EE101: Op Amp circuits (Part 3)

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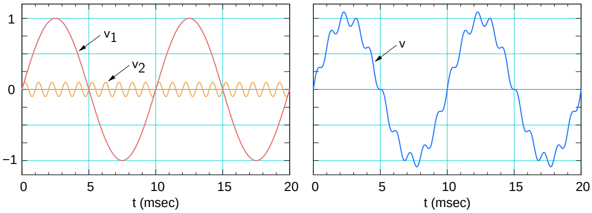
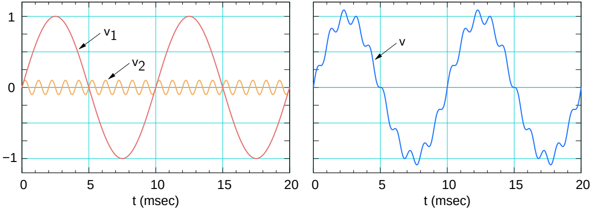
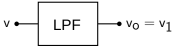
Consider $v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$.



Introduction to filters

Consider $v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$.





A low-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the low-frequency component $v_1(t)$ and remove the high-frequency component $v_2(t)$.

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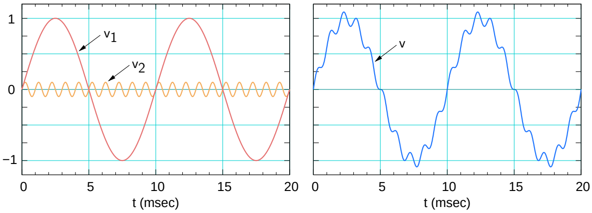

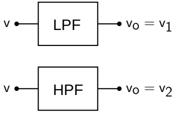

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A high-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the high-frequency component $v_2(t)$ and remove the low-frequency component $v_1(t)$.

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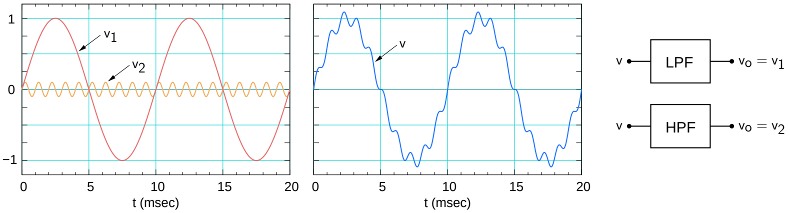

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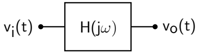
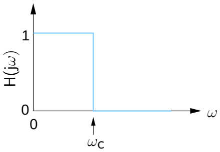
A high-pass filter with a cut-off frequency $\omega_1 < \omega_c < \omega_2$ will pass the high-frequency component $v_2(t)$ and remove the low-frequency component $v_1(t)$.

There are some other types of filters, as we will see.

Ideal low-pass filter





$$V_o(j\omega) = H(j\omega)\, V_i(j\omega)\,.$$

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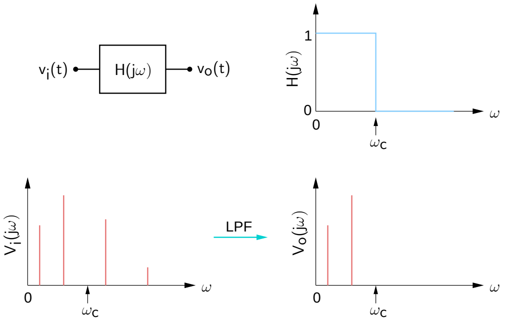

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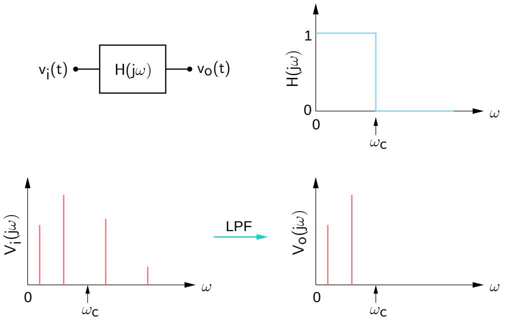


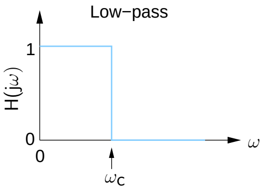
$V_o(j\omega) = H(j\omega)\, V_i(j\omega)\,.$

All components with $\omega < \omega_c$ appear at the output without attenuation.

All components with $\omega > \omega_c$ get eliminated.

(Note that the ideal low-pass filter has $\angle H(j\omega) = 1$, i.e., $H(j\omega) = 1 + j0\,$.)

Ideal filters



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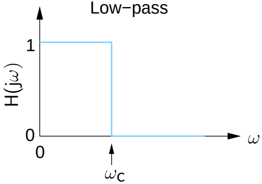

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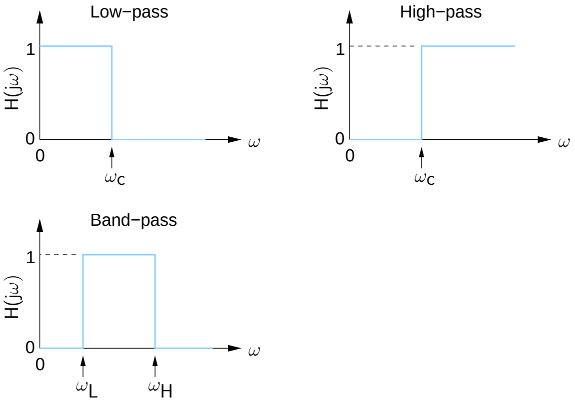

