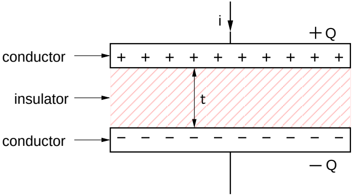
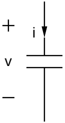
EE101: RC and RL Circuits (with DC sources)

M. B. Patil
mbpatil@ee.iitb.ac.in

Department of Electrical Engineering
Indian Institute of Technology Bombay

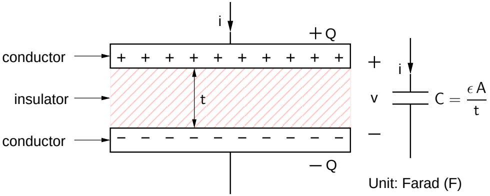




$C = \dfrac{\epsilon A}{t}$

Unit: Farad (F)

Capacitors



* In practice, capacitors are available in a wide range of shapes and values, and they differ significantly in the way they are fabricated. (`http://en.wikipedia.org/wiki/Capacitor`)

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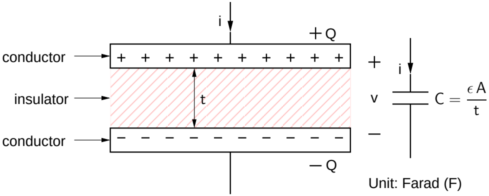

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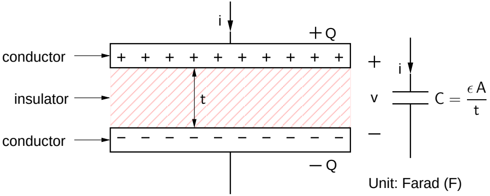

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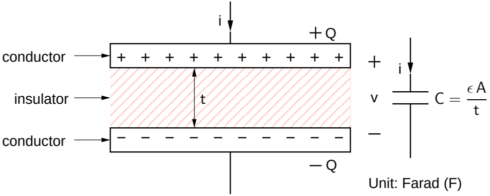
* For a constant capacitance,

$$Q(t) = C\,v(t)\,, \quad \frac{dQ}{dt} = C\,\frac{dv}{dt}\,, \text{ i.e, } i(t) = C\,\frac{dv}{dt}\,.$$

Capacitors



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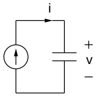
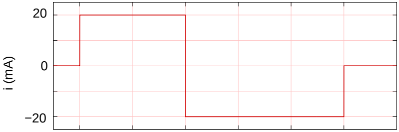
$$Q(t) = C\,v(t)\,, \quad \frac{dQ}{dt} = C\,\frac{dv}{dt}\,, \text{ i.e, } i(t) = C\,\frac{dv}{dt}\,.$$

- If v = constant, $i = 0$, i.e., a capacitor behaves like an open circuit in DC conditions as one would expect from two conducting plates separated by an insulator.

Example

Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.





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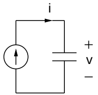

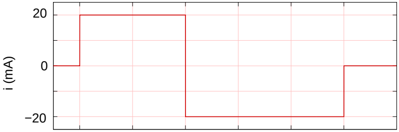

$$i(t) = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int i(t)\,dt$$

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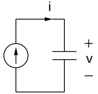

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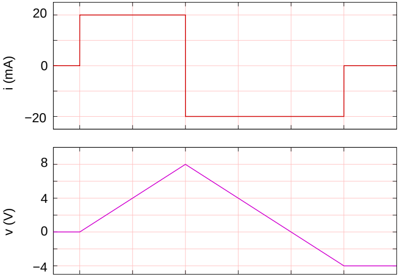

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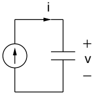

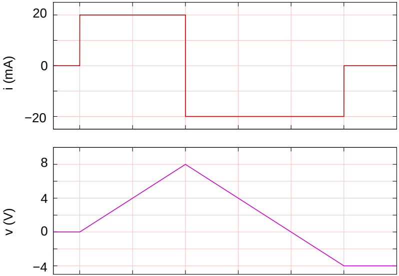

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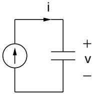

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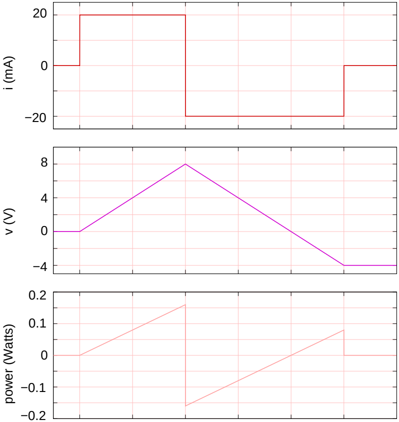

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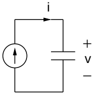

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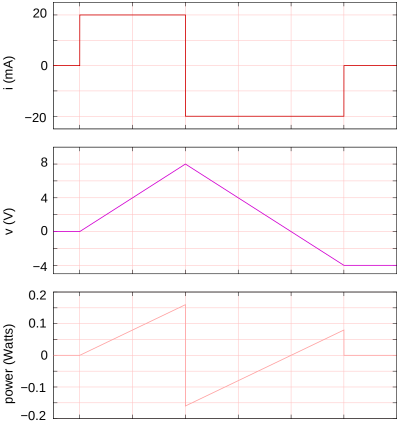

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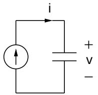

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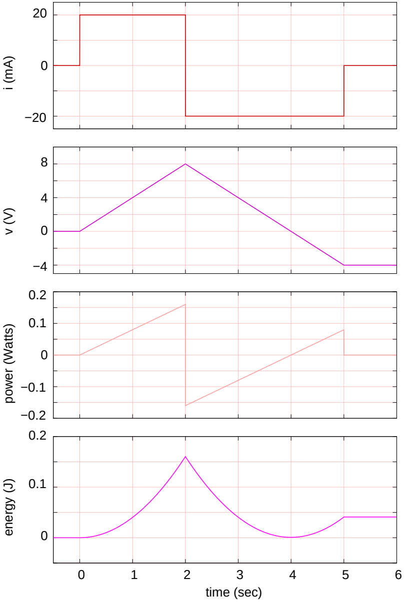

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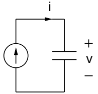

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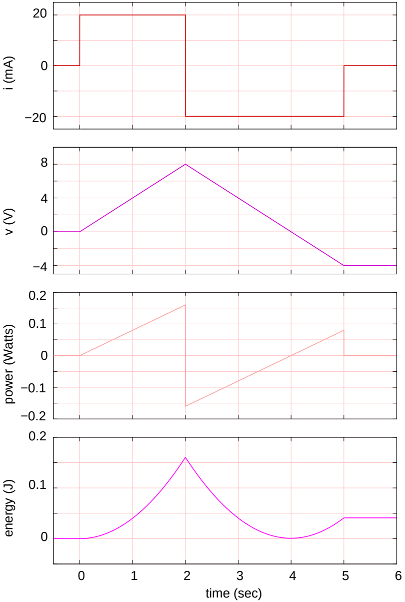
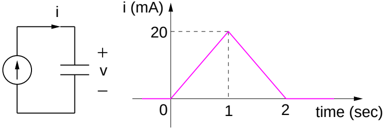
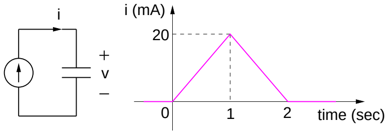
$$\begin{aligned} W(t) &= \int p(t)\,dt \\ &= C\int v\,\frac{dv}{dt}\,dt \\ &= C\int v\,dv \\ &= \frac{1}{2}\,C\,v^2 \end{aligned}$$

