EE101: Sinusoidal steady state analysis

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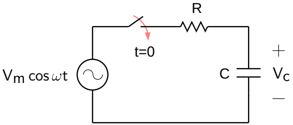
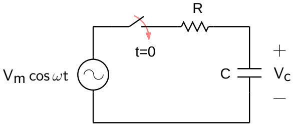
Sinusoidal steady state





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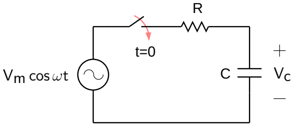

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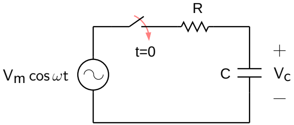

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$V_c^{(h)}(t)$ satisfies the homogeneous differential equation,

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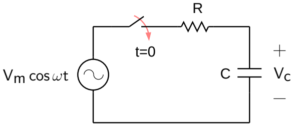
from which, $V_c^{(h)}(t) = A\,\exp(-t/\tau)\,$, with $\tau = RC\,.$



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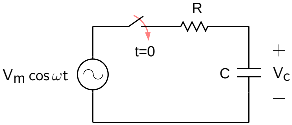

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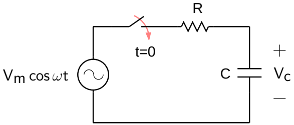

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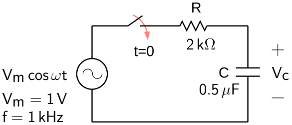
C_1 and C_2 can be found by equating the coefficients of $\sin \omega t$ and $\cos \omega t$ on the left and right sides.

Sinusoidal steady state



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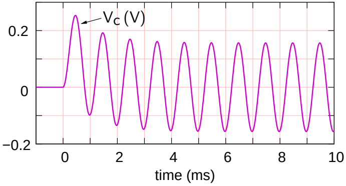

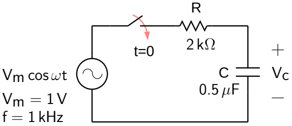

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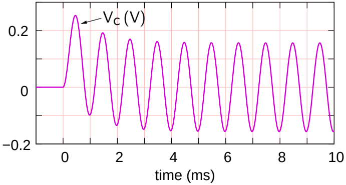

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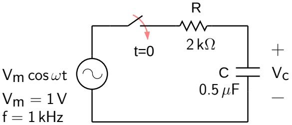

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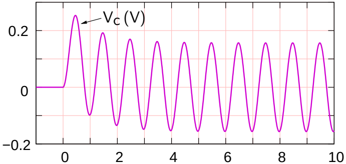

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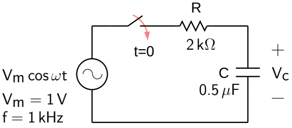

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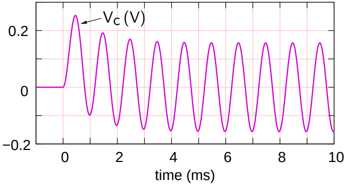

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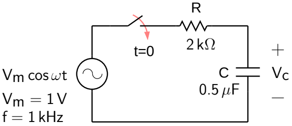

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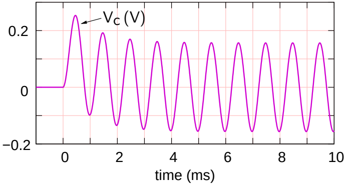

- The complete solution is $V_c(t) = A\,\exp(-t/\tau) + C_1\,\cos\,\omega t + C_2\,\sin\,\omega t\,.$
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- Any circuit containing resistors, capacitors, inductors, sinusoidal voltage and current sources (of the same frequency), dependent (linear) sources behaves in a similar manner, viz., each current and voltage in the circuit becomes purely sinusoidal as $t \rightarrow \infty$.

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Sinusoidal steady state: phasors

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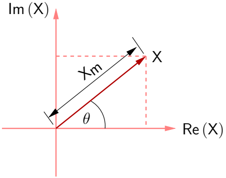
* Note that a phasor can be written in the polar form or rectangular form,
  $\mathbf{X} = X_m \angle\theta = X_m \exp(j\theta) = X_m \cos\theta + j\, X_m \sin\theta$.

  The term ωt is always *implicit*.



Phasors: examples

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Consider addition of two sinusoidal quantities:

$$\begin{aligned} v(t) &= v_1(t) + v_2(t) \\ &= V_{m1}\,\cos(\omega t + \theta_1) + V_{m2}\,\cos(\omega t + \theta_2) \end{aligned}$$

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Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$$
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In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$\tilde{v}(t) = Re\left[\mathbf{V} e^{j\omega t}\right]$$
$$= Re\left[\left(V_{m1} e^{j\theta_1} + V_{m2} e^{j\theta_2}\right) e^{j\omega t}\right]$$
$$= Re\left[V_{m1} e^{j(\omega t + \theta_1)} + V_{m2} e^{(\omega t + j\theta_2)}\right]$$

Consider addition of two sinusoidal quantities:

$$v(t) = v_1(t) + v_2(t)$$
$$= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)$$

Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$$
$$= V_{m1} e^{j\theta_1} + V_{m2} e^{j\theta_2}$$

In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

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$$= Re\left[V_{m1} e^{j(\omega t + \theta_1)} + V_{m2} e^{(\omega t + j\theta_2)}\right]$$
$$= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)$$

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$$= Re\left[V_{m1}e^{j(\omega t+\theta_1)} + V_{m2}e^{(\omega t+j\theta_2)}\right]$$
$$= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)$$

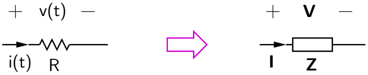
which is the same as $v(t)$.

* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.

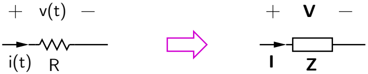
Addition of phasors

* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.
* The KCL and KVL equations,
  $\sum i_k(t) = 0$ at a node, and
  $\sum v_k(t) = 0$ in a loop,
  amount to addition of sinusoidal quantities and can therefore be replaced by the corresponding phasor equations,
  $\sum \mathbf{I}_k = \mathbf{0}$ at a node, and
  $\sum \mathbf{V}_k = \mathbf{0}$ in a loop.

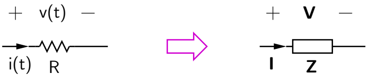
Impedance of a resistor



Impedance of a resistor

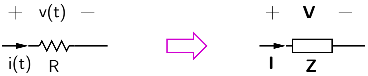


Let $i(t) = I_m \cos(\omega t + \theta)$.



