EE101: Op Amp circuits (Part 2)

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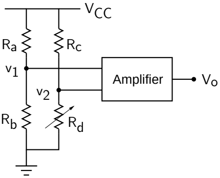
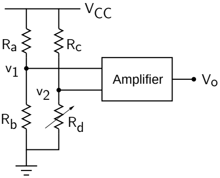

VCC
Ra
Rc
v1
Amplifier
Vo
v2
Rb
Rd



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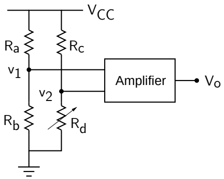
$R_d = R + \Delta R$ varies with the quantity to be measured. Typically, ΔR is a small fraction of R.



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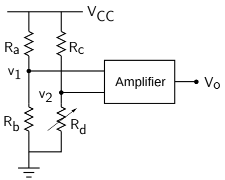

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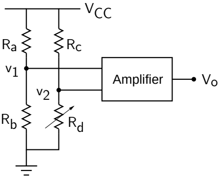

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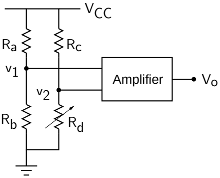

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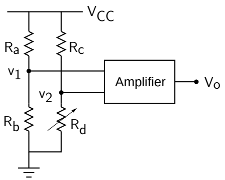
where $x = \Delta R/R$.

Common-mode and differential-mode voltages



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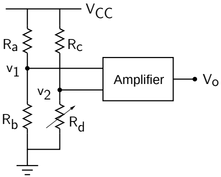
where $x = \Delta R/R$.

For example, with $V_{CC} = 15\,V$, $R = 1\,\mathrm{k}$, $\Delta R = 0.01\,\mathrm{k}$,

$v_1 = 7.5\,V$,

$v_2 = 7.5 + 0.0375\,V$.



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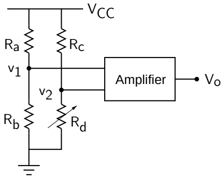
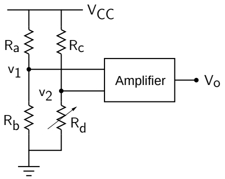


$v_1 = 7.5\ V\ ,\ \ v_2 = 7.5 + 0.0375\ V\ .$

The amplifier should only amplify $v_2 - v_1 = 0.0375\ V$ (since that is the signal arising from ΔR).



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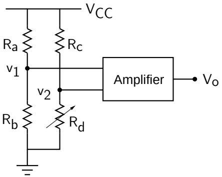
Given v_1 and v_2,

$v_c = \frac{1}{2}(v_1 + v_2)$ = common-mode voltage,

$v_d = (v_2 - v_1)$ = differential-mode voltage.



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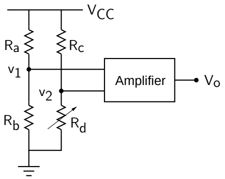

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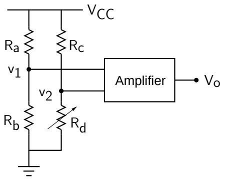

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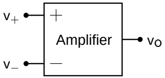
Note that the common-mode voltage is quite large compared to the differential-mode voltage.

This is a common situation in transducer circuits.

Common-Mode Rejection Ratio



$$v_+ = v_c + v_d/2$$
$$v_- = v_c - v_d/2$$

An ideal amplifier would only amplify the difference $(v_+ - v_-)$, giving

$v_o = A_d\,(v_+ - v_-) = A_d\,v_d$,

where A_d is called the “differential gain” or simply the gain (A_V).

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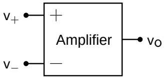

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In practice, the output can also have a common-mode component:

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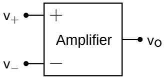

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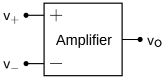
The ability of an amplifier to *reject* the common-mode signal is given by the Common-Mode Rejection Ratio (CMRR):

$$\text{CMRR} = \frac{A_d}{A_c} \,.$$



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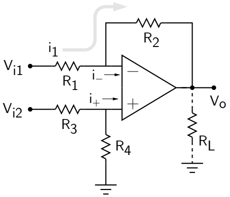
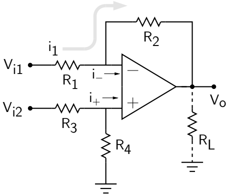
For the 741 Op Amp, the CMRR is 90 dB ($\simeq$ 30,000), which may be considered to be infinite in many applications. In such cases, mismatch between circuit components will determine the overall common-mode rejection performance of the circuit.


R2
i1
Vi1
R1
i−
i+
Vi2
R3
R4
Vo
RL



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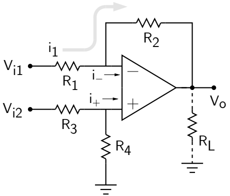
Large input resistance of Op Amp $\rightarrow$ $i_+ = 0$, $V_+ = \dfrac{R_4}{R_3 + R_4}\, V_{i2}$.



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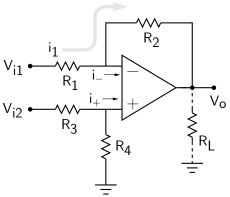

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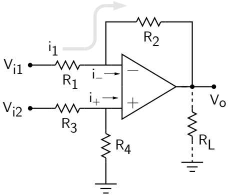

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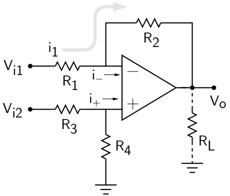
Substituting for V_+ and selecting $R_3/R_4 = R_1/R_2$, we get (show this),

$V_o = \dfrac{R_2}{R_1}(V_{i2} - V_{i1})$.



