Amplifiers

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Chapter 1

Amplifiers

An amplifier essentially takes a small signal as its input and presents a larger replica of the input at its output. This obviously requires some active device, which could be a bipolar junction transistor (BJT), a Junction Field Effect Transistor (JFET), a Metal Oxide Semiconductor Field Effect Transistor (MOSFET) or any other amplifying device.

The amplifying device accepts inputs at one pair of terminals and provides the output at another pair of terminals. However, most amplifying devices like bipolar transistors, JFETs or MOSFETs have 3 terminals. Thus one of the terminals has to be common between the input and the output. Amplifier configurations are often named after this common terminal. For example, using bipolar transistors, we can make common emitter, common base or common collector amplifiers. The corresponding versions using JFETs or MOSFETs are called common source, common gate or common drain configurations.

In fact we can use combinations of devices instead of a single device, as we shall see later in case of cascode and Darlington amplifiers. We shall not worry about the functioning of the devices themselves in this chapter. All we need is that the current-voltage relationship of the device or combination of devices in terms of terminal voltages.

We shall assume that small signal analysis is valid. In our terminology, capital letters like V and I will denote absolute quantities like voltage and Current, whereas the corresponding lower case letters like v and i will denote the *fluctuation* in these quantities.

1.1 A generic amplifier

Consider a generic amplifier as shown below. The black box in the figure could be any amplifying device. In fact, it could be multiple devices connected in some way. All we need is that the current sourced at the output by the black box should be known in terms of the terminal

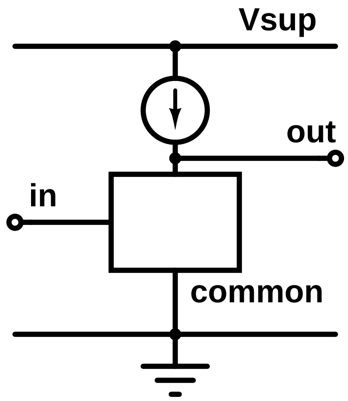


Figure 1.1: A Generic Amplifier

voltages. For this case, we assume that the device output terminal is connected to a current source load, as shown.

The current through the device is:

$$I_{out} = f(V_{in}, V_{out})$$

Then the incremental change in this current is

$$dI_{out} = \frac{\partial I_{out}}{\partial V_{in}} dV_{in} + \frac{\partial I_{out}}{\partial V_{out}} dV_{out}$$

We define the following terms:

$$g_m \equiv \tfrac{\partial I_{out}}{\partial V_{in}} \qquad (= \frac{\mathrm{d}I_{out}}{\mathrm{d}V_{in}} \text{ with } \mathrm{d}V_{out} = 0) \tag{1.1}$$

$$g_o \equiv \tfrac{\partial I_{out}}{\partial V_{out}} \qquad (= \frac{\mathrm{d}I_{out}}{\mathrm{d}V_{out}} \text{ with } \mathrm{d}V_{in} = 0) \tag{1.2}$$

Then we can write the fluctuation in output current, using small signal terminology as

$$i_{out} = g_m v_{in} + g_o v_{out} \tag{1.3}$$

Since the black box is fed by a current source, it does not permit any variation in the output current. Therefore, $dI_{out} = 0$. So,

$$0 = g_m v_i + g_o v_o \tag{1.4}$$

Hence, we can write for the voltage gain (A_o) of the stage,

$$A_o = \frac{v_o}{v_i} = -\frac{g_m}{g_o} = -g_m r_o \tag{1.5}$$

Where r_o, the output resistance of the stage is defined to be $1/g_o$. g_m and g_o depend on the transistor characteristics.

If the amplifying device is fed by a resistor instead of a current source, we cannot assert that $dI_{out} = 0$. When a resistor R_L is used as the load, the output voltage *decreases* by an amount $= R_L dI_{out}$ when the device current *increases* by dI_{out}.

$$v_{out} = -R_L i_{out} \qquad \text{So} \qquad i_{out} = -\frac{v_{out}}{R_L} \tag{1.6}$$

Combining this with eq.1.3,

$$-\frac{v_{out}}{R_L} = g_m v_{in} + g_o v_{out} \qquad \text{or} \qquad 0 = g_m v_{in} + \left(g_o + \frac{1}{R_L}\right) v_{out}$$

Since $g_o \equiv 1/r_o$,

$$0 = g_m v_{in} + \left(\frac{1}{r_o} + \frac{1}{R_L}\right) v_{out} = g_m v_{in} + \left(\frac{r_o + R_L}{r_o R_L}\right) v_{out}$$

$$v_{out} = -g_m v_{in} \frac{r_o R_L}{r_o + R_L} \tag{1.7}$$

Thus the voltage gain in the resistive load case is given by

$$A_o = \frac{v_o}{v_i} = -g_m \frac{r_o R_L}{r_o + R_L} \tag{1.8}$$

This is the same as the gain with a current source load if we replace r_o by a parallel combination of r_o with R_L.

1.2 Bipolar amplifiers

If the active device is a bipolar transistor, we can derive its g_m and g_o values from its characteristics.

1.2.1 Small signal characteristics of a bipolar transistor

For a bipolar transistor, the emitter current is given by

$$I_E = I_0 \left(e^{qV_{BE}/KT} - 1\right) \simeq I_0 e^{qV_{BE}/KT}$$

The emitter current is the sum of base and collector current.

$$I_E = I_B + I_C = I_B + \beta I_B = (1+\beta) I_B \qquad \text{Therefore} \qquad I_B = I_E/(1+\beta)$$

Small signal fluctuation in the emitter current is given by

$$i_E \equiv dI_E = \frac{qI_0}{KT} e^{qV_{BE}/KT} dV_{BE} \simeq \frac{qI_E}{KT} dV_{BE} \tag{1.9}$$

Since $I_B = I_E/(1+\beta)$, we must have $\mathrm{d}I_B = \mathrm{d}I_E/(1+\beta)$. Therefore,

$$i_B \equiv dI_B = \frac{dI_E}{1+\beta} = \frac{qI_E}{(1+\beta)KT} dV_{BE} = \frac{qI_B}{KT} v_B \tag{1.10}$$

Thus, for small signal analysis purposes, it appears that the input at the base sees a resistor given by

$$r_\pi = \frac{KT}{qI_B} \tag{1.11}$$

This base resistor is a *small signal* resistor – hence the lower case 'r'. Its value depends on the DC value of I_B, represented by upper case I.
The simple equivalent small signal equivalent of a bipolar transistor that we derive using this resistor is called the "Pi" model of a bipolar transistor. Hence the subscript π with r. The ohmic resistance associated with the base contact will come in series with it. This resistance is designated as r_B. It is not uncommon to club r_B and r_π into a single resistor.

1.2.2 Early Voltage

Up to now, we have assumed that the collector current is independent of the collector voltage. In fact, the collector current does have a linear dependence on the collector voltage. As the

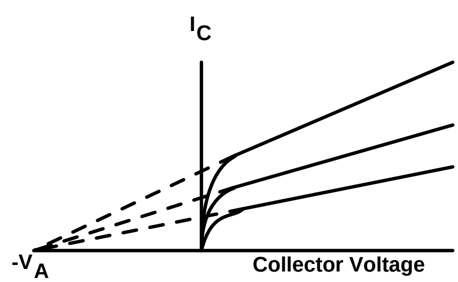


Figure 1.2: Early Voltage

collector voltage is increased, the depletion width of the reverse biased collector-base junction extends into the base, making it narrower, which increase the current. The figure shows a plot of the collector current versus collector voltage for different values of base current. (The slope is rather exaggerated in this figure).

If we extend the I-V characteristics towards negative collector voltages, all of these meet at a single point on the negative collector voltage axis. This point is called the Early voltage.