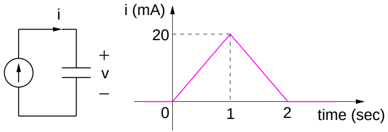


Home work

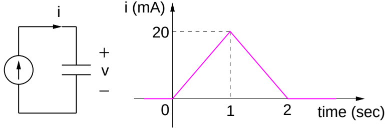




* For the given source current, plot $v(t)$, $p(t)$, and $W(t)$, assuming $v(0) = 0\ V$, $C = 5\ \text{m}F$.



* For the given source current, plot $v(t)$, $p(t)$, and $W(t)$, assuming $v(0) = 0\ V$, $C = 5\ mF$.
* Verify your results with SEQUEL.



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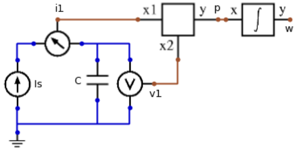
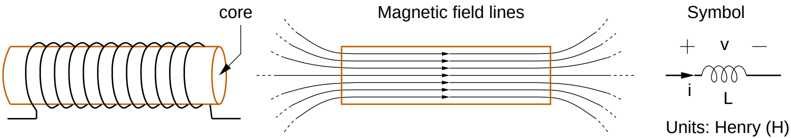
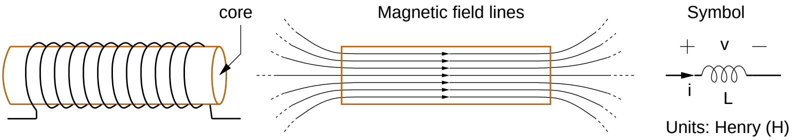
  (file: `ee101_cap_power.sqproj`)




core
Magnetic field lines
Symbol
+
v
−
i
L
Units: Henry (H)



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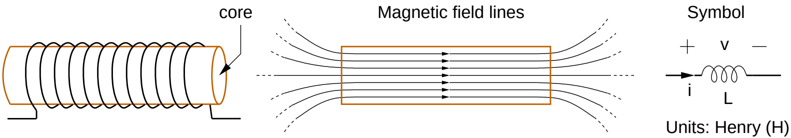

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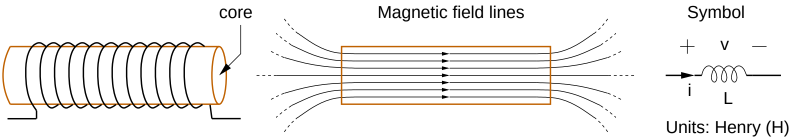
  Compare with $v = L\dfrac{di}{dt}$.

  $\Rightarrow L = \mu\,N^2\,A = \mu_r\mu_0\,N^2\,A\,.$

Inductors



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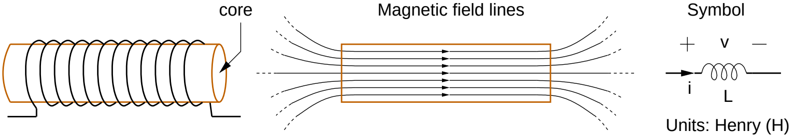

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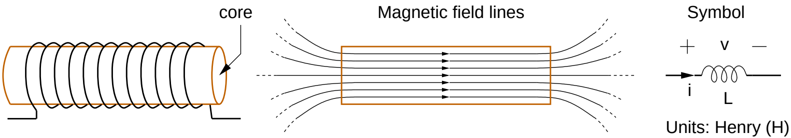
* For 99.8 % pure iron, $\mu_r \simeq 5,000$.

  For "supermalloy" (Ni: 79 %, Mo: 5 %, Fe): $\mu_r \simeq 10^6$.



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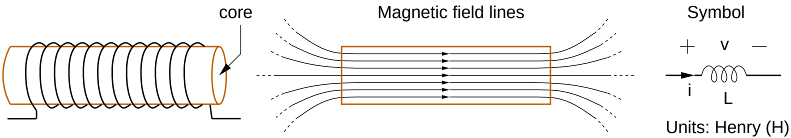
Inductors



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- To make L larger, we need (a) high μ_r, (b) large area, (c) large number of turns.
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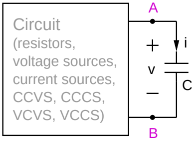
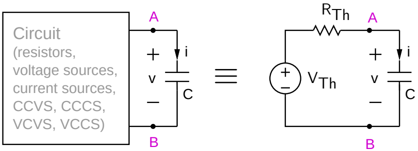
- If i = constant, $v = 0$, i.e., an inductor behaves like a short circuit in DC conditions as one would expect from a highly conducting coil.
- Note: $B = \mu H$ is an approximation. In practice, B may be a nonlinear function of H, depending on the core material.


Circuit
(resistors,
voltage sources,
current sources,
CCVS, CCCS,
VCVS, VCCS)
A
B
i
+
v
−
C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A
B
i
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$\equiv$
R_{Th}
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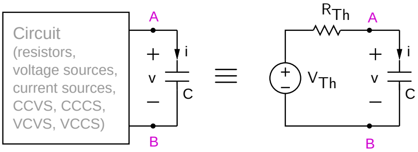
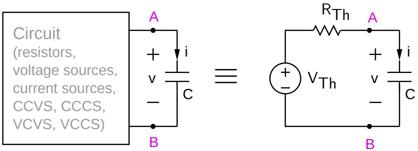
C

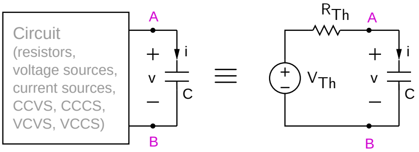


* If all sources are DC (constant), $V_{Th} =$ constant .



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C \dfrac{dv}{dt} + v$.

RC circuits with DC sources



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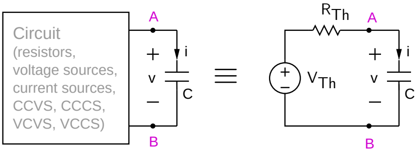
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C\, \dfrac{dv}{dt} + v$.
* Homogeneous solution:

  $\dfrac{dv}{dt} + \dfrac{1}{\tau}\, v = 0$, where $\tau = R_{Th}\, C$ is the "time constant."

  $\rightarrow v^{(h)} = K\, \exp(-t/\tau)$.



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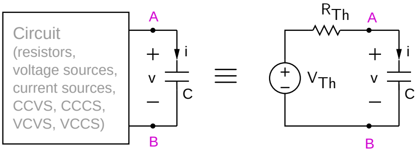
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i = 0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).