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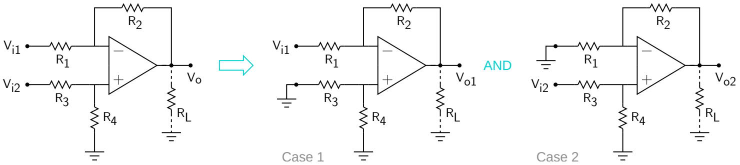
$$V_o = \frac{R_2}{R_1}\,(V_{i2} - V_{i1})\,.$$

The circuit is a "difference amplifier."



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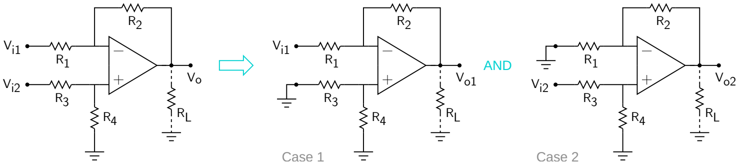

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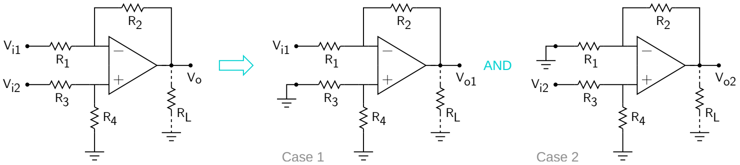
$$\rightarrow V_{o1} = -\frac{R_2}{R_1} V_{i1} .$$



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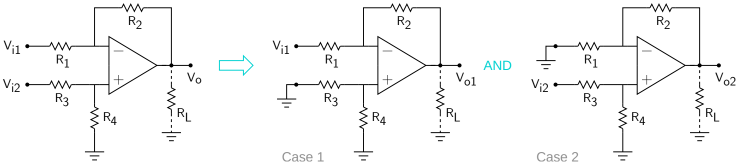
$$\rightarrow V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)\,V_{i2}\,.$$



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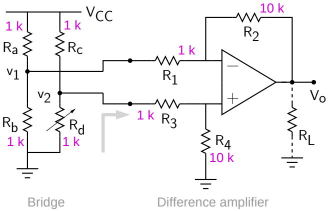
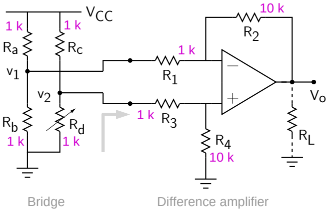
The net result is,

$$V_o = V_{o1} + V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1}\, V_{i1} = \frac{R_2}{R_1}\,(V_{i2} - V_{i1})\,,\ \text{if } R_3/R_4 = R_1/R_2.$$

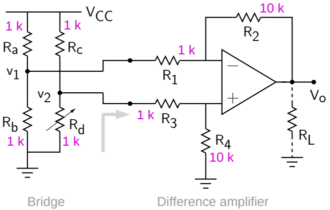
Difference amplifier





The resistance seen from v_2 is $(R_3 + R_4)$ which is small enough to cause v_2 to change.

This is not desirable.



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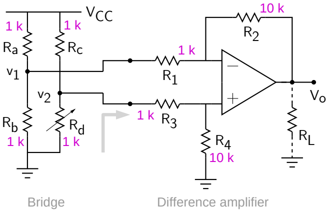

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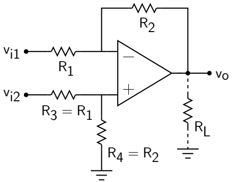
We will discuss an improved difference amplifier later. Before we do that, let us discuss another problem with the above difference amplifier which can be important for some applications (next slide).



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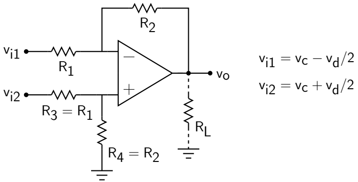
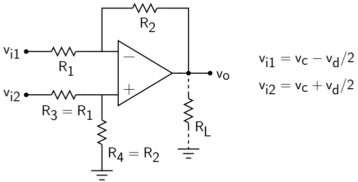


Consider the difference amplifier with $R_3 = R_1$, $R_4 = R_2 \rightarrow V_o = \dfrac{R_2}{R_1}\,(v_{i2} - v_{i1})$.

The output voltage depends only on the differential-mode signal $(v_{i2} - v_{i1})$,
i.e., A_c (common-mode gain) $= 0$.



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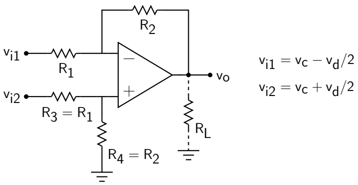
In practice, R_3 and R_1 may not be exactly equal. Let $R_3 = R_1 + \Delta R$.



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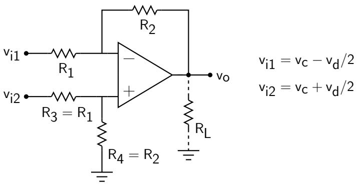
The output voltage depends only on the differential-mode signal $(v_{i2} - v_{i1})$,

i.e., A_c (common-mode gain) $= 0$.

