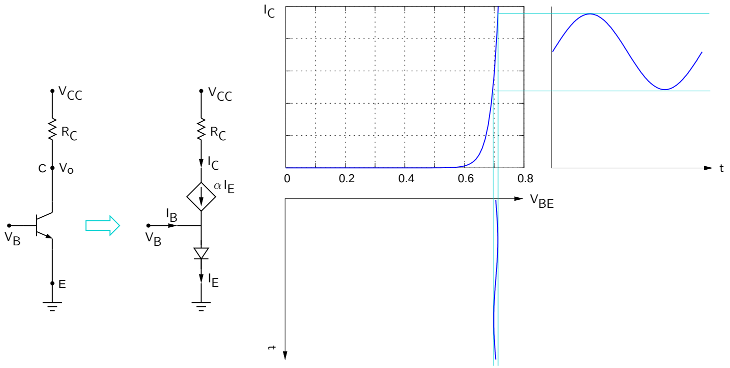
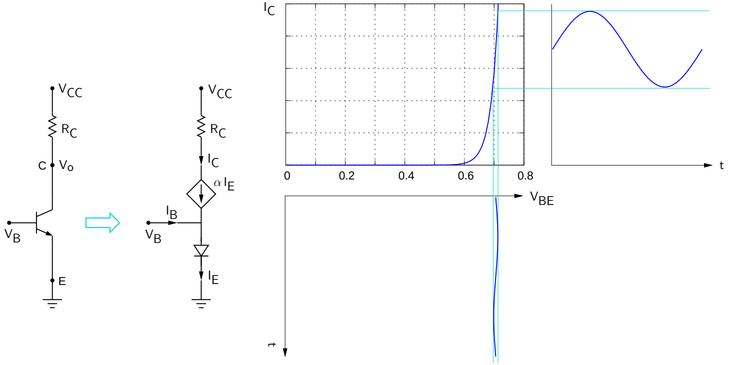
EE101: BJT circuits (Part 1)

M. B. Patil
mbpatil@ee.iitb.ac.in

Department of Electrical Engineering
Indian Institute of Technology Bombay

BJT amplifier: basic operation





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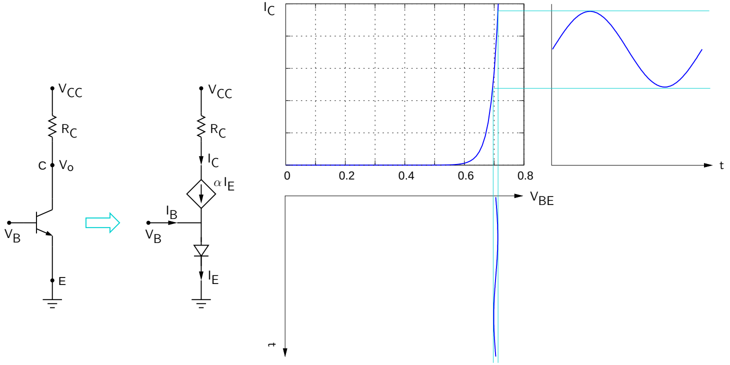

- In the active mode, I_C changes exponentially with V_{BE}.
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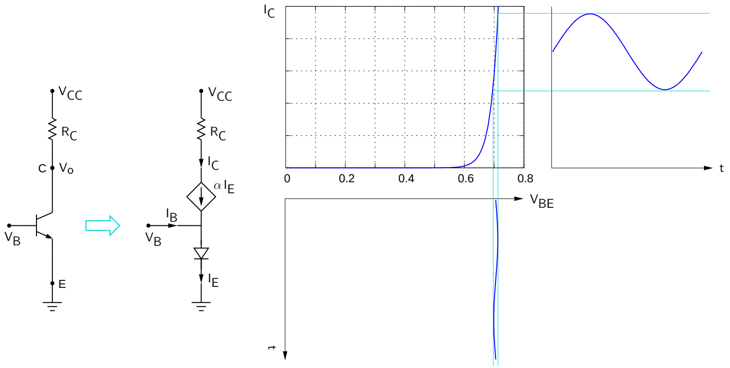

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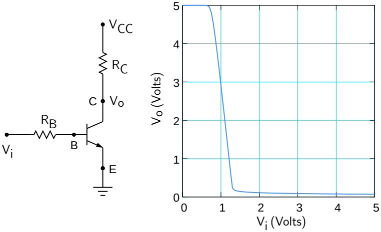
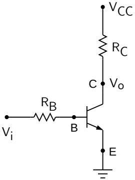
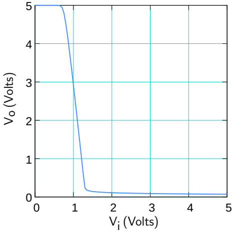
* Note that both the input (V_{BE}) and output (V_o) voltages have DC ("bias") components.



Consider a more realistic BJT amplifier circuit, with R_B added to limit the base current (and thus protect the transistor).