The linear dependence of I_C on V_C can be represented by a small signal resistor r_o connected between the collector and the emitter in the small signal model. The value of r_o is derived from the Early voltage. From fig.1.2 it can be seen that the slope of the collector current-voltage characteristics is given by $\approx I_C/V_A$ where V_A is the Early voltage.

1.2.3 The Pi model for a bipolar transistor

Thus, the small signal equivalent circuit of the bipolar transistor is as shown below:
Here

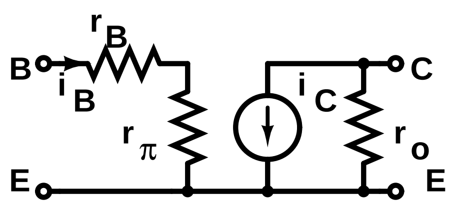


Figure 1.3: Small signal equivalent circuit of the bipolar transistor

$$r_\pi = \frac{KT}{qI_B}, \qquad i_B = \frac{v_B}{r_B + r_\pi}, \qquad i_C = \beta i_B$$

This model is referred to as the "Pi" model, because of its shape.

1.3 Bipolar common emitter configuration

The common emitter configuration is perhaps the most frequently used configuration of bipolar transistor amplifiers.

In the common emitter configuration, the emitter is common to the input and the output. Thus the input is applied between base and emitter, while the output is taken between the collector and the emitter. Since the emitter is grounded, d$V_E = 0$. Therefore, $dV_{BE} = dV_B \equiv v_B$. By definition,

$$g_m = \frac{\partial I_C}{\partial V_B} \,|_{const. V_C}$$

$$g_m = \frac{\beta i_B}{v_B} = \frac{\beta}{r_B + r_\pi} = \frac{\beta}{r_B + KT/qI_B}$$

$$g_m = \frac{I_C}{KT/q + r_B I_B} \tag{1.12}$$

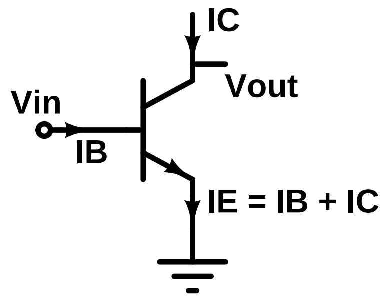


Figure 1.4: The common emitter configuration

$$g_o = \frac{\partial I_C}{\partial V_C} |_{const.V_B} = \frac{I_C}{V_A} \tag{1.13}$$

Therefore the intrinsic voltage gain of the Common Emitter configuration is:

$$A = -\frac{g_m}{g_o} = -\frac{\frac{I_C}{KT/q+r_B I_B}}{\frac{I_C}{V_A}} = -\frac{V_A}{KT/q + r_B I_B} \tag{1.14}$$

If we neglect the drop across the ohmic resistance of the base contact r_B, the gain comes out to be qV_A/KT. Commonly used configurations of common emitter amplifiers use resistors as collector load, rather than current sources. The collector resistor is typically much smaller

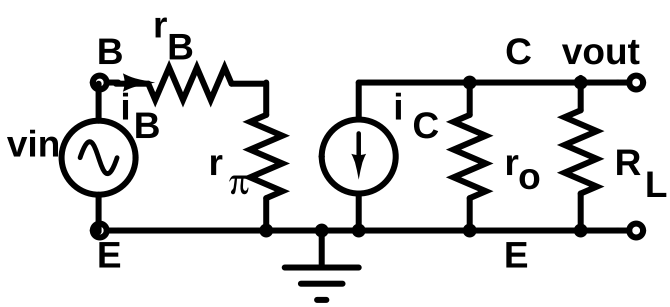


Figure 1.5: Small signal equivalent circuit of common emitter amplifier

than r_o. Thus the parallel combination of r_o with R_L can be taken to be R_L itself. Then the gain is given by:

$$A = -g_m R_L = -\frac{I_C R_L}{KT/q + r_B I_B} \tag{1.15}$$

If we assume that I_B and r_B are small, the voltage gain is the ratio of the DC drop across the collector load resistor and the thermal voltage ($\approx 25mV$). The input impedance of the common emitter amplifier is $r_\pi + r_B$ in parallel with any biasing network connected to the base. The output impedance is the parallel combination of r_o and R_L. The figure below shows a practical common emitter amplifier:

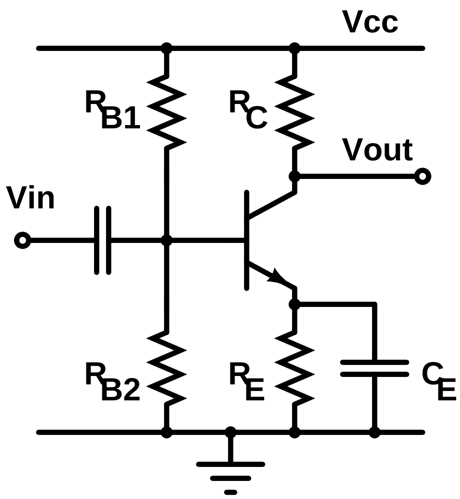


Figure 1.6: A practical Common Emitter amplifier

It is illustrative to see how the component values in the practical design may be derived.

We shall assume the following specifications:

$$V_{cc} = 6V, I_C = 1\text{mA}, \beta \geq 100$$

We would like to operate the transistor so that the quiescent collector voltage is half the supply voltage. We are given that $I_0 = 10^{-14}$A for the base emitter diode and KT/q = 25 mV. This gives

$$V_{BE} \simeq \frac{KT}{q} \ln(I_E/I_0) = 25 \cdot 10^{-3} \ln(10^{11}) = .025 * 25.33 = 0.633\text{V}$$

The role of R_E is to make the biasing robust with respect to temperature changes. The drop across this resistor should be at least an order of magnitude higher than KT/q. Assume that we choose $R_E = 300\Omega$ for this purpose.

$$R_C = \frac{6-3}{10^{-3}} = 3\text{K}\Omega$$

$$V_E = 300\Omega \cdot 1.01 \cdot 10^{-3}\text{A} \; = 0.303\text{V} \quad \text{So} \quad V_B = V_E + V_{BE} = 0.303 + 0.633 = 0.936\text{V}$$

Thus the voltage divider R_{B1}-R_{B2} must be designed such that the base voltage is 0.936V. The base current is:

$$I_B = I_C/\beta \leq 1\text{mA}/100 = 10\mu\text{A}$$

Therefore we would like the current through the voltage divider itself to be an order of magnitude higher – (100μA).
This gives

$$R_{B2} = \frac{0.936}{100 \cdot 10^{-6}} = 9.36\text{K}\Omega$$

Current through R_{B1} is $100\mu\text{A} + I_B = 110\mu\text{A}$.

$$R_{B1} = \frac{6 - 0.936}{110 \cdot 10^{-6}} = \frac{5.064}{1.1 \cdot 10^{-4}} = 46.036\text{K}\Omega$$

To choose practical values, we may take the two resistors to be

$$R_{B1} \simeq 47\text{K}\Omega, \quad R_{B2} \simeq 9.1\text{K}\Omega$$

(These values have been picked from standard values available for 10% tolerance resistors).
With these practical values, the Thevenin equivalent source is

$$V_{Th} = 6\frac{9.1}{47 + 9.1} = 0.973\text{V} \qquad R_{Th} = \frac{9.1 \cdot 47}{9.1 + 47} = 7.624\text{K}\Omega$$

By KVL,

$$V_{Th} = R_{Th}I_B + V_{BE} + (\beta + 1)I_B R_E = \{R_{Th} + (\beta + 1)R_E\}I_B + V_{BE}$$

$$I_B = \frac{V_{Th} - V_{BE}}{R_{Th} + (\beta + 1)R_E}$$

$$\text{and} \qquad V_{BE} = \frac{KT}{q} \ln\left(\frac{(\beta + 1)I_B}{I_0}\right)$$

$$\text{Thus} \qquad I_B = \frac{0.973 - V_{BE}}{7624 + 300 * 101} = \frac{0.973 - V_{BE}}{37924}$$

$$\text{and} \qquad V_{BE} = .025 \ln(101 \cdot 10^{14} \cdot I_B) = .025 \ln(1.01 \cdot 10^{16} \cdot I_B)$$

We can iterate over these two equations to determine I_B and I_E. We can begin with the previously calculated value of V_{BE} which is 0.633 V. This converges quickly to give the base current as 9.02μA. Then the collector current is 902μA, which gives the collector voltage as 6 - 3 * .902 $\approx$ 3.3V.