In practice, R_3 and R_1 may not be exactly equal. Let $R_3 = R_1 + \Delta R\,.$

$$\begin{aligned} v_o &= \frac{R_2}{R_1 + \Delta R + R_2}\left(1 + \frac{R_2}{R_1}\right) v_{i2} - \frac{R_2}{R_1}\, v_{i1} \\ &\simeq \frac{R_2}{R_1}(v_d - x\, v_c)\,, \text{with } x = \frac{\Delta R}{R_1 + R_2} \qquad \text{(show this)} \end{aligned}$$



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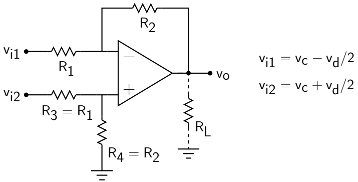
$|A_c| = x\,\dfrac{R_2}{R_1} \ll |A_d| = \dfrac{R_2}{R_1}$, since $x \sim 0.01$ (with 1% tolerance resistors).



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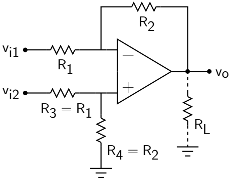
However, since v_c can be large compared to v_d, the effect of A_c cannot be ignored.

Difference amplifier



$v_{i1} = v_c - v_d/2$

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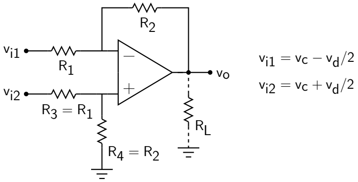
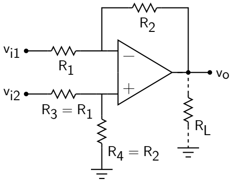


$|A_c| = x \dfrac{R_2}{R_1}$, $|A_d| = \dfrac{R_2}{R_1}$, where $x = \dfrac{\Delta R}{R_1 + R_2}$.

In our earlier example, $v_c = 7.5\ V \quad v_d = 0.0375\ V$.



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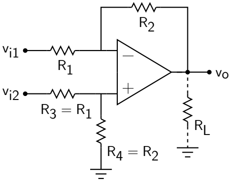
$|A_c\, v_c| = x\,\dfrac{R_2}{R_1}\,v_c = 0.01 \times 10 \times 7.5 = 0.75\,V$.



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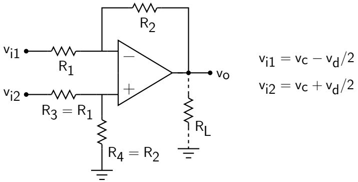

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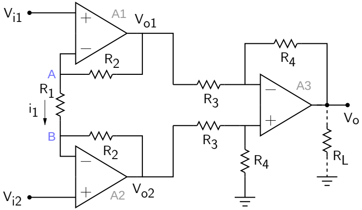
The (spurious) common-mode contribution is substantial.

$\rightarrow$ need a circuit which will reduce the common-mode component at the output.

Improved difference amplifier



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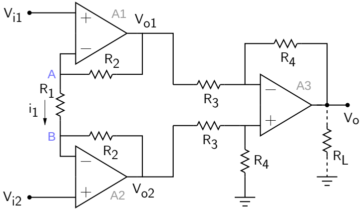

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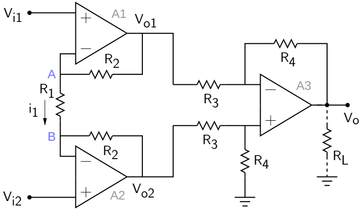

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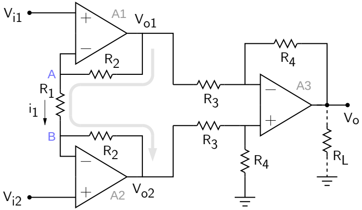

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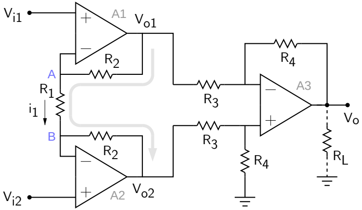

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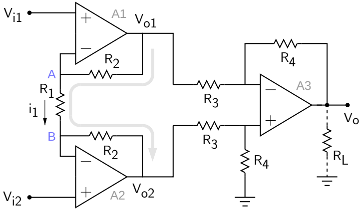

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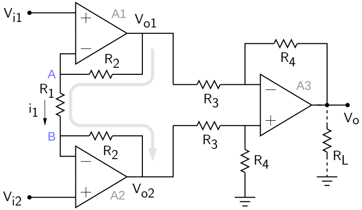

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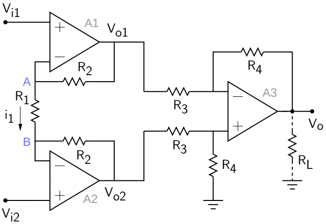
This circuit is known as the "instrumentation amplifier."