BJT amplifier biasing: why?





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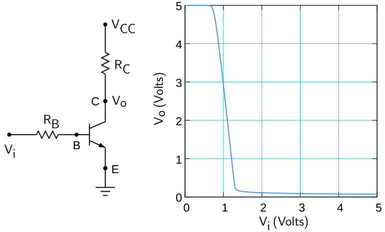

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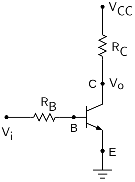

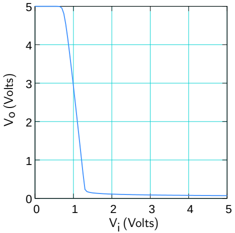

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- Further, to get a large swing in V_o without distortion, the DC bias of V_i should be at the centre of the amplifying region, i.e., $V_i \approx 1\ V$.

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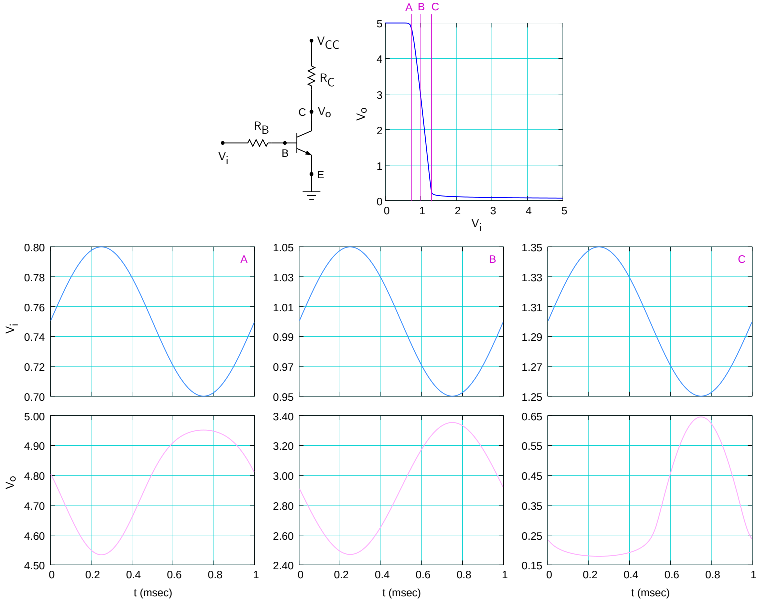
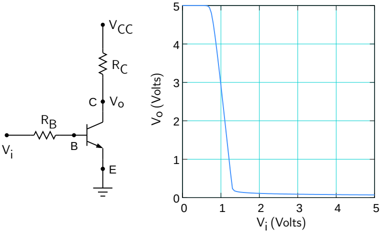


(SEQUEL file: `ee101_bjt_amp1.sqproj`)



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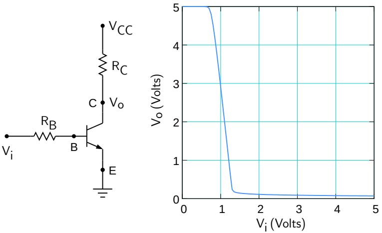
BJT amplifier



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  - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of I_C.

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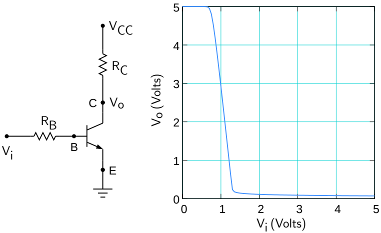

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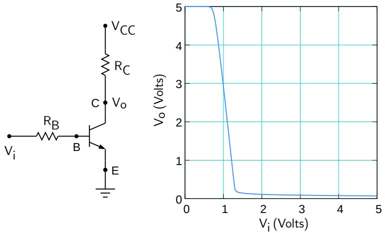

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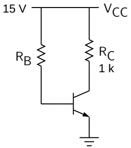
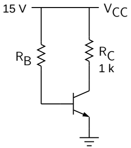
  - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of I_C.
  - mixing the input DC bias with the signal voltage.
* The first issue is addressed by using a suitable biasing scheme, and the second by using "coupling" capacitors.

BJT amplifier: a simple biasing scheme



"Biasing" an amplifier ⇒ selection of component values for a certain DC value of I_C (i.e., when no signal is applied).



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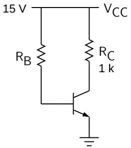

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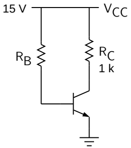

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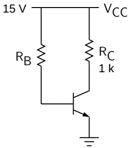

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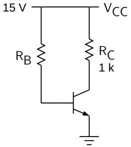
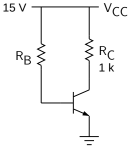
$$\rightarrow R_B = \frac{14.3\,V}{33\,\mu A} = 430\,\text{k}\Omega\,.$$



With $R_B = 430\,\text{k}$, we expect $I_C = 3.3\,\text{m}A$, assuming $\beta = 100$.