We rarely have to make the rigorous calculations described above. Since all components in use have a tolerance of 10%, values with an accuracy of about 20% should be acceptable. If we make the simplifying assumptions that the base current is negligible and V_{BE} remains constant for small changes in I_E, we could have directly calculated the operating point of the transistor as follows:

The potential divider at the base gives 0.97 V. Subtracting the nominal value of $V_{BE} =$ 0.63 V, we get $V_E = 0.34V$, which gives

$$IC \approx I_E = \frac{0.34}{300} \approx 1.1\text{mA}$$

This value is somewhat overestimated because we have neglected the base current causing additional drop in the bias voltage. Notice that this calculation did not use the value of β or KT/q. This shows that this bias scheme is robust with respect to process and temperature variations.

(Suggestion: carry out the rigorous calculation taking $\beta = 150$ and see how much the operating point of the transistor changes.)

1.4 Common Base Amplifier

The common base configuration is not used very widely. This is because it has very low input impedance and very high output impedance. So common base amplifier stages cannot be cascaded as the output impedance of one stage cannot drive the input of the next efficiently. Common base amplifiers are often used in conjunction with other configurations. In this

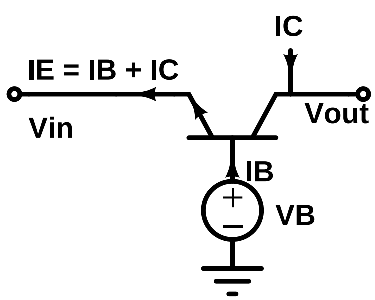


Figure 1.7: Common Base Configuration

configuration, the input is applied to the emitter, while the base is at AC ground i.e. held at a fixed DC voltage. The figure above shows the DC conditions. In a practical case, a resistor will be connected between the emitter and ground to provide a path for the DC current.

As the input voltage applied to the emitter increases, the base emitter diode forward bias will *reduce*, which results in a reduction in the emitter current. Because the base is at AC ground, $dV_B = 0$.

$$dV_{BE} = 0 - dV_E = -dV_E \equiv -v_E$$

The fluctuation in emitter current in response to this fluctuation in the emitter voltage is:

$$dI_E = \frac{\mathrm{d}}{\mathrm{d}V_E} I_0(e^{qV_{BE}/KT} - 1)\mathrm{d}V_{BE} = \frac{qI_0}{KT} e^{qV_{BE}/KT}\mathrm{d}V_{BE} \simeq -\frac{qI_E}{KT} v_E$$

Here we have ignored the effect of the series ohmic resistances associated with the base and emitter contacts. This fluctuation in emitter current is divided between base current and collector current in the ratio $1 : \beta$.

$$dI_B = -\frac{qI_E}{(\beta+1)KT} v_E = -\frac{qI_B}{KT} v_E = -\frac{v_E}{r_\pi} \tag{1.16}$$

$$dI_C = -\frac{\beta qI_E}{(\beta+1)KT} vE = -\frac{qI_C}{KT} v_E = -\beta \frac{v_E}{r_\pi} \tag{1.17}$$

Notice that both base and collector AC currents have a negative sign and must be supplied by the input source for the common base configuration.

The figure on the left above shows the Pi model of a bipolar transistor with the input voltage

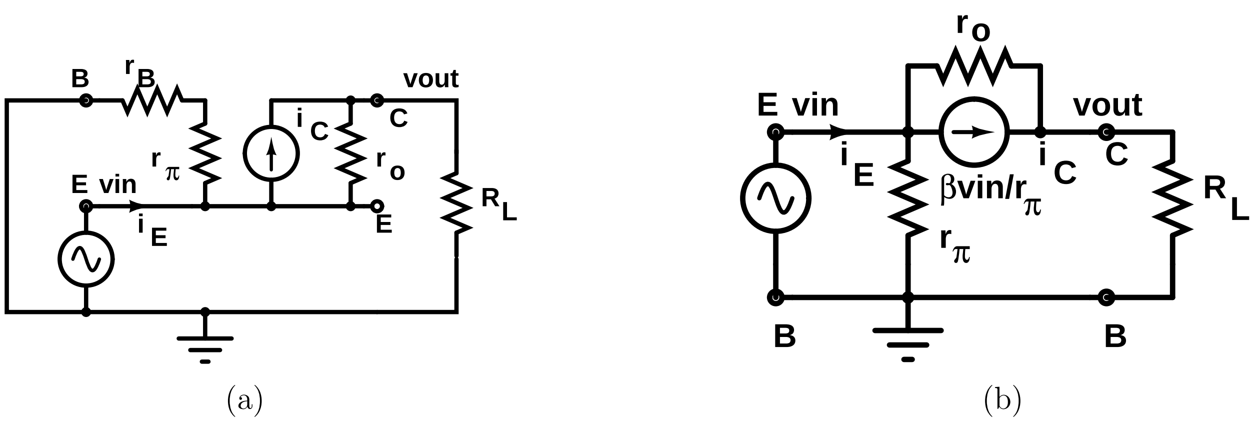


Figure 1.8: Small signal eq. circuit of the Common Base amplifier

connected to emitter and with base grounded. The load resistor is connected between collector and AC ground. The base current fluctuation is in the opposite direction to the common emitter case, as indicated by the negative sign in eq.1.16. Correspondingly, the collector current fluctuation is also in the opposite direction (as indicated by the negative sign in eq.1.17). Notice that the direction of collector current source is marked with the absolute value and its direction has been flipped to show the negative sign in the expression for collector current.

The same circuit is redrawn in Fig.1.8-b so that it looks more conventional with the ground below. We shall ignore r_B in this simplified analysis, so r_B is not shown in the redrawn figure.

Applying KCL at the collector terminal,

$$\frac{v_{in} - v_{out}}{r_o} + \beta\frac{vin}{r_\pi} = \frac{v_{out}}{R_L} \qquad \text{so} \qquad \left(\frac{1}{r_o} + \frac{\beta}{r_\pi}\right) vin = \left(\frac{1}{r_o} + \frac{1}{R_L}\right) v_{out}$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{\frac{1}{r_o} + \frac{\beta}{r_\pi}}{1/r_o + 1/R_L} = \frac{R_L}{r_o + R_L} \cdot \left(1 + \frac{\beta r_o}{r_\pi}\right)$$

So the common base voltage gain is

$$A_v = \frac{R_L}{r_o + R_L} \cdot \left(1 + \frac{\beta r_o}{r_\pi}\right) \tag{1.18}$$

Notice that the common base gain is positive, so the output is in phase with the input.

Applying KCL at the emitter terminal,

$$i_E = \frac{v_E}{r_\pi} + \beta\frac{v_E}{r_\pi} + \frac{v_E - v_{out}}{r_o} = \frac{v_E}{r_\pi} + \frac{v_{out}}{R_L} = \left(\frac{1}{r_\pi} + \frac{A_v}{R_L}\right) v_E$$

The input admittance is given by

$$\frac{i_E}{v_E} = \frac{1}{r_\pi} + \frac{A_v}{R_L}$$

Substituting for A_v/R_L from eq.1.18 the input admittance is:

$$\frac{i_E}{v_E} = \frac{1}{r_\pi} + \frac{1 + \frac{\beta r_o}{r_\pi}}{r_o + R_L}$$

Taking r_o to be very large compared to r_π/β and to R_L, the input admittance reduces to $(1+\beta)/r_\pi$, and therefore the input impedance is:

$$r_{in} = \frac{r_\pi}{1+\beta} = \frac{KT}{qI_B(1+\beta)} = \frac{KT}{qI_E} \tag{1.19}$$

For a transistor with I_E of 1 mA at room temperature, the input impedance will be as low as 25Ω!.