Instrumentation amplifier



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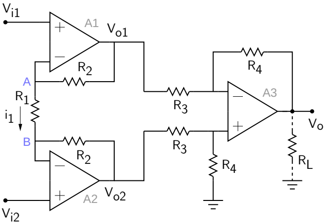

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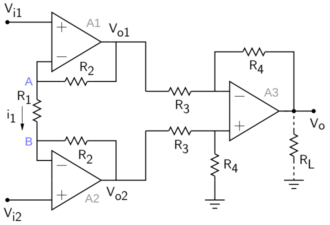

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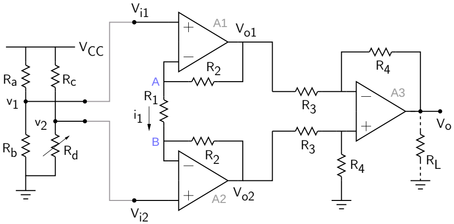

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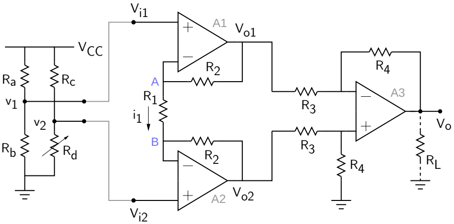

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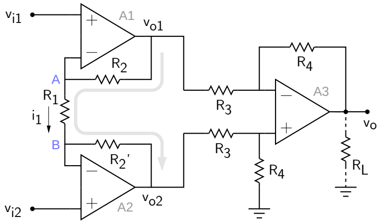
As a result, the voltages v_1 and v_2 in the bridge circuit will remain essentially the same when the bridge circuit is connected to the instrumentation amplifier.



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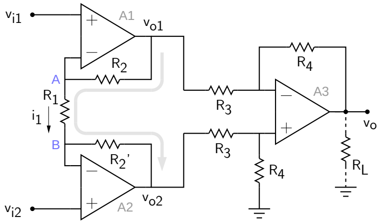

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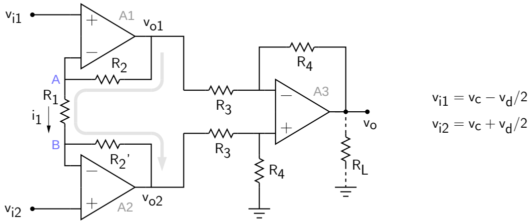

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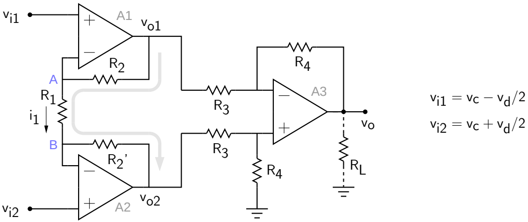

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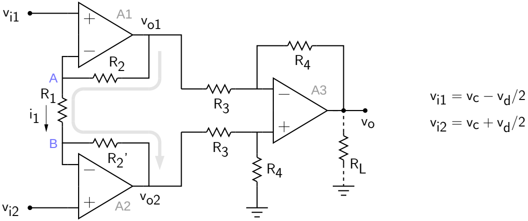

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$\rightarrow$ The instrumentation amplifier is very effective in removing the common-mode signal.

Current-to-voltage conversion

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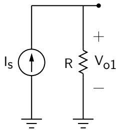

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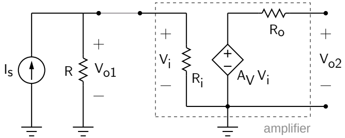
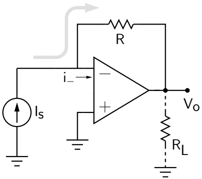


However, this simple approach will not work if the next stage in the circuit (such as an amplifier) has a finite R_i, since it will modify V_{o1} to $V_{o1} = I_s\,(R_i \parallel R)$, which is not desirable.

Current-to-voltage conversion



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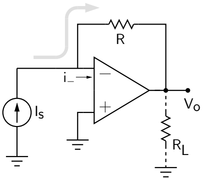

$V_- \approx V_+$, and $i_- \approx 0 \Rightarrow V_o = V_- - I_s\, R = -I_s\, R$.

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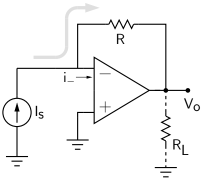

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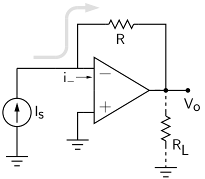

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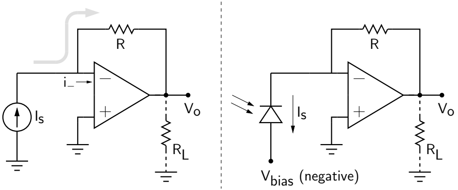

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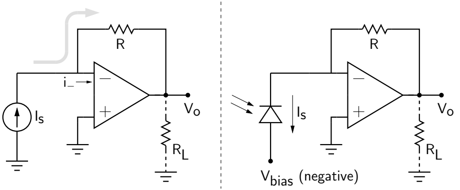

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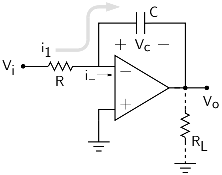
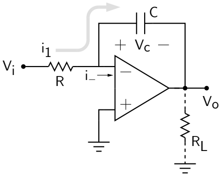
$V_o = I_s R$. The diode is under a reverse bias, with $V_n = 0\,V$ and $V_p = V_{\text{bias}}$.


