ECE/CS 559: Neural Networks **Fall 2024**

Homework 2

Due: **Tuesday** September 10, 2024 (by 9pm, on Gradescope)

Note: *You may discuss these problems in groups. However, you must write up your own solutions and mention the names of the people in your group. Also, please do mention any solution manuals, online material, or other sources you used in a major way.*

1. **[Designing a Feed-Forward NN]**

   Design a neural network that implements the following *logical* statement:

   $$(x_1 \wedge x_2 \wedge \neg x_3) \vee (\neg x_2 \wedge x_3).$$

   Use $+1$ to represent `True` and -1 to represent `False` (this is a bit of a change compared to HW1). Let your activation function be

   $$\varphi(v) = \mathsf{sign}(v) = \begin{cases} +1 & ; \quad v > 0, \\ 0 & ; \quad v = 0, \\ -1 & ; \quad v < 0. \end{cases}$$

   (a) Draw your chosen architecture with all the weights and biases explicitly, and specify what kind of feed forward neural network it is, e.g., 3-5-2-1 is a 3-layer network with 5 neurons in its first layer, 2 in its second, and 1 in its output later.

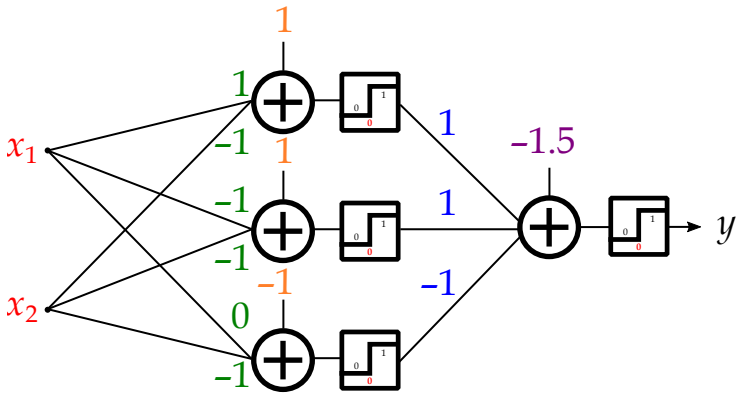
   (b) Write down the input-to-output *analytic* equation that your network represents.

   (c) Write code that produces the truth table (filled with `True/False`) of the logical statement and the input-output table (filled with $+1/-1$) of the analytic equation, and verify that they match. (In your main submission, include the outputted tables. In the code submission, include your code.)

2. **[Decision Boundary of a NN]**

   Consider the following feed-forward neural network that uses the step activation function:

   $$\varphi(v) = \mathsf{step}(v) = \begin{cases} 1 & ; \quad v \geq 0, \\ 0 & ; \quad v < 0. \end{cases}$$



(a) Name and list the parameter matrices and vectors with their dimensions.

(b) Write down the input-to-output *analytic* equation $y = f(x)$ that this network represents, in linear algebraic form.

(c) Write a Python program that does the following:

- Samples 1,000 random points x from a uniform distribution over the square $[-2, 2]^2$.
- Calculates the output of the neural network $y = f(x)$ for each point.
- Plots a scatter plot of all points x, where the points are blue when $y = 0$ and the points are red when $y = 1$.

Submit your code and report its output.

(d) Draw a rough sketch of the *decision boundary* of this neural network, i.e., the curve that separates the $y = 0$ and $y = 1$ regions.

(In your main submission, include the outputted plot and your sketch. In the code submission, include your code.)