MATHEMATICS INTERNAL ASSESSMENT (HL)

INTRODUCTION:

I have always been wanting to investigate further in a topic that has intrigued me. Certainly, when the concept of 'half-life' in radioactive decay was introduced in the physics class to me, this concept got my interest and attention to it and lead me to investigate further about it. Knowing that radioactive decay can be used to find a substance's span, I propose to investigate and study on the modeling of radioactive decay, along with working out real-life problems using the model. Having a science background, I have always admired the significance of applying mathematics in various scientific investigations. I observed that the usage of mathematical models in the examination and application of radioactive decay is essential. By going deeper in the topic, I aim to know how the concept can be used in real-life. Is it possible to find an isotope's half-life using the collected data? Is it possible to anticipate the quantity of left-over radioactive substance? These questions provoke the curiosity in me, and I look forward to solve them using mathematics in this inspection.

BACKGROUND INFORMATION:

Radioactive decay is a process where an unstable nucleus breaks up spontaneously into nuclei of other elements and emits radiation. Alpha, Beta, and Gamma rays are the three types of radiations emitted by the radioactive elements. All substances are effectively isotopes therefore, any radioactive substance is a radioisotope or radioactive isotope. There are several naturally occurring radioisotopes, most of which still exist because they have very long half-lives.[1]

The time taken for half the number of atoms in a sample to undergo radioactive decay, and hence for the radiation emitted to be halved, is known as its half-life. Every kind of radioactive decay – alpha, beta or gamma decay – has half-life. The rate of decay is represented by the decay constant, λ . The correlation between the decay constant and half-life[2]:

$$t_{1/2} = \frac{\ln(2)}{\lambda} ,$$

Where $t_{1/2}$ = half-life, and λ = decay constant.

The current model that is accepted for radioactive decay is: $N(\mathrm{t}) = \mathrm{N}_o e^{-\lambda t}$, where $N(\mathrm{t})$ is the number of particles left after t days; N_o is the original amount of particles.

[1] "Radioactive Decay." *Radioactive Decay*. National Science Foundation, n.d. Web. 21 Oct. 2019. <https://www.nde-ed.org/EducationResources/HighSchool/Radiography/radioactivedecay.htm>.
[2] "The Relation between Half Life (T) and Decay c Toppr.com." *Toppr Ask*, n.d. Web. 21 Oct. 2019. <www.toppr.com/ask/question/the-relation-between-half-life-t-and-decay-constant-lambda/>.

MODELING RADIOACTIVE DECAY:

Below is a table that shows the decay pattern of Vanadium-48 (V-48)[3]:

Table 1: decay pattern of V-48

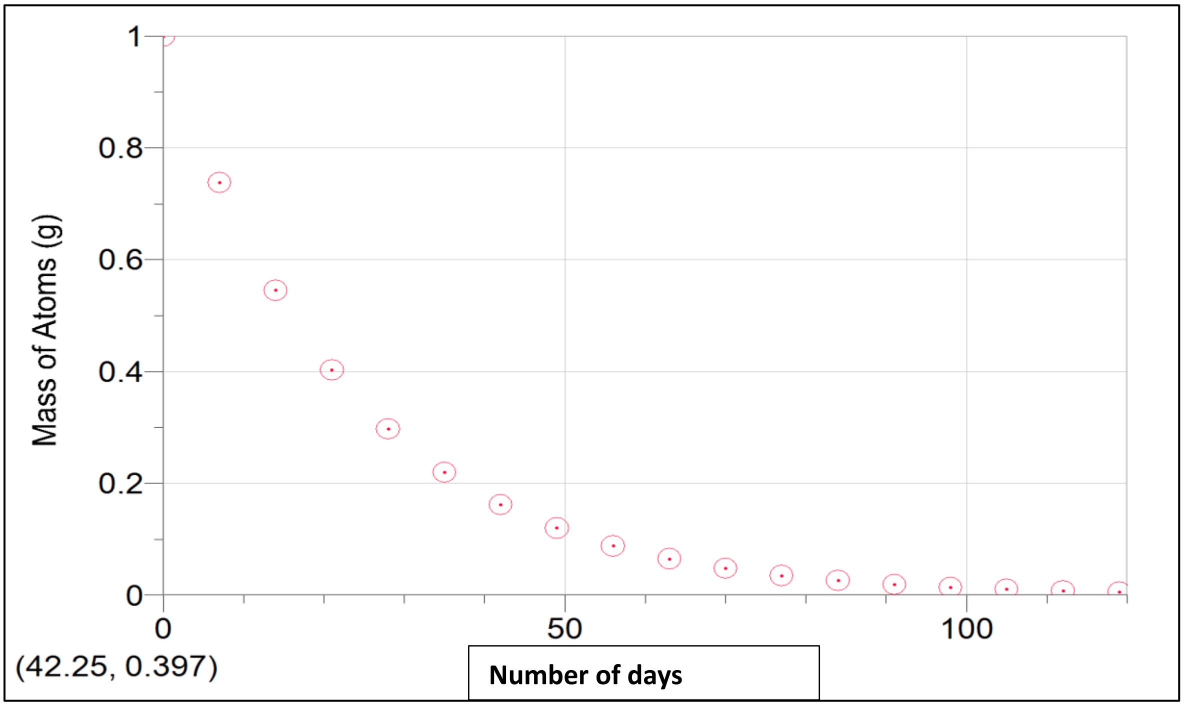
DAYS	DAYS/ WEEKS	1 (pattern starting with 1.000g)	2 (pattern starting with 0.955g)
0	0	1.000	0.955
7	1	0.738	0.705
14	2	0.545	0.521
21	3	0.403	0.385
28	4	0.297	0.284
35	5	0.220	0.210
42	6	0.162	0.155
49	7	0.120	0.114
56	8	0.088	0.084
63	9	0.065	0.062
70	10	0.048	0.046
77	11	0.035	0.034
84	12	0.026	0.025
91	13	0.019	0.018
98	14	0.014	0.014
105	15	0.011	0.010
112	16	0.0078	0.007
119	17	0.0057	0.005

The data above is taken in (g) units. With the data above, it can be predicted that the half-life of vanadium-48 (V-48) is between 14 to 17 days by assuming the half-life activity would be 0.500g for the initial activity of 1.000g. The amount 0.500g lies between the amounts 0.545g (the remaining amount of V-48 on the 14th day) and 0.479g (the remaining amount of V-48 on the 17th day). Further, 0.500g halves to 0.250g. The amount 0.250g is between 0.297g (the remaining amount of V-48 on the 28th day) and 0.220g (the remaining amount of V-48 on the 35th day). As further adding 14-17 days would range between 28-35 days, this makes the prediction true that the half-life of V-48 is between 14 to 17 days.

To trace the data as a graph form, and finding a relation between the remaining amount V-48 and the number of days, I plotted the graph for the first data collected (with initial mass of 1.000g) using LoggerPro:

[3] "Vanadium (V) 48 Radioactive Isotope Decay Calculator." *Vanadium (V) 48 Isotope Decay Calculator | Calculate Radioactivity in Minerals*. N.p., n.d. Web. 28 Oct. 2019. <https://www.easycalculation.com/chemistry/V-48.html>.

Figure 1: Decay graph of V-48 (relation between remaining amount of mass of atoms of V-48 and the number of days)



The graph above showcases a monotone decreasing pattern. The amount of the mass of the atom decreases at a faster rate in the beginning, gradually slowing down later. It can be seen from the graph that it eventually gets closer to y = 0, the horizontal asymptote. There isn't any vertical asymptote. Therefore, the function decreases exponentially[4]:

$f(x) = c \times a^x$, 0<a<1, c > 0[5]

(x is the number of days following the beginning of record of decay for 1.000g of V-48, and c is a positive constant)

When x = 0, $f(x) = c \times 1 = 1 \quad \therefore\ c = 1$

substituting c = 1 and x = 14 in $f(x) = c \times a^x$:

$$1 \times a^{14} = 0.545$$

$$a = \sqrt[14]{0.545}$$

[4] "Exponentially Decreasing Function." *From Wolfram MathWorld*. Wolfram Research, Inc., n.d. Web. 7 Nov. 2019. <https://mathworld.wolfram.com/ExponentiallyDecreasingFunction.html>.