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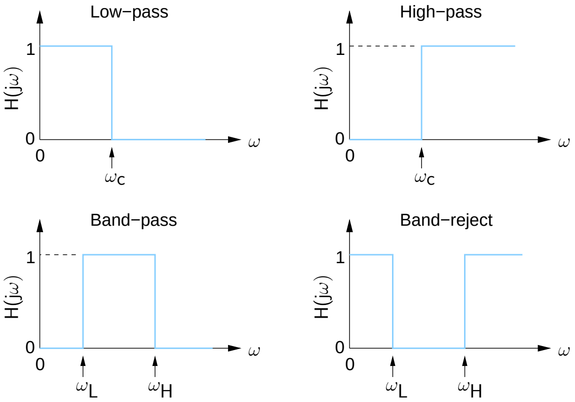

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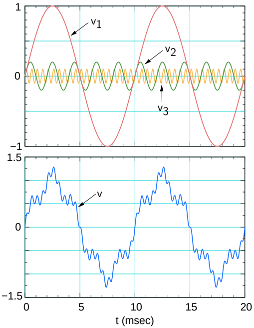

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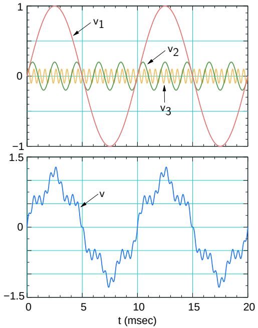

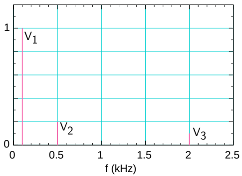

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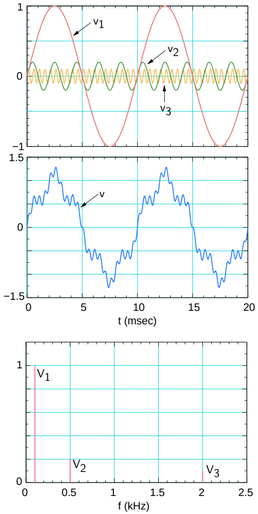
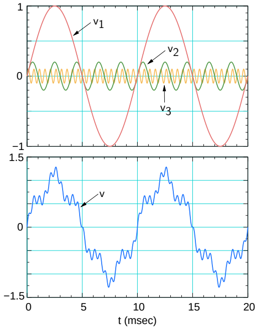
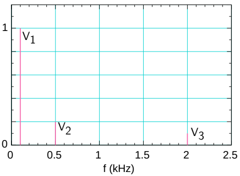
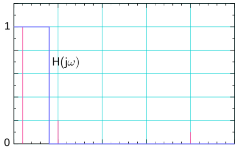


Let us see the effect of a few filters on v(t).







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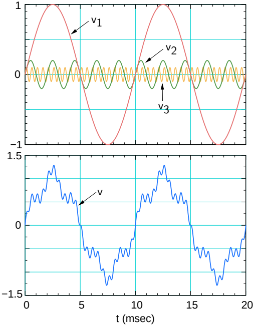

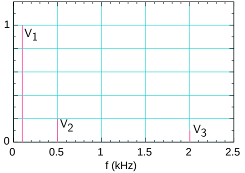

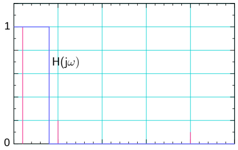

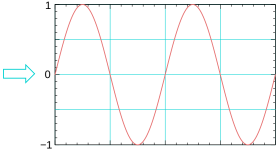

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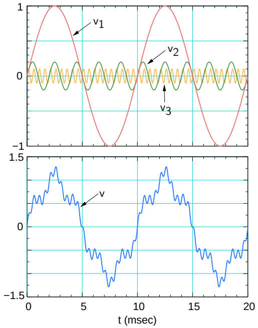

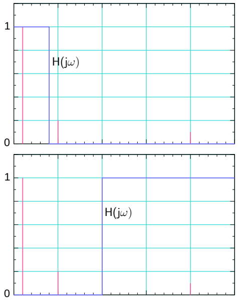

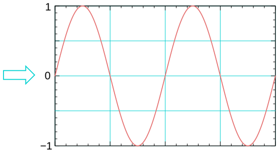

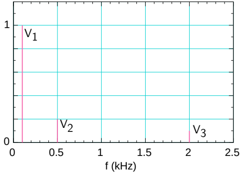

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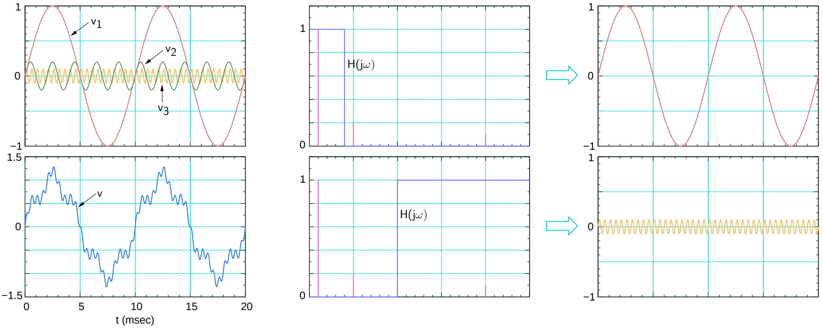