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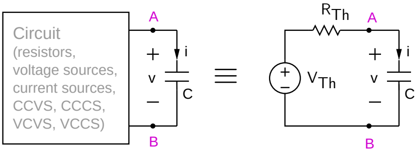
* $v = v^{(h)} + v^{(p)} = K \exp(-t/\tau) + V_{Th}$.



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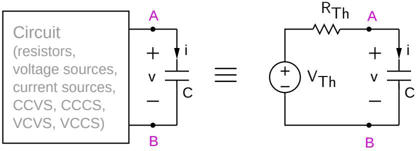
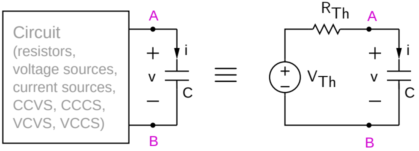
  $\rightarrow v^{(h)} = K \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i = 0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).
* $v = v^{(h)} + v^{(p)} = K \exp(-t/\tau) + V_{Th}$.
* In general, $v(t) = A \exp(-t/\tau) + B$, where A and B can be obtained from known conditions on v.

RC circuits with DC sources (continued)



* If all sources are DC (constant), we have $v(t) = A\,\exp(-t/\tau) + B$, $\tau = RC$.



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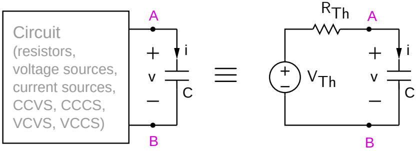

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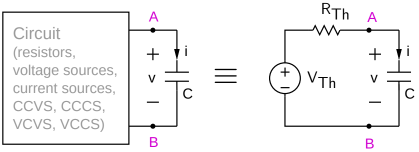

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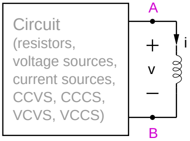
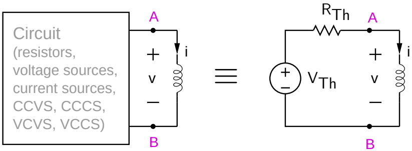
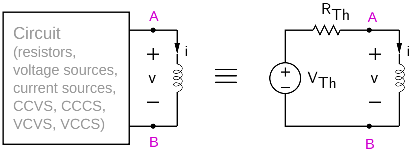
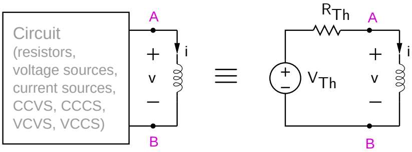
RL circuits with DC sources



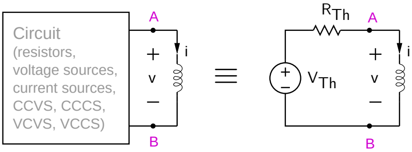

Circuit
(resistors,
voltage sources,
current sources,
CCVS, CCCS,
VCVS, VCCS)
A
B
i
+
v
−
≡
RTh
VTh
A
B
i
+
v
−



∗ If all sources are DC (constant), $V_{Th} =$ constant .



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.



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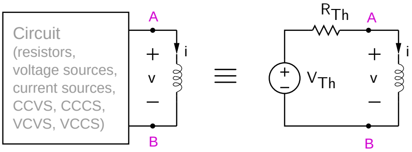
* KVL: $V_{Th} = R_{Th}\, i + L\, \dfrac{di}{dt}$.
* Homogeneous solution:

  $\dfrac{di}{dt} + \dfrac{1}{\tau}\, i = 0$, where $\tau = L/R_{Th}$

  $\rightarrow i^{(h)} = K\, \exp(-t/\tau)$.



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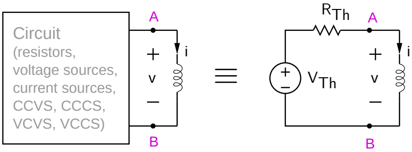
- Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).



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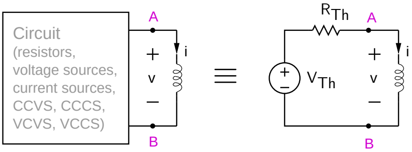

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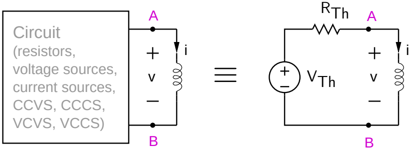
* $i = i^{(h)} + i^{(p)} = K\, \exp(-t/\tau) + V_{Th}/R_{Th}$.
* In general, $i(t) = A\, \exp(-t/\tau) + B$, where A and B can be obtained from known conditions on i.

RL circuits with DC sources (continued)



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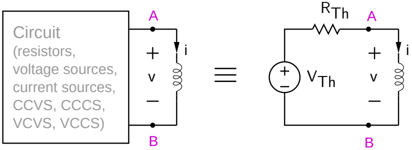

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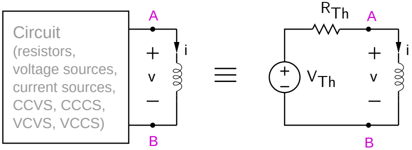

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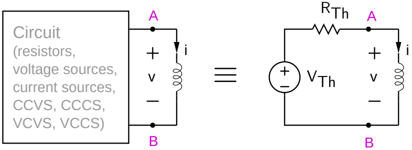

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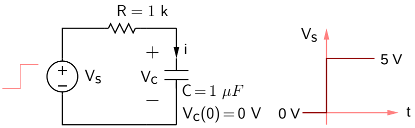
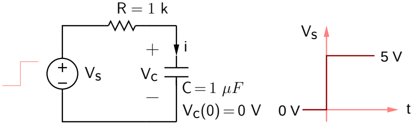
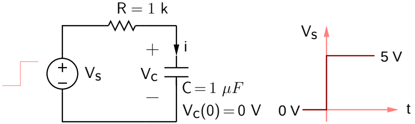
RC circuits: Can V_c change "suddenly?"





* V_s changes from 0 V (at $t = 0^-$), to 5 V (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?



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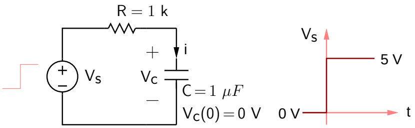

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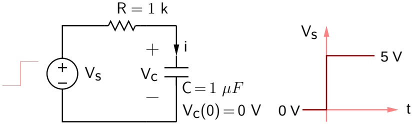

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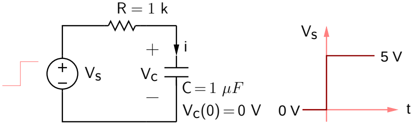
* With $i = 1\ A$, the voltage drop across R would be 1000 V! Not allowed by KVL.



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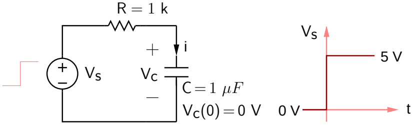
* We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.



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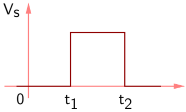
* We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.
* Similarly, an inductor does not allow abrupt changes in i_L.

