EE101: Resonance in *RLC* circuits

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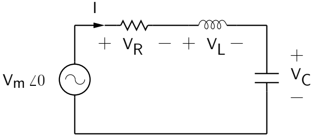
Resonance in series RLC circuits



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

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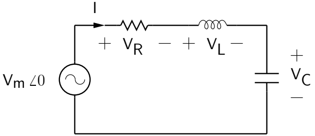

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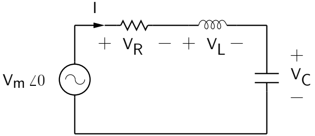

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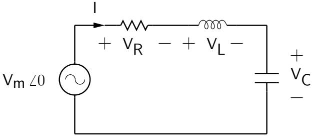

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* The above condition is called "resonance," and the corresponding frequency is called the "resonance frequency" (ω_0).

$$\boxed{\omega_0 = 1/\sqrt{LC}}$$

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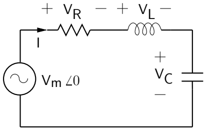

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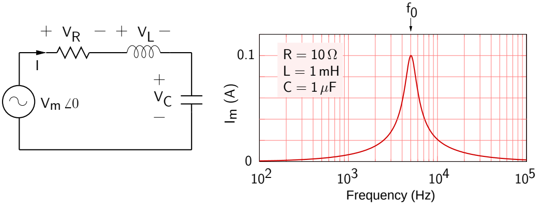

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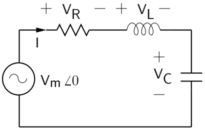

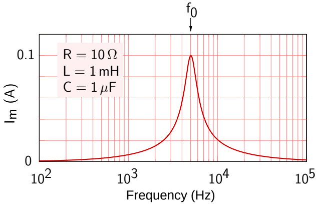

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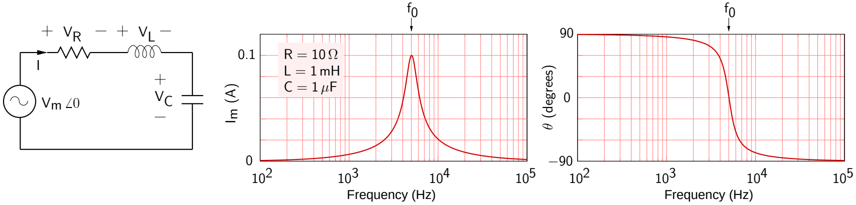

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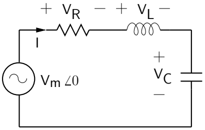

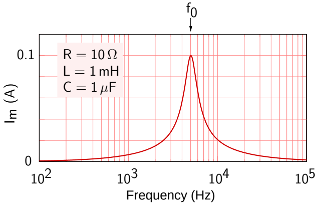

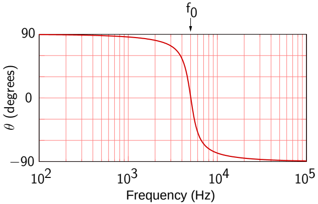

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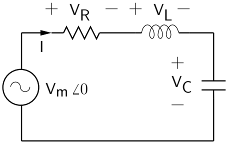

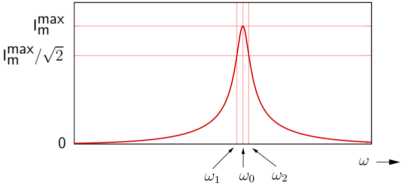

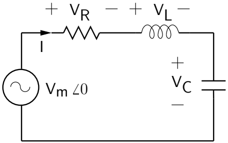

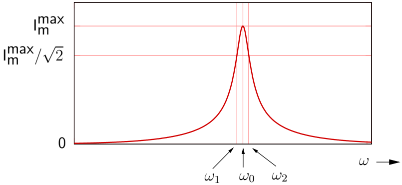

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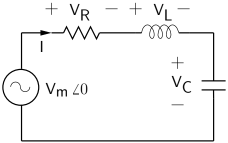

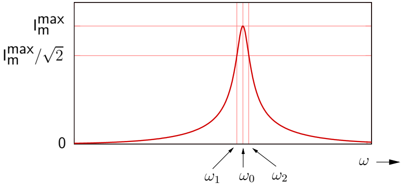

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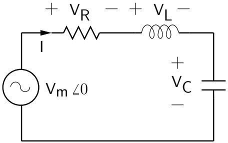

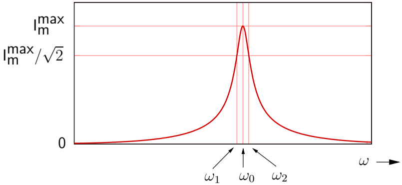

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* Define ω_1 and ω_2 (see figure) as frequencies at which $I_m = I_m^{max}/\sqrt{2}$, i.e., the power absorbed by R is $P^{max}/2$.
* The *bandwidth* of a resonant circuit is defined as $B = \omega_2 - \omega_1$, and the *quality factor* as $Q = \omega_0/B$. Quality is a measure of the sharpness of the I_m versus frequency relationship.