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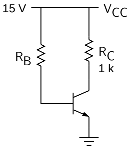

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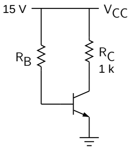
BJT amplifier: a simple biasing scheme (continued)



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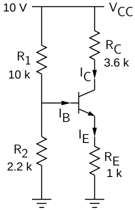
$\Rightarrow$ need a biasing scheme which is not so sensitive to β.

BJT amplifier: improved biasing scheme



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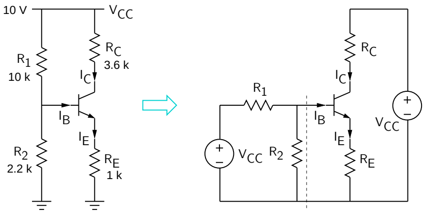

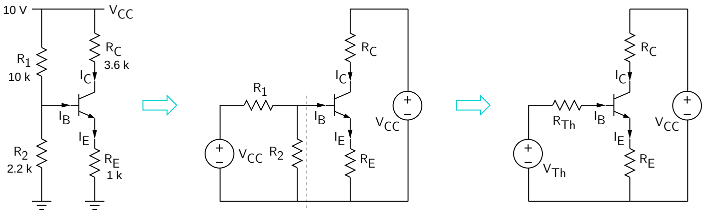
10 V
VCC
R1
10 k
RC
3.6 k
IC
IB
IE
R2
2.2 k
RE
1 k
R1
RC
IC
IB
IE
VCC
VCC
R2
RE
RC
IC
RTh
IB
IE
VCC
VTh
RE

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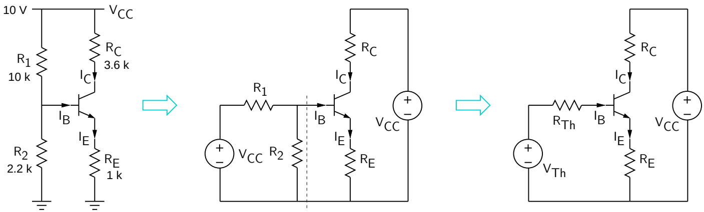

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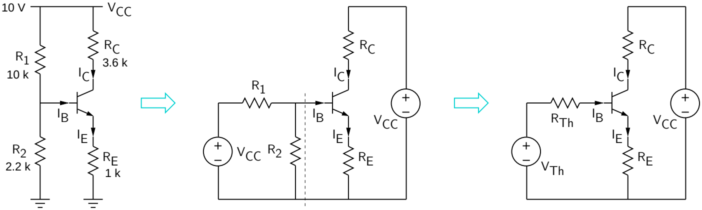

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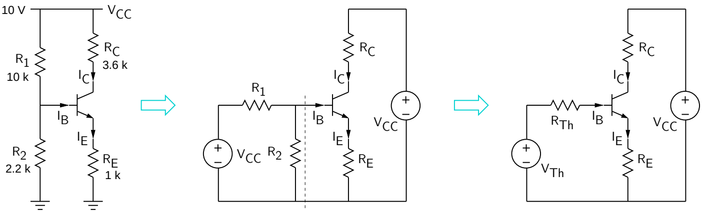

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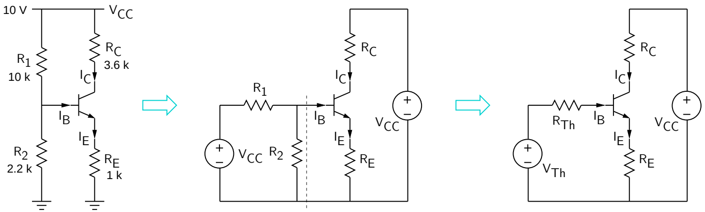

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For $\beta = 100$, I_C=1.07 mA.

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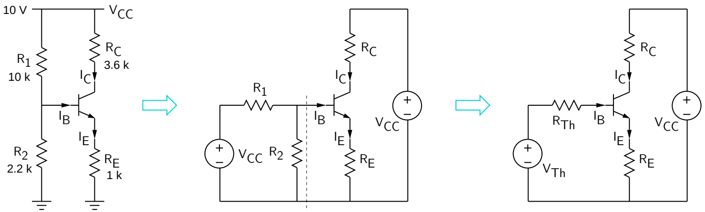

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For $\beta = 200$, I_C=1.085 mA.