* Identify intervals in which the source voltages/currents are constant.
  For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

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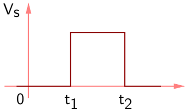

(1) $t < t_1$

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* For *any* quantity of interest $x(t)$, write general expressions such as,
$x(t) = A_1 \exp(-t/\tau) + B_1 , \quad t < t_1 ,$
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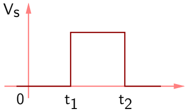

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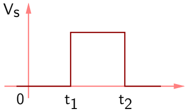

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* Work out suitable conditions on $x(t)$ at specific time points using
  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.

$$\Rightarrow i_C = C\,\frac{dV_c}{dt} = 0\,, \text{ and } V_L = L\,\frac{di_L}{dt} = 0\,.$$

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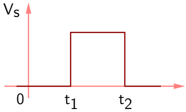

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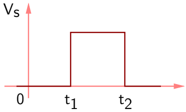

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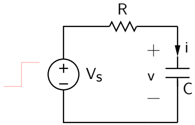
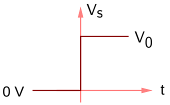
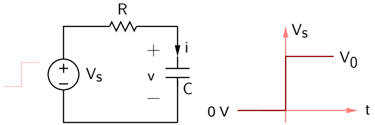
  $$V_c(t_0^+) = V_c(t_0^-)\,,\ \text{and}\ i_L(t_0^+) = i_L(t_0^-)\,.$$

* Compute A_1, B_1, $\cdots$ using the conditions on $x(t)$.


R
i
+
v
−
C
V_s


V_s
V_0
0 V
t

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

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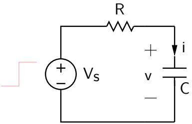
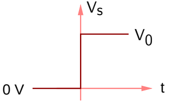




Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

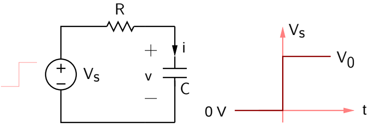
(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty$, $i \to 0 \;\to v(\infty) = V_s(\infty) = V_0$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

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Imposing (1) and (2) on Eq. (A), we get

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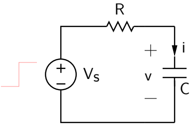
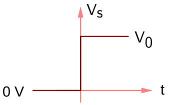
RC circuits: charging and discharging transients





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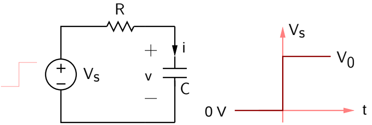

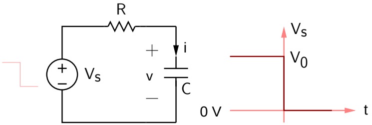

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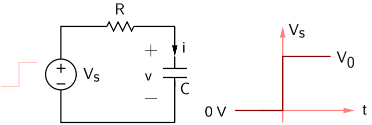
RC circuits: charging and discharging transients



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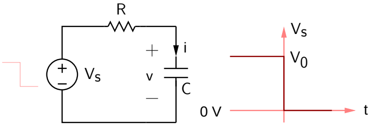

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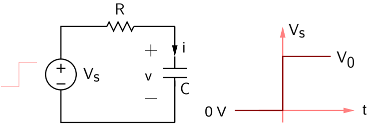

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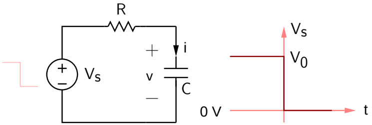

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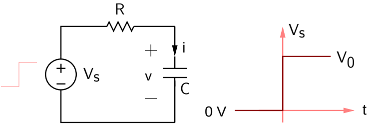

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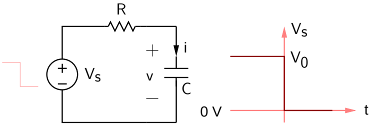

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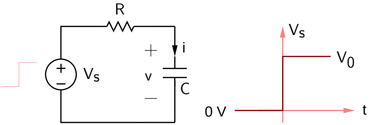
RC circuits: charging and discharging transients



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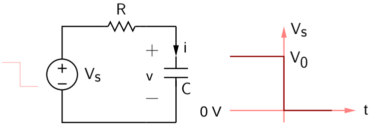

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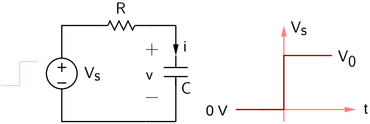
RC circuits: charging and discharging transients



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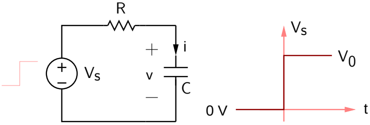

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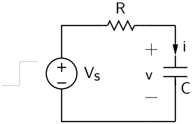

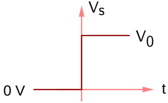

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$t = 0^+$: $v = 0$, $V_s = V_0 \Rightarrow i(0^+) = V_0/R$.

$t \rightarrow \infty$: $i(t) = 0$.

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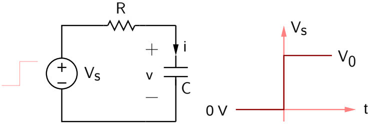
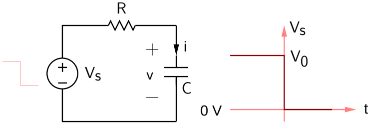
RC circuits: charging and discharging transients





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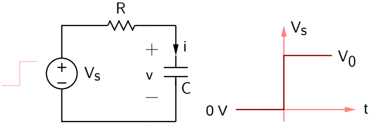
RC circuits: charging and discharging transients



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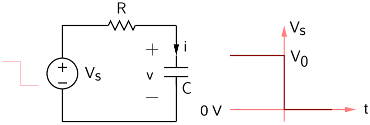

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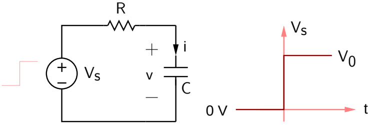

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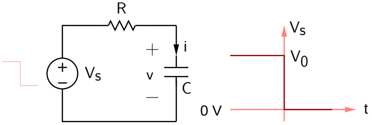

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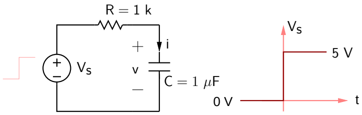
RC circuits: charging and discharging transients



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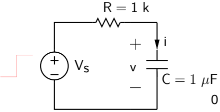

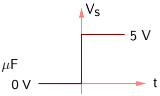

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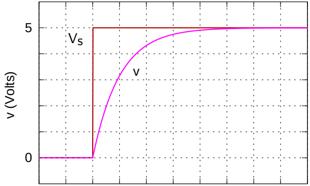

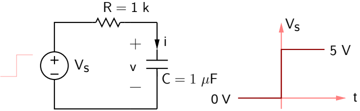

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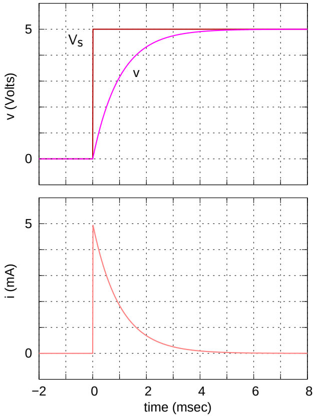
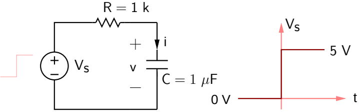


