the series of Tax, Spending and GDP for the sample 1950:Q1 - 1979:Q3

The log ratio of taxes and spending on GDP (Figure 3) are very explanatory regarding the sharp increase of both taxes and government spending in the initial quarters, i.e. 1950:Q2 and 1951:Q1 and the cut of taxes in 1975:Q2. These episodes are extraordinary events that are unrelated to improvements of GDP, in particular the changes in the early fifties' are very persistent and seem uncorrelated to the remaining process, such that Blanchard & Perotti decided to remove this period from their main analysis. However, due to our restricted number of observations, we decided to keep the whole sample period.

We start by computing the Augmented Dickey-Fuller test to understand the nature of the generic variable.

$$y_t = \mu_0 + \mu_1 t + \phi y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + \delta_3 \Delta y_{t-3} + u_t$$

We strongly reject the null $H_0 : \phi = 1$ only for government spending (Table 1), which implies the existence of random walks: however, the test's result is borderline for tax[10]. Hence, we pursue with the study of cointegration, in order to understand how many common stochastic trends lead our variables.
Following Johansen's methodology (1995) [10] the cointegration test is computed under the assumption of restricted trend[11] and the model $H^*(r) = \mu_t = \mu_0 + \alpha\mu_1 t$:

$$\Delta w_t = \mu_0 + \alpha(\beta' w_{t-1} + \mu_1 t) + \Gamma_1 \Delta w_{t-1} + \Gamma_2 \Delta w_{t-2} + \Gamma_3 \Delta w_{t-3} + u_t$$

The Johansen test (Table 2) suggests two alternatives since the null H_0 is not rejected for $r = 1$ and $r = 2$[12]: following the economic framework and our graphical evidence, the variables may be driven by either one or two common stochastic trends, hence we will continue our analysis for both $r = 1$ (one cointegration relation and two unit roots) and $r = 2$ (two cointegration relations and just one unit root).

[10] in the specification considered it does not reject the null but the results are more significant when we implement the test for the simple AR model with drift

[11] this diverges from the assumption of Blanchard & Perotti (2002) but, according to our data, we would avoid the quadratic trends in the levels (Figure 2a) of w_t

[12] given r linear combinations of w_t, this is true for both the Trace test and for the eigenvalue test

Overall, we get $w_t \sim I(1)$ with a drift μ_0 and the cointegrating relations $\beta' w_t \sim I(0)$ with the linear trend term $\mu_1 t$.

Since the cointegrating vector β is not uniquely identified, we need some normalization assumptions which can be achieved from our data or from the economic theory: however, if we add more restrictions to the standard ones (I_r matrix below), we get overidentification and therefore we can test our outcomes.

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} \qquad \text{and} \quad w_t = \begin{bmatrix} TAX_t \\ G_t \\ GDP_t \\ t \end{bmatrix}$$

Following our previous results (Figure 3b) and Angelini et Al. (2022) we can assume a stable and constant long-run relation between *GDP* and *TAX* which implies $b_{31} = -1$ and $b_{41} = 0$ and the cointegrating relations reflect a stationary system (Figure 4a) where the long run elasticity of GDP is 1 with taxes whereas is 1.18 with government spending. In addition, the estimated adjustment vector α suggests short-run movements, for instance an increase in GDP is resulting from a decrease in taxes or an increase in spending.

$$1. \quad TAX_t - GDP_t \sim I(0)$$
$$2. \quad G_t - 1.18 \cdot GDP_t + 0.002 \cdot t \sim I(0)$$

Given the several sharp changes along our sample, which are particularly evident in the 50s (Figure 3b), the *Likelihood-Ratio test* suggests us that the restrictions are consistent with our data[13].
Similarly, if we change the set of restrictions such that $b_{41} = 0$ and $b_{32} = -1$ we would still accept the model as stationary[14] but we don't consider this case since it is much less efficient in capturing the dynamics in our data.