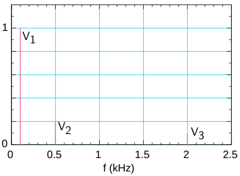

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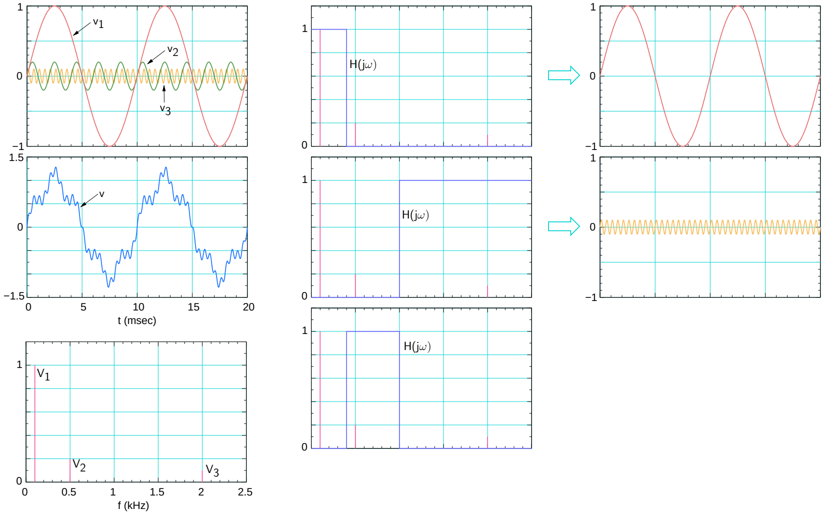

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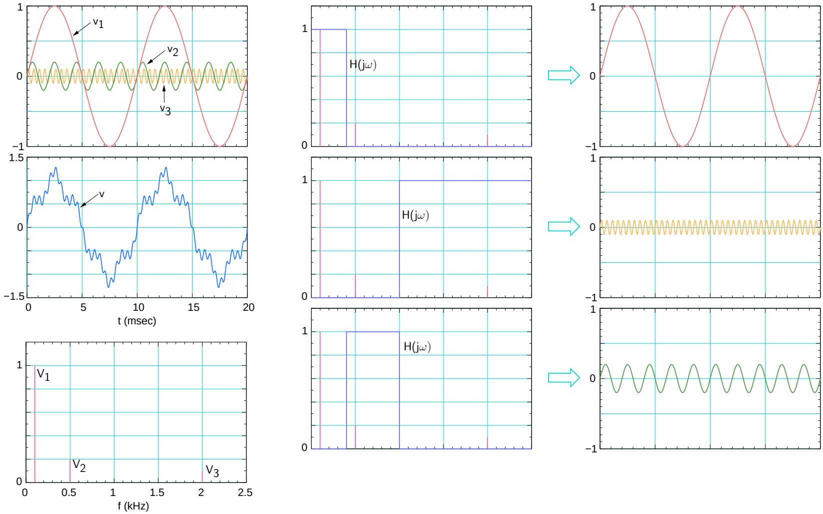

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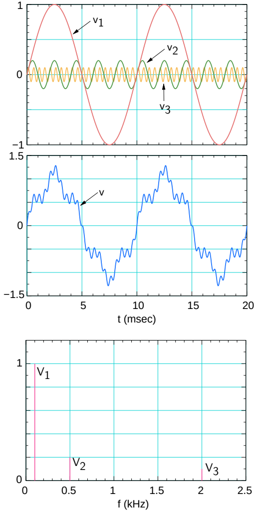

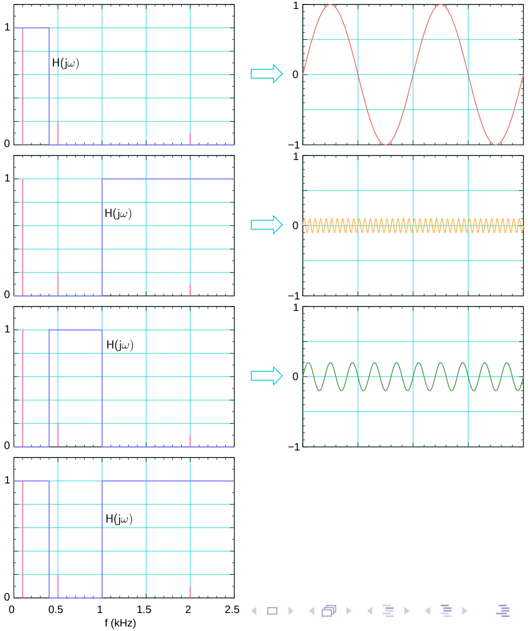

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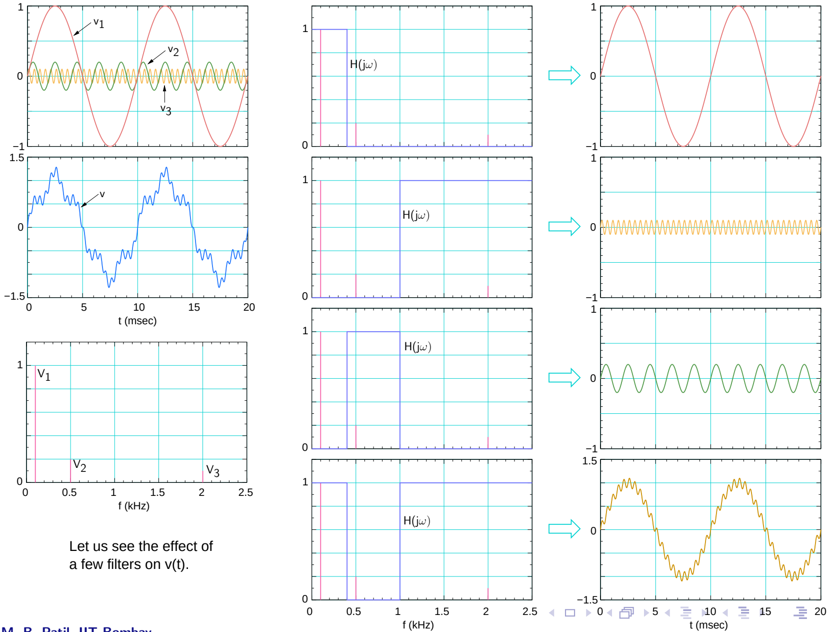

Let us see the effect of a few filters on v(t).