RC circuits: charging and discharging transients



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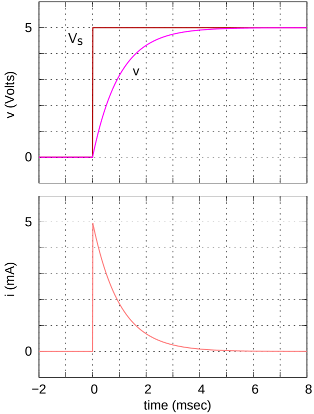

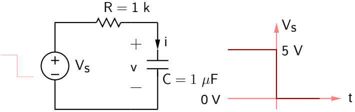

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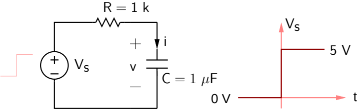

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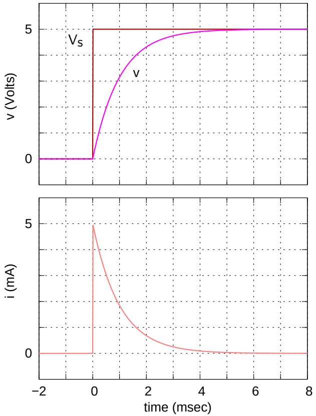

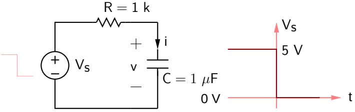

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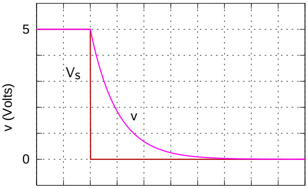

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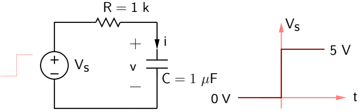

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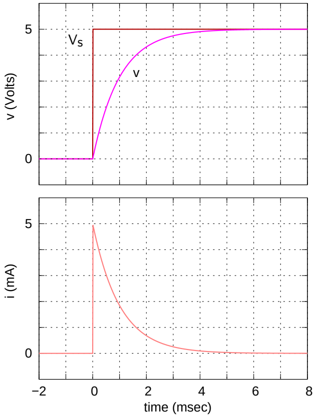
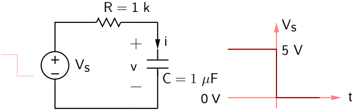
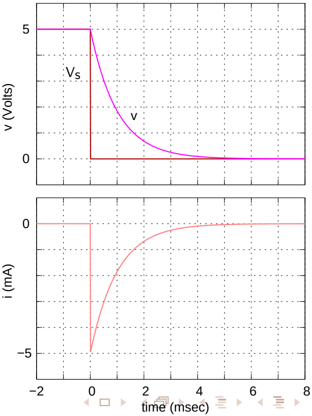




$$v(t) = V_0\exp(-t/\tau)$$

$$i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$



x	e^{-x}	$1 - e^{-x}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933

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* For $x = 5$, $e^{-x} \simeq 0$, $1 - e^{-x} \simeq 1$.

Significance of the time constant (τ)

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* For $x = 5$, $e^{-x} \simeq 0$, $1 - e^{-x} \simeq 1$.
* In RC circuits, $x = t/\tau \Rightarrow$ When $t = 5\,\tau$, the charging (or discharging) process is almost complete.

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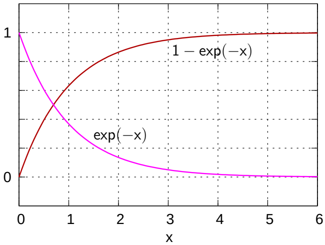
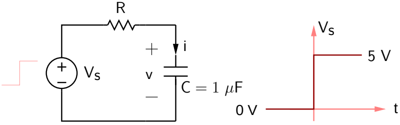
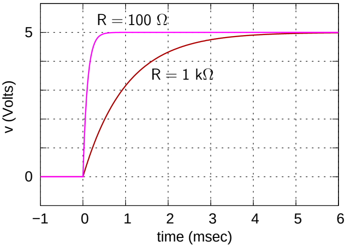
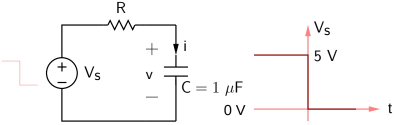


* For $x=5$, $e^{-x}\simeq 0$, $1-e^{-x}\simeq 1$.
* In RC circuits, $x=t/\tau \Rightarrow$ When $t=5\,\tau$, the charging (or discharging) process is almost complete.

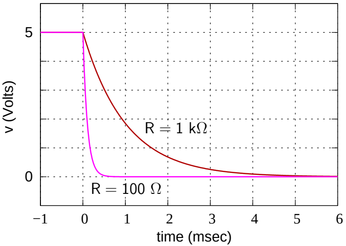


$v(t) = V_0 \left[1 - \exp(-t/\tau)\right]$

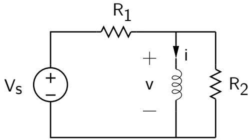




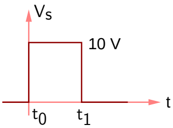
$v(t) = V_0 \exp(-t/\tau)$



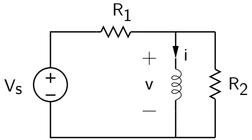
RL circuit: example

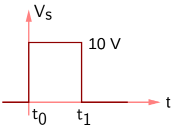


$i(0) = 0\,\text{A}$, Find $i(t)$.



$R_1 = 10\,\Omega$
$R_2 = 40\,\Omega$
$L = 0.8\,\text{H}$
$t_0 = 0$
$t_1 = 0.1\,\text{s}$





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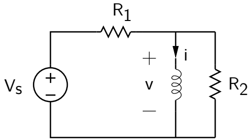
There are three intervals of constant V_s:

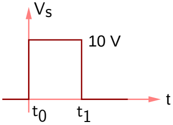
(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

RL circuit: example





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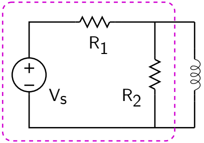
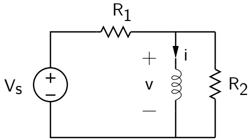
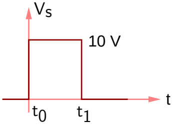
R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$
$= 0.8\ H/8\,\Omega$
$= 0.1\ s$





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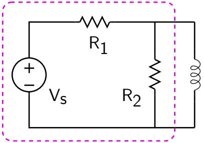

$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

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$= 0.8\ H/8\,\Omega$
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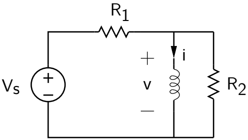
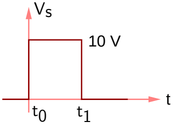
At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V.

$\Rightarrow i(t_0^-) = 0$ A $\Rightarrow i(t_0^+) = 0$ A.