C
i_1
+ V_c −
V_i
R
i_-
−
+
V_o
R_L



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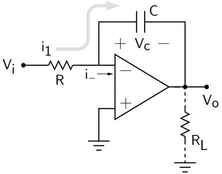

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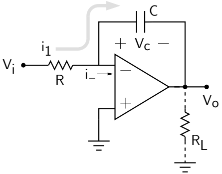

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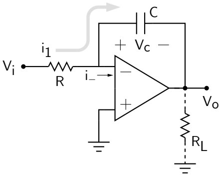

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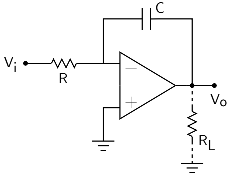
$$\boxed{V_o = -\frac{1}{RC}\int V_i\,dt}$$

The circuit works as an integrator.

Integrator



$R = 1\,\mathrm{k}\Omega\,,\;\; C = 0.2\,\mu F$

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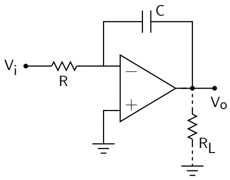

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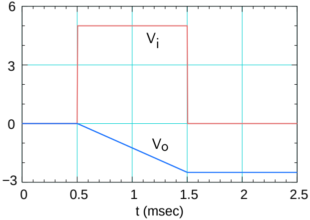

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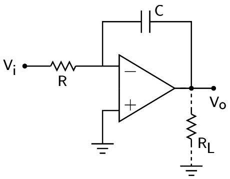

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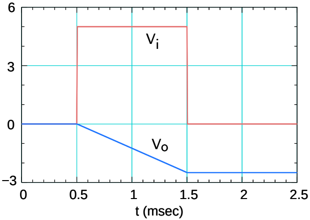
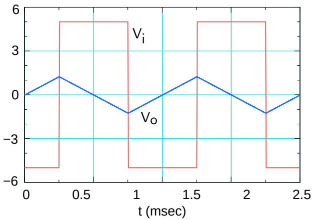
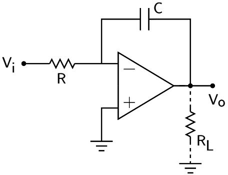




Integrator



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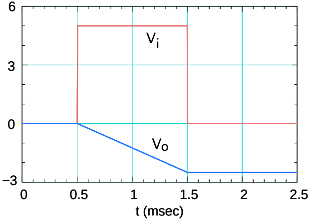
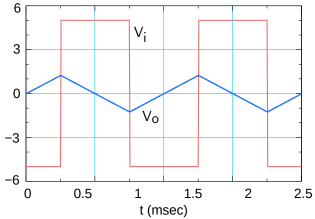
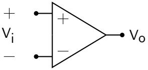
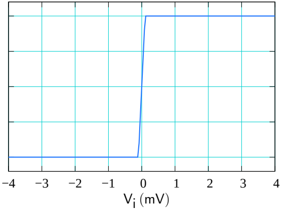




SEQUEL files: `ee101_integrator_1.sqproj`, `ee101_integrator_2.sqproj`

Offset voltage





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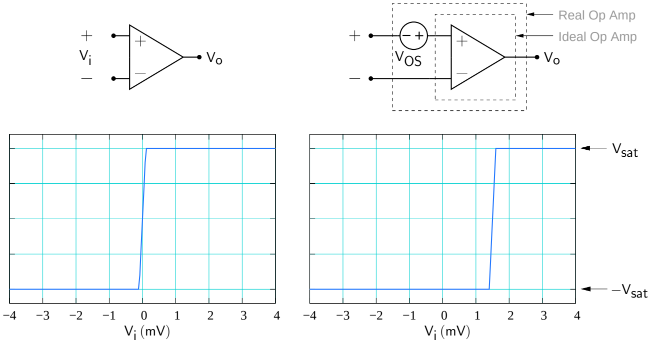

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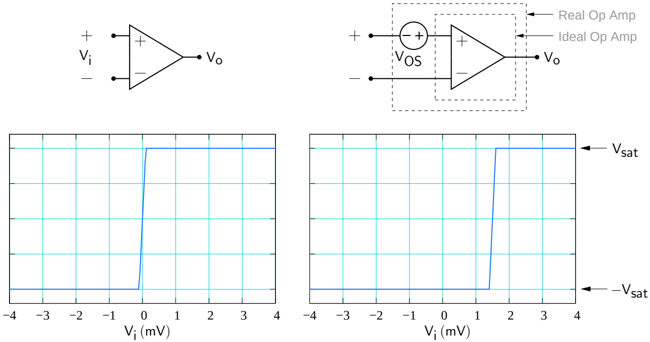

For the real Op Amp, $V_o = A_V((V_+ + V_{OS}) - V_-)$.

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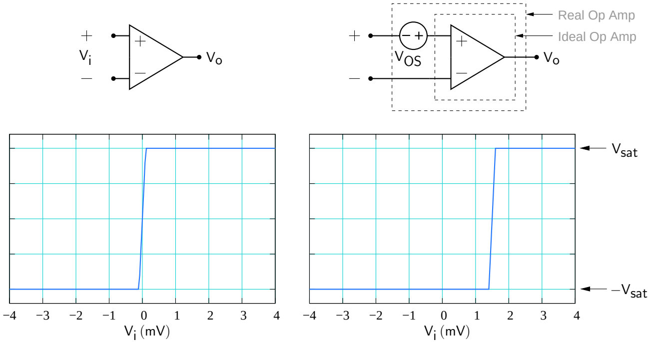

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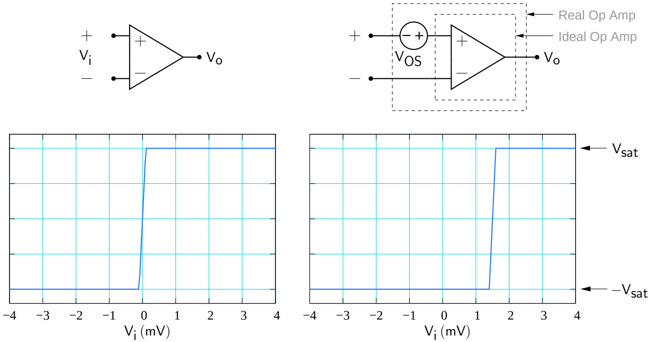

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V_o versus V_i curve gets shifted.