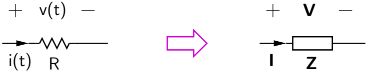
Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = R\, i(t)$



Let $i(t) = I_m \cos(\omega t + \theta)$.

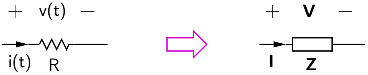
$$v(t) = R\,i(t)$$
$$= R\,I_m \cos(\omega t + \theta)$$



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$
\begin{aligned}
v(t) &= R\, i(t) \\
&= R\, I_m \cos(\omega t + \theta) \\
&\equiv V_m \cos(\omega t + \theta),
\end{aligned}
$$

Impedance of a resistor



Let $i(t) = I_m \cos(\omega t + \theta)$.

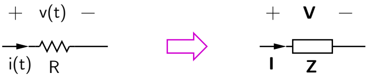
$v(t) = R\, i(t)$

$\quad = R\, I_m \cos(\omega t + \theta)$

$\quad \equiv V_m \cos(\omega t + \theta)$,

which can be rewritten as,

$Re\left[V_m\, e^{j(\omega t + \theta)}\right] = Re\left[R\, I_m\, e^{j(\omega t + \theta)}\right]$,



Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = R\, i(t)$

$\quad = R\, I_m \cos(\omega t + \theta)$

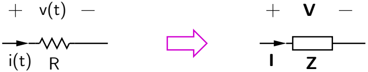
$\quad \equiv V_m \cos(\omega t + \theta)$,

which can be rewritten as,

$Re\left[V_m\, e^{j(\omega t+\theta)}\right] = Re\left[R\, I_m\, e^{j(\omega t+\theta)}\right]$,

i.e., $Re\left[V_m\, e^{j\theta}\, e^{j\omega t}\right] = R \times Re\left[I_m\, e^{j\theta}\, e^{j\omega t}\right]$,

Impedance of a resistor



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$\begin{aligned} v(t) &= R\, i(t) \\ &= R\, I_m \cos(\omega t + \theta) \\ &\equiv V_m \cos(\omega t + \theta), \end{aligned}$$

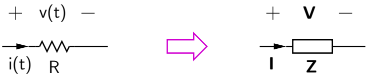
which can be rewritten as,

$Re\left[V_m\, e^{j(\omega t+\theta)}\right] = Re\left[R\, I_m\, e^{j(\omega t+\theta)}\right]$,

i.e., $Re\left[V_m\, e^{j\theta}\, e^{j\omega t}\right] = R \times Re\left[I_m\, e^{j\theta}\, e^{j\omega t}\right]$,

corresponding to the phasor relationship,

$\mathbf{V} = R\,\mathbf{I}$.



Let $i(t) = I_m \cos(\omega t + \theta)$.
$v(t) = R\,i(t)$
$= R\,I_m \cos(\omega t + \theta)$
$\equiv V_m \cos(\omega t + \theta)$,
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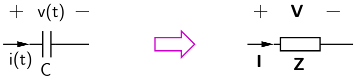
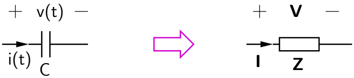
$Re\left[V_m\,e^{j(\omega t+\theta)}\right] = Re\left[R\,I_m\,e^{j(\omega t+\theta)}\right]$,
i.e., $Re\left[V_m\,e^{j\theta}\,e^{j\omega t}\right] = R \times Re\left[I_m\,e^{j\theta}\,e^{j\omega t}\right]$,
corresponding to the phasor relationship,
$\mathbf{V} = R\,\mathbf{I}$.
Thus, the *impedance* of a resistor, defined as, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is

$$\boxed{\mathbf{Z} = R + j\,0}$$

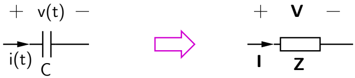
Impedance of a capacitor



Impedance of a capacitor



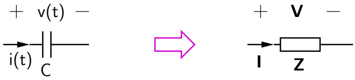
Let $v(t) = V_m \cos(\omega t + \theta)$.



Let $v(t) = V_m \cos(\omega t + \theta)$.

$$i(t) = C\,\frac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta).$$

Impedance of a capacitor



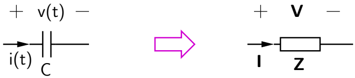
Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$i(t) = C\,\omega\,V_m\ \cos(\omega t + \theta + \pi/2)$.

Impedance of a capacitor



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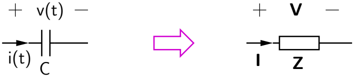
Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$i(t) = C\,\omega\,V_m\,\cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{V} = V_m \angle \theta$, $\mathbf{I} = \omega C V_m \angle(\theta + \pi/2)$.



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Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

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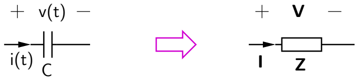
In terms of phasors, $\mathbf{V} = V_m \angle \theta$, $\mathbf{I} = \omega C V_m \angle(\theta + \pi/2)$.

$\mathbf{I}$ can be rewritten as,

$\mathbf{I} = \omega C V_m\, e^{j(\theta+\pi/2)} = \omega C V_m\, e^{j\theta}\, e^{j\pi/2} = j\omega C\, \left(V_m\, e^{j\theta}\right) = j\omega C\, \mathbf{V}$



Let $v(t) = V_m \cos(\omega t + \theta)$.

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Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

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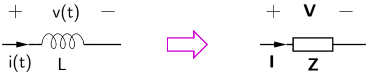
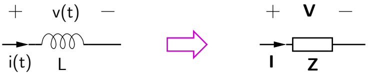
Thus, the *impedance* of a capacitor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = 1/(j\omega C)}$,

and the *admittance* of a capacitor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = j\omega C}$.

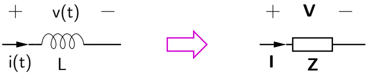
Impedance of an inductor





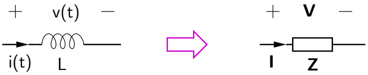
Let $i(t) = I_m \cos(\omega t + \theta)$.

Impedance of an inductor



Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = L\,\dfrac{di}{dt} = -L\,\omega\,I_m \sin(\omega t + \theta)$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

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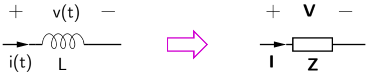
Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$v(t) = L\,\omega\, I_m\ \cos(\omega t + \theta + \pi/2)$.



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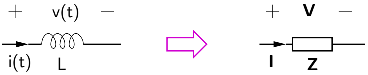
Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$v(t) = L\,\omega\, I_m\, \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = \omega L I_m \angle(\theta+\pi/2)$.