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There are three intervals of constant V_s:

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(2) $t_0 < t < t_1$

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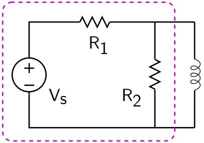
R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

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$\;\;= 0.8\ \text{H}/8\,\Omega$
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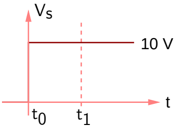
At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V.

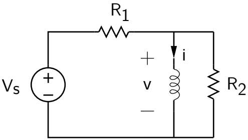
$\Rightarrow i(t_0^-) = 0\ \text{A} \Rightarrow i(t_0^+) = 0\ \text{A}$.

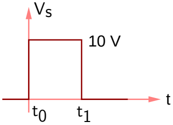
If V_s did not change at $t = t_1$, we would have



$v(\infty) = 0$ V, $i(\infty) = 10\ \text{V}/10\ \Omega = 1$ A.

RL circuit: example





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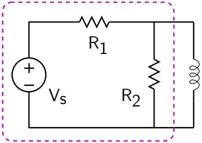

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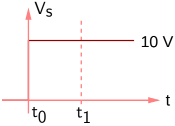
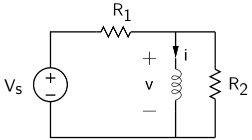
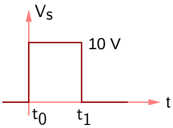


$v(\infty) = 0$ V, $i(\infty) = 10\text{ V}/10\ \Omega = 1$ A.

Using $i(t_0^+)$ and $i(\infty)$, we can obtain $i(t)$, $t > 0$ (See next slide).





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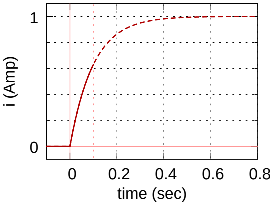
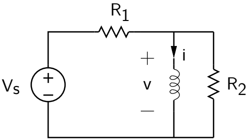
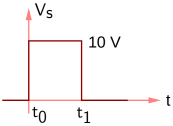


RL circuit: example





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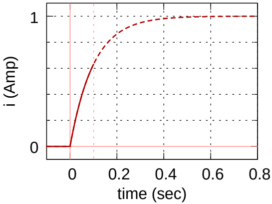
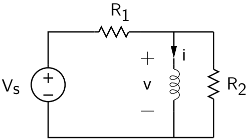
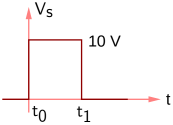


In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

RL circuit: example





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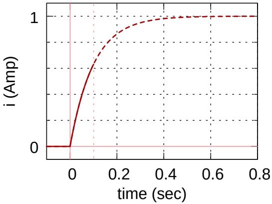
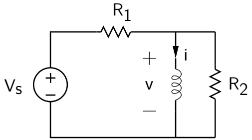
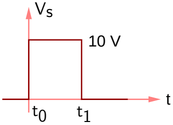


In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.
Consider $t > t_1$.

RL circuit: example





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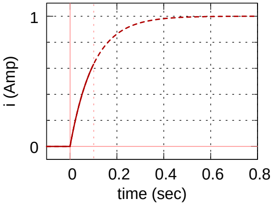

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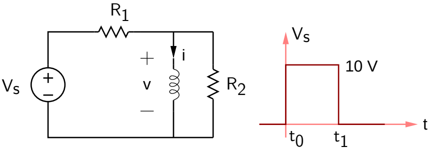
For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.
Consider $t > t_1$.
$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).
$i(\infty) = 0$ A.

RL circuit: example



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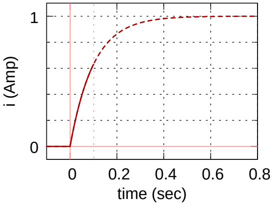

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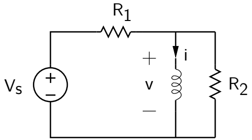
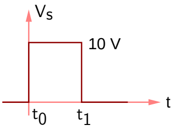
Let $i(t) = A\exp(-t/\tau) + B$.

RL circuit: example





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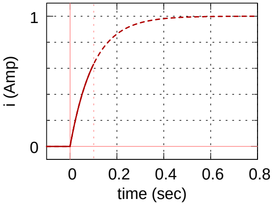

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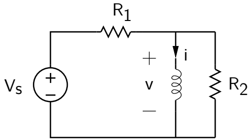
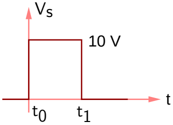
$i(\infty) = 0$ A.

Let $i(t) = A \exp(-t/\tau) + B$.

It is convenient to rewrite $i(t)$ as
$i(t) = A' \exp[-(t - t_1)/\tau] + B$.





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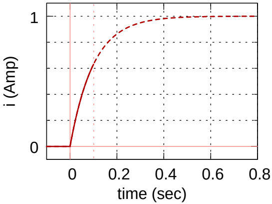

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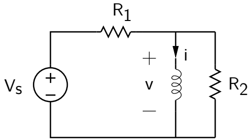
It is convenient to rewrite $i(t)$ as

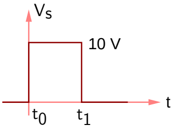
$i(t) = A'\exp[-(t - t_1)/\tau] + B$.

Using $i(t_1^+)$ and $i(\infty)$, we get

$i(t) = 0.693\exp[-(t - t_1)/\tau]$ A.

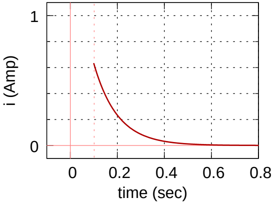
RL circuit: example



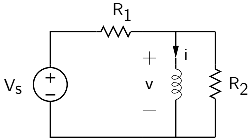


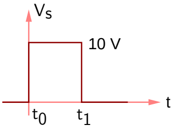
$R_1 = 10\,\Omega$
$R_2 = 40\,\Omega$
$L = 0.8\,H$
$t_0 = 0$
$t_1 = 0.1\,s$

$i(t) = 0.693\,\exp[-(t - t_1)/\tau]$ A.



RL circuit: example





$R_1 = 10\,\Omega$
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$t_0 = 0$
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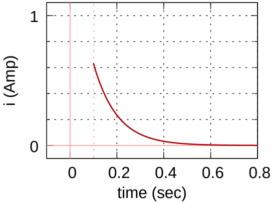
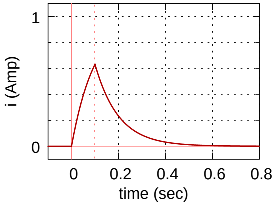
$i(t) = 0.693\,\exp[-(t - t_1)/\tau]$ A.

