$$\mu = \mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_P + p}\right)$$

$$\frac{dx}{dt} = \mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_P + p}\right) x$$

$$\frac{ds}{dt} = -\left(\frac{1}{\gamma_{\frac{x}{S}}} \mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p+p}\right) + m_s\right) x$$

$$\frac{dp}{dt} = (K_1(\mu_m \left(\frac{s}{s + K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_p + p}{K_p + p}\right)) + K_2)x$$

$$\mu = \mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right)$$

$$\frac{dx}{dt} = \mu_m \left[\left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_n + p}\right) \right] x - \frac{F}{V} x$$

$$\frac{ds}{dt} = -\left(\frac{1}{\gamma_{\frac{x}{s}}} \mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p+p}\right) + m_s\right) x + \frac{F}{V} \left(Sr - S\right)$$

$$\frac{dp}{dt} = \left[K_1 \left(\mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_n+p}\right)\right) + K_2\right] x - \frac{F}{V} p$$

$$\frac{dV}{dt} = 0$$

$$\frac{dV}{dt} = F$$

$$\frac{dx}{dt} = \mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right) x - Dx$$

$$\frac{ds}{dt} = -\left(\frac{1}{\gamma_{\frac{s}{s}}} \mu_m \left(\frac{s}{s+K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_p + p}\right) + m_s\right) x + D\left(Sr - s\right)$$

 $\frac{dp}{dt} = \left(K_1 \left(\mu_m \left(\frac{s}{s + K_s}\right) \left(1 - \frac{s}{s_m}\right)^a \left(\frac{K_P}{K_D + p}\right)\right) + K_2\right) x - Dp$

 $\mu = \mu_m \left(\frac{s}{s+K} \right) \left(1 - \frac{s}{s} \right)^a \left(\frac{K_P}{K_P + n} \right)$

$$\mu = \mu_m \left(\frac{s_1}{s_1 + K_{s1}} \right) \left(\frac{s_2}{s_2 + K_{s2}} \right) \left(\frac{(s_3)^n}{(s_3)^n + (K_{s3})^n} \right) \left(\frac{KI_1}{KI_1 + S_1} \right) \left(\frac{KI_2}{KI_2 + S_2} \right) \left(\frac{KI_3}{KI_3 + S_3} \right)$$

$$\frac{dX}{dt} = \mu X$$

$$\frac{dS_1}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{s}}} + m_{s1}\right)X$$

$$dt \qquad \left(\frac{Y_{\frac{x}{S_1}}}{S_1} \right)$$

$$dS_2 \qquad \left(u \right)$$

$$\frac{dS_2}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{S_2}}} + m_{S2}\right)X$$

$$\frac{dS_3}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{S_3}}} + m_{S3}\right)X$$

$$\frac{dP}{dt} = (K_1\mu + K_2)X$$

$$\mu = \mu_m \left(\frac{s_1}{s_1 + K_{s1}} \right) \left(\frac{s_2}{s_2 + K_{s2}} \right) \left(\frac{(s_3)^n}{(s_3)^n + (K_{s3})^n} \right) \left(\frac{KI_1}{KI_1 + S_1} \right) \left(\frac{KI_2}{KI_2 + S_2} \right) \left(\frac{KI_3}{KI_3 + S_3} \right)$$

$$F = F_1 + F_2 + F_3$$

$$\frac{dX}{dt} = \mu X - \left(\frac{F}{V}X\right)$$

$$\frac{dS_1}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{S_1}}} + m_{s1}\right)X + \left(\frac{F_1S_{01}}{V}\right) - \left(\frac{FS_1}{V}\right)$$

$$\frac{dS_2}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{S_2}}} + m_{s2}\right)X + \left(\frac{F_2S_{02}}{V}\right) - \left(\frac{FS_2}{V}\right)$$

$$\frac{dS_3}{dt} = -\left(\frac{\mu}{Y_{\frac{x}{S_2}}} + m_{s3}\right)X + \left(\frac{F_3S_{03}}{V}\right) - \left(\frac{FS_3}{V}\right)$$