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,.$$

For $\omega = \omega_0,\ I_m = I_m^{max} = V_m/R\,.$

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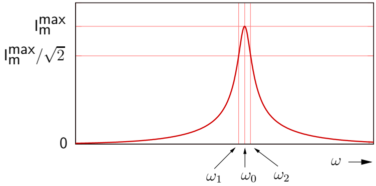

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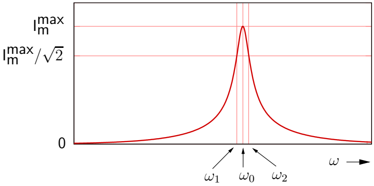

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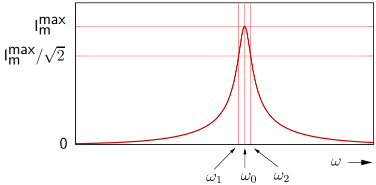

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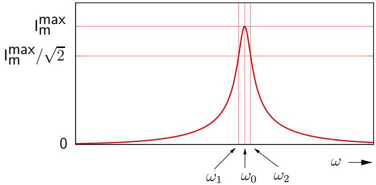

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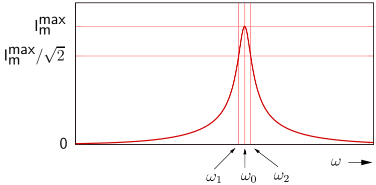

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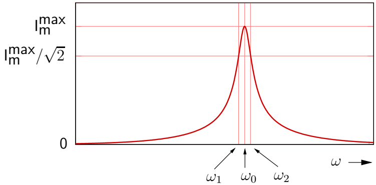

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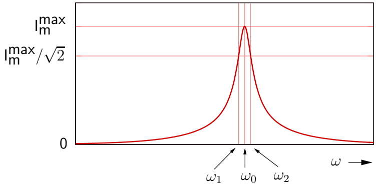

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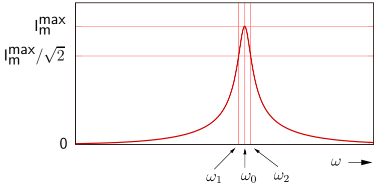

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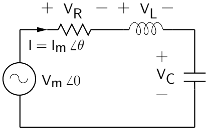

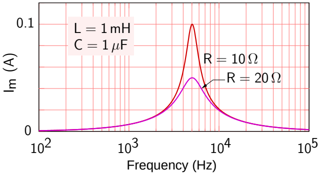

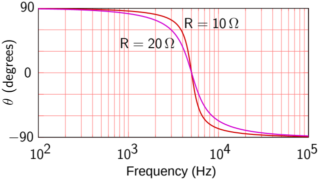

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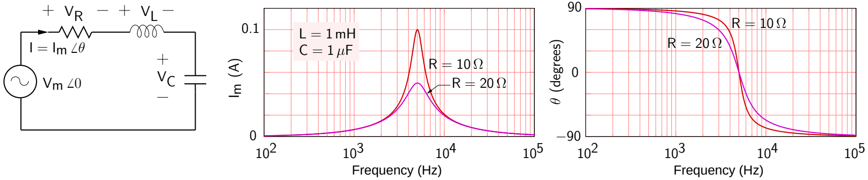
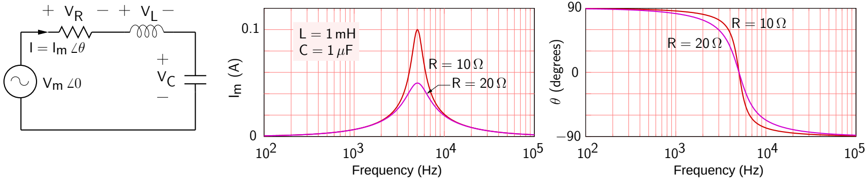


As R is increased,

* The quality factor $Q = \omega_0 L/R$ decreases, i.e., I_m versus ω curve becomes broader.



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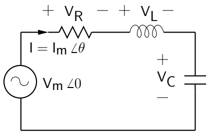

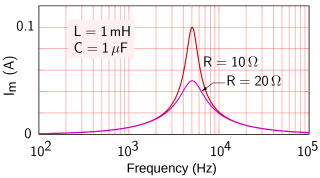

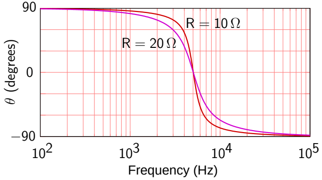

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* The resonance frequency ($\omega_0 = 1/\sqrt{LC}$) is not affected.

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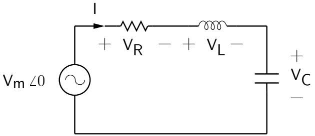

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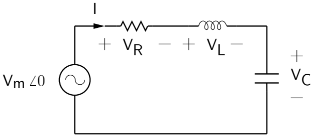

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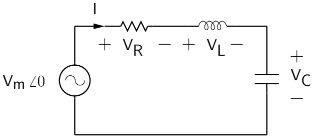
$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.