1.5 Common Collector Amplifier

The common collector amplifier is better known as the emitter follower. The collector is connected to the supply voltage. It is thus at AC ground. The input is between the base and ground, while the output is taken from the emitter (with respect to ground). The collector, at AC ground, is the common terminal. The small signal equivalent circuit can be derived

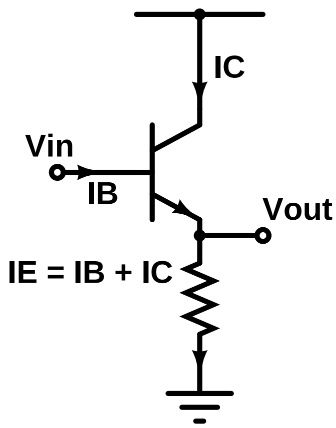


Figure 1.9: The common collector configuration

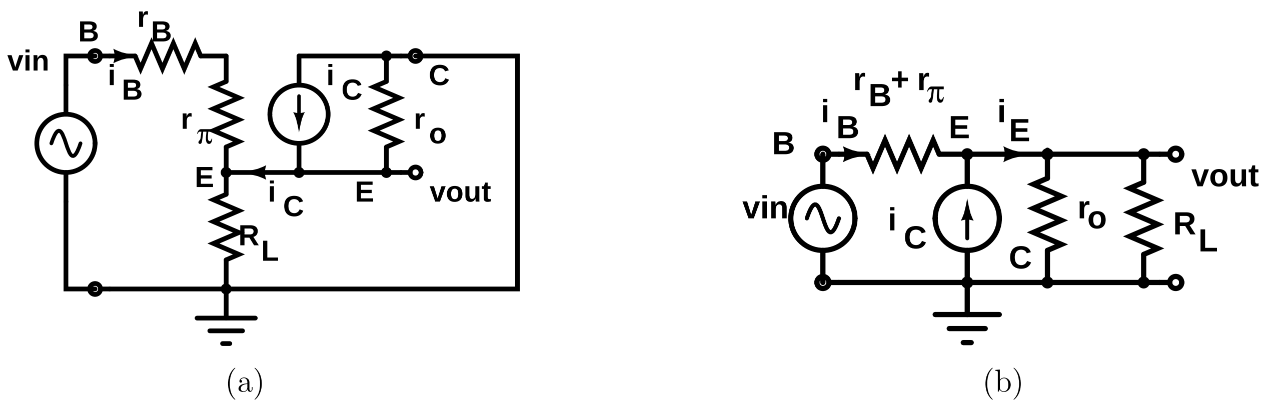


Figure 1.10: Small signal equivalent of the Common Collector amplifier

from the small signal PI model of the transistor.

The figure on the left above shows the collector grounded, input applied between base and ground and output taken from the emitter. The circuit has been redrawn on the right for greater clarity. Here the collector current source and r_o have been flipped down to show the ground connection at the bottom as is customary.

In the analysis below, we shall ignore the effect of r_B.

$$\text{Then} \quad i_B = \frac{v_{in} - v_{out}}{r_\pi} \tag{1.20}$$

Applying KCL at the output terminal,

$$i_B + i_C = \frac{v_{out}}{r_0} + \frac{v_{out}}{R_L}$$

$$\text{So} \qquad (1+\beta)i_B = \frac{v_{out}}{r_0} + \frac{v_{out}}{R_L}$$

$$\text{Therefore} \qquad i_B = \frac{v_{out}}{1+\beta}\left(\frac{1}{r_o} + \frac{1}{R_L}\right)$$

We define

$$\frac{1}{R'_L} \equiv \frac{1}{r_o} + \frac{1}{R_L}$$

Then

$$i_B = \frac{v_{out}}{(1+\beta)R'_L} \tag{1.21}$$

Substituting for i_B from eq.1.20,

$$\frac{v_{in} - v_{out}}{r_\pi} = \frac{v_{out}}{(1+\beta)R'_L}$$

Separating terms in v_{in} and v_{out},

$$\frac{v_{in}}{r_\pi} = \left(\frac{1}{r_\pi} + \frac{1}{(1+\beta)R'_L}\right) v_{out}$$

$$\text{So} \qquad v_{in} = \left(1 + \frac{r_\pi}{(1+\beta)R'_L}\right) v_{out} = \left(\frac{(1+\beta)R'_L + r_\pi}{(1+\beta)R'_L}\right) v_{out}$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{(1+\beta)R'_L}{(1+\beta)R'_L + r_\pi} \approx 1 \text{ for large } \beta$$

From eq.1.21

$$i_B = \frac{v_{out}}{(1+\beta)R'_L}$$

substituting for $v_{out} = A_v v_{in}$

$$i_B = \frac{(1+\beta)R'_L}{(1+\beta)R'_L + r_\pi} \frac{v_{in}}{(1+\beta)R'_L} = \frac{v_{in}}{(1+\beta)R'_L + r_\pi}$$

$$r_{in} = \frac{v_{in}}{i_B} = (1+\beta)R'_L + r_\pi \tag{1.22}$$

This shows that the input impedance of the emitter follower is quite high and is given approximately by the product of the load resistor and the current gain.

The output impedance can be evaluated by applying a test source to the output (emitter) and grounding the input (for AC). A small positive fluctuation of the emitter voltage *reduces* V_{BE} and hence the base and emitter current. Thus, the current *fluctuation* is from emitter to collector. Threfore the collector current source points *down* in the equivalent test circuit. This results in a base current v_{test}/r_π *towards ground* through r_π. Correspondingly, a current $\beta v_{test}/r_\pi$ flows through the collector current source. Applying KCL at the emitter,

$$i_{test} = (1+\beta)\frac{v_{test}}{r_\pi} + \frac{v_{test}}{R'_L}$$

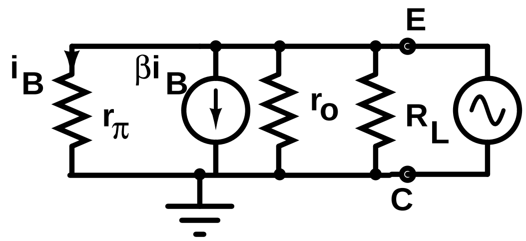


Figure 1.11: Emitter Follower output impedance

$$\frac{i_{test}}{v_{test}} = \frac{1+\beta}{r_\pi} + \frac{1}{R'_L} = \frac{(1+\beta)R'_L + r_\pi}{r_\pi R'_L}$$

Threfore the output impedance is:

$$R_{out} = \frac{r_\pi R'_L}{(1+\beta)R'_L + r_\pi} \tag{1.23}$$

For high values of β, this reduces to

$$\frac{r_\pi}{1+\beta} = \frac{KT}{qI_B(1+\beta)} = \frac{KT}{qI_E}$$

This is a very small value. Therefore, even though the emitter follower offers no voltage gain, its input impedance is very high, while the output impedance is quite low. Therefore, it will not load a signal source and can feed a low impedance without substantial drop in the output voltage. For this reason, emitter followers are widely used as buffer stages.

Chapter 2

Operational Amplifiers

2.1 Single Ended Amplifiers

A *single ended* linear amplifier is a circuit with a single input and a single output, where the output is linearly proportional the input:

$$v_{out} = A \cdot v_{in}$$

A is called the amplifier gain. With proper design, very large values of gain can be realized. however, high gain is not of much use if the input signal is mixed with noise, as the noise will be amplified as much as the signal. An amplifier configuration which improves the signal to noise ratio is clearly desirable.