Despite these promising results, we can follow the approach suggested by Blanchard & Perotti (2002) which is still coherent with the previous outcomes: since the *Johansen test* do not reject the null for one cointegrating relation, i.e. $r = 1$ (Table 2) and from the *Augmented Dickey-Fueller test* we could allow for two unit roots (Table 1), i.e. taxes and GDP.

$$\beta = \begin{bmatrix} 1 \\ -1 \\ 0 \\ b_{41} \end{bmatrix} \qquad \text{and} \quad w_t = \begin{bmatrix} TAX_t \\ G_t \\ GDP_t \\ t \end{bmatrix}$$

The estimate of the restricted system is still consistent with our data[15] and provides a significant relation between taxes and government spending $TAX_t - G_t - 0.0014 \cdot t \sim I(0)$.
The intuition is analysed more in detail from Hamilton & Flavin (1986) [9] and Bohn (1991) and illustrate the stationarity of the deficit: since the government is subject to an intertemporal budget constraint, changes in the deficit are followed by modifications of either taxes or spending in order to maintain their connection constant. However, since large deficit is frequent in the short term, Bohn (1991) additionally stresses that a long horizon is necessary to study this correlation since it will be more apparent how the government is likely to adjust the deficit to the constraint. As a consequence of this, given the very short period of our sample and the numerous disturbances of taxes and spending, we are not surprised that our relation looks very volatile (Figure 4b).

[13] given a p-value of 0.105472 we do not reject the null at 10% significance level
[14] given a p-value of 0.737962 we do not reject the null of the Likelihood-Ratio test
[15] the Likelihood-Ratio test do not reject the null given a p-value of 0.0867032

Shock Effects

The last step of our project is the analysis of the impulse responses of both tax and government spending on GDP, with the relatives dynamic multipliers.
A correct and significant estimate of the dynamic causal effect that is present among these variables is crucial for policymakers to elaborate and propose efficient fiscal policies.
During the entire analysis, our baseline assumption on IRFs is that they represent the response to a one standard deviation shock: hence, this estimate is showed in all the graphs by the solid line, while dashed lines gives the 90% nonparametric residual-based iid bootstrap confidence bands based on 999 iterations.

Blanchard & Perotti (2002) analysis of the dynamic effects of a fiscal shock can be summarise as follow:

- taxes' shocks reduce the output that stabilises by slowly increasing after it had reached the trough at around six quarters;
- spending shock increases the output that afterward steadily returns to the stationary level

Overall, our results are comparable with Blanchard & Perotti's (2002) regarding the sign but the dynamics differ when we account for cointegration and when we don't: note that our sample includes the 50s that is an eventful period with many abrupt fiscal changes.

When we don't account for cointegration (Figure 5) the GDP's response to a tax shock is immediate and reaches the trough in five quarters and then is restored in less than twenty quarters, after exceeding the initial value. However, as we can see from the confidence bands, only for a small period around the fifth quarter the effect is significantly different from zero (even though the confidence interval is close to it) suggesting that in this model specification GDP may not be affected by shocks of taxes. If we move to analyse the tax multiplier (Figure 9a), we get the same results: the estimates are negative but the surrounding uncertainty makes them not reliable and not statistically different from zero.
On the other hand, the positive effect of a spending shock is stronger on impact but less persistent and requires under ten quarters to stabilises (Figure 5b). Nevertheless, the effect is not significant after a year, suggesting that the only way to significantly affect GDP in the short run may be by taking decisions about spending and not taxes. When looking at the spending multiplier (Figure 9b we notice that it follows almost the same path as its IRF, with the greatest influence significant within two years, hence confirming the theory aforementioned (Table 3).
It is worth noticing that in the no-cointegration framework all the impulse responses tend to zero, which implies that the effects are not persistent and vanishes after an adequate number of quarters: this is a consequence of the methodology approach, which takes the assumption of stationarity and no unit roots in the data.

When we account for cointegration our findings are very comparable across the three specifications, i.e. no restrictions imposed, Angelini et Al. (2022) and Blanchard & Perotti (2002): overall, the shocks look very persistent since the GDP doesn't manage to come back to the stabilised level.
Starting with two relations (Figures 6 and 7), a tax shock brings a slight, but significant, reduction of GDP that gets stable right after five quarters. Similarly, a spending shock rise the GDP on impact, but after the peak is reached the effect becomes non significant when we impose restrictions from Angelini et Al. (2022).
The multipliers for tax are always significant and lower than the ones derived without cointegration, whereas government spending seems to have a stronger impact which is not significant for the second specification (Tables 4 and 5): overall, a fiscal policy implemented through taxes looks more efficient.
Finally, the results are alike for the last specification (Figure 8), except for the IRF of tax which is more similar to the case of no-cointegration since the estimates are not statistically different from zero for almost the entire period and the spending multiplier (Figure 12 and Table 6) which is more elevated and leads to a stronger long

run effect on GDP: indeed, it gets stable at a higher value than the one from the previous models.
It is interesting to notice that unlike the stationary case, the Impulse Response Functions computed under cointegration don't revert to the preshock values: in addition, we produce a larger multiplier[16], for spending than the one computed without cointegration whereas the effect from taxes yields more divisive results.