$$\frac{dP}{dt} = (K_1\mu + K_2)X - \frac{F}{V}P \qquad \qquad \frac{dV}{dt} = Fx$$

$$\frac{dP_1}{dt} = (K_1 r_s + m_1)x$$

$$\frac{dP_2}{dt} = (K_2 r_s + m_2)x - \frac{K_3 P_2}{P_2 + K_{s_1}}x$$

$$\frac{dP_3}{dt} = (K_4 r_s + m_3)x + \frac{K_5 P_2}{P_2 + K_s}x$$

 $r_{\rm S} = r_{\rm S}^{max} \left(\frac{\rm S}{\rm S+K}\right) \left(\frac{\rm K I_1}{\rm K I_2 + P_2}\right) \left(\frac{\rm K I_2}{\rm K I_2 + P_2}\right)$

 $\frac{ds}{dt} = -r_s^{max} \left(\left(\frac{s}{s + K_s} \right) \left(\frac{KI_1}{KI_1 + P_1} \right) \left(\frac{KI_2}{KI_2 + P_3} \right) \right) x$

 $\frac{dx}{dt} = \left(r_{S}Y_{\underline{x}}^{max} - m_{S}Y_{\underline{x}}^{max}\right)x$

 $\frac{dP_4}{dt} = (K_6 r_s + m_4) x + \frac{K_7 P_2}{P_2 + K_3} x$

$$\mu = \mu_m \left(\frac{s}{s + K_s}\right) \left(\frac{K_I}{K_I + B}\right) - K_d B$$

$$\frac{dx}{dt} = \left(\mu_m \left(\frac{s}{s + K_s}\right) \left(\frac{K_I}{K_I + B}\right) - K_d B\right) x$$

$$\frac{dx}{dt} = \left(\mu_m \left(\frac{s}{s+K_s}\right) \left(\frac{K_I}{K_I+B}\right) - K_d B\right) x$$

$$r_s = \frac{1}{x} \frac{ds}{dt} = \left(\frac{1}{Y} \mu_m \left(\frac{s}{s+K_s}\right) \left(\frac{K_I}{K_I+B}\right) - K_d B\right) + m\right)$$

$$r_{s} = \frac{1}{x} \frac{ds}{dt} = \left(\frac{1}{Y} \mu_{m} \left(\left(\frac{s}{s + K_{s}} \right) \left(\frac{r_{I}}{K_{I} + B} \right) - K_{d}B \right) + m \right)$$

$$\frac{ds}{dt} = -r_{s}x$$

$$\frac{dBA}{dt} = \left(K_1 r_s - K_2 \left(\frac{BA}{BA + K_{BA}}\right) \left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dAA}{dt} = \left(K_1 r_s - K_2 \left(\frac{AA}{BA + K_{BA}}\right) \left(\frac{K_I}{K_I}\right)\right) x$$

$$\frac{dAA}{dt} = \left(K_3 r_s - K_4 \left(\frac{AA}{AA + K_{AA}}\right) \left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dAA}{dt} = \left(K_3 r_s - K_4 \left(\frac{AA}{AA + K_{AA}}\right) \left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dB}{dt} = \left(K_5 r_s + K_6 \left(\frac{BA}{BA + K_{BA}}\right) \left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dA}{dt} = \left(K_7 r_s + K_8 \left(\frac{AA}{AA + K_{AA}}\right) \left(\frac{K_I}{K_I + B}\right)\right) x$$

$$\frac{dE}{dt} = (K_0 r_s) x$$

$$\frac{dE}{dt} = (K_9 r_s) x$$

$$F_{s1} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{s2} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{II} = \left(1 - \left(\frac{L_{ac}}{L_{ac_{max}}}\right)^{n}\right)$$

$$F_{I2} = \left(1 - \left(\frac{A_{mm}}{A_{mm_{max}}}\right)^{n1}\right)$$

$$\mu = \mu_{m} F_{s1} F_{s2} F_{I1} F_{I2}$$

$$K_{d} = K_{dmax} A_{mm}$$

$$\frac{dX_{d}}{dt} = K_{d} X_{v}$$

$$\frac{dX_{v}}{dt} = (\mu - K_{d}) X_{v}$$

 $q_{Glc} = \left(\frac{\mu}{Y_{\frac{x_{v}}{C}}}\right) + mG_{lc}$

 $\frac{dG_{ln}}{dt} = -q_{Gln} X_{v}$ $q_{Lac} = \left(\frac{\mu}{Y_{x_{v}}}\right)$