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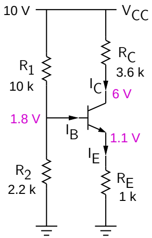

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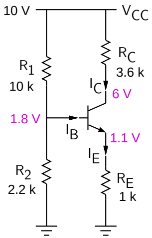

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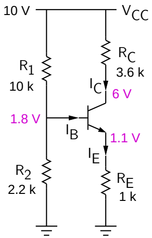

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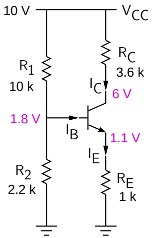

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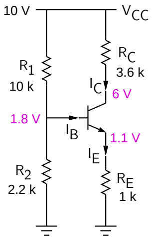

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$V_{CE} = V_C - V_E = 6 - 1.1 = 4.9\,V$.

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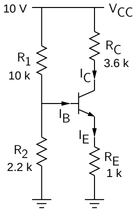

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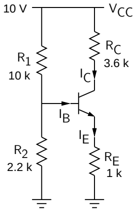

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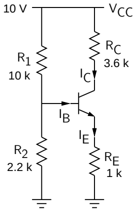
BJT amplifier: improved biasing scheme (continued)



A quick estimate of the bias values can be obtained by ignoring I_B (which is fair if β is large). In that case,

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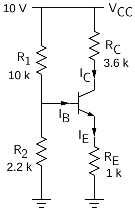

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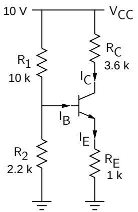

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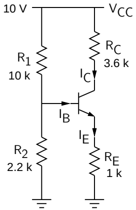

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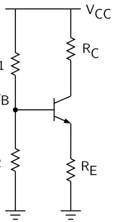
$$V_{CE} = V_{CC} - I_C\, R_C - I_E\, R_E = 10\; V - (3.6\,\text{k} \times 1.1\,\text{m}A) - (1\,\text{k} \times 1.1\,\text{m}A) \approx 5\; V.$$

Adding signal to bias



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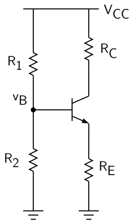

* As we have seen earlier, the input signal $v_s(t) = \hat{V}\,\sin\,\omega t$ (for example) needs to be mixed with the desired bias value V_B so that the net voltage at the base is $v_B(t) = V_B + \hat{V}\,\sin\,\omega t$.

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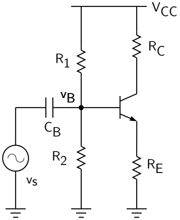

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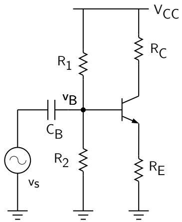

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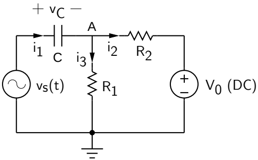
* Let us take a simple circuit to illustrate how a coupling capacitor works.

Coupling capacitor: example



Let v_A be the instantaneous node voltage at A. KCL gives,

$$-i_1 + i_3 + i_2 = 0 \rightarrow -C\,\frac{dv_C}{dt} + \frac{v_A}{R_1} + \frac{v_A - V_0}{R_2} = 0.$$

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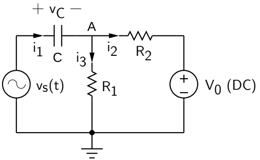

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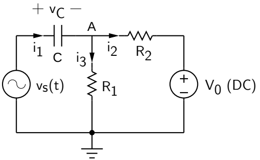

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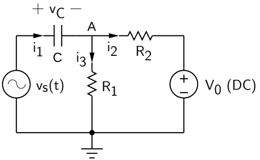

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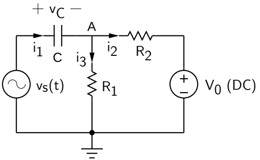

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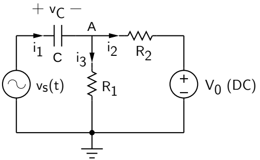

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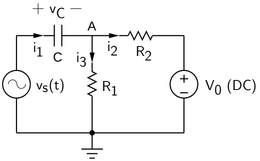
Coupling capacitor: example (continued)



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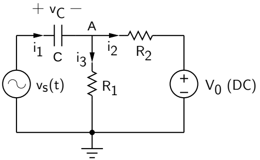

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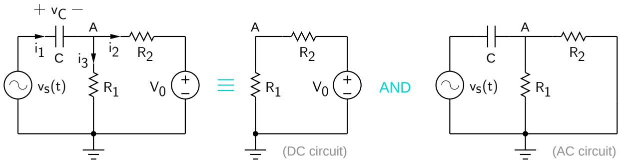
In other words, the original circuit can be thought of as,



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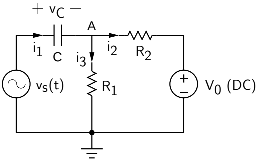

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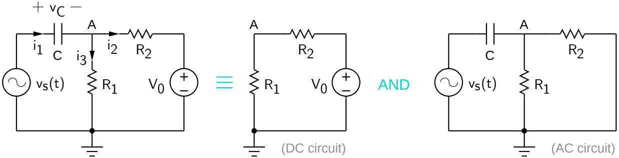