Impedance of an inductor



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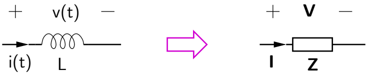
In terms of phasors, $\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = \omega L I_m \angle(\theta + \pi/2)$.

$\mathbf{V}$ can be rewritten as,

$\mathbf{V} = \omega L I_m\, e^{j(\theta+\pi/2)} = \omega L I_m\, e^{j\theta}\, e^{j\pi/2} = j\omega L\, \left(I_m\, e^{j\theta}\right) = j\omega L\, \mathbf{I}$



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$v(t) = L\,\dfrac{di}{dt} = -L\,\omega\,I_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$v(t) = L\,\omega\,I_m\ \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = \omega L I_m\ \angle(\theta+\pi/2)$.

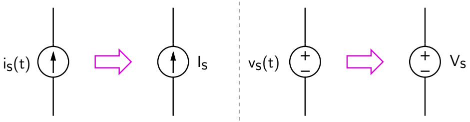
$\mathbf{V}$ can be rewritten as,

$\mathbf{V} = \omega L I_m\, e^{j(\theta+\pi/2)} = \omega L I_m\, e^{j\theta}\, e^{j\pi/2} = j\omega L\,\left(I_m\, e^{j\theta}\right) = j\omega L\,\mathbf{I}$

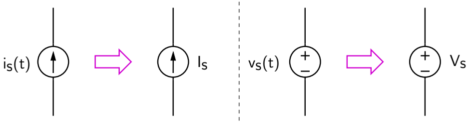
Thus, the *impedance* of an indcutor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = j\omega L}$,

and the *admittance* of an inductor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = 1/(j\omega L)}$.

Sources

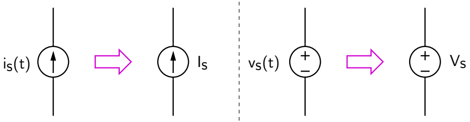


Sources



* An independent sinusoidal current source, $i_s(t) = I_m \cos(\omega t + \theta)$, can be represented by the phasor $I_m \angle \theta$ (i.e., a *constant* complex number).

Sources



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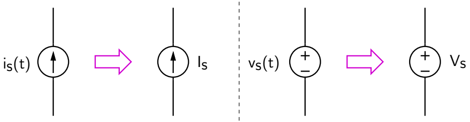

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* An independent sinusoidal voltage source, $v_s(t) = V_m \cos(\omega t + \theta)$, can be represented by the phasor $V_m \angle \theta$ (i.e., a *constant* complex number).
* Dependent (linear) sources can be treated in the sinusoidal steady state in the same manner as a resistor, i.e., by the corresponding phasor relationship.
  For example, for a CCVS, we have,
  $v(t) = r\, i_c(t)$ in the time domain.
  $\mathbf{V} = r\, \mathbf{I_c}$ in the frequency domain.

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.

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* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).

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* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = \mathit{constant}$ (a complex number).

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* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).
* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = \textit{constant}$ (a complex number).
* For dependent sources, the time-domain relationships such as $i(t) = \beta\, i_c(t)$ translate to $\mathbf{I} = \beta\,\mathbf{I_c}$ in the frequency domain.

Use of phasors in circuit analysis

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.
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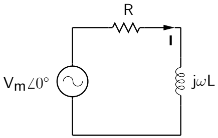
* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = \mathit{constant}$ (a complex number).
* For dependent sources, the time-domain relationships such as $i(t) = \beta\, i_c(t)$ translate to $\mathbf{I} = \beta\,\mathbf{I_c}$ in the frequency domain.
* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.

Use of phasors in circuit analysis

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.
* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).
* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = constant$ (a complex number).
* For dependent sources, the time-domain relationships such as $i(t) = \beta\, i_c(t)$ translate to $\mathbf{I} = \beta\,\mathbf{I_c}$ in the frequency domain.
* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.
* Series/parallel formulas for resistors, nodal analysis, mesh analysis, Thevenin's and Norton's theorems can be directly applied to circuits in the sinusoidal steady state.

RL circuit



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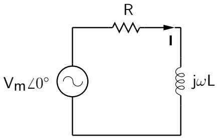

$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle(-\theta),$$

where $I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$, and $\theta = \tan^{-1}(\omega L/R)$.

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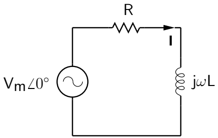

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In the time domain, $i(t) = I_m \cos(\omega t - \theta)$, which *lags* the source voltage since the peak (or zero) of $i(t)$ occurs $t = \theta/\omega$ seconds after that of the source voltage.

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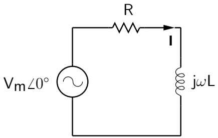

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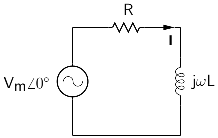
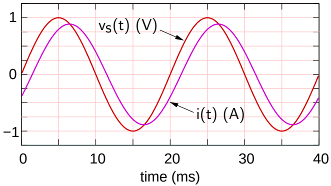
For $R = 1\,\Omega$, $L = 1.6\,\text{mH}$, $f = 50\,\text{Hz}$, $\theta = 26.6^\circ$, $t_{\text{lag}} = 1.48$ ms.

(SEQUEL file: `ee101_rl_ac_1.sqproj`)





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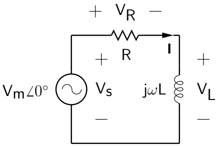

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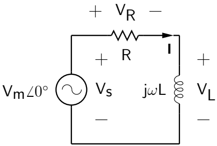

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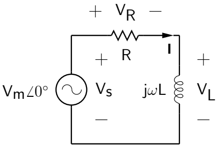

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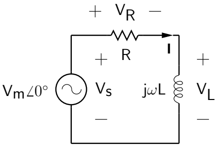

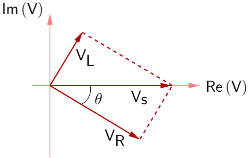

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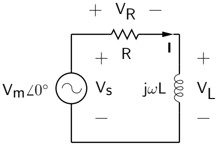

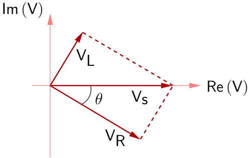

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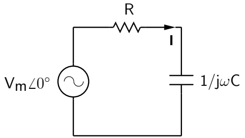
RC circuit



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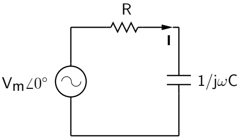
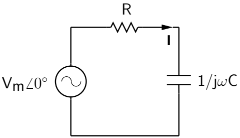


$$\mathbf{I} = \frac{V_m \angle 0}{R + 1/j\omega C} \equiv I_m \angle \theta,$$

where $I_m = \dfrac{\omega C V_m}{\sqrt{1 + (\omega R C)^2}}$, and $\theta = \pi/2 - \tan^{-1}(\omega R C)$.