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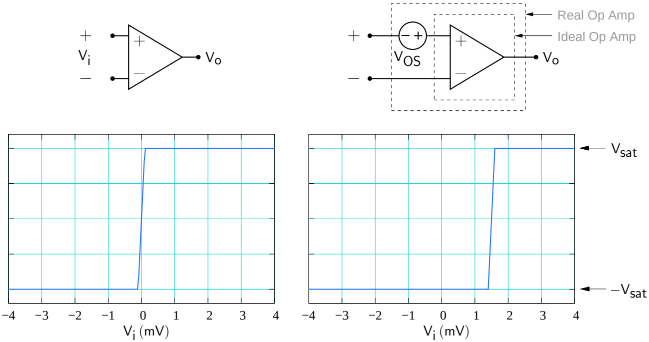

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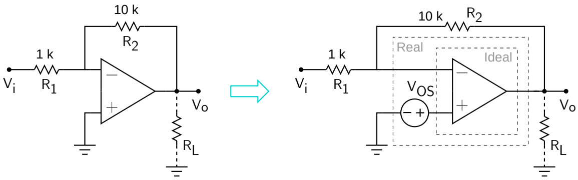
V_o versus V_i curve gets shifted.

741: $-6\,\mathrm{m}V < V_{OS} < 6\,\mathrm{m}V$.

OP-77: $-50\,\mu V < V_{OS} < 50\,\mu V$.

Effect of V_{OS}



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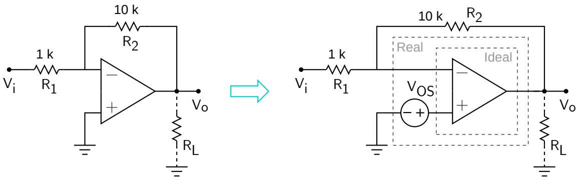

By superposition, $V_o = -\dfrac{R_2}{R_1}\,V_i + V_{OS}\left(1 + \dfrac{R_2}{R_1}\right).$

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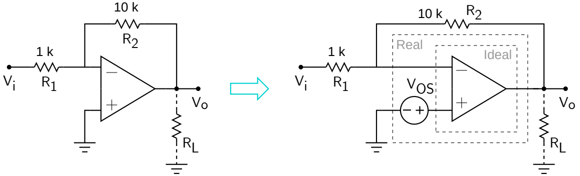

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For $V_{OS} = 2\,\text{m}V$, the contribution from V_{OS} to V_o is $22\,\text{m}V$,

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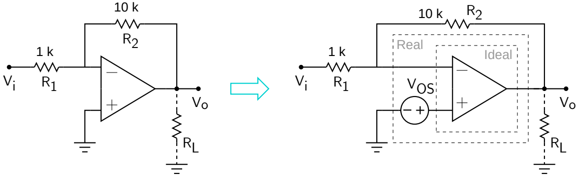

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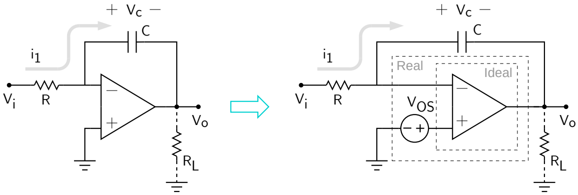
i.e., a DC shift of $22\,\mathrm{m}V$.

Effect of V_{OS}



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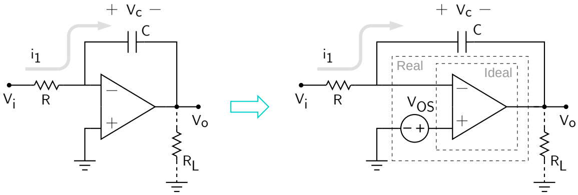

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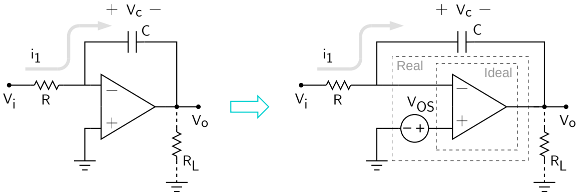

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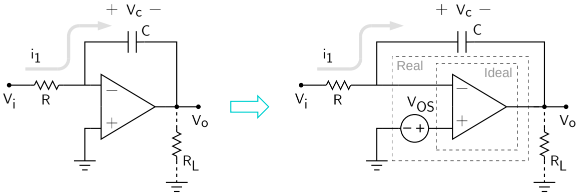

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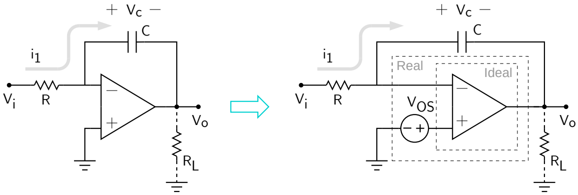
Eventually, the Op Amp will be driven into saturation.