[5] "Use the Formula Y = Ca^x to Determine the Constants C and A." *Use the Formula Y = Ca^x to Determine the Constants C and A. | Wyzant Ask An Expert*. Wyzant, Inc., 11 Feb. 2014. Web. 10 Nov. 2019. <https://www.wyzant.com/resources/answers/26801/use_the_formula_y_ca_x_to_determine_the_constants_c_and_a>.

$$a = \pm 0.958$$

$\because\ a > 0$

$$a = 0.958$$

$$f(x) = 0.958^x$$

To check if this model is valid, data from the table can be put into the equation formed. Coordinates (63, 0.065) and (77, 0.035) are substituted and checked:

When x_1 = 63,

$$f(63) = 0.958^{63} = 0.067 \approx 0.065$$

When x_2 = 77,

$$f(77) = 0.958^{77} = 0.037 \approx 0.035$$

By this, it can be said that the exponential function, $f(x) = 0.958^x$, approximately represents V-48's decay pattern. But the model doesn't fit if 2.000g is taken as the original mass for V-48 because then the starting points will vary the model.

Therefore, taking 2.000g as original mass instead of 1.000g:

$f(x) = c \times a^x$, 0<a<1, c > 0[6] (0,2.000)

$\therefore$ when x = 0 and y = 2,

$$2 = c \times a^0$$

$$2 = c \times 1$$

$\therefore$ c = 2

Knowing the fact, half-life of V-48 is 16 days, we can interpret that in 16 days, 2.000g of V-48 will be halved, that is 1.000g.

If, $x = 16$

Then, $f(x) = 2 \times a^{16}$

$$\therefore 2 \times a^{16} = 1$$

$$a^{16} = \frac{1}{2}$$

$a = \sqrt[16]{\frac{1}{2}} = 0.958$ (as a is positive)

[6] Ibid.

Hence, $f(x) = 2 \times 0.958^x$

It can be seen that the original amount of substance does not affect the base of the function, that is, a = 0.958. In the function, $f(x) = c \times a^x$, c is a coefficient that equals to the initial quantity of the substance.

Therefore, the model that can represent V-48 decay is,

$f(x) = c \times 0.958^x$ (c = initial amount of V-48)

The pattern that is generally followed, for radioactive decay, is $f(x) = c \times a^x$. In this, 'c' is the initial amount of radioactive decay and 'a' is a constant that depends on the type of radioactive atom.

Relation of half-life to base function a:

Let there be a radioactive atom, having d number of days as half-life,

$$f(d) = a^d$$
$$f(2d) = a^{2d} = (a^d)^2$$

The mass halves as d to 2d is another half-life.

[For example, If d is the number of days that represents the half-life of a radioactive atom and x ($f(d)$) is the remaining amount of that radioactive atom, then as d doubles (2d), x halves to $\frac{x}{2}$; further, if d triples (3d), $\frac{x}{2}$ halves to $\frac{x}{4}$]

$$\therefore \quad f(d) = 2f(2d)$$
$$a^d = 2(a^d)^2$$
$$a^d = \frac{1}{2} \text{ -------------------------- (1)}$$

Hence, $a = \sqrt[d]{\frac{1}{2}}$

Therefore, $f(x) = C \times \left(\sqrt[d]{\frac{1}{2}}\right)^x$, 'c' being the initial number of atoms and 'd' being the half-life of the atom.

Now, with the half-life and the initial amount of an atom, model for recording radioactive decay can be easily worked out according to, $a = \sqrt[d]{\frac{1}{2}}$.