We can get V_A from the first circuit, $v_a(t)$ from the second, and then combine them to get the actual $v_A(t)$: $v_A(t) = V_A + v_a(t)$

Capacitors in an amplifier circuit

* Split the original circuit into two circuits:
  - DC circuit: replace each capacitor with an open circuit.
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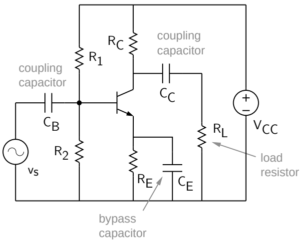
* The procedure described above also applies to a non-linear circuit such as a BJT amplifier. We have already looked at DC analysis of an amplifier; we will now look at how to handle the time-dependent (AC) part.

Common-emitter amplifier



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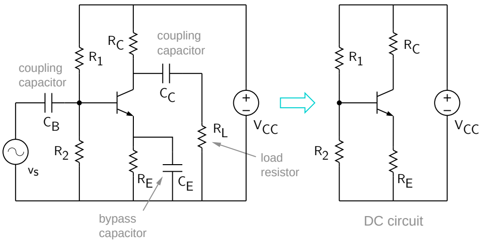

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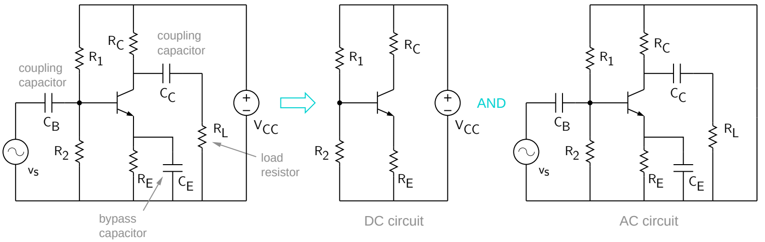

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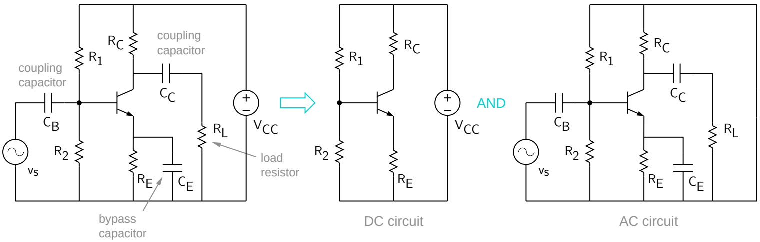
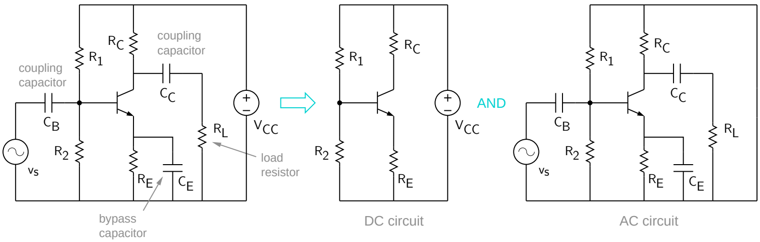


* The coupling capacitors ensure that the singal source and the load resistor do not affect the DC bias of the amplifier. (We will see the purpose of C_E a little later.)

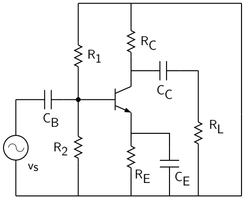


DC circuit

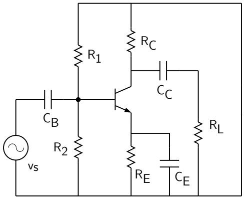
AC circuit

* The coupling capacitors ensure that the singal source and the load resistor do not affect the DC bias of the amplifier. (We will see the purpose of C_E a little later.)
* This enables us to bias the amplifier without worrying about what load it is going to drive.

Common-emitter amplifier: AC circuit



Common-emitter amplifier: AC circuit



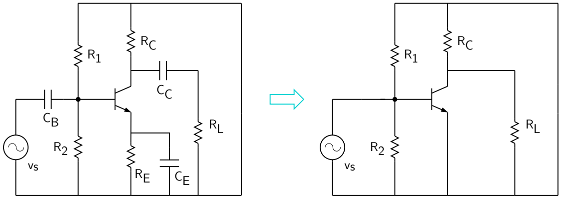
* The coupling and bypass capacitors are "large" (typically, a few μF), and at frequencies of interest, their impedance is small.

  For example, for $C = 10\,\mu F$, $f = 1\,\text{kHz}$,

  $$Z_C = \frac{1}{2\pi \times 10^3 \times 10 \times 10^{-6}} = 16\,\Omega,$$

  which is much smaller than typical values of R_1, R_2, R_C, R_E (a few kΩ).