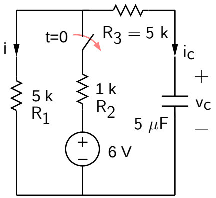


Combining the solutions for $t_0 < t < t_1$ and $t > t_1$, we get

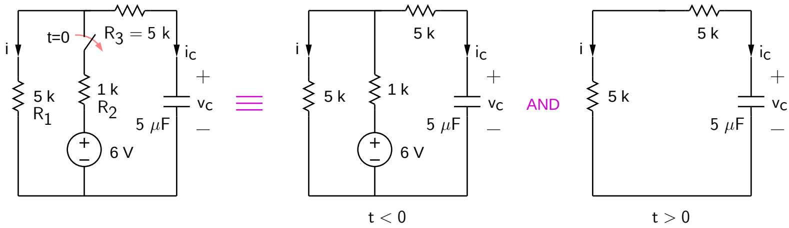


(SEQUEL file: ee101_rl1.sqproj)

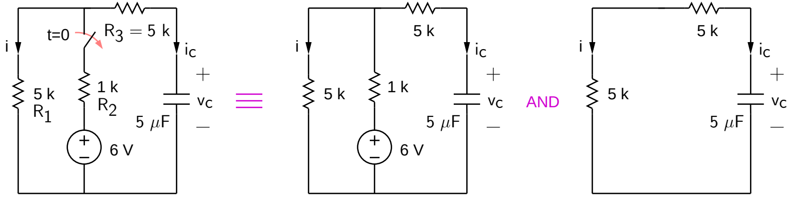
RC circuit: example



RC circuit: example



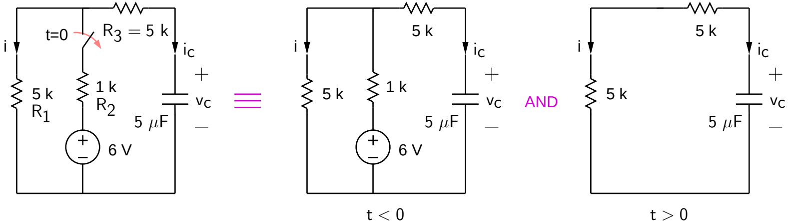
RC circuit: example



$t < 0$ $t > 0$

$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

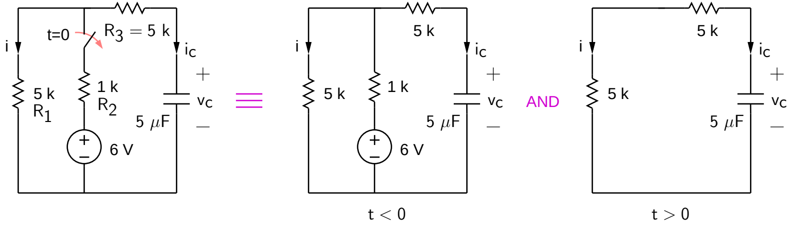
RC circuit: example



$t = 0^{-}$: capacitor is an open circuit, $\Rightarrow i(0^{-}) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_C(0^{-}) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_C(0^{+}) = 5\text{ V}$.

RC circuit: example

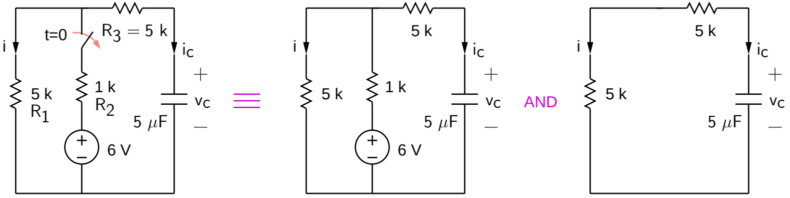


$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_C(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_C(0^+) = 5\text{ V}$.

$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

RC circuit: example



$t < 0$ $\qquad$ $t > 0$

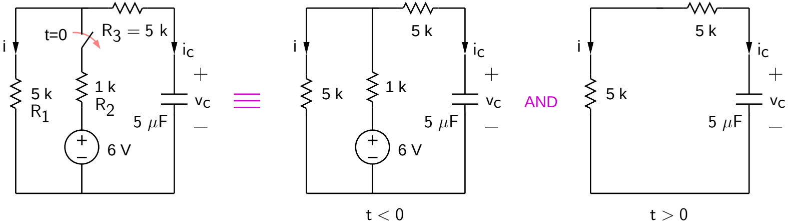
$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_C(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_C(0^+) = 5\text{ V}$.

$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50\text{ ms}$.

RC circuit: example

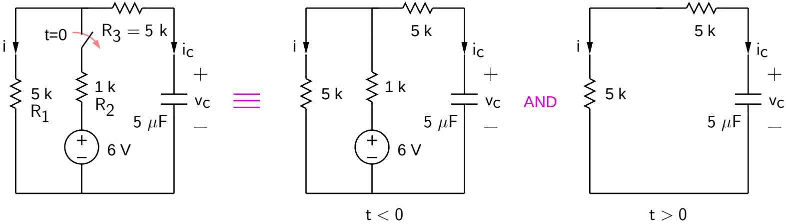


$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\ \text{V}/(5\ \text{k} + 1\ \text{k}) = 1$ mA.
$v_C(0^-) = 6\ \text{V} - 1\ \text{mA} \times R_2 = 5\ \text{V} \Rightarrow v_C(0^+) = 5$ V.
$\Rightarrow i(0^+) = 5\ \text{V}/(5\ \text{k} + 5\ \text{k}) = 0.5$ mA.

Let $i(t) = A \exp(\text{-}t/\tau) + B$ for $t > 0$, with $\tau = 10\ \text{k} \times 5\,\mu\text{F} = 50$ ms.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get
$i(t) = 0.5 \exp(\text{-}t/\tau)$ mA.

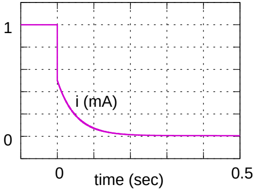
RC circuit: example



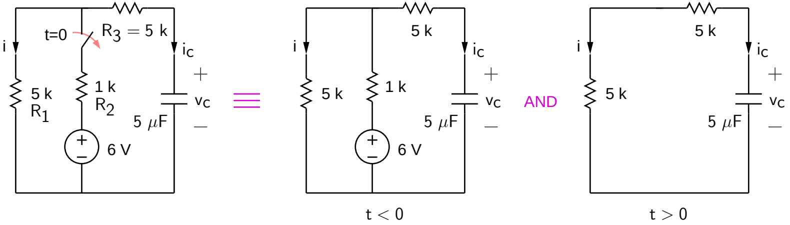
$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.
$v_C(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_C(0^+) = 5\text{ V}$.
$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50\text{ ms}$.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get
$i(t) = 0.5\exp(-t/\tau)$ mA.



RC circuit: example



t = 0^-: capacitor is an open circuit, $\Rightarrow$ $i(0^-) = 6$ V$/(5$ k $+ 1$ k$) = 1$ mA.

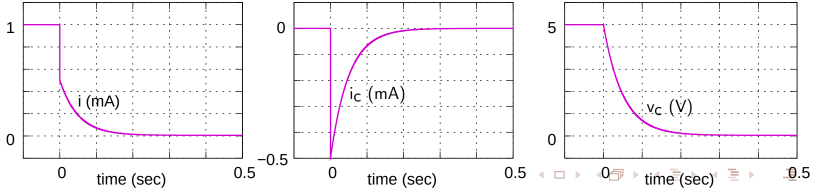
$v_C(0^-) = 6$ V $- 1$ mA $\times R_2 = 5$ V $\Rightarrow v_C(0^+) = 5$ V.