APPLICATIONS OF THE MODEL:

This model's use of anticipating the measure of the undecayed radioactive substances in the coming years is most widely recognized. The Chernobyl disaster can be taken as an application and an example for showing how this model can come into use for the scientists.[7]

On April 26, 1986, Cesium-137 (Cs-137), a type of radioactive atom that has a half-life of 30.17 years, was released in the disaster in an amount of around 2.5MCi (megaCuries[8]).[9]

For Cs-137,

1Ci = 12mg, and 1MCi = 1,000,000Ci.[10]

Therefore, 2.5MCi = 2,500,000Ci which is equal to 30,000,000mg = 30,000g.

$$f(x) = C \times \left(\sqrt[d]{\frac{1}{2}}\right)^x$$

$= 30,000 \times (\sqrt[30.17]{\frac{1}{2}})^x$

= $30,000 \times 0.977^x$ (x = no. of years from April 26, 1986)

Thus, the use of this model can be predicting the time period for the radiation in Chernobyl to be normal, that is a safe level. Chernobyl is predicted by scientists all over the world to be an unsafe place to live in the future and so, this could be one of the cases to mathematical model and solve Environmental Sciences problems.

One use for this approach is to calculate a radioactive atom's uncertain half-life. The table below contains the remaining masses for 1.000g of decaying Phosphorus-32 (P-32)[11]:

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[7] OpenStax. "Physics." *Half-Life and Activity | Physics*. N.p., n.d. Web. 13 Nov. 2019. <https://courses.lumenlearning.com/physics/chapter/31-5-half-life-and-activity/>.

[8] Rouse, Margaret. "What Is Curie? - Definition from WhatIs.com." *WhatIs.com*. TechTarget, 27 Dec. 2005. Web. 18 Nov. 2019. <https://whatis.techtarget.com/definition/curie>.

[9] Ragheb, M. "Chernobyl Accident." *Chernobyl Accident.pdf*. N.p., 13 Oct. 2018. Web. 23 Nov. 2019. <https://mragheb.com/NPRE%20457%20CSE%20462%20Safety%20Analysis%20of%20Nuclear%20Reactor%20Systems/Chernobyl%20Accident.pdf>.

[10] Khan, Mansoor, and Indra Reddy. "Pharmaceutical and Clinical Calculations." *Pharmaceutical and Clinical Calculations Ed 2.pdf*. N.p., n.d. Web. 25 Nov. 2019. <http://www.anhihs.com/up/Pharmaceutical%20and%20Clinical%20Calculations%20ed%202.PDF>.

[11] "Phosphorus (P) 32 Radioactive Isotope Decay Calculator." *Phosphorus (P) 32 Isotope Decay Calculator | Calculate Radioactivity in Minerals*. N.p., n.d. Web. 26 Nov. 2019. <https://www.easycalculation.com/chemistry/P-32.html>.

Table 2: Decay pattern of P-32

DAYS	WEEKS	1 (pattern with initial mass, 1.000g)	2 (pattern with initial mass, 0.950g)
0	0	1.000	0.950
7	1	0.712	0.676
14	2	0.507	0.481
21	3	0.361	0.342
28	4	0.257	0.244
35	5	0.183	0.173
42	6	0.131	0.123
49	7	0.093	0.088
56	8	0.066	0.062
63	9	0.047	0.044
70	10	0.034	0.031
77	11	0.024	0.022
84	12	0.017	0.016
91	13	0.012	0.011
98	14	0.008	0.008
105	15	0.006	0.005
112	16	0.004	0.004
119	17	0.003	0.003

In reference to the above table, we can estimate that the half-life of P-32 is somewhat between 14 and 15 days, for this is the time it takes for 1.000g of P-32 to half to 0.500g. We don't have a way to find out the exact value without proper calculations though. Moreover, this method requires that the experimenters wait until the original amount of atoms have halved. This approach is limited in terms of both the precision of the tests and the amount of time it may take. Problem arises when the half-life of atoms is, for example, 8.00×10^{7} years (Plutonium-244). This is the reason why observations can't replace mathematic estimations in scientific investigations. Assume, the first week of the decay of P-32 is all the data we have:

Table 2.1: first week of Decay of P-32

DAYS	WEEKS	0	1
0	0	1.000	0.950
7	1	0.712	0.676

Working of the model for this decay:

$$f(x) = a^x$$ [12]

Take a random coordinate (7, 0.712), and substitute it in the function above,

$$0.712 = a^7$$

$$\therefore a = 0.953$$

$$a^d = \frac{1}{2} \text{ --------------- (1)}$$

$$\therefore d = \log_a \frac{1}{2}$$

$$d = \log_{0.953} \frac{1}{2} \quad \text{(substituting the value of } a \text{ from above)}$$

$$d = 14.39$$

Therefore, the model, $a^d = \frac{1}{2}$, enables us to get the exact amount of P-32's half-life as well as saving the time consumed in waiting the substance to half. This model is very useful when it comes to atoms that have extremely long half-lives, such as Plutonium-244.

