$$\mu = \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)$$

$$\frac{dx}{dt} = \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right) x$$

$$\frac{ds}{dt} = -(\frac{1}{\gamma_{\frac{x}{s}}} \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right) + m_s)x$$

$$\frac{dp}{dt} = (K_1(\mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)) + K_2)x$$

$$\mu = \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)$$

$$\frac{dx}{dt} = \mu_m \left[\left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)\right] x - \frac{F}{V} x$$

$$\frac{ds}{dt} = -\left(\frac{1}{\gamma_{\frac{x}{s}}} \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right) + m_s\right) x + \frac{F}{V} (Sr - S)$$

$$\frac{dp}{dt} = \left[K_1 \left(\mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)\right) + K_2\right] x - \frac{F}{V} p$$

$$\frac{dV}{dt} = F$$

$$\mu = \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)$$

$$\frac{dx}{dt} = \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right) x - Dx$$

$$\frac{ds}{dt} = -\left(\frac{1}{\gamma_{\frac{x}{s}}} \mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right) + m_s\right) x + D\ (Sr - s)$$

$$\frac{dp}{dt} = \left(K_1 \left(\mu_m \left(\frac{s}{s + K_s}\right)\left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)\right) + K_2\right) x - Dp$$

$$\mu = \mu_m \left(\frac{s_1}{s_1 + K_{s1}}\right)\left(\frac{s_2}{s_2 + K_{s2}}\right)\left(\frac{(s_3)^n}{(s_3)^n + (K_{s3})^n}\right)\left(\frac{KI_1}{KI_1 + S_1}\right)\left(\frac{KI_2}{KI_2 + S_2}\right)\left(\frac{KI_3}{KI_3 + S_3}\right)$$

$$\frac{dX}{dt} = \mu X$$

$$\frac{dS_1}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{s_1}}} + m_{s1}\right)X$$

$$\frac{dS_2}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{s_2}}} + m_{s2}\right)X$$

$$\frac{dS_3}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{s_3}}} + m_{s3}\right)X$$

$$\frac{dP}{dt} = (K_1\mu + K_2)X$$

$$\mu = \mu_m \left(\frac{s_1}{s_1 + K_{s1}}\right)\left(\frac{s_2}{s_2 + K_{s2}}\right)\left(\frac{(s_3)^n}{(s_3)^n + (K_{s3})^n}\right)\left(\frac{KI_1}{KI_1 + S_1}\right)\left(\frac{KI_2}{KI_2 + S_2}\right)\left(\frac{KI_3}{KI_3 + S_3}\right)$$

$$F = F_1 + F_2 + F_3$$

$$\frac{dX}{dt} = \mu X - \left(\frac{F}{V}X\right)$$

$$\frac{dS_1}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{s_1}}} + m_{s1}\right)X + \left(\frac{F_1 S_{01}}{V}\right) - \left(\frac{FS_1}{V}\right)$$

$$\frac{dS_2}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{s_2}}} + m_{s2}\right)X + \left(\frac{F_2 S_{02}}{V}\right) - \left(\frac{FS_2}{V}\right)$$

$$\frac{dS_3}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{s_3}}} + m_{s3}\right)X + \left(\frac{F_3 S_{03}}{V}\right) - \left(\frac{FS_3}{V}\right)$$

$$\frac{dP}{dt} = (K_1\mu + K_2)X - \frac{F}{V}P \qquad \frac{dV}{dt} = Fx$$

$$r_s = r_s^{max}\left(\frac{s}{s+K_s}\right)\left(\frac{KI_1}{KI_1+P_1}\right)\left(\frac{KI_2}{KI_2+P_3}\right)$$

$$\frac{ds}{dt} = -r_s^{max}\left(\left(\frac{s}{s+K_s}\right)\left(\frac{KI_1}{KI_1+P_1}\right)\left(\frac{KI_2}{KI_2+P_3}\right)\right)x$$

$$\frac{dx}{dt} = \left(r_s Y_{\frac{x}{s}}^{max} - m_s Y_{\frac{x}{s}}^{max}\right)x$$

$$\frac{dP_1}{dt} = (K_1 r_s + m_1)x$$

$$\frac{dP_2}{dt} = (K_2 r_s + m_2)x - \frac{K_3 P_2}{P_2 + K_{s_1}}x$$

$$\frac{dP_3}{dt} = (K_4 r_s + m_3)x + \frac{K_5 P_2}{P_2 + K_{s_1}}x$$

$$\frac{dP_4}{dt} = (K_6 r_s + m_4)x + \frac{K_7 P_2}{P_2 + K_{s_1}}x$$

$$\mu = \mu_m \left(\frac{s}{s + K_s}\right)\left(\frac{K_I}{K_I + B}\right) - K_d B$$