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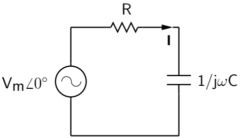

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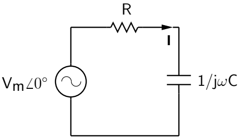
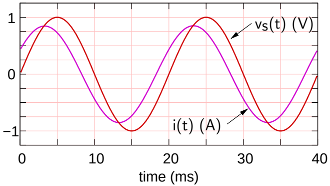
For $R = 1\,\Omega$, $L = 5.3\,\text{mF}$, $f = 50\,\text{Hz}$, $\theta = 31^\circ$, $t_{\text{lead}} = 1.72$ ms.

(SEQUEL file: `ee101_rc_ac_1.sqproj`)





$R = 1\,\Omega$
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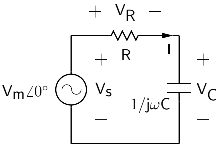

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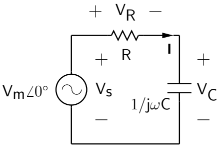

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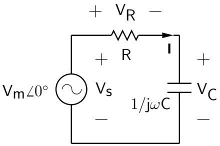

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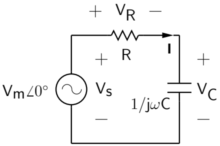

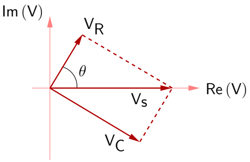

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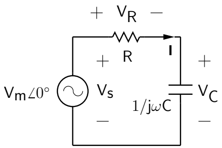

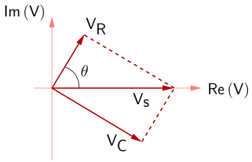

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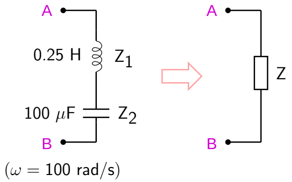
Series/parallel connections



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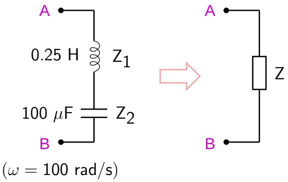
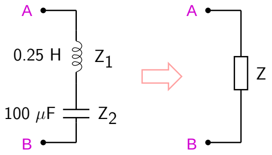


$Z_1 = \mathrm{j} \times 100 \times 0.25 = \mathrm{j}\,25\,\Omega$

$Z_2 = -\mathrm{j}/(100 \times 100 \times 10^{-6}) = -\mathrm{j}\,100\,\Omega$

$Z = Z_1 + Z_2 = -\mathrm{j}\,75\,\Omega$



$(\omega = 100$ rad/s$)$

$$Z_1 = j \times 100 \times 0.25 = j\,25\,\Omega$$

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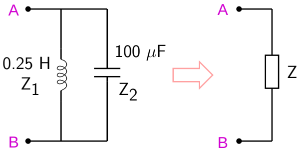

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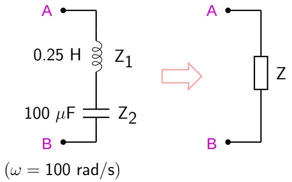

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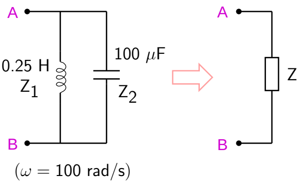
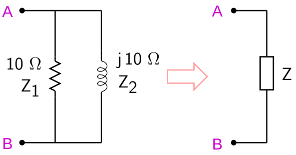


$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(j\,25) \times (-j\,100)}{j\,25 - j\,100} = \frac{25 \times 100}{-j\,75} = j\,33.3\,\Omega$$

Impedance example

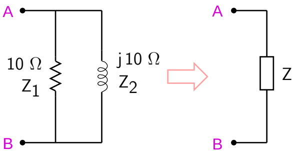
Obtain Z in polar form.



($\omega = 100$ rad/s)

Impedance example

Obtain Z in polar form.



$(\omega = 100$ rad/s$)$

Method 1:

$$Z = \frac{10 \times j10}{10 + j10} = \frac{j10}{1 + j}$$

$$= \frac{j10}{1 + j} \times \frac{1 - j}{1 - j}$$

$$= \frac{10 + j10}{2} = 5 + j5\,\Omega$$

Convert to polar form $\rightarrow Z = 7.07\,\angle\,45^\circ\ \Omega$

Impedance example

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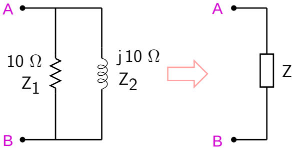

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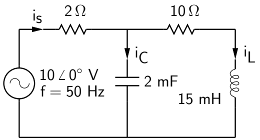
Convert to polar form $\rightarrow Z = 7.07\,\angle\,45^\circ\,\Omega$

Method 2:

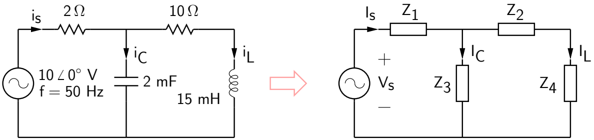
$$Z = \frac{10 \times j10}{10 + j10} = \frac{100\,\angle\,\pi/2}{10\sqrt{2}\,\angle\,\pi/4}$$