Effect of V_{OS}



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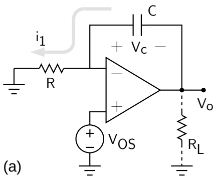
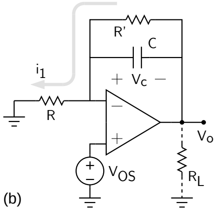
Even with $V_i = 0\,V$, V_c will keep rising or falling (depending on the sign of V_{OS}).

Eventually, the Op Amp will be driven into saturation.

$\longrightarrow$ need to address this issue!

Integrator with $V_i = 0\ V$:





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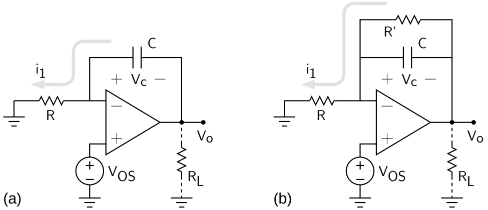

$$\text{(a)} \qquad i_1 = \frac{V_{OS}}{R} = -C\,\frac{dV_c}{dt}$$

$$V_c = -\frac{1}{RC}\int V_{OS}\,dt \rightarrow \text{Op Amp saturates.}$$

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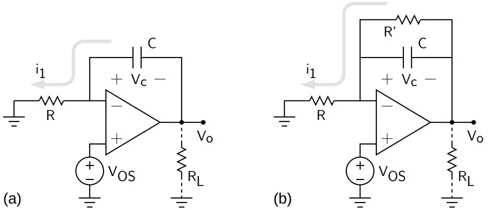

(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

$V_c = -\dfrac{1}{RC}\displaystyle\int V_{OS}\,dt \rightarrow$ Op Amp saturates.

(b) There is a DC path for the current.

$\rightarrow V_o = \left(1 + \dfrac{R'}{R}\right) V_{OS}\,.$

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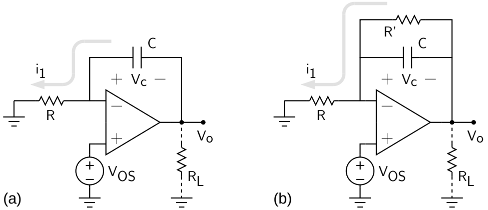

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R' should be small enough to have a negligible effect on V_o.

Effect of V_{OS}

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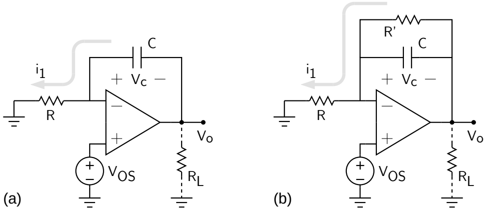

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However, R' must be large enough to ensure that the circuit still functions as an integrator.

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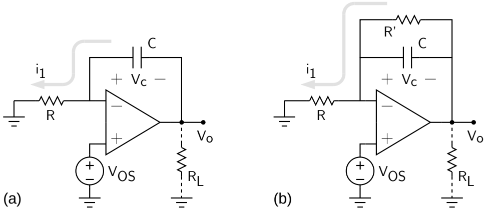

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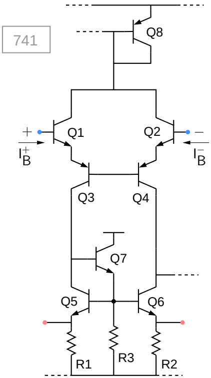
R' should be small enough to have a negligible effect on V_o.

However, R' must be large enough to ensure that the circuit still functions as an integrator.

$\rightarrow R' \gg 1/\omega C$ at the frequency of interest.

Input bias currents



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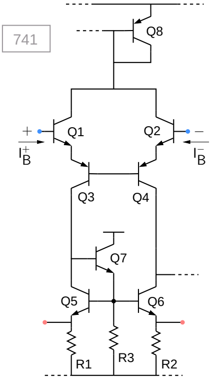

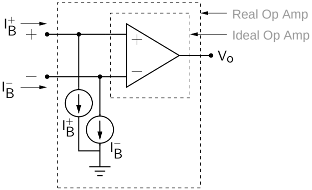

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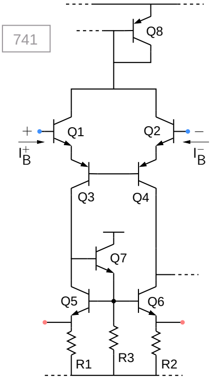

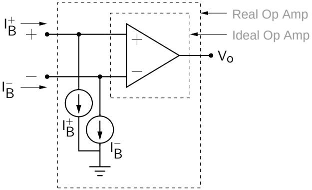

I_B^+ and I_B^- are generally not exactly equal.

$|I_B^+ - I_B^-|$: "offset current" (I_{OS})

$(I_B^+ + I_B^-)/2$: "bias current" (I_B).

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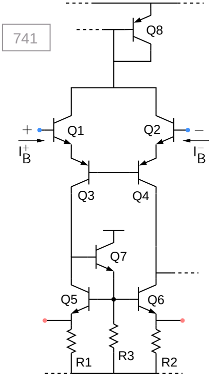

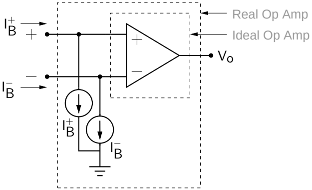

Op Amp	I_B	I_{OS}	V_{OS}	
741	80 nA	20 nA	1 mV	BJT input
OP77	1.2 nA	0.3 nA	10 μV	BJT input
411	50 pA	25 pA	0.8 mV	FET input

I_B^+ and I_B^- are generally not exactly equal.