However, things such as radiocarbon dating, which is a method that provides objective age estimated for carbon-based materials that originated from living organisms, can't be done using the formula, $f(x) = C \times \left(\sqrt[d]{\frac{1}{2}}\right)^x$, unless the original amount of substance is known. Having 3 unknowns in a single formula – x, d, and c – makes this formula limited. Thereby, one of the extensions of the radioactive decay model's application is radiocarbon dating.

After exploring this, I have observed that the radioactive model's application is an accurate and quality way of showing the use of math in physics. I've also come up with the realization of the significance of math models, that represent tools, in the scientific studies. Mathematics plays a major role in the field of science. However, arithmetic, which is used in building up model and expounding data, uses logic more than math. Therefore, we can say that the building blocks of sciences is math.

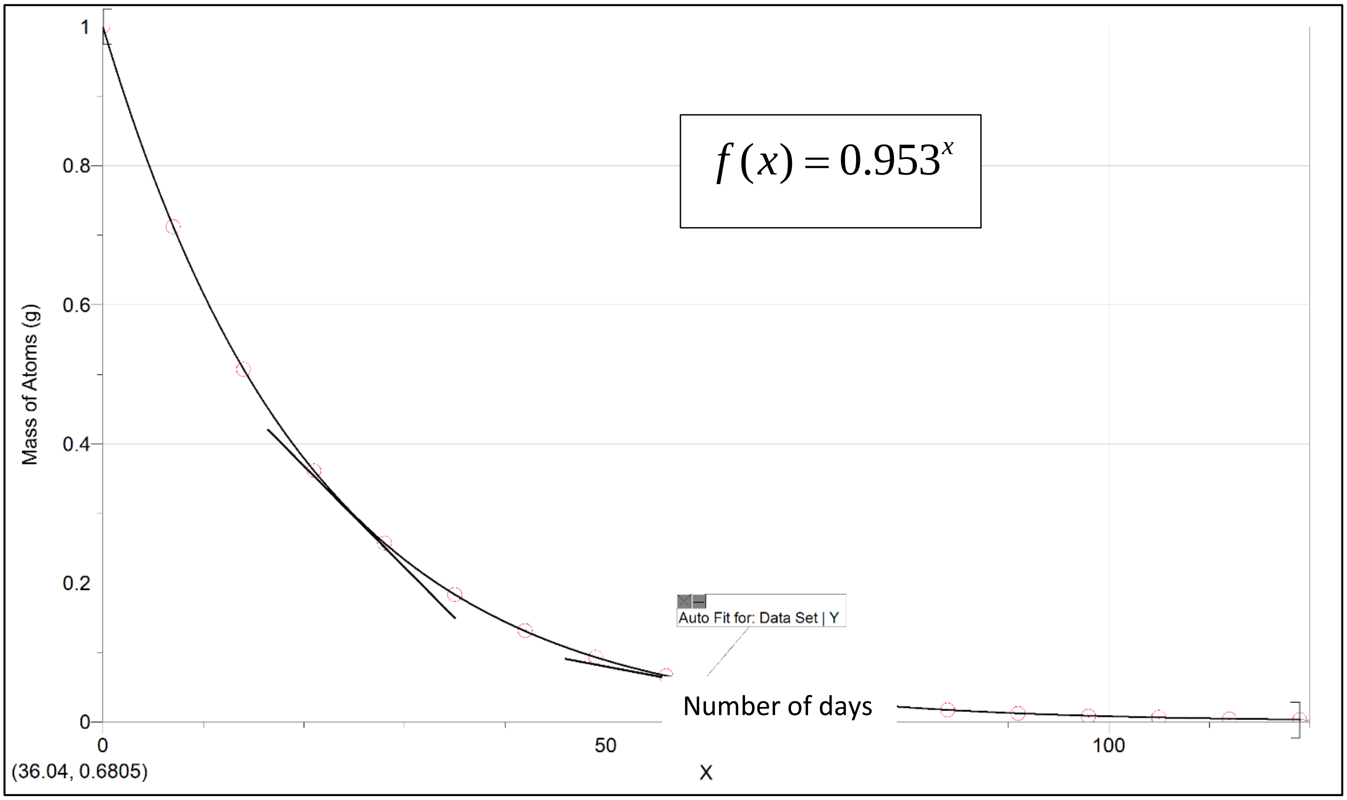
TOTAL ACTIVITY:

Just like how we use instantaneous speed to calculate the pace of travelling while driving, 'instantaneous rate' can be used to denote the rate of decay. The number of unstable atomic

[12] "Exponential Function Reference." *Exponential Function Reference*. N.p., n.d. Web. 2 Dec. 2019. <https://www.mathsisfun.com/sets/function-exponential.html>.

nuclei that decay per second in a given sample is the total activity, A.[13] I will be using the decay of P-32 for example.

Figure 2: Decay graph of P-32



The instantaneous changing rate of y equals to the slope of the tangent to the graph. Therefore, I will be differentiating the function, $f(x) = 0.953^x$, in order to get the derivative function $f'(x)$.

$f'(x) = f'(0) \times 0.953^x$ (where $f'(0)$ is the initial rate of decay)

By this we know that for knowing the instantaneous decay rate, $f'(0)$, the initial rate of decay, has to be measured. Because $f'(x)$ is needed to calculate $f'(0)$, it can only be acquired through experiment.

As A (total activity) is the rate of change of radioactive decay,

$A = \pm f'(x)$

[13] "Activity." *Encyclopædia Britannica*. Encyclopædia Britannica, Inc., 01 Aug. 2018. Web. 4 Dec. 2019. <https://www.britannica.com/science/activity-radioactivity>.

The gradient for a monotone decreasing curve must always be negative; therefore, the gradient of $f(x) = 0.953^x$ is negative as it is a monotone decreasing curve.

$$f'(x) < 0$$
$$\because A > 0,$$
$$\therefore A = -f'(x) = -f'(0) \times 0.953^x$$

For radioactive decay of another substance:

$$f(x) = \mathrm{c} \times (\sqrt[d]{\frac{1}{2}})^x$$

$$A = -f'(x) = c \times (\sqrt[d]{\frac{1}{2}})^x \times f'(0)$$

Now, because $f(x) = \mathrm{c} \times (\sqrt[d]{\frac{1}{2}})^x$

$$A = -f'(x) = -f'(0) \times f(x)$$

$\because -f'(0)$ is a constant

$$\therefore A \propto f(x)$$

This shows that the instant rate of decay ∝ the remaining number of atoms, given that $f(x)$ is the amount of decaying substance left. Therefore, the formula above proves the law that if a quantity decreases at a rate ∝ its current value, it experiences an exponential decay.

PROVE: $N(\mathrm{t}) = \mathrm{N}_0 e^{-\lambda t}$

Now that I have a model of my own for radioactive decay, I will link the model I built to the one accepted and see if the model that is pre-existed could be derived from the model I worked out. $f(x) = \mathrm{c} \times (\sqrt[d]{\frac{1}{2}})^x$ indicates an exponential relationship similar to the model, $N(\mathrm{t}) = \mathrm{N}_0 e^{-\lambda t}$[14]. The symbols that are used in these two models differentiate, f(x) indicates N(t), c indicates N_0, and x indicates t. Below, are the differences in the equations:

[14] Svirin, Alex. "Radioactive Decay." *Math24*. N.p., n.d. Web. 9 Dec. 2019. <https://www.math24.net/radioactive-decay/>.

Table 3: Differences between the two equations.

$f(x) = c \times (\sqrt[d]{\frac{1}{2}})^x$	$N(t) = N_0 e^{-\lambda t}$
Number of days, d, that represent the half-life is included.	Number of days that represent the half-life not included.
λ not included	Decay constant, λ, included
Base: $\sqrt[d]{\frac{1}{2}}$	Base: e

The method to find a relationship between these two functions is to solve the connection between e, T, and λ. Here, we will use the formula $t_{1/2} = \frac{\ln(2)}{\lambda}$.