2.2 Differential Amplifiers

Circuits which amplify the *difference* of two input voltages

$$v_{out} = A(v_{in1} - v_{in2})$$

have many advantages over single ended amplifiers.

- Noise picked up by both inputs gets canceled in the output.
- If both inputs have the same DC bias, the output is insensitive to changes in the bias.
- We often want to feed back the output of an amplifier to its input. In a difference amplifier, the input and feedback can be given to two different inputs, so the input and feedback paths can be isolated.

2.2.1 Differential and common mode quantities

Consider an amplifier with two inputs and one output.

It is more convenient to represent the two input voltages by their mean and difference values.

$$v_{id} \equiv v_{in1} - v_{in2} \tag{2.1}$$
$$v_{icm} \equiv \frac{v_{in1} + v_{in2}}{2} \tag{2.2}$$

Here v_{id} is the **differential** input voltage.
and v_{icm} is the **common mode** input voltage.

In general, the output voltage can be expressed as

$$v_{out} = A_{diff} v_{id} + A_{cm} v_{icm}$$

Where A_{diff} is the **Differential Gain** and A_{cm} is the **Common Mode Gain**.

An ideal difference amplifier will be sensitive only to its differential input. Its output should not depend on the common mode input.

Therefore, for an ideal difference amplifier,

$$A_{cm} = 0, \qquad \text{and} \qquad v_{out} = A_{diff} v_{id}$$

for a practical difference amplifier, the common mode gain is small, but non zero.

We define a figure of merit for a difference amplifier, which represents how close a practical difference amplifier is to an ideal one.

This figure of merit is the **common mode rejection ratio**:

$$\text{CMRR} \equiv 20 \log \frac{a_{diff}}{a_{cm}} \quad (db)$$

The term common mode rejection ratio is often abbreviated to **CMRR**.

2.2.2 The ideal difference amplifier

an ideal difference amplifier will have

- very high differential gain ($A_{diff} \to \infty$),
- very low common mode gain ($A_{cm} \to 0$),

- very high input impedance, so that it does not disturb the input source ($Z_{in} \to \infty$),
- very low output impedance, so that its output voltage is independent of the load ($Z_{out} \to 0$),
- frequency independent gain, so that the output is independent of frequency.

such amplifiers are called operational amplifiers.

2.3 operational amplifiers

operational amplifiers are represented by a triangular symbol as shown below:

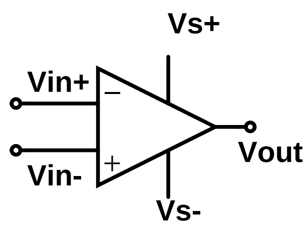


Figure 2.1: Symbol for Operational Amplifiers

in the diagram above, v_{in+} is the positive differential input, v_{in-} is the negative differential input and v_{out} is the output voltage.

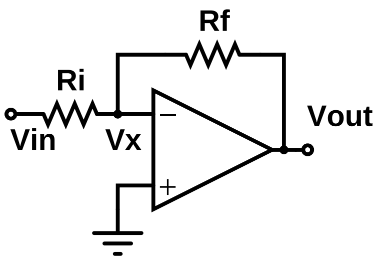
$$v_{out} = A(v_{in+} - v_{in-}) \quad \text{with} \quad A \to \infty$$

Since v_{out} changes in the opposite direction to v_{in-}, it is called the **inverting input**. v_{out} changes in the same direction as v_{in+}, Therefore v_{in+} is called the **non-inverting input**

$$v_{out} = A(v_{in+} - v_{in-}) \quad \text{with} \quad A \to \infty$$

V_{s+} and V_{s-} are the positive and negative supply voltages. An opamp usually requires bipolar supply voltage because the output can be positive or negative, depending on which of v_{in+} and v_{in-} is larger.

The power supply connection are often not shown in circuit diagrams to reduce clutter. These are taken for granted.



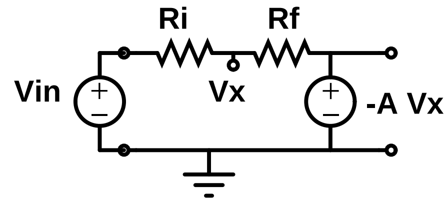


Figure 2.2: Inverting configuration

2.4 Inverting Configuration

Consider the following circuit configuration: For circuit analysis, an op-amp may be considered as a voltage dependent voltage source, whose output is given by

$$v_{out} = A(v_{in+} - v_{in-}) \quad \text{with} \quad A \to \infty$$

Since the input impedance of the opamp is infinite, no current flows into its inputs.

$$\text{By KCL at X,} \quad \frac{V_{in} - V_x}{R_i} = \frac{V_x + AV_x}{R_f} = \frac{1+A}{R_f} V_x$$

$$\text{So} \quad \frac{V_{in}}{R_i} = \left(\frac{1+A}{R_f} + \frac{1}{R_i} \right) V_x = \frac{(1+A)R_i + R_f}{R_i R_f} V_x$$

$$V_x = \frac{R_f V_{in}}{(1+A)R_i + R_f} \quad \text{Therefore} \quad V_x \to 0 \text{ as } A \to \infty$$

2.4.1 Virtual Short

$$V_x = \frac{R_f V_{in}}{(1+A)R_i + R_f} \quad \Rightarrow \quad V_x \to 0 \text{ as } A \to \infty$$

Because of negative feedback, the output tries to bring the inverting input terminal to the same voltage as the non-inverting terminal.

This is called a **virtual short**.

2.4.2 Closed Loop Gain

$$V_x = \frac{R_f V_{in}}{(1+A)R_i + R_f} \quad \Rightarrow \quad V_{out} = -AV_x = -V_{in} \frac{AR_f}{(1+A)R_i + R_f}$$

$$\text{Therefore} \qquad \frac{V_{out}}{V_{in}} = -\frac{R_f}{(1+1/A)R_i + R_f/A} \rightarrow -\frac{R_f}{R_i} \text{ as } A \rightarrow \infty$$

V_{out}/V_{in} is called the **closed loop gain** of this amplifier.

The closed loop gain V_{out}/V_{in} is negative. This is because the input voltage is applied to the inverting input and the output voltage falls as the input voltage rises.

This configuration with negative gain is known as the **inverting configuration**.

2.5 Non-inverting Configuration

Now consider the following configuration: Since the input impedance of the opamp is infinite,

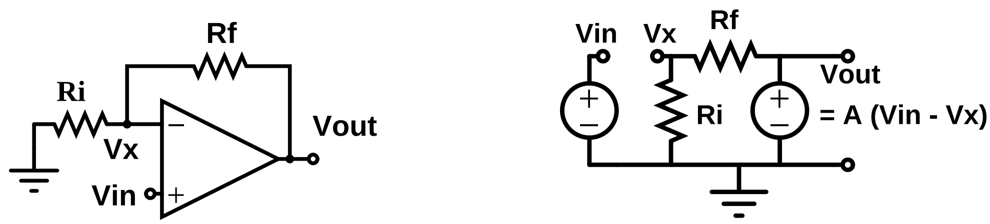


Figure 2.3: Non inverting configuration for OpAmps

no current flows into its inputs.

Therefore the input voltage is shown hanging and the current path to ground at the inverting input is through R_i.