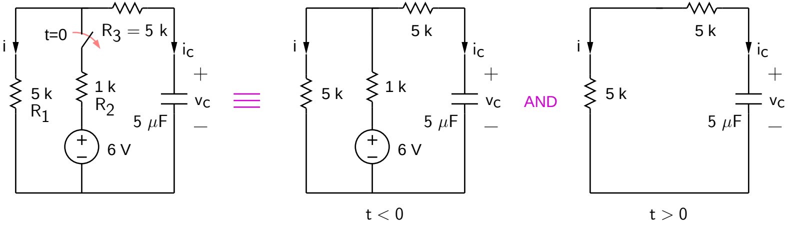
$\Rightarrow i(0^+) = 5$ V$/(5$ k $+ 5$ k$) = 0.5$ mA.

Let $i(t) = A \exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10$ k $\times 5\,\mu$F $= 50$ ms.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get

$i(t) = 0.5 \exp(-t/\tau)$ mA.



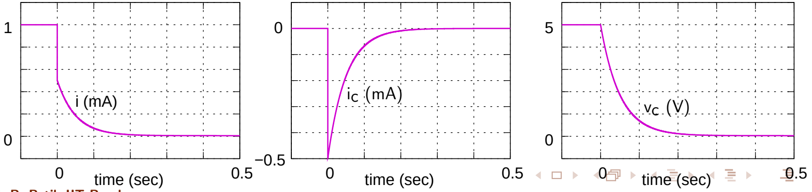


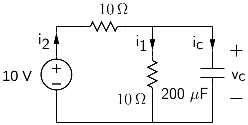
$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1$ mA.
$v_C(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_C(0^+) = 5$ V.
$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5$ mA.

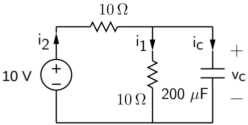
Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50$ ms.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get
$i(t) = 0.5\exp(-t/\tau)$ mA.

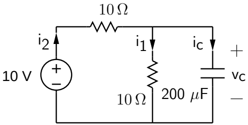
(SEQUEL file: ee101_rc2.sqproj)



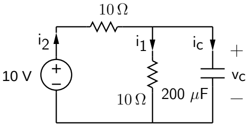

10 Ω
i_2
i_1
i_c
+
10 V
v_c
10 Ω
200 μF
−



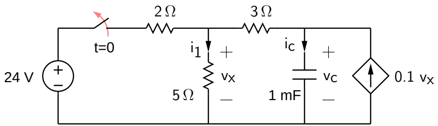
* Given $v_c(0) = 0\ V$, find $v_c(t)$ for $t > 0$. Using this $v_c(t)$, find i_1, i_2, i_c for $t > 0$. Plot v_c, i_1, i_2, i_c versus t.



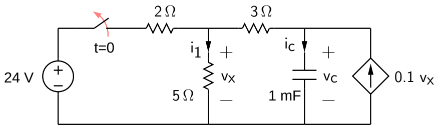
* Given $v_c(0) = 0~V$, find $v_c(t)$ for $t > 0$. Using this $v_c(t)$, find i_1, i_2, i_c for $t > 0$. Plot v_c, i_1, i_2, i_c versus t.
* Find i_1, i_2, i_c directly (i.e., without getting v_c) by finding the initial and final conditions for each of them ($i_1(0^+)$ and $i_1(\infty)$, etc.) and then using them to compute the coefficients in the general expression, $x(t) = A\,\exp(-t/\tau) + B$.



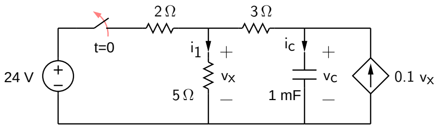
* Given $v_c(0) = 0\ V$, find $v_c(t)$ for $t > 0$. Using this $v_c(t)$, find i_1, i_2, i_c for $t > 0$. Plot v_c, i_1, i_2, i_c versus t.
* Find i_1, i_2, i_c directly (i.e., without getting v_c) by finding the initial and final conditions for each of them ($i_1(0^+)$ and $i_1(\infty)$, etc.) and then using them to compute the coefficients in the general expression, $x(t) = A\,\exp(-t/\tau) + B$.
* Verify your results with SEQUEL (file: `ee101_rc3.sqproj`).



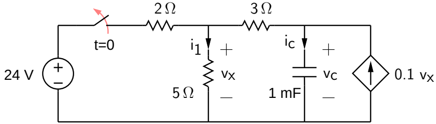
2 Ω
3 Ω
t=0
24 V
i1
vx
5 Ω
iC
vC
1 mF
0.1 vx



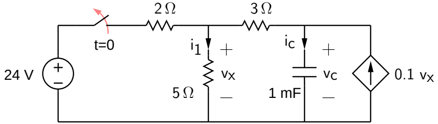
* Find $v_c(0^-)$, $v_c(\infty)$.



* Find $v_c(0^-)$, $v_c(\infty)$.
* Find R_{Th} as seen by the capacitor for $t > 0$.

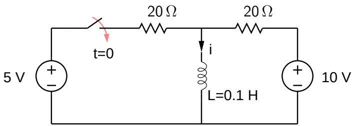


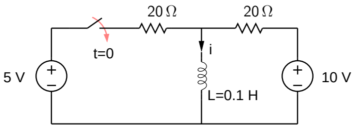
* Find $v_c(0^-)$, $v_c(\infty)$.
* Find R_{Th} as seen by the capacitor for $t > 0$.
* Solve for $v_c(t)$ and $i_1(t)$, $t > 0$.



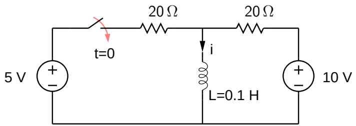
* Find $v_c(0^-)$, $v_c(\infty)$.
* Find R_{Th} as seen by the capacitor for $t > 0$.
* Solve for $v_c(t)$ and $i_1(t)$, $t > 0$.
* Verify your results with SEQUEL (file: `ee101_rc4.sqproj`).

RL circuits: home work



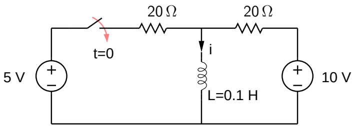


* Find $i(0^-)$, $i(\infty)$.

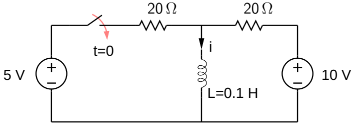


* Find $i(0^-)$, $i(\infty)$.
* Find R_{Th} as seen by the inductor for $t > 0$.

RL circuits: home work



* Find $i(0^-)$, $i(\infty)$.
* Find R_{Th} as seen by the inductor for $t > 0$.
* Solve for $i(t)$, $t > 0$.



* Find $i(0^-)$, $i(\infty)$.
* Find R_{Th} as seen by the inductor for $t > 0$.
* Solve for $i(t)$, $t > 0$.
* Verify your results with SEQUEL (file: `ee101_rl2.sqproj`).