* In practical filter circuits, the ideal filter response is approximated with a suitable $H(j\omega)$ that can be obtained with circuit elements. For example,

  $$H(s) = \frac{1}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

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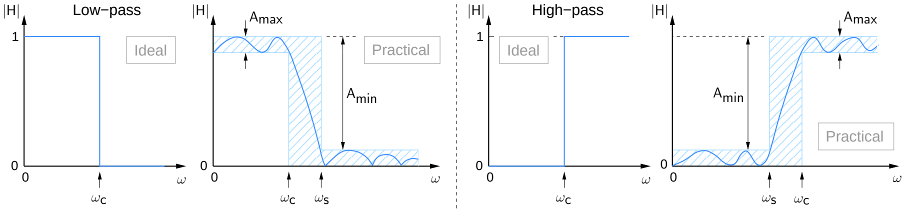
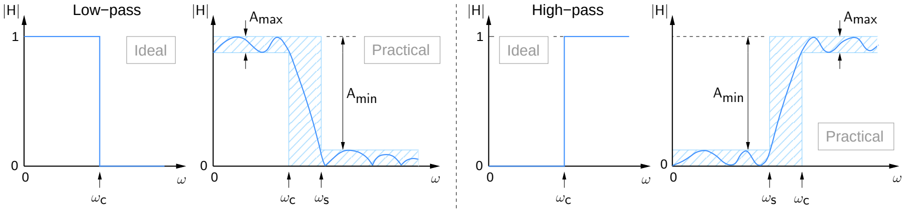
* Coefficients for these filters listed in filter handbooks. Also, programs for filter design are available on the internet.

Practical filters





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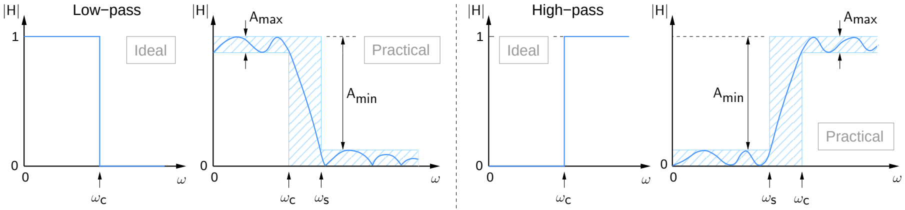

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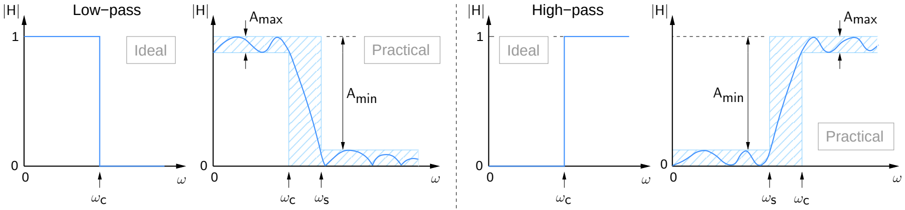
Practical filters



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∗ ω_s: edge of the stop band.

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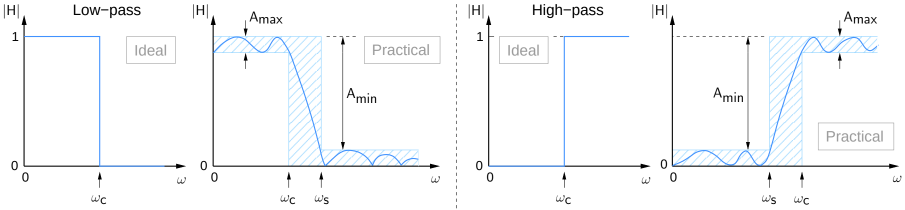

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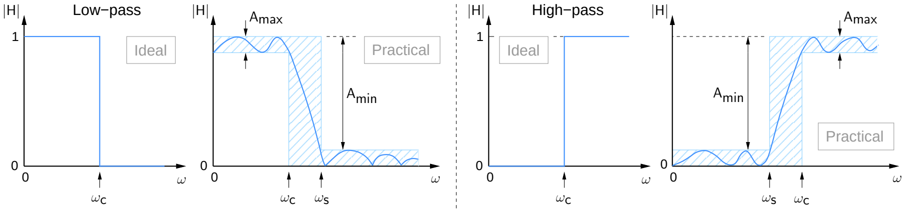
* ω_s/ω_c (for a low-pass filter): selectivity factor, a measure of the sharpness of the filter.



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* ω_s: edge of the stop band.
* ω_s/ω_c (for a low-pass filter): selectivity factor, a measure of the sharpness of the filter.
* $\omega_c < \omega < \omega_s$: transition band.

For a low-pass filter, $H(s) = \dfrac{1}{\displaystyle\sum_{i=0}^{n} a_i (s/\omega_c)^i}$.

Coefficients (a_i) for various types of filters are tabulated in handbooks. We now look at $|H(j\omega)|$ for two commonly used filters.