  $\Rightarrow$ C_B, C_C, C_E can be replaced by short circuits at the frequencies of interest.



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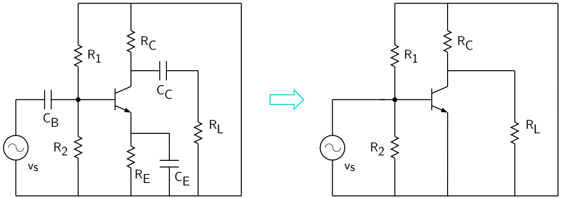

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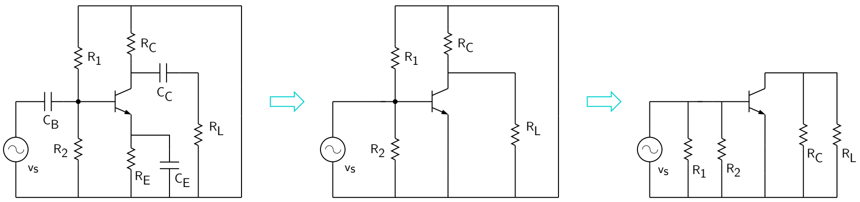
* The circuit can be re-drawn in a more friendly format.

Common-emitter amplifier: AC circuit



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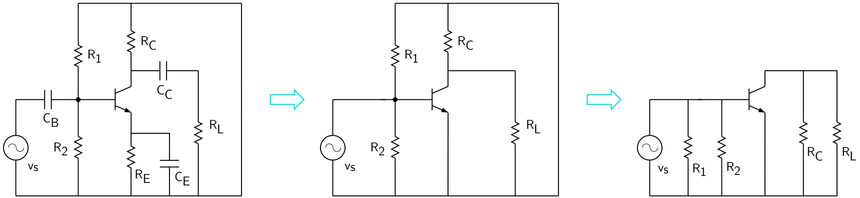

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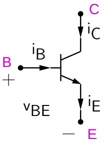
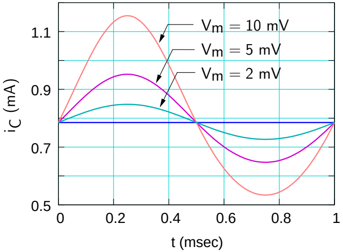
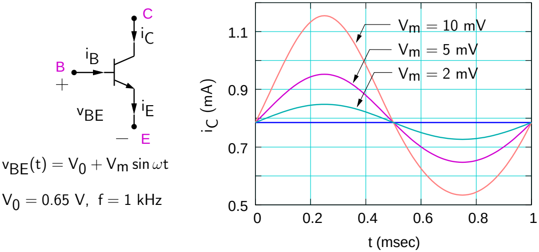
* The circuit can be re-drawn in a more friendly format.
* We now need to figure out the AC description of a BJT.



$v_{BE}(t) = V_0 + V_m \sin \omega t$

$V_0 = 0.65$ V, $f = 1$ kHz





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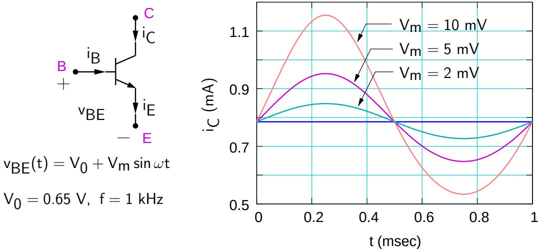
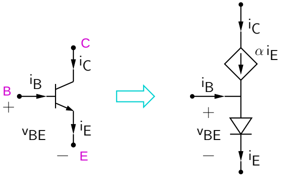
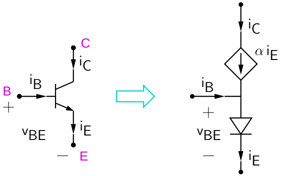


* As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid $\rightarrow$ distortion.
* If $v_{be}(t)$, i.e., the time-varying part of v_{BE}, is kept small, i_C varies linearly with v_{BE}. How small? Let us look at this in more detail.

BJT: small-signal model



Let $v_{BE}(t) = V_{BE} + v_{be}(t)$ (bias+signal), and $i_C(t) = I_C + i_c(t)$.



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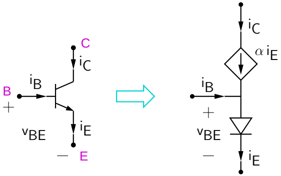
Assuming active mode, $i_C(t) = \alpha\, i_E(t) = \alpha\, I_{ES} \left[\exp\left(\frac{v_{BE}(t)}{V_T}\right) - 1\right]$.