The output voltage is $A(V_{in} - V_x)$.

$$V_x = \frac{R_i}{R_i + R_f} V_{out} = A(V_{in} - V_x)\frac{R_i}{R_i + R_f}$$

$$\text{So} \qquad (R_i + R_f)V_x = AR_i(V_{in} - V_x)$$

Collecting terms containing V_x

$$(R_i + R_f + AR_i)V_x = AR_i V_{in} \quad \text{Therefore} \quad V_x = \frac{AR_i}{AR_i + R_i + R_f} V_{in}$$

$$V_x = V_{in}\frac{AR_i}{AR_i + R_i + R_f} = V_{in}\frac{R_i}{R_i + (R_i + R_f)/A}$$

$$\text{So} \quad V_x \to V_{in} \quad \text{as } A \to \infty \qquad \text{(Virtual Short)}$$

$$V_{in} - V_x = V_{in}\left(1 - \frac{AR_i}{AR_i + R_i + R_f}\right) = V_{in}\frac{R_i + R_f}{R_i + R_f + AR_i}$$

$$V_{out} = A(V_{in} - V_x) = AV_{in}\frac{R_i + R_f}{R_i + R_f + AR_i}$$

$$V_{out} = AV_{in}\frac{R_i + R_f}{R_i + R_f + AR_i}$$

$$\text{So} \qquad \frac{V_{out}}{V_{in}} = \frac{A(R_i + R_f)}{R_i + R_f + AR_i} = \frac{R_i + R_f}{R_i + (R_i + R_f)/A}$$

$$\text{Therefore,} \quad \frac{V_{out}}{V_{in}} \to \frac{R_i + R_f}{R_i} = 1 + \frac{R_f}{R_i} \quad \text{as} \quad A \to \infty$$

2.6 Inverting Adder

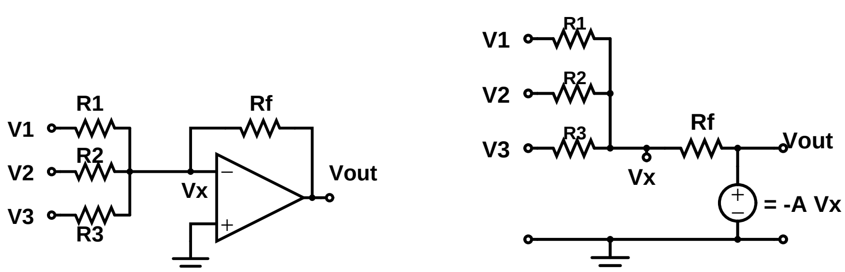


Figure 2.4: Opamp based analog adder

We can modify the inverting adder to connect multiple inputs, each with its own series resistor.

The circuit has negative feed back and therefore, the potential at the summing node X should be the same as the potential at the non-inverting node (virtual short).

$$\text{From virtual short: } V_x = 0$$

$$\text{Applying KCL at X: } \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_{out}}{R_f}$$

$$\text{So,} \qquad V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

In the special case, where $R_f = R_1 = R_2 = R_3$

$$V_{out} = -\left(V_1 + V_2 + V_3\right)$$

So the circuit acts as an inverting analog adder.
By using other values for resistors, we can use it for evaluating a weighted sum, with the weights determined by resistor values.

2.7 Differentiator

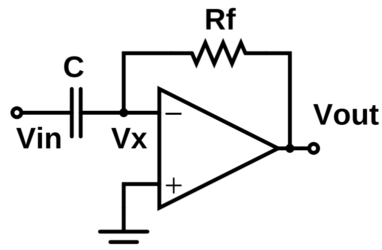


Figure 2.5: A differentiator using OpAmps

$$\text{From virtual short: } V_x = 0$$

Applying KCL at X:

$$C\frac{\mathrm{d}V_{in}}{\mathrm{d}t} = -\frac{V_{out}}{R_f}$$

$$\text{So,} \qquad V_{out} = -R_f C\frac{\mathrm{d}V_{in}}{\mathrm{d}t}$$

So the output of the circuit is proportional to the time derivative of the input waveform.

2.8 Integrator

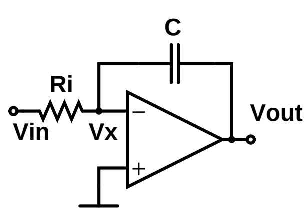


Figure 2.6: Operational Amplifier based integrator

$$\text{From virtual short: } V_x = 0$$

$$\text{Applying KCL at X: } \frac{V_{in}}{R} = -C\frac{\mathrm{d}V_{out}}{\mathrm{d}t}$$
$$\text{So, } \quad \frac{\mathrm{d}V_{out}}{\mathrm{d}t} = -\frac{V_{in}}{RC}$$
$$V_{out} = -\frac{1}{RC}\int V_{in}\mathrm{d}t$$

So the output of the circuit is proportional to the integral of the input waveform.

2.9 Logarithmic Amplifier

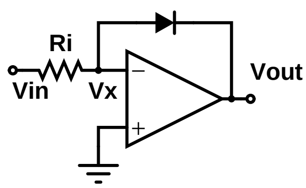


Figure 2.7: Logarithmic amplifier for positive inputs

Assume that the input voltage is always positive. Because the input is given to the inverting terminal, the output will be negative and so the diode will be in forward bias. From virtual short: $V_x = 0$, and voltage across the diode is
$V_d = 0 - V_{out} = -V_{out}$. If the diode current is much larger than its leakage current I_0,

$$I_d = I_0\left(e^{qV_d/KT} - 1\right) \simeq I_0 e^{-\frac{qV_{out}}{KT}} \quad \text{(ignoring the -1 term)}$$

$$\text{Applying KCL at X: } \frac{V_{in}}{R} = I_0 e^{-\frac{qV_{out}}{KT}} \quad \text{So,} \quad V_{out} = -\frac{KT}{q}\ln\left(\frac{V_{in}}{RI_0}\right)$$

The output voltage of the circuit is proportional to the logarithm of the the input voltage, with a negative sign.

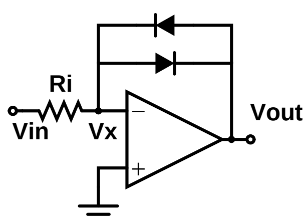


Figure 2.8: Opamp based logarithmic amplifier

By using two diodes as shown, the logarithmic amplifier can be made to work for both positive and negative inputs.
For either polarity, one of the diodes is reverse biased, while the other is forward biased. Virtual short still applies, and for a positive input voltage, the lower diode pointing right is forward biased, while the other is reverse biased. The reverse is true for a negative input.

$$\text{In either case,} \quad Id = I_0\left(e^{q|V_{out}|/KT} - 1\right) + I_0 = I_0 e^{q|V_{out}|/KT}$$
$$\text{and} \quad V_{out} = -\mathrm{sgn}(V_{in})\frac{KT}{q}\ln\left(\frac{|V_{in}|}{RI_0}\right)$$

Chapter 3

Feedback and Oscillators

3.1 Introduction

Amplifiers produce a larger replica of the input signals at their output. It is interesting to see what happens when we mix fractions of this output with the original input. Let us first look at generic feedback. For simplicity, we shall assume that the input as well as the output are voltages. (In a more general case, either or both of these could be currents).

3.2 Negative Feedback

Let us say that we take a fraction f of the output and subtract it from the input.

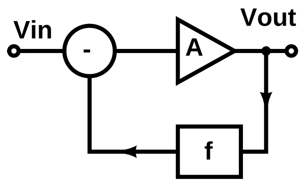


Figure 3.1: Amplifier with negative feedback

The amplifier with Gain A sees $V_{in} - fV_{out}$ at its input. Therefore:

$$V_{out} = A(V_{in} - fV_{out}) \qquad \text{So} \qquad (1 + fA)V_{out} = AV_{in}$$

Which gives

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + fA} \tag{3.1}$$

V_{out}/V_{in} is called the *closed loop* gain of the amplifier. This is to distinguish it from A, which is the *open loop* gain.

$$\text{As } A \to \infty, \qquad \frac{V_{out}}{V_{in}} \to \frac{1}{f}$$

This means that as long as A is large, The effective gain, which is the *closed loop* gain V_{out}/V_{in}, is independent of A! Thus we can design robust amplifiers whose gains are decided by linear passive components and which are therefore unaffected by changes in parameters of amplifying devices, power supply or temperature. This is a useful attribute and because of this, amplifiers are generally used with negative feedback. Notice, that since f is a fraction produced by passive devices, the magnitude of f is less than one, and consequently, the closed loop gain is greater than one – which is what we want.