Conclusion

The aim of this project was to replicate the analysis performed by Blanchard & Perotti by accounting for possible cointegration: since this is related to long run connections among the variables, we should be able to provide a detailed description of the dynamics resulting from a fiscal shock.
After a preliminary analysis of the variables' characteristics and their trends, we chose an identification strategy for the cointegrating vector, i.e. the leading tool to ensure stationarity in the system, which is based on current literature: in particular, we relied on Angelini et Al. (2022) and Blanchard & Perotti (2002) to detect the correct relations.
Hence, through this approaches we tried to capture the effects of fiscal shocks and find out theirs implications on the economic activity.

Our results can be summarised as follows:

- The outcomes are consistent with Blanchard & Perotti (2002) and qualitatively with what we can expect, i.e an increase in government spending rises GDP, whereas when taxes grow the output falls.
  Generally, when including cointegration in the estimate the trend of the response doesn't change, however, it is essential to stress that the non stationarity leads to high persistence of the shocks and therefore the affected variable is unable to restore the initial value.
  Despite this, the significance of the multipliers may suggest some suitable policies: for instance, we noted that a tax shock is significant along the whole period only in the first and second specification under cointegration, i.e. no restrictions and Angelini et Al. (2022), whereas its effect is not particularly relevant for the other two methods.
  On the contrary, the spending multiplier is always significant at its highest value in the first periods, whereas as time goes by its effect vanishes.
  Overall, we can't recognise the best policy to implement and, recalling the findings of Favero and Giavazzi (2007), our results may be biased since we didn't account for the response to the level of the debt[17]

- Our results are slightly different depending on the number of cointegrating relations and thus, the number of unit roots in our data. In the latter specification with cointegration, Blanchard & Perotti (2002), we considered r = 1 allowing for both taxes and GDP to be random walks. This feature affects the outcome by reducing, in absolute value with respect to r = 2, both the IRF of GDP to tax and its dynamic multiplier. This may be because we considered taxes as unit roots and therefore the relation with GDP may be more challenging to find, causing an increased uncertainty and a loss of significance.
  On the other hand, we noticed that both the IRF and the dynamic multiplier of spending are higher and strongly significant: as a result of this, it may be that the unexplained part of the movements of GDP is captured by the dynamics of government spending, rising both the response to a spending shock and the value of the multiplier.

[16]empirically, we can't conclude that the multiplier is larger in absolute value for spending than for tax in all our specifications, as the Keynesian theory would suggests

[17]despite this, we proved that the feedback of tax from its own shock is consistent with the theory, and the same happens for spending

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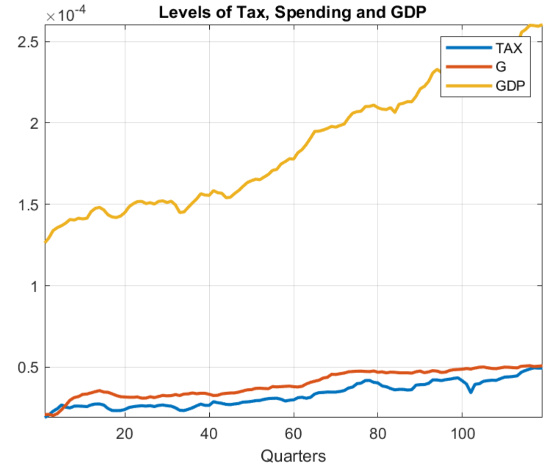
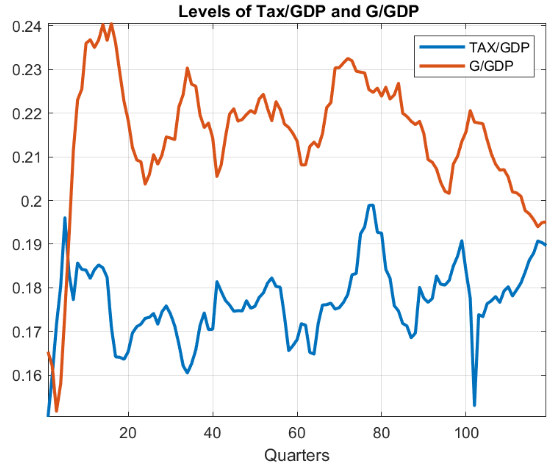
Appendix - Graphs