$$= 5\sqrt{2}\,\angle\,(\pi/2 - \pi/4) = 7.07\,\angle\,45^\circ\,\Omega$$

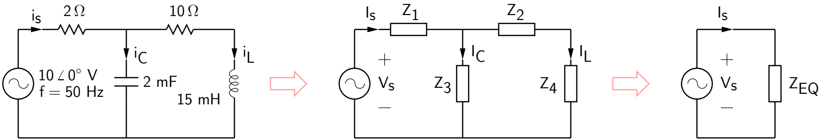
Circuit example

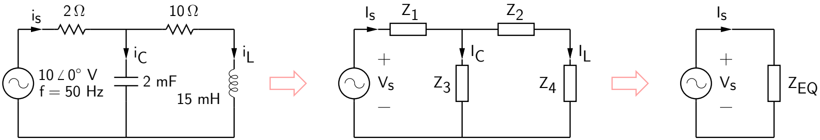


Circuit example

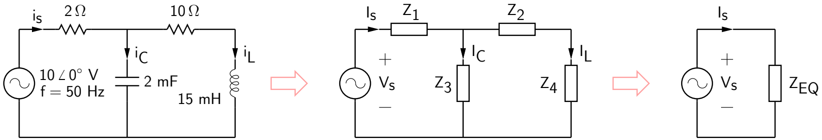


Circuit example



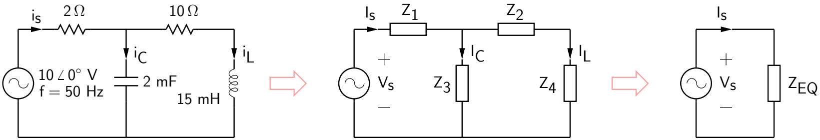


$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

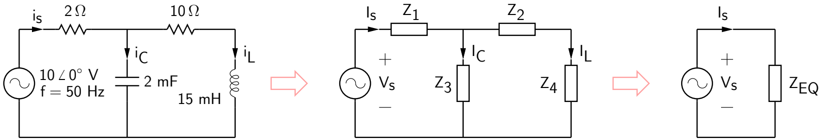
$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$



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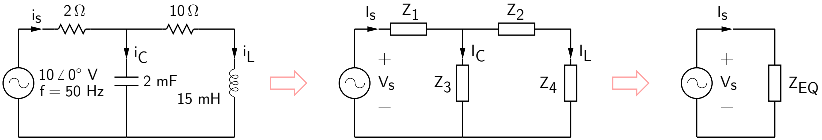
$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

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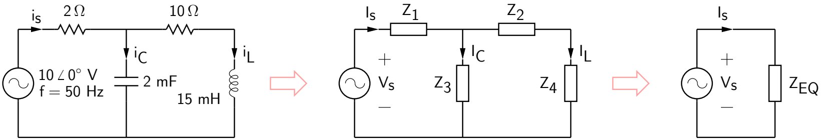
$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

$$= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)}$$



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

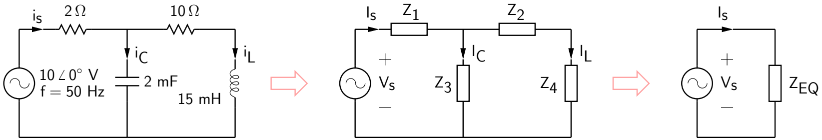
$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

$$= 2 + \frac{1.6\angle(-90°) \times 11.05\angle(25.2°)}{10.47\angle(17.2°)} = 2 + \frac{17.7\angle(-64.8°)}{10.47\angle(17.2°)}$$

$$= 2 + 1.69\angle(-82°) = 2 + (0.235 - j\,1.67)$$

Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

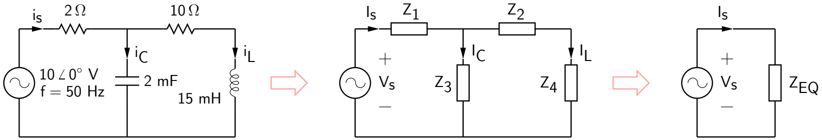
$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

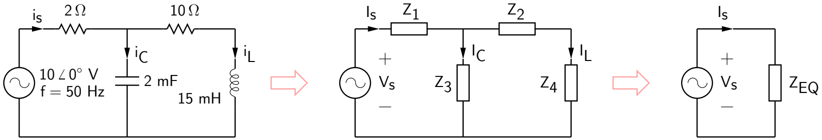
$$= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)}$$

$$= 2 + 1.69\angle(-82^\circ) = 2 + (0.235 - j\,1.67)$$

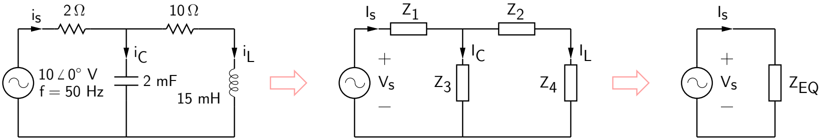
$$= 2.235 - j\,1.67 = 2.79\angle(-36.8^\circ)\ \Omega$$

Circuit example (continued)





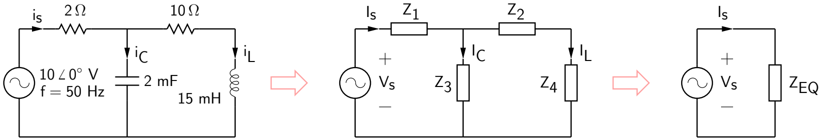
$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\,\angle\,(0^\circ)}{2.79\,\angle\,(-36.8^\circ)} = 3.58\,\angle\,(36.8^\circ)\ A$$



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\,\angle\,(0^\circ)}{2.79\,\angle\,(-36.8^\circ)} = 3.58\,\angle\,(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\,\angle\,(44.6^\circ)\ A$$

Circuit example (continued)

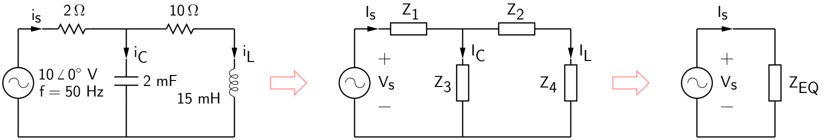


$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_3}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 0.546\angle(-70.6^\circ)\ A$$

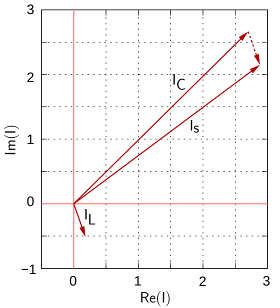
Circuit example (continued)



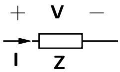
$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

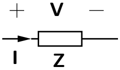
$$\mathbf{I}_C = \frac{(\mathbf{Z}_2+\mathbf{Z}_4)}{\mathbf{Z}_3+(\mathbf{Z}_2+\mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_3}{\mathbf{Z}_3+(\mathbf{Z}_2+\mathbf{Z}_4)} \times \mathbf{I}_s = 0.546\angle(-70.6^\circ)\ A$$



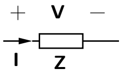
Sinusoidal steady state: power computation





Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

$i(t) = I_m \cos(\omega t + \phi)$, i.e., $\mathbf{I} = I_m \angle \phi$.

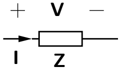


Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

$i(t) = I_m \cos(\omega t + \phi)$, i.e., $\mathbf{I} = I_m \angle \phi$.