Thus, we can design amplifiers with predictable and useful gains, the only requirement being

$$fA = \frac{A}{1/f} = \frac{\text{Open loop Gain}}{\text{Closed Loop Gain}} >> 1$$

Notice that f could, in general, be complex. This would be the case if the voltage divider used to implement f contains reactive elements like capacitors or inductors. In such a case, the gain can be made frequency dependent by properly choosing f. We can design amplifiers which amplify only certain frequencies or which *attenuate* certain frequencies. Such amplifiers are called filters. We shall see an example of such filters in the Baxendall tone control.

3.3 Positive Feedback

Instead of subtracting the fraction f of the output from the input, we could actually add it to the input. This is equivalent to changing the sign of f to negative in eq.3.1. This gives

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 - fA} \tag{3.2}$$

as long as fA is smaller than one, it has the effect of providing a closed loop gain which is greater than the open loop gain A. This technique is called gain boosting. However, one has to be careful with gain boosting, as the amplifier becomes unstable if we boost the gain too much.

3.4 Oscillators

An interesting case arises as $fA \to 1$. The closed loop gain becomes infinite, which means it can give a finite output for zero input. This kind of circuit is an oscillator.

To understand the behaviour of an oscillator, let us create an artificial construct. Suppose we have an amplifier with gain A. We take a fraction f of its output using a divider. if $fA = 1$, it means that this fraction is an exact replica of the input. In fact, we could switch the input of the amplifier to this replica and then remove the input source altogether, and the amplifier will never know the difference! It will continute to produce the same ouput, even though the input has been removed.

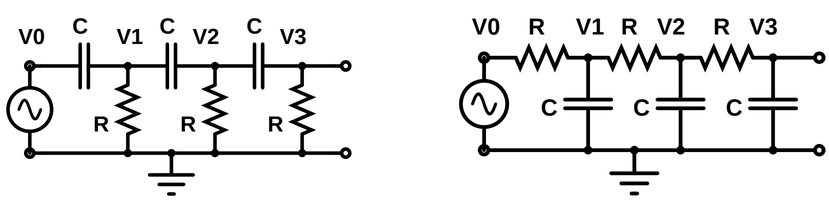
In a practical case, of course, there is no input. The feedback is fequency sensitive, so that $fA = 1$ only for a *specific frequency*. The closed loop gain approaches infinity only for this frequency. There is always some noise at the input of the amplifier due to the statistical nature of current conduction. This noise is random, which can be modeled as a sum of equal inifinitesimal amplitudes of all frequencies. (This is called "white" noise). The amplifier has a gain approaching infinity for a specific frequency. The infinitesimal amplitude of this frequency is therefore amplified to a finite value, while other frequencies of the noise remain at very low levels. Thus we have an electronic circuit, which produces an output at a definite frequency (that is, a sinusoid output).

To make an oscillator, we need to produce an *exact* replica of the input, in amplitude as well as phase. This means that the open loop gain A combined with the feedback network f, should produce a signal with $fA = 1$ and a phase difference through A and f of $2n\pi$ where n is an integer. These are known as Barkhousen's criteria.

3.4.1 Phase shift oscillator

An inverting amplifier produces a phase difference of π. If we can construct a feed back network which introduces another phase differencr of *pi*, we can meet Barkhausen's criteria by setting the gain of the amplifier to be the reciprocal of the attenuation caused by our feedback oscillator.

Consider the following RC circuits as candidates for producing a phase difference of π. The phase difference will be π when the real component of V_3/V_0 is zero. Let Zi be the



(a) (b)

Figure 3.2: Feedback networks for Phase shift oscillator

impedance to ground from Vi for either network.

For the CR circuit shown in (a)

$$\frac{V3}{V2} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR}$$

Z2 is R in parallel with the series combination of C and R.

$$Z2 = \frac{R \cdot \left(R + \frac{1}{j\omega C}\right)}{R + R + \frac{1}{j\omega C}} = R\frac{1 + j\omega CR}{1 + 2j\omega CR}$$

$$\frac{V2}{V1} = \frac{Z2}{Z2 + \frac{1}{j\omega C}} = \frac{j\omega CZ2}{1 + j\omega CZ2} = \frac{j\omega CR\frac{1+j\omega CR}{1+2j\omega CR}}{1 + j\omega CR\frac{1+j\omega CR}{1+2j\omega CR}}$$

$$\frac{V2}{V1} = \frac{j\omega CR(1 + j\omega CR)}{1 + 2j\omega CR + j\omega CR(1 + j\omega CR)} = \frac{j\omega CR(1 + j\omega CR)}{1 - \omega^2C^2R^2 + 3j\omega CR}$$

Therefore

$$\frac{V3}{V1} = \frac{V3}{V2} \cdot \frac{V2}{V1} = \frac{j\omega CR}{1 + j\omega CR} \cdot \frac{j\omega CR(1 + j\omega CR)}{1 - \omega^2C^2R^2 + 3j\omega CR} = \frac{-\omega^2C^2R^2}{1 - \omega^2C^2R^2 + 3j\omega CR}$$

Z1 is R in parallel with the series combination of C with Z2.

$$Z1 = \frac{R \cdot \left(\frac{1}{j\omega C} + Z2\right)}{R + \frac{1}{j\omega C} + Z2} = R\frac{1 + j\omega CZ2}{1 + j\omega C(R + Z2)}$$

$$Z1 = R\frac{1 + j\omega CR\frac{1+j\omega CR}{1+2j\omega CR}}{1 + j\omega C\left(R + R\frac{1+j\omega CR}{1+2j\omega CR}\right)} = R\frac{1 + j\omega CR\frac{1+j\omega CR}{1+2j\omega CR}}{1 + j\omega CR\left(1 + \frac{1+j\omega CR}{1+2j\omega CR}\right)}$$

$$Z1 = R\frac{1 + 2j\omega CR + j\omega CR(1 + j\omega CR)}{1 + 2j\omega CR + j\omega CR(2 + 3j\omega CR)} = R\frac{1 - \omega^2C^2R^2 + 3j\omega CR}{1 - 3\omega^2C^2R^2 + 4j\omega CR}$$

$$\frac{V1}{V0} = \frac{Z1}{\frac{1}{j\omega C} + Z1} = \frac{j\omega CZ1}{1 + j\omega CZ1} = \frac{j\omega CR\frac{1-\omega^2C^2R^2+3j\omega CR}{1-3\omega^2C^2R^2+4j\omega CR}}{1 + j\omega CR\frac{1-\omega^2C^2R^2+3j\omega CR}{1-3\omega^2C^2R^2+4j\omega CR}}$$

$$\frac{V1}{V0} = \frac{j\omega CR\left(1 - \omega^2C^2R^2 + 3j\omega CR\right)}{1 - 3\omega^2C^2R^2 + 4j\omega CR + j\omega CR\left(1 - \omega^2C^2R^2 + 3j\omega CR\right)}$$

$$= \frac{j\omega CR\left(1 - \omega^2C^2R^2 + 3j\omega CR\right)}{1 - 6\omega^2C^2R^2 + j\omega CR(5 - \omega^2C^2R^2)}$$

$$\frac{V3}{V0} = \frac{V3}{V1} \cdot \frac{V1}{V0} = \frac{-\omega^2C^2R^2}{1 - \omega^2C^2R^2 + 3j\omega CR} \cdot \frac{j\omega CR\left(1 - \omega^2C^2R^2 + 3j\omega CR\right)}{1 - 6\omega^2C^2R^2 + j\omega CR(5 - \omega^2C^2R^2)}$$

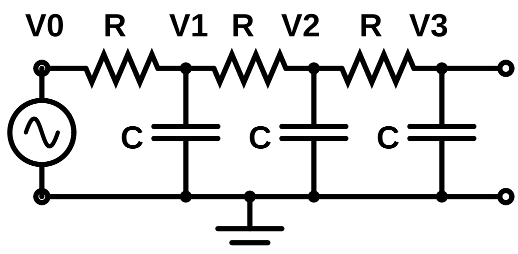
$$\frac{V3}{V0} = \frac{-j\omega^3 C^3 R^3}{1 - 6\omega^2 C^2 R^2 + j\omega CR(5 - \omega^2 C^2 R^2)}$$

For V3/V0 to have no imaginary term,

$$1 - 6\omega^2 C^2 R^2 = 0 \qquad \text{So} \qquad \omega = \frac{1}{\sqrt{6}CR}$$

Thus the C-R circuit in A will produce a phase difference of π at a frequency given by $1/(2\pi\sqrt{6}CR)$.

