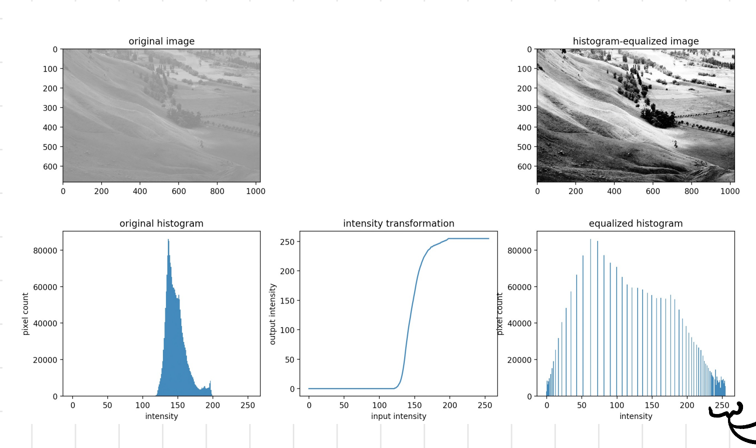
Histogram matching – 1/28/2020

Review of homework



"uniform distribution"
not flat b/c discretization

closer together → shorter
further apart → taller

} cause dist to be on avg uniform

Histogram matching:

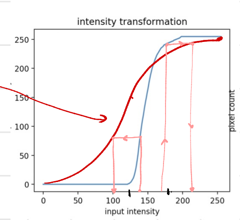
- we've done hist eq (where transformation is cdf)

① Map histogram onto uniform dist

② What is map from Gaussian → uniform?

↳ use this function inversely

to uniform → inverted to uniform of desired →



- in discrete world, this gets tricky but it's ok

Done with point processes!

Noise - 1/28/2020

- always have noise
- "electronic noise" generally
- ideally voltage $\propto$ # photons, but there is some noise

$$I(u,v) = g \cdot (S(u,v) + N(u,v))$$

I: image pixel intensity; u: row; v: col; S: signal, actual thing in world; N: some random noise

$$N(u,v) \sim N(0, \sigma_n^2)$$

variance; assuming gaussian noise; std deviation σ_n (property of camera)

$$S(u,v) \propto \text{illuminance} * dt * \pi\left(\frac{D}{2}\right)^2$$

g = gain (analog b/c digital would lose info b/c discretization)

Light level	Shutter speed ΔT	aperature D	noise
high ↑ · illuminance high · S is high relative to noise	—	—	low ↓
low ↓	—	—	high ↑
low ↓	↑ · bring S up to improve SNR · limited b/c long exp. blurs	or ↑ · bring S up to improve SNR · limited physically	avg —
high ↑	↓ · desired fast shutter · moving subject	↓ · say you have small aperature	average —

Why we can't just apply gain to fix problems? Gain amplifies the signal as well as the noise!

Denoising problem

- We know $I(u,v)$
  - we don't know $N(u,v)$ or $S(u,v)$ but we would like $N(u,v)$

$$SNR = \frac{E(S)}{E(N)} = \frac{\text{avg energy of signal}}{\text{avg energy of noise}}$$

$$E(S) = \frac{1}{NM} \sum_{u,v} S(u,v)^2 \qquad E(N) = \frac{1}{NM} \sum_{u,v} N(u,v)^2$$

N, M: dimensions of image

another way to think about noise:

$RMSE(I, \hat{I})$

"root mean squared error"

$$\underbrace{RMSE(I,\hat{I}) = \sqrt{}}_{\text{root}} \!\!\! \underbrace{\frac{1}{NM}\sum_{u,v}}_{\text{mean}} \underbrace{\left[I(u,v) - \hat{I}(u,v)\right]}_{\text{error}}{}^{2}$$

squares

- good at describing how close two images are
- compare:
  image + added noise to image + added noise − denoising alg

$$RMSE(I, S) = \sqrt{\frac{1}{NM}\sum (I_{uv} - S_{uv})^2}$$

$$= \sqrt{\frac{1}{NM}\sum N_{uv}^2} = \sqrt{E(N)}$$ ← prev. page.

= sqrt of energy of noise

- Noise of each pixel is independent & normally distributed

$\mathbb{E}[E(N)] \equiv$ expectation of energy of noise

$\mathbb{E}[E(N)] = f_1(\sigma_n)$

$\mathbb{E}(E(N)) = \frac{1}{MN}\sum_{u,v}\mathbb{E}[N(u,v)^2]$

$\mathbb{E}(E(N)) = \mathbb{E}(N(u,v)^2)$

- $N(u,v)$ is same for every u,v, so avg

$N \sim \mathcal{N}(0, \sigma_n^2)$

$\mathbb{E}(E(N)) = \underbrace{\mathbb{E}((N-\overline{N_{uv}})^2)}_{\text{def of var}} + \underbrace{\mathbb{E}(N)^2}_{0}$

$\mathbb{E}(E(N)) = \sigma_n^2$

$$\underbrace{\mathbb{E}[(X-\bar{X})^2]}_{var(X)} = E(X^2) - \underbrace{E(X)}_{RV}{}^2$$

- Expectation of energy of noise is <u>variance</u>
- $RMSE(I, S)$ is std. dev of noise