The *instantaneous* power absorbed by $\mathbf{Z}$ is,

$$\begin{aligned} P(t) &= v(t)\, i(t) \\ &= V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi) \\ &= \frac{1}{2} V_m I_m \left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right] \end{aligned} \quad (1)$$



Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

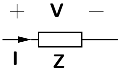
$i(t) = I_m \cos(\omega t + \phi)$, i.e., $\mathbf{I} = I_m \angle \phi$.

The *instantaneous* power absorbed by **Z** is,

$$\begin{aligned} P(t) &= v(t)\, i(t) \\ &= V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi) \\ &= \frac{1}{2} V_m I_m \left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right] \end{aligned} \tag{1}$$

The *average* power absorbed by **Z** is

$P = \dfrac{1}{T}\displaystyle\int_0^T P(t)\, dt$, where $T = 2\pi/\omega$.



Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

$i(t) = I_m \cos(\omega t + \phi)$, i.e., $\mathbf{I} = I_m \angle \phi$.

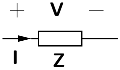
The *instantaneous* power absorbed by **Z** is,

$$\begin{aligned} P(t) &= v(t)\, i(t) \\ &= V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi) \\ &= \frac{1}{2} V_m I_m \left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right] \end{aligned} \qquad (1)$$

The *average* power absorbed by **Z** is

$P = \dfrac{1}{T} \displaystyle\int_0^T P(t)\, dt$, where $T = 2\pi/\omega$.

The first term in Eq. (1) has an average value of zero and does not contribute to P.
Therefore,



Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle\theta$,

$i(t) = I_m \cos(\omega t + \phi)$, i.e., $\mathbf{I} = I_m \angle\phi$.

The *instantaneous* power absorbed by **Z** is,

$$\begin{aligned} P(t) &= v(t)\, i(t) \\ &= V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi) \\ &= \frac{1}{2} V_m I_m \left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right] \end{aligned} \tag{1}$$

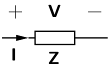
The *average* power absorbed by **Z** is

$$P = \frac{1}{T}\int_0^T P(t)\, dt, \text{ where } T = 2\pi/\omega.$$

The first term in Eq. (1) has an average value of zero and does not contribute to P. Therefore,

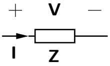
$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta - \phi)}$ gives the average power absorbed by **Z**.

Average power for R, L, C



General formula:

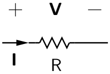
$$V = V_m \angle\theta, I = I_m \angle\phi$$
$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$



General formula:

$V = V_m \angle \theta \,, I = I_m \angle \phi$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

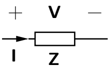


$V = R\,I$

For $I = I_m \angle \alpha$, $V = R\,I_m \angle \alpha$,

$$P = \frac{1}{2}(R\,I_m)\,I_m \cos(\alpha - \alpha) = \frac{1}{2} I_m^2\, R = \frac{1}{2} V_m^2/R$$

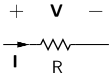
Average power for R, L, C



General formula:

$$V = V_m \angle \theta \,, I = I_m \angle \phi$$

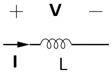
$$P = \frac{1}{2} V_m \, I_m \cos(\theta - \phi)$$



$$V = R\, I$$

For $I = I_m \angle \alpha,\ \ V = R\, I_m \angle \alpha \,,$

$$P = \frac{1}{2}(R\, I_m)\, I_m \cos(\alpha - \alpha) = \frac{1}{2} I_m^2\, R = \frac{1}{2} V_m^2 / R$$

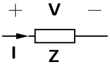


$$V = j\omega L\, I$$

For $I = I_m \angle \alpha,\ \ V = \omega L\, I_m \angle (\alpha + \pi/2) \,,$

$$P = \frac{1}{2} V_m\, I_m \cos\, [(\alpha + \pi/2) - \alpha] = 0$$

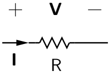
Average power for R, L, C



General formula:

$V = V_m \angle \theta\,,\, I = I_m \angle \phi$

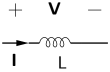
$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$



$V = R\,I$

For $I = I_m \angle \alpha,\;\; V = R\,I_m \angle \alpha\,,$

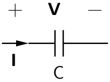
$P = \frac{1}{2}(R\,I_m)\,I_m \cos(\alpha - \alpha) = \frac{1}{2} I_m^2\,R = \frac{1}{2} V_m^2/R$



$V = j\omega L\,I$

For $I = I_m \angle \alpha,\;\; V = \omega L\,I_m \angle (\alpha + \pi/2)\,,$

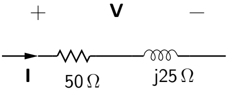
$P = \frac{1}{2} V_m I_m \cos\,[(\alpha + \pi/2) - \alpha] = 0$



$I = j\omega C\,V$

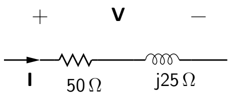
For $V = V_m \angle \alpha,\;\; I = \omega C\,V_m \angle (\alpha + \pi/2)\,,$

$P = \frac{1}{2} V_m I_m \cos\,[\alpha - (\alpha + \pi/2)] = 0$



Given: $\mathrm{I} = 2\angle 45^\circ$ A

Find the average power absorbed.

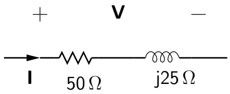


Given: $\mathrm{I} = 2\,\angle\,45^\circ$ A

Find the average power absorbed.

Method 1:

$$\mathbf{V} = (50 + j25) \times 2\,\angle\,45^\circ$$
$$= 55.9\,\angle\,26.6^\circ \times 2\,\angle\,45^\circ$$
$$= 111.8\,\angle\,(45^\circ + 26.6^\circ)$$



Given: $\mathbf{I} = 2\,\angle\,45^{\circ}$ A

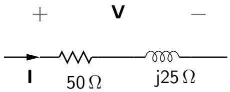
Find the average power absorbed.

Method 1:

$$\begin{aligned}\mathbf{V} &= (50 + j25) \times 2\,\angle\,45^{\circ}\\ &= 55.9\,\angle\,26.6^{\circ} \times 2\,\angle\,45^{\circ}\\ &= 111.8\,\angle\,(45^{\circ} + 26.6^{\circ})\end{aligned}$$

$$P = \frac{1}{2} \times 111.8 \times 2 \times \cos\,(26.6^{\circ}) = 100\;\; W.$$

Average power: example



Given: $\mathbf{I} = 2\angle 45^\circ$ A

Find the average power absorbed.