$$\frac{dx}{dt} = \left(\mu_m \left(\frac{s}{s + K_s}\right)\left(\frac{K_I}{K_I + B}\right) - K_d B\right) x$$

$$r_s = \frac{1}{x}\frac{ds}{dt} = \left(\frac{1}{Y}\mu_m(\left(\frac{s}{s + K_s}\right)\left(\frac{K_I}{K_I + B}\right) - K_d B) + m\right)$$

$$\frac{ds}{dt} = -r_s x$$

$$\frac{dBA}{dt} = \left(K_1 r_s - K_2 \left(\frac{BA}{BA + K_{BA}}\right)\left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dAA}{dt} = \left(K_3 r_s - K_4 \left(\frac{AA}{AA + K_{AA}}\right)\left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dB}{dt} = \left(K_5 r_s + K_6 \left(\frac{BA}{BA + K_{BA}}\right)\left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dA}{dt} = \left(K_7 r_s + K_8 \left(\frac{AA}{AA + K_{AA}}\right)\left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dE}{dt} = (K_9 r_s) x$$

$$F_{s1} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{s2} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{II} = (1 - \left(\frac{L_{ac}}{L_{ac_{max}}}\right)^{n})$$

$$F_{I2} = \left(1 - \left(\frac{A_{mm}}{A_{mm_{max}}}\right)^{n1}\right)$$

$$\mu = \mu_m F_{S1} F_{S2} F_{I1} F_{I2}$$

$$K_d = K_{dmax} A_{mm}$$

$$\frac{dX_d}{dt} = K_d X_v$$

$$\frac{dX_v}{dt} = (\mu - K_d) X_v$$

$$q_{Glc} = \left(\frac{\mu}{Y_{\frac{x_v}{G_{lc}}}}\right) + mG_{lc}$$

$$\frac{dG_{lc}}{dt} = -q_{Glc} X_v$$

$$q_{Gln} = \left(\frac{\mu}{Y_{\frac{x_v}{G_{ln}}}}\right) + mG_{ln}$$

$$\frac{dG_{ln}}{dt} = -q_{Gln} X_v$$

$$q_{Lac} = \left(\frac{\mu}{Y_{\frac{x_v}{L_{ac}}}}\right)$$

$$\frac{dL_{ac}}{dt} = q_{Lac} X_v$$

$$q_{Amm} = \left(\frac{\mu}{Y_{\frac{x_v}{Amm}}}\right)$$

$$\frac{dA_{mm}}{dt} = q_{Amm} X_v$$

$$D = \frac{F}{V}$$

$$F_{s1} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{s2} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{I1} = \left(1 - \left(\frac{L_{ac}}{L_{ac_{max}}}\right)^{n}\right)$$

$$F_{I2} = \left(1 - \left(\frac{A_{mm}}{A_{mm_{max}}}\right)^{n1}\right)$$

$$\mu = \mu_m F_{S1} F_{S2} F_{I1} F_{I2}$$

$$K_d = K_{dmax} A_{mm}$$

$$\frac{dX_d}{dt} = K_d X_v - D X_d$$

$$\frac{dX_v}{dt} = (\mu - K_d) X_v - D X_v$$

$$q_{Glc} = \left(\frac{\mu}{Y_{\frac{x_v}{G_{lc}}}}\right) + m G_{lc}$$

$$\frac{dG_{lc}}{dt} = -q_{Glc} X_v + D(G_{Lcin} - G_{Lc})$$

$$q_{Gln} = \left(\frac{\mu}{Y_{\frac{x_v}{G_{ln}}}}\right) + m G_{ln}$$

$$\frac{dG_{ln}}{dt} = -q_{Gln} X_v + D(G_{Lnin} - G_{Ln})$$

$$q_{Lac} = \left(\frac{\mu}{Y_{\frac{x_v}{L_{ac}}}}\right)$$

$$\frac{dL_{ac}}{dt} = q_{Lac} X_v - D L_{ac}$$

$$q_{Amm} = \left(\frac{\mu}{Y_{\frac{x_v}{Amm}}}\right)$$

$$\frac{dA_{mm}}{dt} = q_{Amm} X_v - D A_{mm}$$

$$\frac{dV}{dt} = F$$