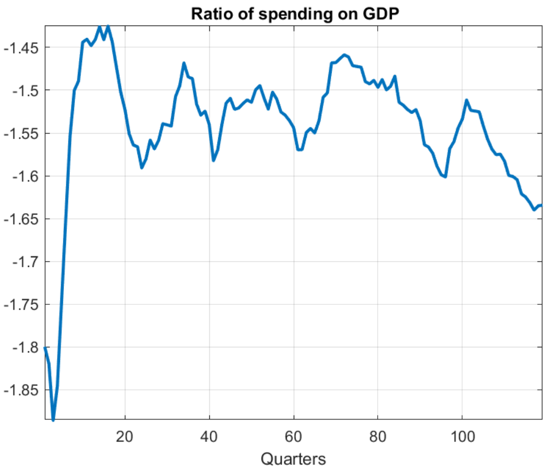


(a) Levels of Tax, G and GDP

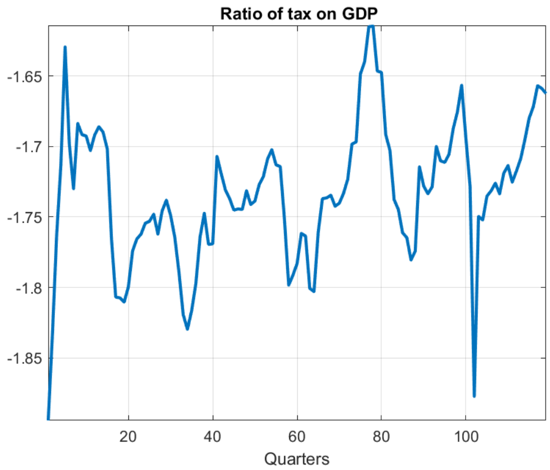


(b) Ratio of the variables in levels

Figure 2: Graphical analysis for variables in levels for the sample 1950:Q1 - 1979:Q3

