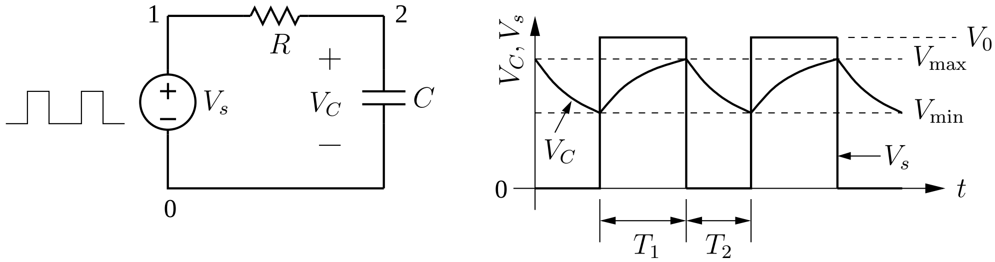
ee101_rc1b.sqproj

Description



The RC circuit shown in the figure is driven by a clock, with T_1 and T_2 as the high and low interval, respectively (and period $T = T_1 + T_2$). Show that the following results hold in the steady state:

(a) $V_{\max} = V_0 \dfrac{1-k_1}{1-k_0}$, $V_{\min} = k_2\, V_{\max}$, where $k_1 = e^{-T_1/\tau}$, $k_2 = e^{-T_2/\tau}$, $k_0 = k_1\, k_2$, $\tau = R\, C$.
Hint: Obtain $V_C(t)$ in the T_1 and T_2 intervals, use the condition of periodicity of V_C in the steady state.

(b) The average value of V_C is the same as the average value of V_s. i.e.,
$\dfrac{1}{T}\displaystyle\int_0^T V_s\, dt = \dfrac{1}{T}\displaystyle\int_0^T V_C\, dt.$
Hint: write KVL for the circuit and integrate.

Exercise Set

1. For $R = 1\,\text{k}$, $C = 1\,\mu F$, $T = 2\,\text{ms}$, simulate the circuit for different values of T_1 and T_2 (but keeping the period T the same), e.g., ($T_1 = 1\,\text{ms}$, $T_2 = 1\,\text{ms}$), ($T_1 = 0.2\,\text{ms}$, $T_2 = 1.8\,\text{ms}$), ($T_1 = 0.5\,\text{ms}$, $T_2 = 1.5\,\text{ms}$), etc. In each case, compare the simulation result with the expressions given above.

2. Derive an expression for the current $i(t)$ in steady state. For the conditions in (1), validate your analytic result with simulation.

3. For ($T_1 = 0.5\,\text{ms}$, $T_2 = 1.5\,\text{ms}$), work out the minimum and maximum values of V_C for the following combinations:

(i) $R = 1\,\mathrm{k}\Omega$, $C = 0.2\,\mu F$.

(ii) $R = 0.2\,\mathrm{k}\Omega$, $C = 1\,\mu F$.

(iii) $R = 0.2\,\mathrm{k}\Omega$, $C = 0.2\,\mu F$.

(iv) $R = 5\,\mathrm{k}\Omega$, $C = 5\,\mu F$.

Compare your values with simulation results.

4. Repeat for the current $i(t)$.