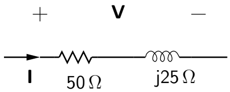
Method 1:

$$\mathbf{V} = (50 + j25) \times 2\angle 45^\circ$$
$$= 55.9\angle 26.6^\circ \times 2\angle 45^\circ$$
$$= 111.8\angle (45^\circ + 26.6^\circ)$$
$$P = \frac{1}{2} \times 111.8 \times 2 \times \cos(26.6^\circ) = 100\ W.$$

Method 2:

No average power is absorbed by the inductor.

$\Rightarrow P = P_R$ (average power absorbed by R)



Given: $\mathrm{I} = 2\angle 45^\circ$ A

Find the average power absorbed.

Method 1:

$$\mathbf{V} = (50 + j25) \times 2\angle 45^\circ$$
$$= 55.9\angle 26.6^\circ \times 2\angle 45^\circ$$
$$= 111.8\angle(45^\circ + 26.6^\circ)$$
$$P = \frac{1}{2} \times 111.8 \times 2 \times \cos(26.6^\circ) = 100\ W.$$

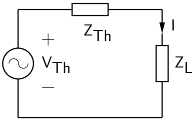
Method 2:

No average power is absorbed by the inductor.

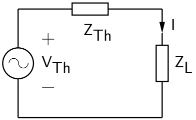
$\Rightarrow P = P_R$ (average power absorbed by R)

$$= \frac{1}{2} I_m^2 R = \frac{1}{2} \times 2^2 \times 50$$
$$= 100\ W.$$

Maximum power transfer



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.



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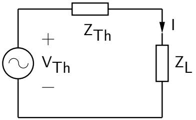
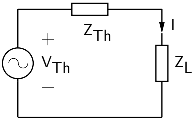
The power absorbed by $\mathbf{Z}_L$ is,

$$P = \frac{1}{2}\, I_m^2 R_L$$
$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$
$$= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\, R_L\,.$$





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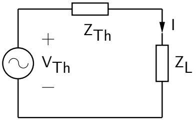
For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.



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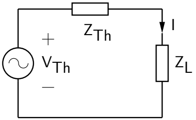
With $X_L = -X_{Th}$, we have,

$$P = \frac{1}{2}\, \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2}\, R_L\, ,$$

which is maximum for $R_L = R_{Th}$.



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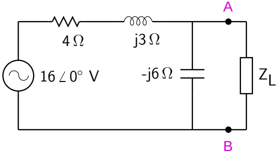
which is maximum for $R_L = R_{Th}$.

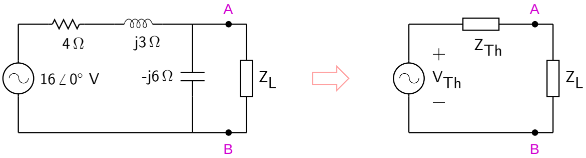
Therefore, for maximum power transfer to the load $\mathbf{Z}_L$, we need,

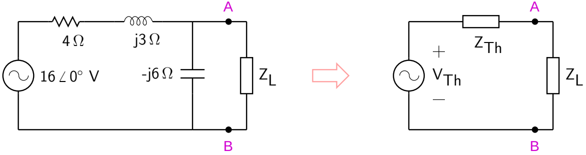
$R_L = R_{Th}$, $X_L = -X_{Th}$, i.e., $\boxed{\mathbf{Z}_L = \mathbf{Z}_{Th}^*.}$

Maximum power transfer: example



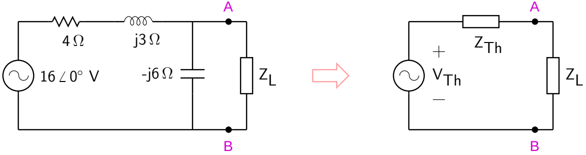
Maximum power transfer: example





$\mathbf{Z}_{Th} = (-j\,6) \parallel (4 + j\,3) = 5.76 - j\,1.68\,\Omega\,.$

Maximum power transfer: example



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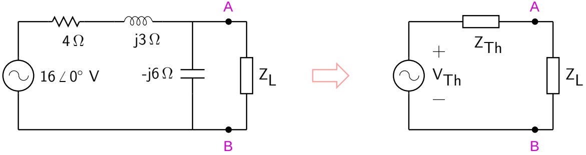
For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}^*_{Th} = 5.76 + j\,1.68\,\Omega \equiv R_L + j\,X_L\,.$



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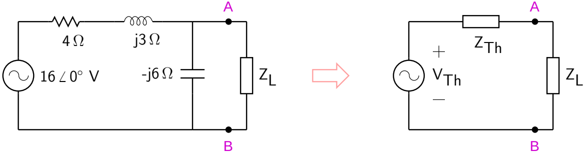
$$\mathbf{V}_{Th} = 16\angle 0^{\circ} \times \frac{-j\,6}{(4+j\,3)+(-j\,6)} = 19.2\angle(-53.13^{\circ})\,.$$



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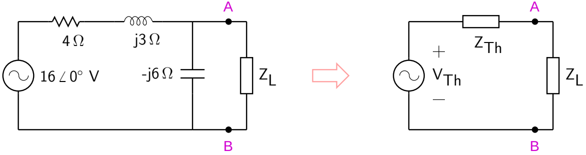

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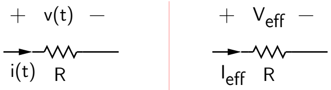
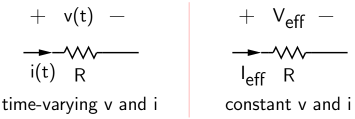
$$P = \frac{1}{2}\,I_m^2 R_L = \frac{1}{2}\left(\frac{19.2}{2\,R_L}\right)^2 \times R_L = \frac{1}{2}\,\frac{(19.2)^2}{4\,R_L} = 8\;W.$$



time-varying v and i | constant v and i

Consider a periodic current $i(t)$ passing through R.



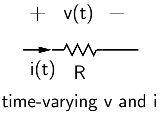
Consider a periodic current $i(t)$ passing through R.

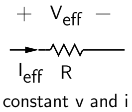
The *average* power absorbed by R is,

$$P_1 = \frac{1}{T}\int_{t_0}^{t_0+T} [i(t)]^2\, R\, dt\,,$$

where t_0 is some reference time (we will take t_0 to be 0).

Effective (rms) values of voltage/current





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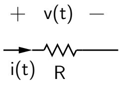
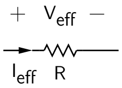
The *average* power absorbed by R in the DC case is,

$P_2 = I_{eff}^2 \, R$.