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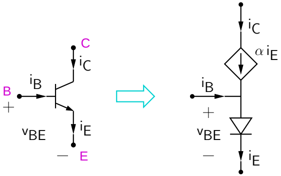
Since the B-E junction is forward-biased, $\exp\left(\dfrac{v_{BE}(t)}{V_T}\right) \gg 1$, and we get

$$\begin{aligned} i_C(t) &= \alpha\, I_{ES}\, \exp\left(\frac{v_{BE}(t)}{V_T}\right) = \alpha\, I_{ES}\, \exp\left(\frac{V_{BE} + v_{be}(t)}{V_T}\right) \\ &= \alpha\, I_{ES}\, \exp\left(\frac{V_{BE}}{V_T}\right) \times \exp\left(\frac{v_{be}(t)}{V_T}\right) \end{aligned}$$

BJT: small-signal model



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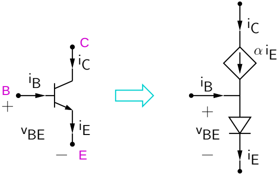
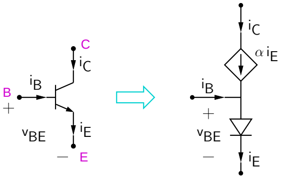
If $v_{be}(t) = 0$, $i_C(t) = I_C$ (the bias value of i_C), i.e.,

$I_C = \alpha\, I_{ES}\,\exp\left(\dfrac{V_{BE}}{V_T}\right) \Rightarrow i_C(t) = I_C\,\exp\left(\dfrac{v_{be}(t)}{V_T}\right)$.

BJT: small-signal model



$$i_C(t) = I_C \, \exp\left(\frac{v_{be}(t)}{V_T}\right) = I_C \, \left[1 + x + x^2 + \cdots\right], \quad x = v_{be}(t)/V_T\,.$$



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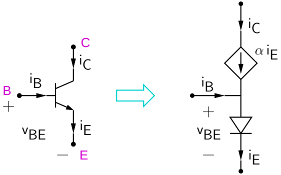
If x is small, i.e., if the amplitude of $v_{be}(t)$ is small compared to the thermal voltage V_T, we get

$$i_C(t) = I_C \, \left[1 + \frac{v_{be}(t)}{V_T}\right].$$



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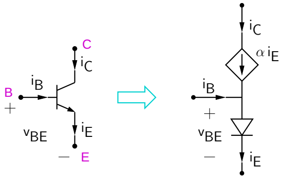

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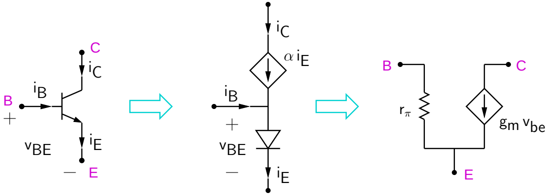
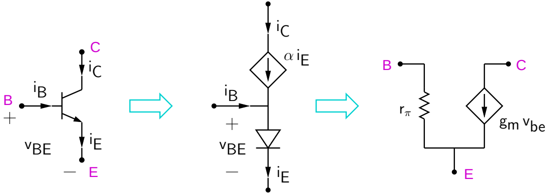
$$i_C(t) = I_C + i_c(t) = I_C \, \left[1 + \frac{v_{be}(t)}{V_T}\right] \Rightarrow \boxed{i_c(t) = \frac{I_C}{V_T}\, v_{be}(t)}$$

BJT: small-signal model



The relationship, $i_c(t) = \dfrac{I_C}{V_T}\, v_{be}(t)$ can be represented by a VCCS, $i_c(t) = g_m\, v_{be}(t)$, where $g_m = I_C/V_T$.



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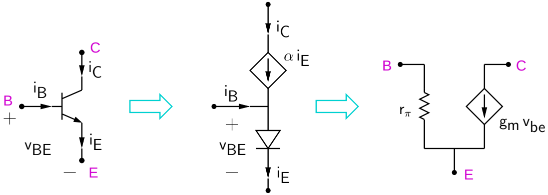
For the base current, we have,

$$i_B(t) = I_B + i_b(t) = \frac{1}{\beta}\, [I_C + i_c(t)]$$

$$\rightarrow i_b(t) = \frac{1}{\beta}\, i_c(t) = \frac{1}{\beta}\, g_m\, v_{be}(t) \rightarrow v_{be}(t) = (\beta / g_m)\, i_b(t).$$

BJT: small-signal model



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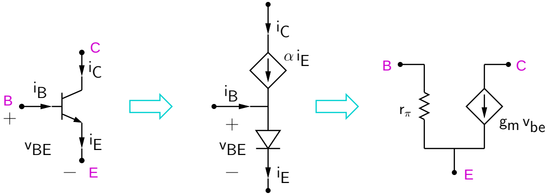
The above relationship is represented by a resistance, $r_\pi = \beta/g_m$, connected between B and E.