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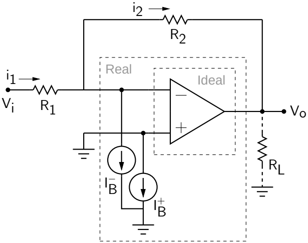
Inverting amplifier:



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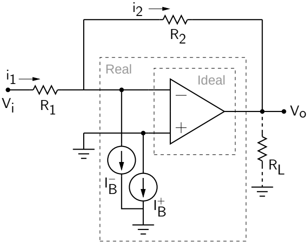

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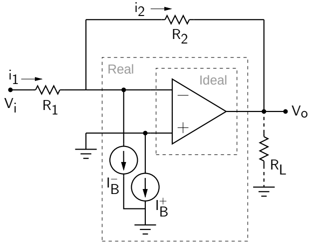

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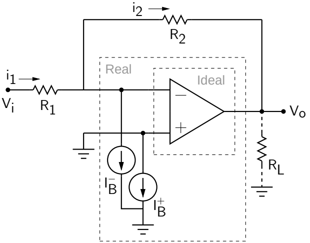

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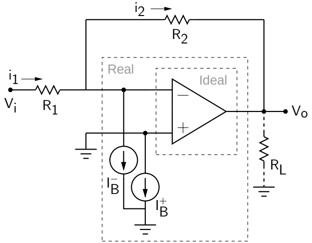

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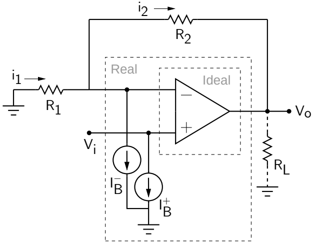
i.e., the bias current causes a DC shift in V_o.

For $I_B^- = 80\,\mathrm{n}A$, $R_2 = 10\,\mathrm{k}$, $\Delta V_o = 0.8\,\mathrm{m}V$.

Non-nverting amplifier:



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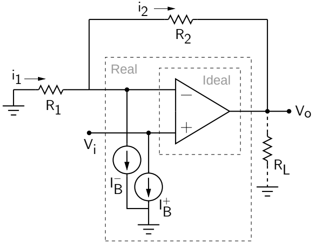

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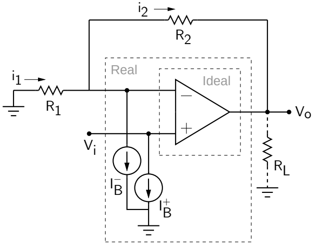

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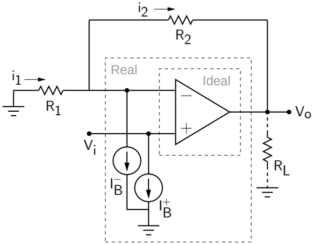

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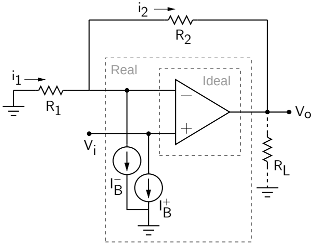

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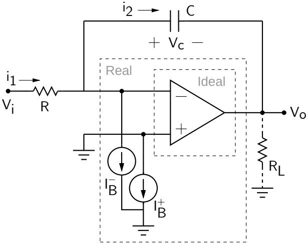
$$i_2 = i_1 - I_B^- = -\frac{V_i}{R_1} - I_B^-\,.$$

$$V_o = V_i - i_2\,R_2 = V_i - \left(-\frac{V_i}{R_1} - I_B^-\right) R_2 = V_i\left(1 + \frac{R_2}{R_1}\right) + I_B^-\,R_2\,.$$

$\rightarrow$ Again, a DC shift ΔV_o.

Effect of bias currents

Integrator:



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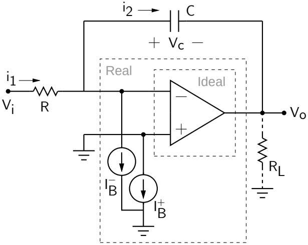

Even with $V_i = 0\ V$, $V_c = \dfrac{1}{C}\displaystyle\int -I_B^-\, dt$ will drive the Op Amp into saturation.

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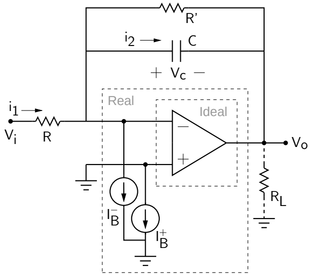

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Connecting R' across C provides a DC path for the current, and results in a DC shift $\Delta V_o = I_B^- R'$ at the output.

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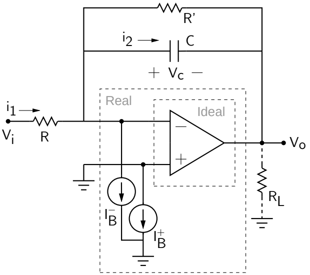

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As we have discussed earlier, R' should be small enough to have a negligible effect on V_o. However, R' must be large enough to ensure that the circuit still functions as an integrator.