Virginia Pan
ECE 488
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Image Noise Homework

$I_1[u,v] = S_1[u,v] + N_1[u,v]$

$I_2[u,v] = S_2[u,v] + N_2[u,v]$

$S_2[u,v] = \frac{1}{2}S_1[u,v]$

$N_i[u,v] \sim N(0,\sigma_n^2)$

Part 1

a)

SNR $= \frac{E(S)}{E(N)}$

SNR$_1 = \frac{E(S_1)}{E(N_1)}$

SNR$_2 = \frac{E(S_2)}{E(N_2)} = \frac{\frac{1}{4}E(S_1)}{E(N_2)} = \frac{\frac{1}{4}E(S_1)}{E(N_1)}$

$E(N_1) = E(N_2)$

$\boxed{\frac{SNR_1}{SNR_2} = \frac{\frac{E(S_1)}{E(N_1)}}{\frac{\frac{1}{4}E(S_1)}{E(N_1)}} = 4}$

the SNR of I_2 is one quarter the SNR of I_1

b)

if$E(I) = E(S) + E(N)$note, middle term goes away because mean of N is 0.

$E(I_1) = E(S_1) + E(N_1)$

$E(I_2) = E(S_2) + E(N_2)$

$E(I_2) = E(\frac{1}{2}S_1) + E(N_2)$

$E(I_2) = \frac{1}{4}E(S_1) + E(N_2)$

$E(N_1) = E(N_2)$

$E(I_2) = \frac{1}{4}E(S_1) + E(N_1)$

$\boxed{\frac{E(I_1)}{E(I_2)} = \frac{E(S_1)+E(N_1)}{\frac{1}{4}E(S_1)+E(N_1)}}$

c)

$f(I) = I \times \sqrt{\frac{E(S_1)+E(N_1)}{\frac{1}{4}E(S_1)+E(N_1)}}$

$\hat{I}_2 = f(I_2) = \sqrt{\frac{E(S_1)+E(N_1)}{\frac{1}{4}E(S_1)+E(N_1)}}\,(S_2[u,v] + N_2[u,v]) = \sqrt{\frac{E(S_1)+E(N_1)}{\frac{1}{4}E(S_1)+E(N_1)}}\,(\frac{1}{2}S_1[u,v] + N_1[u,v])$

$E(I_1) = E\left(\hat{I}_2\right)$

$E(S_1) + E(N_1) = E\left(\sqrt{\frac{E(S_1)+E(N_1)}{\frac{1}{4}E(S_1)+E(N_1)}}\,(\frac{1}{2}S_1[u,v] + N_1[u,v])\right)$

$= \sqrt{\frac{E(S_1)+E(N_1)}{\frac{1}{4}E(S_1)+E(N_1)}}\,\frac{1}{M}\sum_{u,v}(\frac{1}{2}S_1[u,v] + N_1[u,v])^2$

$$= \sqrt{\frac{E(S_1)+E(N_1)}{\frac{1}{4}E(S_1)+E(N_1)}}^{\,2}\left(\tfrac{1}{4}E(S_1)+E(N_1)\right) = E(S_1)+E(N_1) = E(S_1)+\sigma_n^2$$

$$E[E(N)] = \sigma_n^2$$

$$\boxed{f(I) = I \times \sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}}$$

d)

$$RMSE(I_i,S_i) = \left(\frac{1}{M}\sum\nolimits_{u,v}(I_i[u,v]-S_i[u,v])^2\right)^{\frac{1}{2}}$$

$$RMSE(I_1,S_1) = \left(\frac{1}{M}\sum\nolimits_{u,v}(I_1[u,v]-S_1[u,v])^2\right)^{\frac{1}{2}}$$

$$I_1[u,v] = S_1[u,v]+N_1[u,v]$$

$$RMSE(I_1,S_1) = \left(\frac{1}{M}\sum\nolimits_{u,v}(S_1[u,v]+N_1[u,v]-S_1[u,v])^2\right)^{\frac{1}{2}}$$

$$RMSE(I_1,S_1) = \left(\frac{1}{M}\sum\nolimits_{u,v}(N_1[u,v])^2\right)^{\frac{1}{2}}$$

$$RMSE(I_1,S_1) = (E(N_1))^{\frac{1}{2}} = (\sigma_n^2)^{\frac{1}{2}} = \sigma_n$$

$$E[E(N)] = \sigma_n^2$$

$$RMSE(I_2,S_2) = \left(\frac{1}{M}\sum\nolimits_{u,v}(I_2[u,v]-S_2[u,v])^2\right)^{\frac{1}{2}}$$

$$I_2[u,v] = S_2[u,v]+N_2[u,v]$$

$$RMSE(I_2,S_2) = \left(\frac{1}{M}\sum\nolimits_{u,v}(S_2[u,v]+N_2[u,v]-S_2[u,v])^2\right)^{\frac{1}{2}}$$

$$RMSE(I_2,S_2) = \left(\frac{1}{M}\sum\nolimits_{u,v}(N_2[u,v])^2\right)^{\frac{1}{2}}$$

$$RMSE(I_2,S_2) = (E(N_2))^{\frac{1}{2}} = (\sigma_n^2)^{\frac{1}{2}} = \sigma_n$$

$$E[E(N)] = \sigma_n^2$$

$$RMSE\left(\hat{I}_2,S_1\right) = \left(\frac{1}{M}\sum\nolimits_{u,v}\left(\hat{I}_2[u,v]-S_1[u,v]\right)^2\right)^{\frac{1}{2}}$$

$$\hat{I}_2 = f(I_2) = \sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}\,(S_2[u,v]+N_2[u,v]) = \sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}\,\left(\tfrac{1}{2}S_1[u,v]+N_1[u,v]\right)$$

$$RMSE\left(\hat{I}_2,S_1\right) = \left(\frac{1}{M}\sum\nolimits_{u,v}\left(\sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}\,\left(\tfrac{1}{2}S_1[u,v]+N_1[u,v]\right)-S_1[u,v]\right)^2\right)^{\frac{1}{2}}$$

$$RMSE\left(\hat{I}_2,S_1\right) = \left(\frac{1}{M}\sum\nolimits_{u,v}\left(\sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}\,\tfrac{1}{2}S_1[u,v]+\sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}\,N_1[u,v]-S_1[u,v]\right)^2\right)^{\frac{1}{2}}$$

$$\boxed{\frac{RMSE\left(\hat{I}_2,S_1\right)}{RMSE(I_1,S_1)} = \frac{\left(\frac{1}{M}\sum\nolimits_{u,v}\left(\sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}\,\frac{1}{2}S_1[u,v]+\sqrt{\frac{E(S_1)+\sigma_n^2}{\frac{1}{4}E(S_1)+\sigma_n^2}}\,N_1[u,v]-S_1[u,v]\right)^2\right)^{\frac{1}{2}}}{\sigma_n}}$$