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$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2(\omega/\omega_c)^{2n}}}.$$

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Chebyshev filters:

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_n^2(\omega/\omega_c)}} \quad \text{where}$$

$$C_n(x) = \cos\left[n \cos^{-1}(x)\right] \quad \text{for } x \leq 1,$$

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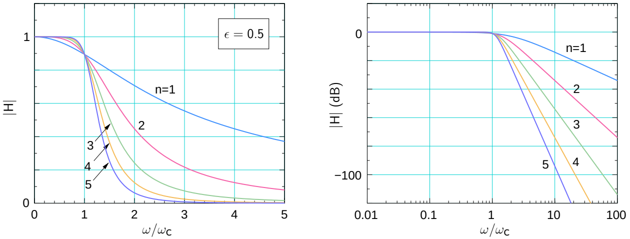
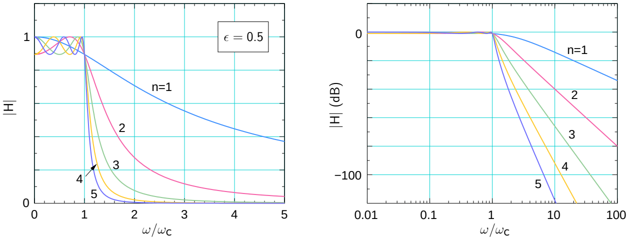
$H(s)$ for a high-pass filter can be obtained from $H(s)$ of the corresponding low-pass filter by $(s/\omega_c) \rightarrow (\omega_c/s)$.

Practical filters (low-pass)

Butterworth filters:

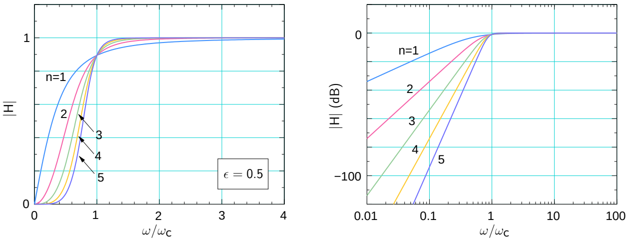


Chebyshev filters:

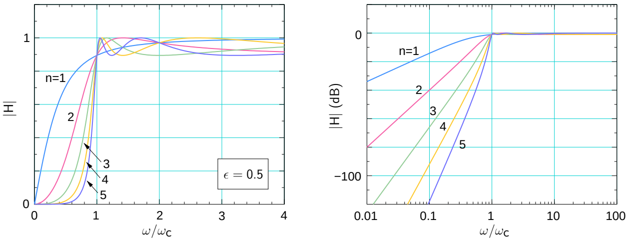


Practical filters (high-pass)

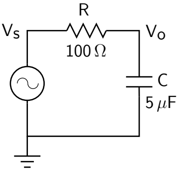
Butterworth filters:



Chebyshev filters:



Passive filter example



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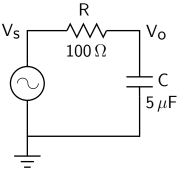
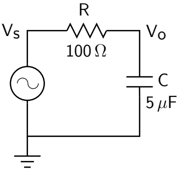


$$H(s) = \frac{(1/sC)}{R + (1/sC)} = \frac{1}{1 + (s/\omega_0)},$$

with $\omega_0 = 1/RC$.

(Low−pass filter)



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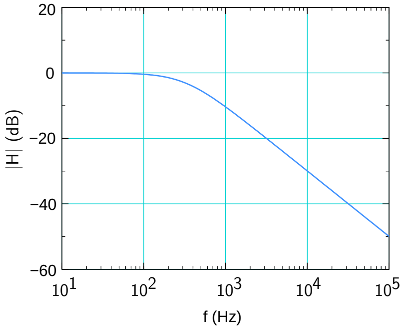
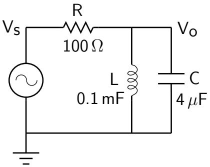
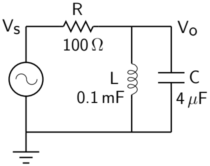


(SEQUEL file: ee101_rc_ac_2.sqproj)

Passive filter example



Passive filter example



$$H(s) = \frac{(sL) \parallel (1/sC)}{R + (sL) \parallel (1/sC)} = \frac{s(L/R)}{1 + s(L/R) + s^2 LC}$$

with $\omega_0 = 1/\sqrt{LC}$.

(Band−pass filter)

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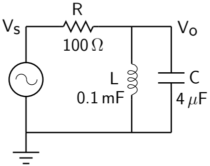

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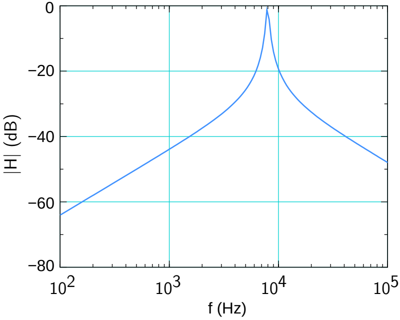

(SEQUEL file: ee101_lc_1.sqproj)

Op Amp filters ("Active" filters)

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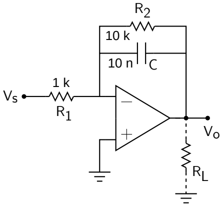
* Op Amp filters can be easily incorporated in an integrated circuit.
* However, at high frequencies ($\sim$ MHz), Op Amps no longer have a high gain $\longrightarrow$ passive filters.
* Also, if the power requirement is high, Op Amp filters cannot be used $\longrightarrow$ passive filters.