 $\frac{aL_{ac}}{A_{L}} = q_{Lac} X_{v}$

 $q_{Amm} = \left(\frac{\mu}{Y_{\underline{x_v}}}\right)$

 $\frac{dA_{mm}}{dt} = q_{Amm} X_v$

 $\frac{dG_{lc}}{dt} = -q_{Glc} X_v$

 $q_{Gln} = \left(\frac{\mu}{Y_{\frac{x_{v}}{G_{l}}}}\right) + mG_{ln}$

$$F_{s1} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{s2} = \frac{G_{Lc}}{K_{GLC} + G_{LC}}$$

$$F_{II} = \left(1 - \left(\frac{L_{ac}}{L_{ac_{max}}}\right)^{n}\right)$$

$$F_{I2} = \left(1 - \left(\frac{A_{mm}}{A_{mm_{max}}}\right)^{n1}\right)$$

$$\mu = \mu_{m} F_{S1} F_{S2} F_{I1} F_{I2}$$

$$K_{d} = K_{dmax} A_{mm}$$

 $D = \frac{F}{V}$

$$\left(\frac{A_{mm}}{A_{mm_{max}}}\right)^{n1}$$

$$\left(1 + F_{S2}F_{I1}F_{I2}\right)$$

$$K_d = K_{dmax} A_{mm}$$

$$\frac{K_d}{t} = K_d X_v - DX_d$$

 $q_{Glc} = \left(\frac{\mu}{Y_{\frac{x_v}{G}}}\right) + mG_{lc}$

$$K_d = K_{dmax} A_{mm}$$

$$\frac{dX_d}{dt} = K_d X_v - DX_d$$

$$= \mu_m F_{S1} F_{S2} F_{I1} F_{I2}$$

$$K_d = K_{dmax} A_{mm}$$

$$\frac{X_d}{t} = K_d X_v - DX_d$$

$$\frac{1}{d} = K_{dmax} A_{mm}$$

$$\frac{1}{d} = K_d X_v - DX_d$$

$$K_d = K_{dmax} A_{mm}$$

$$\frac{dX_d}{dt} = K_d X_v - DX_d$$

$$\frac{dX_v}{dt} = (\mu - K_d) X_v - DX_v$$

$$K_d(X_v - DX_d)$$

$$\frac{dG_{ln}}{dt} = -q_{Gln}X_v + D(G_{Lnin} - G_{Ln})$$
$$q_{Lac} = \left(\frac{\mu}{\frac{Y_{x_v}}{L_{ac}}}\right)$$

$$q_{Lac} = \left(\frac{\mu}{Y_{\frac{x_{v}}{L_{ac}}}}\right)$$

 $\frac{dV}{dt} = F$

 $\frac{dG_{lc}}{dt} = -q_{Glc}X_v + D(G_{Lcin} - G_{Lc})$

 $q_{Gln} = \left(\frac{\mu}{Y_{\frac{x_v}{G}}}\right) + mG_{ln}$

$$\frac{dL_{ac}}{dt} = q_{Lac}X_v - DL_{ac}$$

$$q_{Amm} = \left(\frac{\mu}{Y_{\frac{x_v}{Amm}}}\right)$$

$$\left(\frac{\mu}{x_v}\right)$$

$$\left(\frac{\mu}{x_v}\right)$$

$$\left(\frac{x_v}{Amm}\right)$$

$$\left(\frac{x_v}{Amm}\right)$$

$$\overline{Amm}$$
/
 $X_v - DA_{mm}$

$$\frac{\overline{Amm}}{Amm}$$
/ $X_{ij} - DA_{mm}$

$$\frac{dA_{mm}}{dt} = q_{Amm} X_v - DA_{mm}$$

$$X_v - DA_{mm}$$