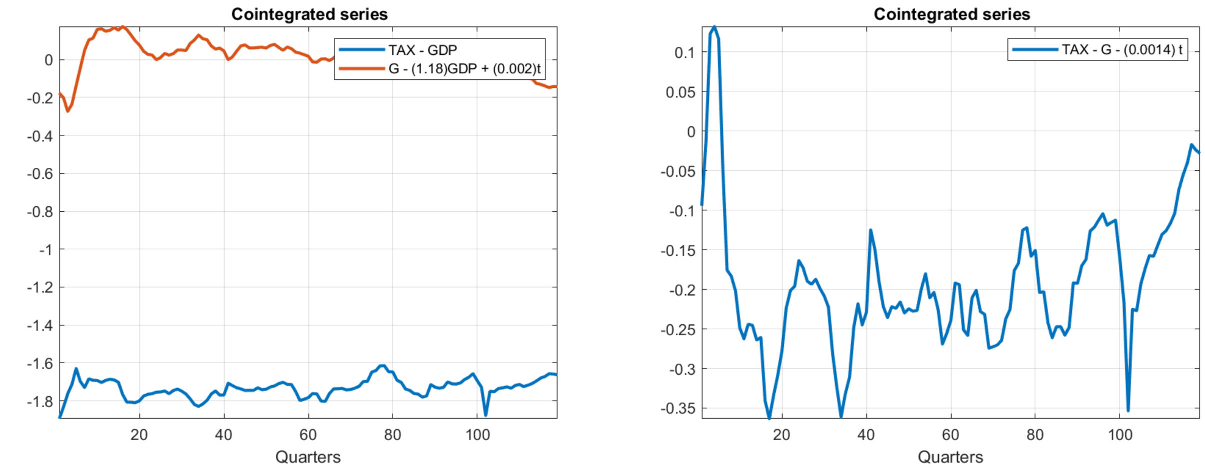


(a) Ratio of government spending on GDP



(b) Ratio of taxes on GDP

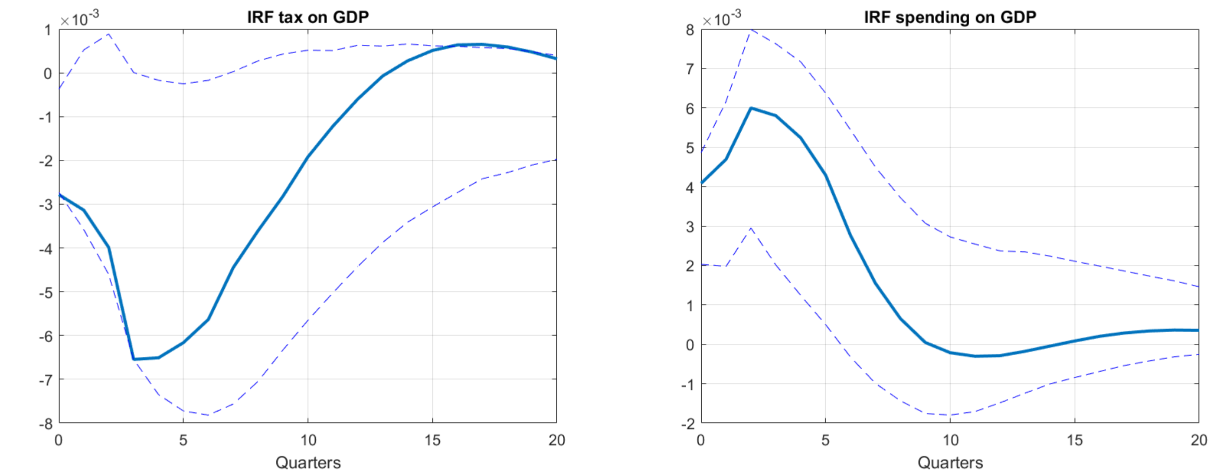
Figure 3: Graphical analysis for variables in logs for the sample 1950:Q1 - 1979:Q3



(a) Restrictions from Angelini et Al. (2022)

(b) Restrictions from Blanchard & Perotti, 2002

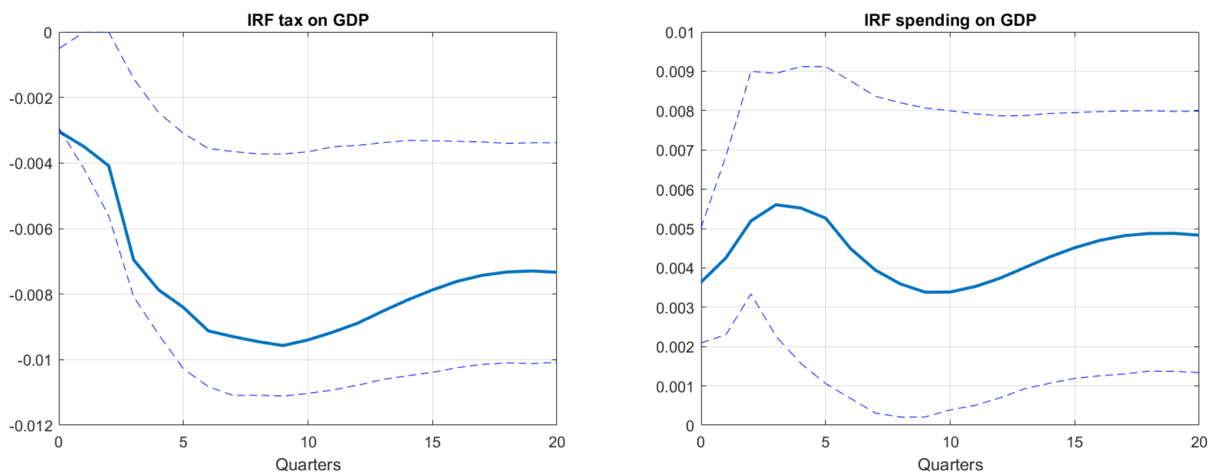
Figure 4: Cointegrated series for the sample 1950:Q1 - 1979:Q3