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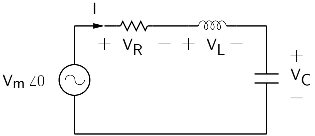
* For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.



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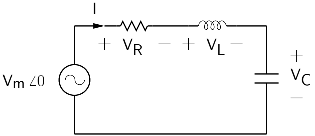
* For $\omega > \omega_0$, $\omega L > 1/\omega C$, the net impedance is inductive, and the current lags the applied voltage.



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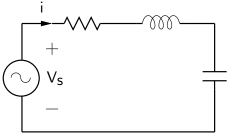
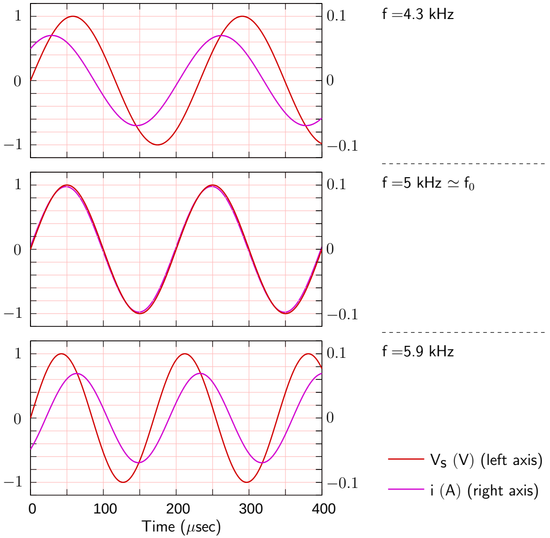
* Let us look at an example (next slide).



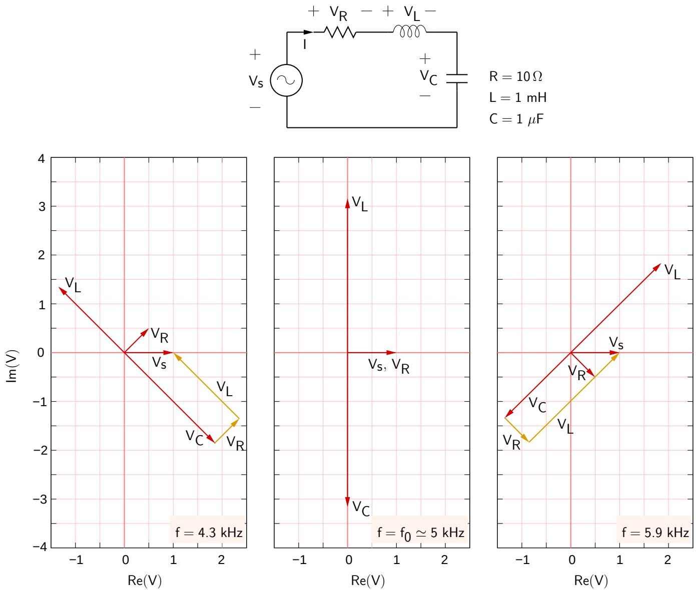
$R = 10\,\Omega$

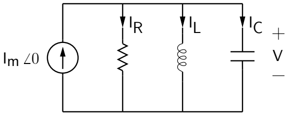
$L = 1$ mH

$C = 1\ \mu$F



Resonance in series *RLC* circuits: phasor diagrams





$$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}, \text{ where } \mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)\,.$$

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta\,, \text{ where}$$

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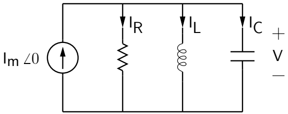

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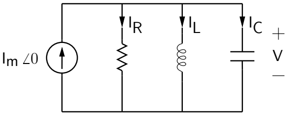
* As ω is varied, both V_m and θ change.



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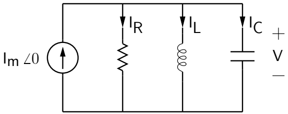

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* The above condition is called "resonance," and the corresponding frequency is called the "resonance frequency" (ω_0).

$$\boxed{\omega_0 = 1/\sqrt{LC}}$$

Resonance in parallel RLC circuits

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right].$

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right].$

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* The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
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* Show that $\omega_{1,2} = \mp\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 + \dfrac{1}{LC}}$
  $\Rightarrow$ Bandwidth $B = 1/RC$.

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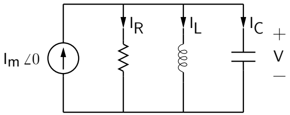
* Show that $\omega_0 = \sqrt{\omega_1 \omega_2}\,.$



$I_m = 50$ mA
$R = 2\,k\Omega$
$L = 40$ mH
$C = 0.25\ \mu F$

* Calculate ω_0, f_0, B, Q.
* Calculate $\mathbf{I}_R$, $\mathbf{I}_L$, $\mathbf{I}_C$ at $\omega = \omega_0, \omega_1, \omega_2$.
* Verify graphically that $\mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C = \mathbf{I}_s$ in each case.
* Plot the power absorbed by R as a function of frequency for $f_0/10 < f < 10\, f_0$.