BJT: small-signal model



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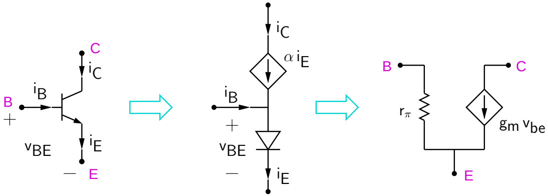
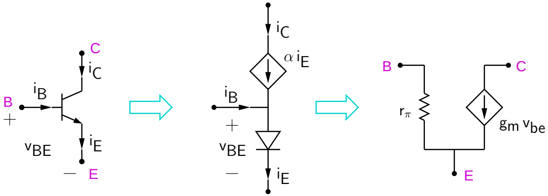
The resulting model is called the π-model for small-signal description of a BJT.



* The transconductance g_m depends on the biasing of the BJT, since $g_m = I_C/V_T$. For $I_C = 1\,\mathrm{m}A$, $V_T \approx 25\,\mathrm{m}V$ (room temperature), $g_m = 1\,\mathrm{m}A/25\,\mathrm{m}V = 40\,\mathrm{m}\mho$.



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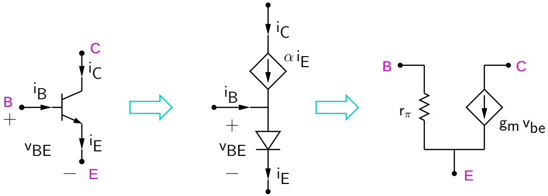
* r_π also depends on I_C, since $r_\pi = \beta/g_m = \beta\, V_T/I_C$. For $I_C = 1\,\mathrm{m}A$, $V_T \approx 25\,\mathrm{m}V$, $\beta = 100$, $r_\pi = 2.5\,\mathrm{k}\Omega$.

BJT: small-signal model



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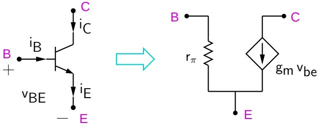
* Note that the small-signal model is valid only for small v_{be} (compared to V_T), as we have seen earlier.

BJT: small-signal model



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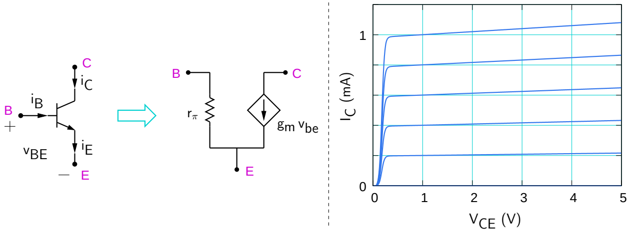
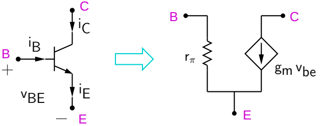
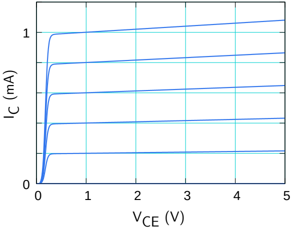
BJT: small-signal model



* In the above model, note that i_c is independent of v_{ce}.
* In practice, i_c does depend on v_{ce} because of the Early effect, and $\dfrac{dI_C}{dV_{CE}} \approx$ constant $= 1/r_o$, where r_o is called the output resistance.

BJT: small-signal model





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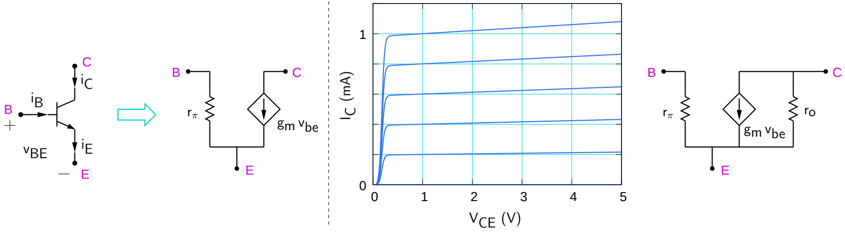
- A more accurate model includes r_o as well.

BJT: small-signal model



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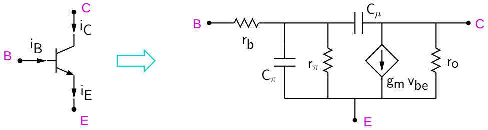
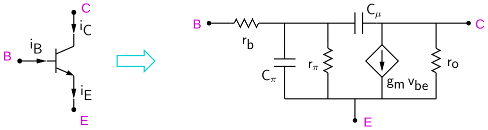


* A few other components are required to make the small-signal model complete:
  r_b: base spreading resistance
  C_π: base charging capacitance + B-E junction capacitance
  C_μ: B-C junction capacitance



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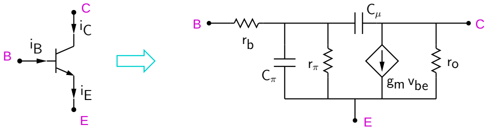

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* Note that the small-signal models we have described are valid in the active region only.