(a) IRF of GDP to a shock of Tax

(b) IRF of GDP to a shock of Spending

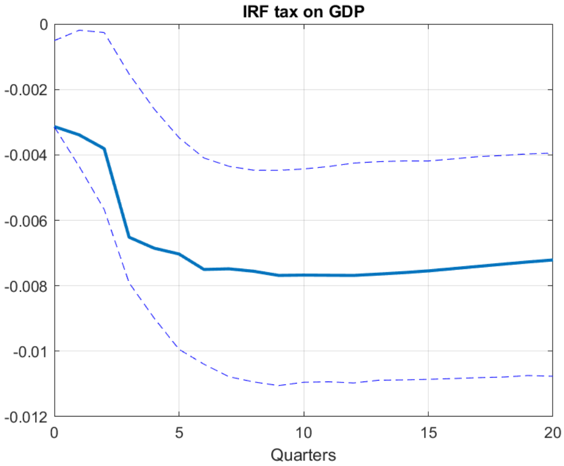
Figure 5: Impulse Response Functions with no cointegration



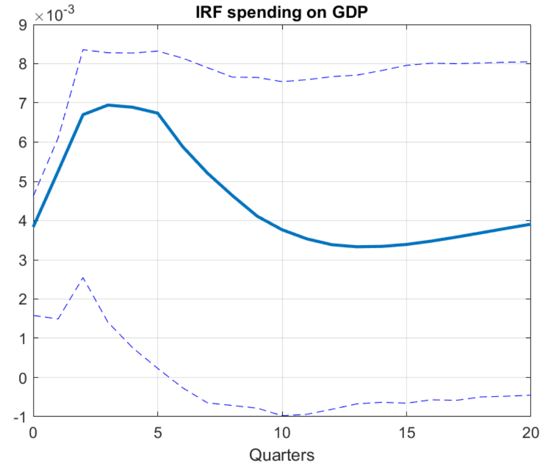
(a) IRF of GDP to a shock of Tax

(b) IRF of GDP to a shock of Spending

Figure 6: Impulse Response Functions with cointagration and no restrictions

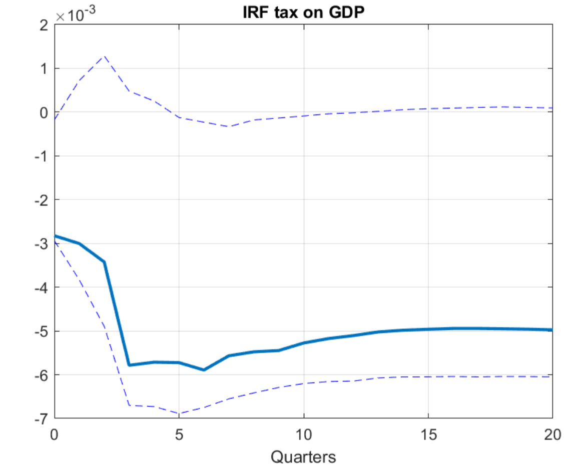


(a) IRF of GDP to a shock of Tax

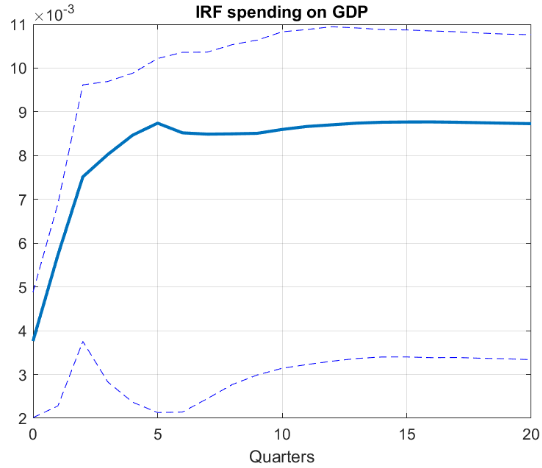


(b) IRF of GDP to a shock of Spending

Figure 7: Impulse Response Functions with cointagration and restrictions from Angelini et Al., 2022