For the R-C case shown in b)



$$\frac{V3}{V2} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR}$$

$$\frac{V2}{V1} = \frac{Z2}{R + Z2}$$

where Z2 is C in parallel with a series combination of R and C.

$$Z2 = \frac{\frac{1}{j\omega C}\left(R + \frac{1}{j\omega C}\right)}{\frac{1}{j\omega C} + R + \frac{1}{j\omega C}} = \frac{1}{j\omega C} \cdot \frac{1 + j\omega CR}{2 + j\omega CR}$$

$$\text{Therefore} \qquad j\omega C Z2 = \frac{1 + j\omega CR}{2 + j\omega CR}$$

$$\frac{V2}{V1} = \frac{Z2}{R + Z2} = \frac{\frac{1}{j\omega C} \cdot \frac{1+j\omega CR}{2+j\omega CR}}{R + \frac{1}{j\omega C} \cdot \frac{1+j\omega CR}{2+j\omega CR}}$$

$$= \frac{1 + j\omega CR}{j\omega CR(2 + j\omega CR) + 1 + j\omega CR} = \frac{1 + j\omega CR}{1 - \omega^2 C^2 R^2 + 3j\omega CR}$$

$$\frac{V1}{V0} = \frac{Z1}{R + Z1} = \frac{1}{1 + R/Z1}$$

Z1 is the impedance of C in parallel with a series combination of R with Z2.

$$Z1 = \frac{\frac{1}{j\omega C} \cdot (R + Z2)}{\frac{1}{j\omega C} + R + Z2} = \frac{R + Z2}{1 + j\omega CR + j\omega CZ2} = \frac{R + \frac{1}{j\omega C} \cdot \frac{1+j\omega CR}{2+j\omega CR}}{1 + j\omega CR + \frac{1+j\omega CR}{2+j\omega CR}}$$

$$= \frac{1}{j\omega C} \cdot \frac{j\omega CR + \frac{1+j\omega CR}{2+j\omega CR}}{(1 + j\omega CR)\left(1 + \frac{1}{2+j\omega CR}\right)} = \frac{1}{j\omega C} \cdot \frac{j\omega CR(2 + j\omega CR) + 1 + j\omega CR}{(1 + j\omega CR)(3 + j\omega CR)}$$

$$\text{So} \qquad Z1 = \frac{1}{j\omega C} \cdot \frac{1 - \omega^2 C^2 R^2 + 3j\omega CR}{(1 + j\omega CR)(3 + j\omega CR)}$$

Therefore

$$\frac{V1}{V0} = \frac{1}{1 + R/Z1} = \frac{1}{1 + j\omega CR\frac{(1+j\omega CR)(3+j\omega CR)}{(1-\omega^2C^2R^2)+3j\omega CR}}$$

$$= \frac{(1 - \omega^2 C^2 R^2) + 3j\omega CR}{1 - \omega^2 C^2 R^2 + 3j\omega CR + j\omega CR(3 - \omega^2 C^2 R^2 + 4j\omega CR)}$$

$$= \frac{1 - \omega^2 C^2 R^2 + 3j\omega CR}{1 - \omega^2 C^2 R^2 + 6j\omega CR - j\omega^3 C^3 R^3 - 4\omega^2 C^2 R^2}$$

$$= \frac{1 - \omega^2 C^2 R^2 + 3j\omega CR}{1 - 5\omega^2 C^2 R^2 + j\omega CR(6 - \omega^2 C^2 R^2)}$$

$$\frac{V3}{V0} = \frac{V3}{V2} \cdot \frac{V2}{V1} \cdot \frac{V1}{V0}$$

$$= \frac{1}{1 + j\omega CR} \cdot \frac{1 + j\omega CR}{1 - \omega^2 C^2 R^2 + 3j\omega CR} \cdot \frac{1 - \omega^2 C^2 R^2 + 3j\omega CR}{1 - 5\omega^2 C^2 R^2 + j\omega CR(6 - \omega^2 C^2 R^2)}$$

$$= \frac{1}{1 - 5\omega^2 C^2 R^2 + j\omega CR(6 - \omega^2 C^2 R^2)}$$

If we assert that $V3/V0$ should have no imaginary part, we must have

$$6 - \omega^2 C^2 R^2 = 0 \qquad \text{So} \qquad \omega = \frac{\sqrt{6}}{CR}$$

Thus the R-C circuit in B will produce a phase difference of π at a frequency given by $\sqrt{6}/(2\pi CR)$.

We should evaluate the amplitude attenuation at the frequency which produces a phase difference of π in the two cases.

CR circuit: $6\omega^2C^2R^2 = 1$

$$\frac{V3}{V0} = \frac{-j\omega^3C^3R^3}{j\omega CR(5-\omega^2C^2R^2)} = -\frac{\omega^2C^2R^2}{5-\omega^2C^2R^2}$$

Therefore

$$\frac{V3}{V0} = -\frac{1/6}{5-1/6} = -\frac{1}{29}$$

RC circuit: $\omega^2C^2R^2 = 6$

$$\frac{V3}{V0} = \frac{1}{1-5\omega^2C^2R^2+j\omega CR(6-\omega^2C^2R^2)} = \frac{1}{1-5\omega^2C^2R^2}$$

$$\text{So} \qquad \frac{V3}{V0} = \frac{1}{1-5\times 6} = -\frac{1}{29}$$

So both the networks are capable of providing a phase difference of π and produce an attenuation of 1/29. Therefore the amplifier should provide a minimum gain of 29. An

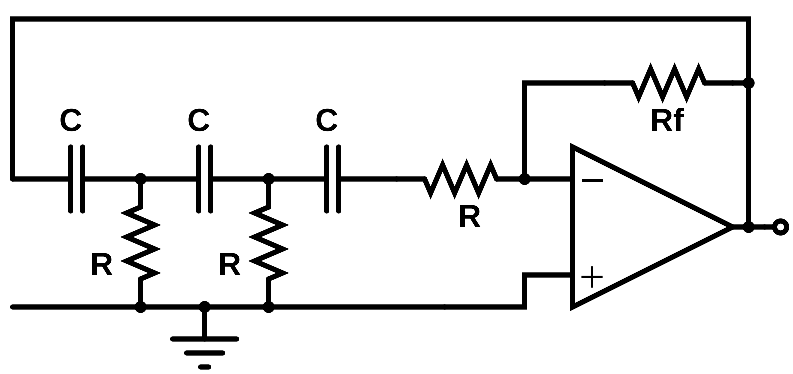


Figure 3.3: Phase shift oscillator

oscillator using the C-R phase shift network (a) is shown in the figure. Notice that the last of the resistors in the phase shift network goes to the virtual ground rather than to ground itself. The feedback resistor must be greater than 29R, so that the insertion loss of the feedback network is made up by the amplifier.