



Figure 1: (a)  $RC$  phase-shift network ( $R_1 = R_2 = R_3 = R$ ,  $C_1 = C_2 = C_3 = C$ ), (b) Op Amp based phase-shift oscillator.

Fig. 1 (a) shows an  $RC$  network for which,

$$\frac{V_1}{V_i} = \frac{(\omega RC)^3}{\omega RC [(\omega RC)^2 - 5] + j [1 - 6(\omega RC)^2]} . \quad (1)$$

This network can be used to make an oscillator (see Fig. 1 (b)) if the Barkhausen criterion of unity loop gain is satisfied. Since the inverting terminal of the Op Amp is at  $0V$  (virtual ground), the  $RC$  networks in Figs. 1 (a) and (b) are identical, and Eq. 1 for  $V_1/V_i$  in Fig. 1 (a) also holds for  $V_B/V_A$  in Fig. 1 (b). The loop gain is therefore

$$A\beta = -\frac{R_f}{R_3} \times \frac{(\omega RC)^3}{\omega RC [(\omega RC)^2 - 5] + j [1 - 6(\omega RC)^2]} . \quad (2)$$

For the loop gain to be  $1 \angle 0$ , we require the denominator to be real, i.e.,

$$\omega_0 = \frac{1}{\sqrt{6} RC} . \quad (3)$$

At this frequency,

$$A\beta = -\frac{R_f}{R_3} \left( -\frac{1}{29} \right) , \quad (4)$$

thus requiring  $R_f = 29 R$  (a little larger than that in practice) for oscillations to be sustained. The output amplitude depends on the nonlinearities in the circuit due to the diode network (consisting of  $D_1, D_2, R_{1A}, R_{2A}, R_{1B}, R_{2B}$ ).

## Exercise Set

1. Simulate the circuit of Fig. 1 (a) with  $\mathbf{V}_i = 1\angle 0 V$ ,  $R = 10\text{ k}\Omega$ ,  $C = 16\text{ nF}$ . In this case, the output  $\mathbf{V}_1$  is the same as  $\mathbf{V}_1/\mathbf{V}_i$ .

- (a) Compute  $\omega_0$  (see Eq. 3).
- (b) Plot  $|\mathbf{V}_1|$  and  $\angle \mathbf{V}_1$  versus  $\log f$ , and verify that the gain at  $\omega_0$  is  $-1/29$ , i.e., the magnitude is  $1/29$  and the phase is  $\pi$  (same as  $-\pi$ ).
- (c) Increase  $R$  by a factor of 2 and verify that  $\omega_0$  shifts appropriately.

(Circuit file: `ee101_osc_2.sqproj`)

2. Simulate the oscillator circuit of Fig. 1 (b) with the same values of  $R$  and  $C$  as in (1) above, and with  $R_f = 310\text{ k}$ .
  - (a) Verify that the frequency of oscillations observed is consistent with that predicted by Eq. 3.
  - (b) Change  $R_f$  to  $280\text{ k}$ , simulate the circuit, and obtain the output waveform. Explain your observations.