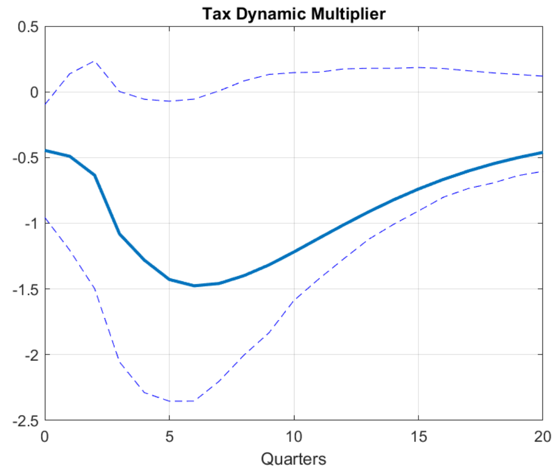


(a) IRF of GDP to a shock of Tax

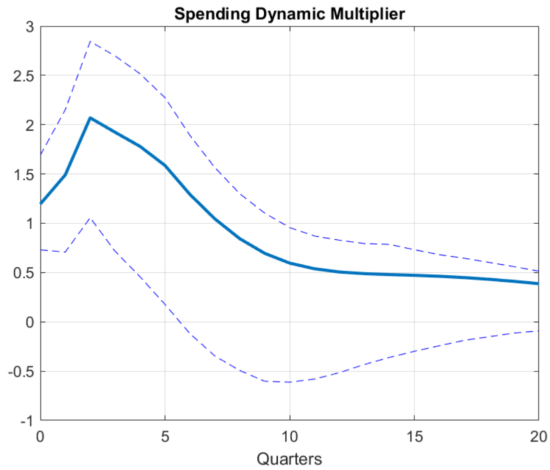


(b) IRF of GDP to a shock of Spending

Figure 8: Impulse Response Functions with cointagration and restrictions from Blanchard & Perotti, 2002

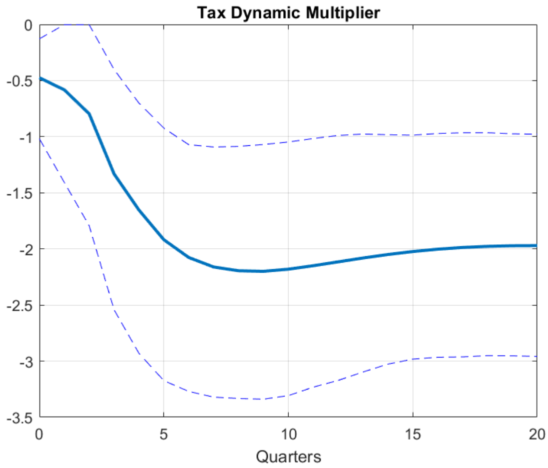


(a) Tax multiplier

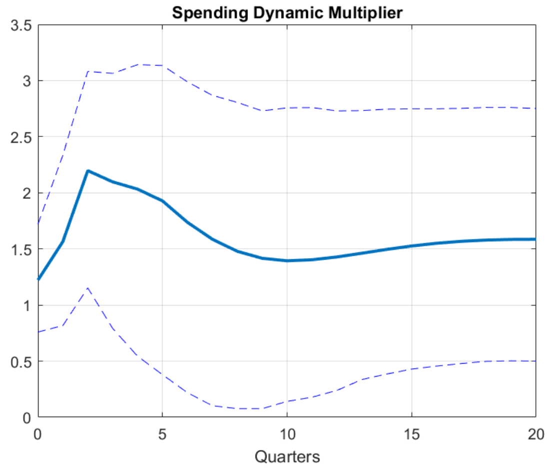


(b) Spending multiplier

Figure 9: Dynamic Multipliers of Tax and Spending with no cointegration

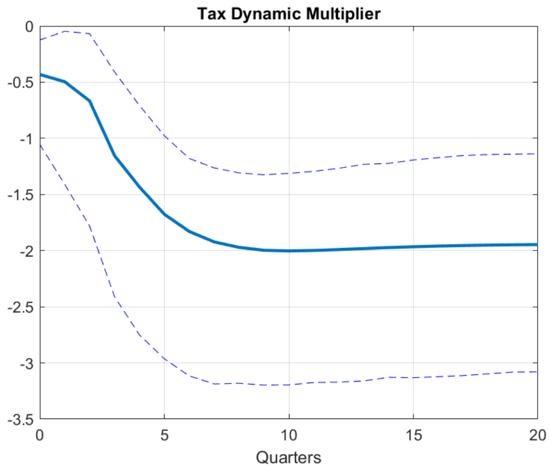


(a) Tax multiplier

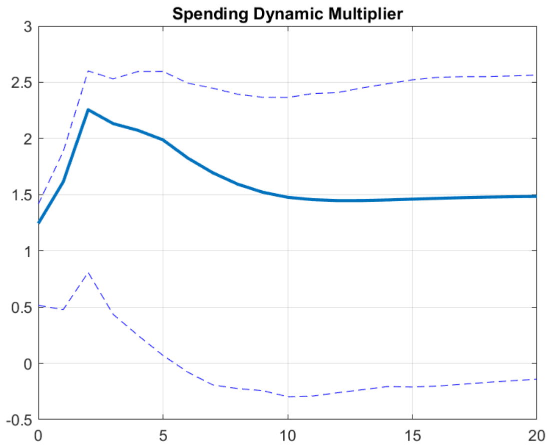


(b) Spending multiplier

Figure 10: Dynamic Multipliers of Tax and Spending for unrestricted cointegration