Effective (rms) values of voltage/current



time-varying v and i



constant v and i

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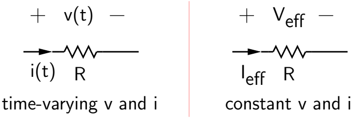
I_{eff}, the *effective* value of $i(t)$, is defined such that $P_1 = P_2$, i.e.,

$$I_{eff}^2 \, R = \frac{1}{T} \int_0^T [i(t)]^2 \, R \, dt \,,$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T [i(t)]^2 \; dt} \,.$$

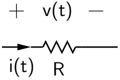
Effective (rms) values of voltage/current



$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T [i(t)]^2 \, dt} \, .$$

I_{eff} is called the root-mean-square (rms) value of $i(t)$ because of the operations (root, mean, and square) involved in its computation.

Effective (rms) values of voltage/current



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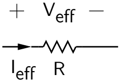

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$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T [i(t)]^2\, dt}\,.$$

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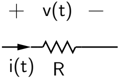
If $i(t)$ is sinusoidal, i.e., $i(t) = I_m \cos(\omega t + \phi)$,

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\, dt} = I_m \sqrt{\frac{1}{T}\int_0^T \frac{1}{2}\left[1 + \cos(2\omega t + \phi)\right] dt}$$

$$= I_m \sqrt{\frac{1}{T}\,\frac{1}{2}\,T} = I_m/\sqrt{2}\,.$$

Effective (rms) values of voltage/current



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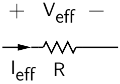

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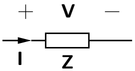
Similarly, $V_{eff} = V_m/\sqrt{2}\,.$

Apparent power, real power, and power factor



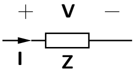
$\mathrm{V} = \mathrm{V_m} \angle \theta$

$\mathrm{I} = \mathrm{I_m} \angle \phi$

The average ("real") power absorbed by **Z** is,

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta - \phi)$$

$$= V_{eff} I_{eff} \cos(\theta - \phi) \text{ (Watts)}$$



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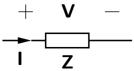
Apparent power is defined as $P_{app} = V_{eff}\, I_{eff}$ (Volt-Amp).



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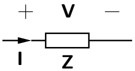
Power factor is defined as $P.F. = \dfrac{\text{Average power}}{\text{Apparent power}} = \cos(\theta - \phi)$.



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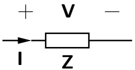
$(\theta - \phi) > 0$: $i(t)$ lags $v(t)$, the P. F. is called a *lagging* P. F.
(inductive impedance)

Apparent power, real power, and power factor



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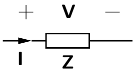
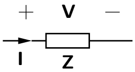
$(\theta - \phi) < 0$: $i(t)$ leads $v(t)$, the P. F. is called a *leading* P. F. (capacitive impedance)



1. Given: $\mathbf{V} = 120\,\angle 0^{\circ}$ V (rms), $\mathbf{I} = 2\,\angle(-36.9^{\circ})$ A (rms).
   Find P_{app}, P.F., and P.



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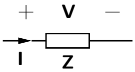
   $P_{app} = 120 \times 2 = 240\ V\text{-}A.$



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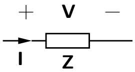
   P.F. $= \cos(0^{\circ} - (-36.9^{\circ})) = 0.8$ lagging (since $\mathbf{I}$ lags $\mathbf{V}$).



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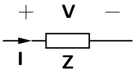
   $P = P_{app} \times \text{P.F.} = 192\ W.$

Power factor: examples



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   $P = P_{app} \times$ P.F. $= 192$ W.

2. Given: $P = 50$ kW, P.F. $= 0.95$ (lagging), $\mathbf{V} = 480\,\angle 0^{\circ}$ V (rms).
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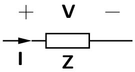
Power factor: examples



1. Given: $\mathbf{V} = 120\,\angle 0^\circ$ V (rms), $\mathbf{I} = 2\,\angle(-36.9^\circ)$ A (rms).
   Find P_{app}, P.F., and P.

   $P_{app} = 120 \times 2 = 240\ V\text{-}A.$

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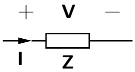
   $V_{eff} \times I_{eff} \times$ P.F $= 50 \times 10^3$



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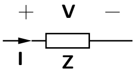
   $V_{eff} \times I_{eff} \times \text{P.F} = 50 \times 10^3$

   $$I_{eff} = \frac{50 \times 10^3}{480 \times 0.95} = 109.6\ A.$$



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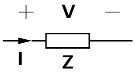
   Since P.F. is 0.95 (lagging), **I** lags **V** by $\cos^{-1}(0.95) = 18.2^{\circ}$.



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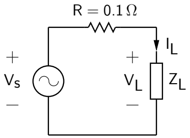
   $\Rightarrow \mathbf{I} = 109.6\,\angle(-18.2^\circ)$ A (rms).

Why is power factor important?



Consider a simplified model of a power system consisting of a generator ($\mathbf{V}_s$), transmission line (R), and load ($\mathbf{Z}_L$).

The load is specified as $P = 50$ kW, P.F.$=$ 0.6 (lagging), $\mathbf{V}_L = 480\angle 0^\circ\ V$ (rms).

Note: lagging power factors are typical of industrial loads (motors).

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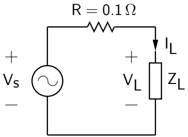

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$$P = 50 \times 10^3 \; W = |\mathbf{V}_L| \times |\mathbf{I}_L| \times \text{P.F.} \Rightarrow |\mathbf{I}_L| = \frac{50 \times 10^3}{480 \times 0.6} = 173.6 \; A \text{ (rms)}.$$

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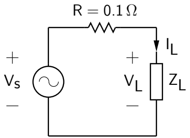

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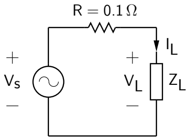

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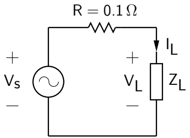

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Thus, a higher power factor can substantially reduce transmission losses.

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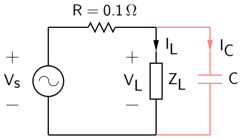

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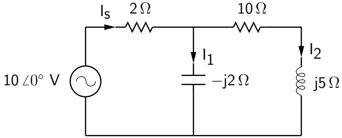
Thus, a higher power factor can substantially reduce transmission losses.
The effective power factor of an inductive load can be improved by connecting a suitable capacitance in parallel.



* Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_s$.
* Compute the average power absorbed by each element.
* Verify power balance.

(SEQUEL file: `ee101_phasors_2.sqproj`)