Prove[15]: $N(t) = N_0 e^{-\lambda t}$

$$d = \frac{\ln(2)}{\lambda}$$

$$\therefore \ln(2) = d\lambda$$

$$\because \ln(\frac{1}{2}) = \ln(2^{-1}) = -\ln(2)$$

$$\therefore -d\lambda$$

$$\frac{1}{d} \times \ln(\frac{1}{2}) = -\lambda$$

$$\Rightarrow \ln(\frac{1}{2})^{\frac{1}{d}} = -\lambda$$

$$\because (\frac{1}{2})^{\frac{1}{d}} = \sqrt[d]{\frac{1}{2}}$$

$$\therefore \ln(\sqrt[d]{\frac{1}{2}}) = \ln(\frac{1}{2})^{\frac{1}{d}} = -\lambda$$

$$e^{-\lambda} = \sqrt[d]{\frac{1}{2}}$$

15 "Nuclear Physics Derivations." *Radioactive Decay, Nuclear Physics - Derivations from A-level Physics Tutor*. N.p., n.d. Web. 9 Dec. 2019. <https://a-levelphysicstutor.com/deriv-nuc-radioactive-decay.php>.

$\because t = x$

$\therefore (e^{-\lambda})^t = e^{-\lambda t} = (\sqrt[d]{\frac{1}{2}})^x$

$\because c = N_0,$

$\therefore c \times (\sqrt[d]{\frac{1}{2}})^x = N_0 e^{-\lambda t}$

$N(t) = N_0 \, e^{-\lambda t}$

This proves that using the exponential function y = e^x, the function $N(t) = N_0 \, e^{-\lambda t}$ can be obtained which is just an altered form of the function $f(x) = c \times (\sqrt[d]{\frac{1}{2}})^x$. The natural exponential function can be used frequently in subjects other than mathematics and science as well: in business studies, we can calculate the compound interest using the formula, $u_n = u_0 e^{rt}$, where u_0 is the principal amount, r is the interest rate, and t is the time in years.

Also, I figured out that the reason the original model, $f(x) = c \times (\sqrt[d]{\frac{1}{2}})^x$, is transformed into the natural exponential function, $N(t) = N_0 \, e^{-\lambda t}$ [16], is because the natural exponential function is comparatively easier to differentiate over all other exponential functions.

$$\frac{dN}{dt} = N_0 e^{-\lambda t} \times (-\lambda)$$
$$\frac{dN}{dt} = -\lambda N$$

When compared to the differentiation of $f(x) = c \times (\sqrt[r]{\frac{1}{2}})^x$:

$A = -f'(x) = -f'(0) \times f(x)$ (where A is the total activity)

Differentiating function $N(t) = N_0 \, e^{-\lambda t}$, and getting $\frac{dN}{dt} = -\lambda N$, is much simpler than measuring $f'(0)$ in some experiments. Also, λ is a known constant. Differentiating y = e^x into a simpler form will be helpful when I will calculate the exponential growth or decay's instant rate. This

[16] "Exponential Decay." *Exponential Decay - from Wolfram MathWorld*. Wolfram Research, Inc., n.d. Web. 10 Dec. 2019. <http://mathworld.wolfram.com/ExponentialDecay.html>.

makes me think, maybe all the supposed "scientific conventions" exist for good reasons, for example making data interpretation and modeling easier.

COLNCLUSION:

As mentioned before, I will be building a model that can predict the number of remaining radioactive atoms in the future and calculate the half-life of an atom. For this, I have used the decay pattern of V-48 and used the model, $f(x) = c \times a^x$, to predict the half-life of V-48. I was able to build a model using the original model of radioactive decay, and analyzing the uses and pattern of the decay. The model I built is,

$$f(x) = \mathrm{c} \times (\sqrt[d]{\frac{1}{2}})^x .$$

Then, I have used this model in the application of the Chernobyl disaster and showed how scientists can use this model in real life to measure the remaining amount of radioactive atoms. However, this model isn't capable of doing radioactive dating.