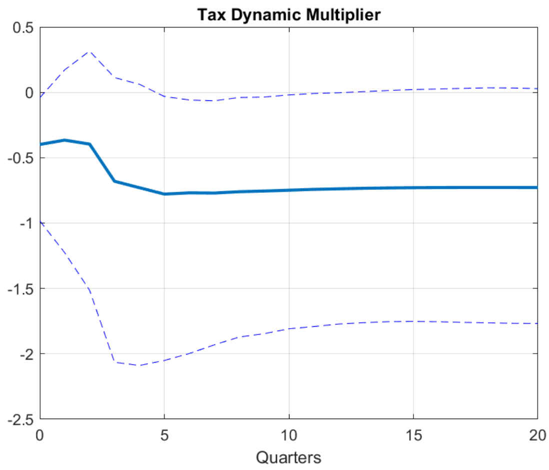


(a) Tax multiplier

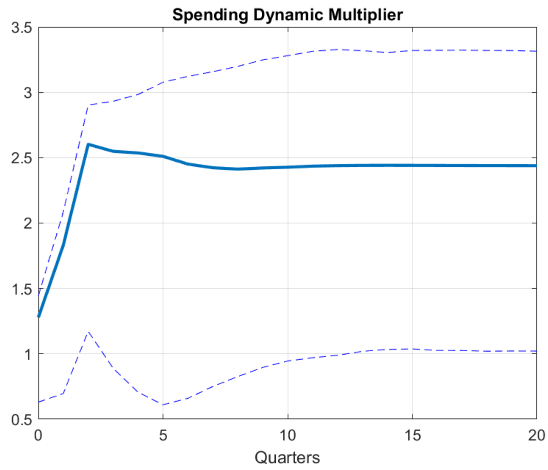


(b) Spending multiplier

Figure 11: Dynamic Multipliers of Tax and Spending with cointegration and restrictions from Angelini et Al., 2022



(a) Tax multiplier



(b) Spending multiplier

Figure 12: Dynamic Multipliers of Tax and Spending with cointegration and restrictions from Blanchard & Perotti, 2002

Appendix - Tables

w_t	**p-value**	**Result**
Tax	0.063	do not reject
G	0.001	reject
GDP	0.383	do not reject

(a) test with three lags in the difference terms and trend stationary option

(b) significance level $\alpha = 5\%$

Table 1: Augmented Dickey-Fuller test for $H_0 : \pi = 1$

r	**result**	**p-value**
		Trace test
0	reject	0.0010
1	do not reject	0.2799
2	do not reject	0.7787
		Eigenvalue test
0	reject	0.0.0010
1	do not reject	0.1613
2	do not reject	0.7787

Table 2: Cointegration test for $H_0 : r \leq j$

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	peak
Tax Multiplier	-0.490	-1.280*	-1.398	-1.011	-0.460	-1.476*
Spending Multiplier	1.490*	1.779*	0.843	0.505	0.387	2.069*

Table 3: Tax and Spending multiplier for no cointegration

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	peak
Tax Multiplier	-0.582	-1.654*	-2.194*	-2.115*	-1.972*	-2.20*
Spending Multiplier	1.564*	2.032*	1.478*	1.428*	1.584*	2.197*

Table 4: Tax and Spending multiplier for unrestricted cointegration

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	peak
Tax Multiplier	-0.432*	-1.153*	-1.922*	-1.999*	-1.948*	-2.003*
Spending Multiplier	1.244*	2.132*	1.695	1.456	1.482	2.256*

Table 5: Tax and Spending multiplier for restricted cointegration, Angelini et Al. (2022)

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	peak
Tax Multiplier	-0.399	-0.680	-0.771*	-0.742	-0.728	-0.778
Spending Multiplier	1.277*	2.549*	2.423*	2.435*	2.440*	2.602*

Table 6: Tax and Spending multiplier for restricted cointegration, Blanchard & Perotti (2002)