Op Amp filters: example



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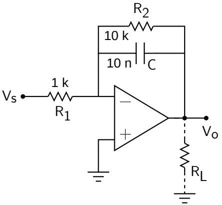

Op Amp filters are designed for Op Amp operation in the linear region
$\rightarrow$ Our analysis of the inverting amplifier applies, and we get,

$$\mathbf{V_o} = -\frac{R_2 \parallel (1/sC)}{R_1}\,\mathbf{V_s} \quad (\mathbf{V_s} \text{ and } \mathbf{V_o} \text{ are phasors})$$

$$H(s) = -\frac{R_2}{R_1}\,\frac{1}{1 + sR_2C}$$

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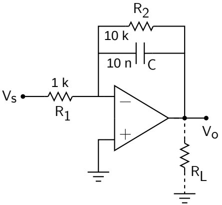

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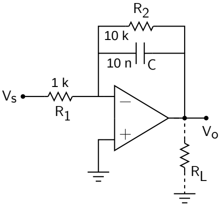

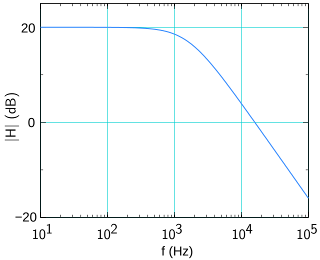

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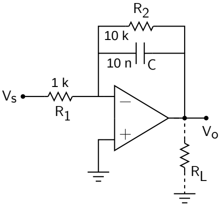

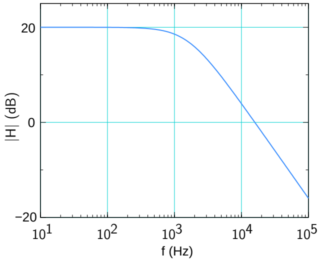

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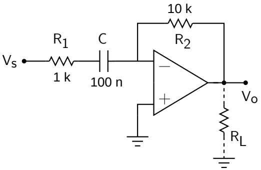
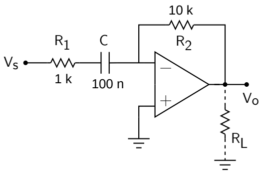
This is a low-pass filter, with $\omega_0 = 1/R_2C$.

(SEQUEL file: `ee101_op_filter_1.sqproj`)

Op Amp filters: example

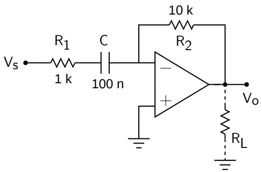


Op Amp filters: example



$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}\,.$$

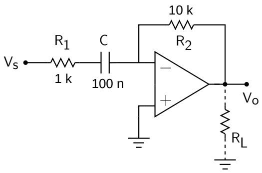
Op Amp filters: example

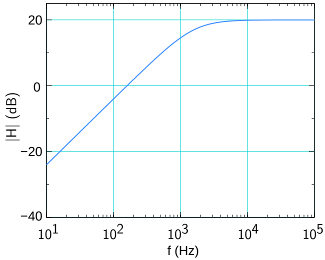


$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = \frac{sR_2C}{1 + sR_1C}\,.$$

This is a high-pass filter, with $\omega_0 = 1/R_1C$.

Op Amp filters: example





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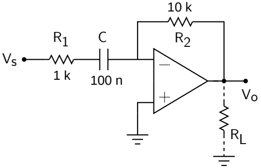

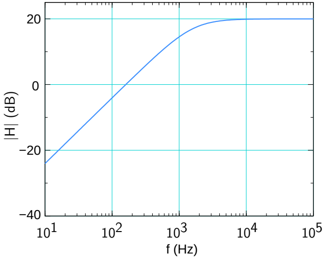

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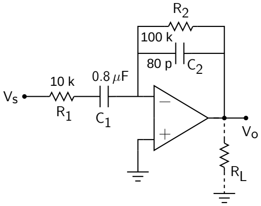
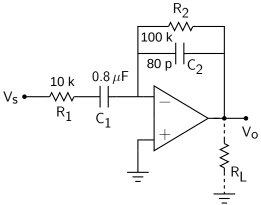
This is a high-pass filter, with $\omega_0 = 1/R_1C$.

(SEQUEL file: `ee101_op_filter_2.sqproj`)

Op Amp filters: example

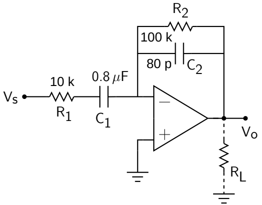


Op Amp filters: example



$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1}\,\frac{sR_1C_1}{(1+sR_1C_1)(1+sR_2C_2)}\,.$$

Op Amp filters: example



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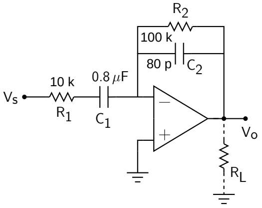
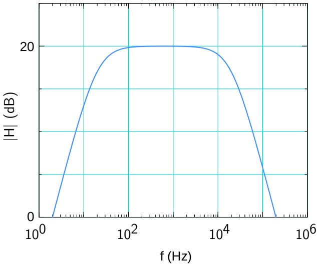
This is a band-pass filter, with $\omega_L = 1/R_1C_1$ and $\omega_H = 1/R_2C_2$.