The formula that deals with the total activity, A, of radioactive decay is,

$$A \propto f(x)$$

This proves the proportionality of the instantaneous decay rate to the present number of radioactive atoms. I have successfully derived the model, $N(\mathrm{t}) = \mathrm{N}_0 \, e^{-\lambda t}$, from the original model

$$f(x) = \mathrm{c} \times (\sqrt[d]{\frac{1}{2}})^x .$$

As mentioned before, I converted the original model into the natural exponential form because the natural exponential form $N(\mathrm{t}) = \mathrm{N}_0 \, e^{-\lambda t}$, which can be transformed to $y=e^x$, is easier to differentiate with its use in solving instant rate.

BIBLIOGRAPHY:

- "Activity." *Encyclopædia Britannica*. Encyclopædia Britannica, Inc., 01 Aug. 2018. Web. 4 Dec. 2019. <https://www.britannica.com/science/activity-radioactivity>.
- "Exponential Decay." *Exponential Decay - from Wolfram MathWorld*. Wolfram Research, Inc., n.d. Web. 10 Dec. 2019. <http://mathworld.wolfram.com/ExponentialDecay.html>.
- "Exponential Function Reference." *Exponential Function Reference*. N.p., n.d. Web. 2 Dec. 2019. <https://www.mathsisfun.com/sets/function-exponential.html>.
- "Exponentially Decreasing Function." *From Wolfram MathWorld*. Wolfram Research, Inc., n.d. Web. 7 Nov. 2019. <https://mathworld.wolfram.com/ExponentiallyDecreasingFunction.html>.
- Khan, Mansoor, and Indra Reddy. "Pharmaceutical and Clinical Calculations." *Pharmaceutical and Clinical Calculations Ed 2.pdf*. N.p., n.d. Web. 25 Nov. 2019. <http://www.anhihs.com/up/Pharmaceutical%20and%20Clinical%20Calculations%20ed%202.PDF>.
- "Nuclear Physics Derivations." *Radioactive Decay, Nuclear Physics - Derivations from A-level Physics Tutor*. N.p., n.d. Web. 9 Dec. 2019. <https://a-levelphysicstutor.com/deriv-nuc-radioactive-decay.php>.
- OpenStax. "Physics." *Half-Life and Activity | Physics*. N.p., n.d. Web. 13 Nov. 2019. <https://courses.lumenlearning.com/physics/chapter/31-5-half-life-and-activity/>.
- "Phosphorus (P) 32 Radioactive Isotope Decay Calculator." *Phosphorus (P) 32 Isotope Decay Calculator | Calculate Radioactivity in Minerals*. N.p., n.d. Web. 26 Nov. 2019. <https://www.easycalculation.com/chemistry/P-32.html>.
- "Radioactive Decay." *Radioactive Decay*. National Science Foundation, n.d. Web. 21 Oct. 2019. <https://www.nde-ed.org/EducationResources/HighSchool/Radiography/radioactivedecay.htm>.
- Ragheb, M. "Chernobyl Accident." *Chernobyl Accident.pdf*. N.p., 13 Oct. 2018. Web. 23 Nov. 2019. <https://mragheb.com/NPRE%20457%20CSE%20462%20Safety%20Analysis%20of%20Nuclear%20Reactor%20Systems/Chernobyl%20Accident.pdf>.
- Svirin, Alex. "Radioactive Decay." *Math24*. N.p., n.d. Web. 9 Dec. 2019. <https://www.math24.net/radioactive-decay/>.
- "Use the Formula Y = Ca^x to Determine the Constants C and A." *Use the Formula Y = Ca^x to Determine the Constants C and A. | Wyzant Ask An Expert*. Wyzant, Inc., 11 Feb. 2014. Web. 10 Nov. 2019. <https://www.wyzant.com/resources/answers/26801/use_the_formula_y_ca_x_to_determine_the_constants_c_and_a>.
- "Vanadium (V) 48 Radioactive Isotope Decay Calculator." *Vanadium (V) 48 Isotope Decay Calculator | Calculate Radioactivity in Minerals*. N.p., n.d. Web. 28 Oct. 2019. <https://www.easycalculation.com/chemistry/V-48.html>.