Op Amp filters: example





$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1} \frac{sR_1C_1}{(1 + sR_1C_1)(1 + sR_2C_2)} .$$

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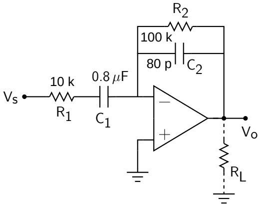
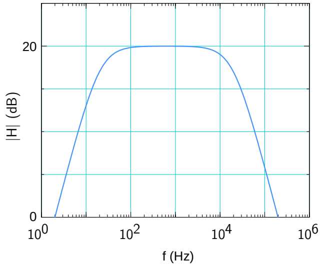
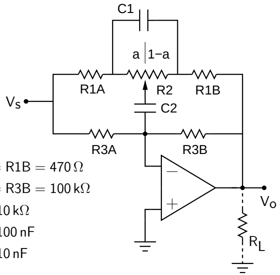




$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1} \frac{sR_1C_1}{(1+sR_1C_1)(1+sR_2C_2)} \,.$$

This is a band-pass filter, with $\omega_L = 1/R_1C_1$ and $\omega_H = 1/R_2C_2$.

(SEQUEL file: `ee101_op_filter_3.sqproj`)



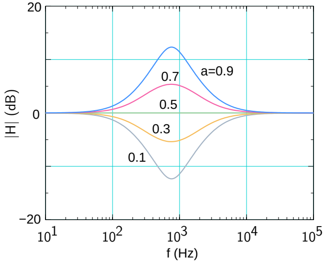
R1A = R1B = 470 Ω

R3A = R3B = 100 kΩ

R2 = 10 kΩ

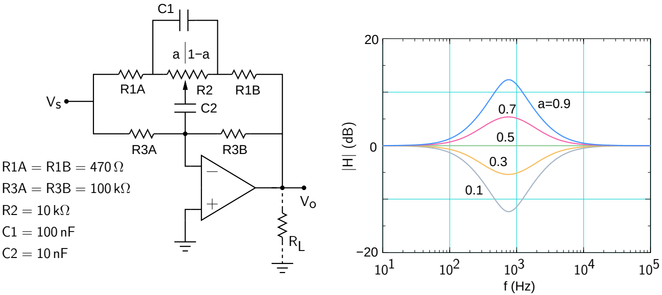
C1 = 100 nF

C2 = 10 nF



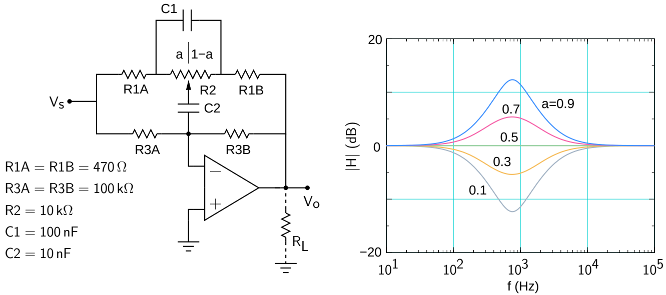
(Ref.: S. Franco, "Design with Op Amps and analog ICs")

Graphic equalizer



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

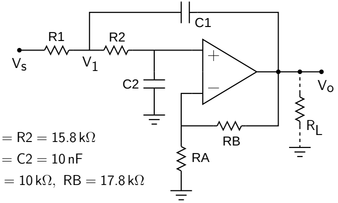
* Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

* Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.
* The circuit shown above represents one of the equalizer sections. (SEQUEL file: `ee101_op_filter_4.sqproj`)

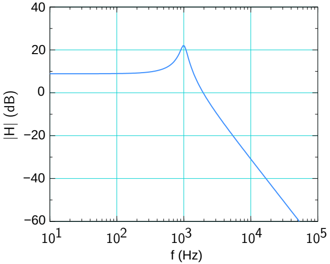
Sallen-Key filter example (2^{nd} order, low-pass)



R1 = R2 = 15.8 kΩ

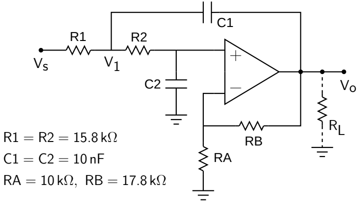
C1 = C2 = 10 nF

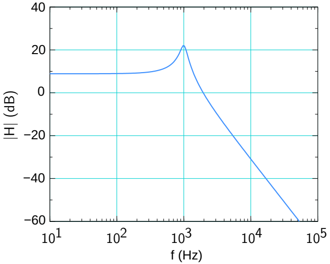
RA = 10 kΩ, RB = 17.8 kΩ



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

Sallen-Key filter example (2nd order, low-pass)

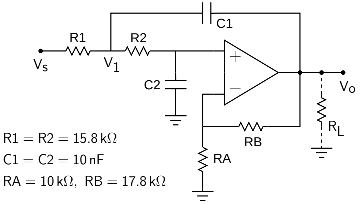


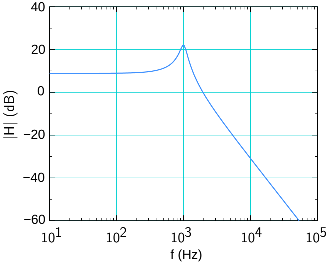


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o/K\,.$$

Sallen-Key filter example (2^{nd} order, low-pass)



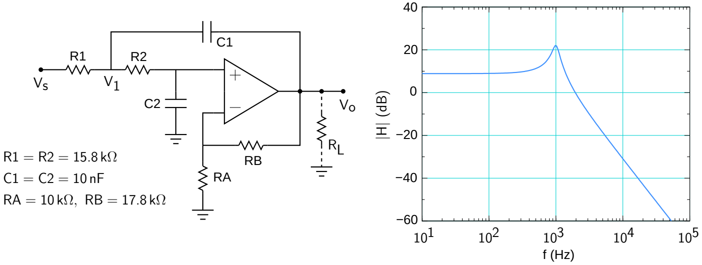


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o/K\ .$$

$$\text{Also, } V_+ = \frac{(1/sC_2)}{R_2 + (1/sC_2)}\, V_1 = \frac{1}{1 + sR_2 C_2}\, V_1\ .$$

Sallen-Key filter example (2nd order, low-pass)



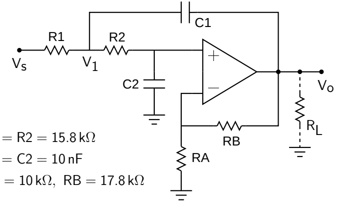
(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \, \frac{R_A}{R_A + R_B} \equiv V_o/K \, .$$

Also, $V_+ = \dfrac{(1/sC_2)}{R_2 + (1/sC_2)} \, V_1 = \dfrac{1}{1 + sR_2 C_2} \, V_1 \, .$

KCL at $V_1 \rightarrow \dfrac{1}{R_1}(V_s - V_1) + sC_1(V_o - V_1) + \dfrac{1}{R_2}(V_+ - V_1) = 0 \, .$

Sallen-Key filter example (2^{nd} order, low-pass)



R1 = R2 = 15.8 kΩ
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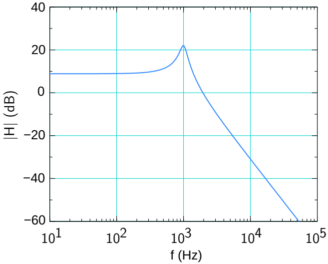


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

$$V_+ = V_- = V_o \frac{R_A}{R_A + R_B} \equiv V_o/K\,.$$

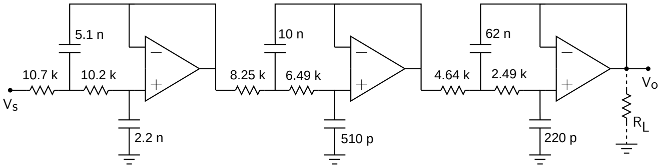
Also, $V_+ = \dfrac{(1/sC_2)}{R_2 + (1/sC_2)}\,V_1 = \dfrac{1}{1 + sR_2C_2}\,V_1\,.$

KCL at $V_1 \rightarrow \dfrac{1}{R_1}(V_s - V_1) + sC_1(V_o - V_1) + \dfrac{1}{R_2}(V_+ - V_1) = 0\,.$

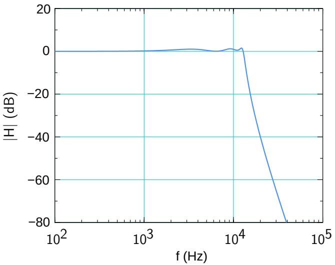
Combining the above equations, $H(s) = \dfrac{K}{1 + s\,[(R_1 + R_2)C_2 + (1-K)R_1C_1] + s^2R_1C_1R_2C_2}\,.$

(SEQUEL file: `ee101_op_filter_5.sqproj`)

Sixth-order Chebyshev low-pass filter (cascade design)

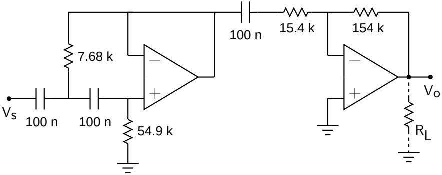


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

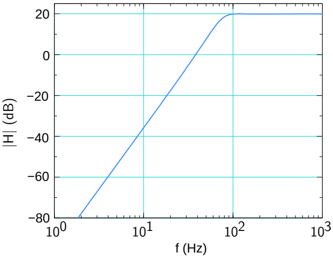


SEQUEL file: ee101_op_filter_6.sqproj

Third-order Chebyshev high-pass filter

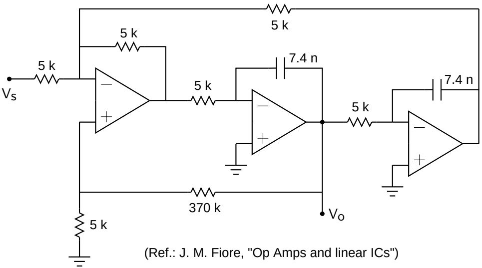


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

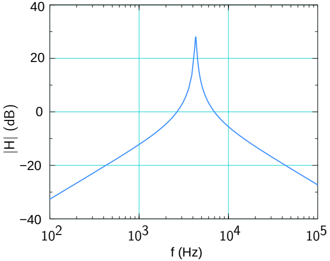


SEQUEL file: ee101_op_filter_7.sqproj

Band-pass filter example

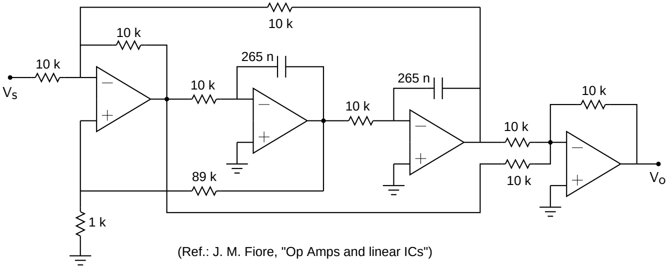


(Ref.: J. M. Fiore, "Op Amps and linear ICs")

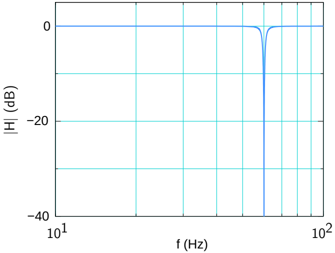


SEQUEL file: ee101_op_filter_8.sqproj

Notch filter example



(Ref.: J. M. Fiore, "Op Amps and linear ICs")



SEQUEL file: ee101_op_filter_9.sqproj