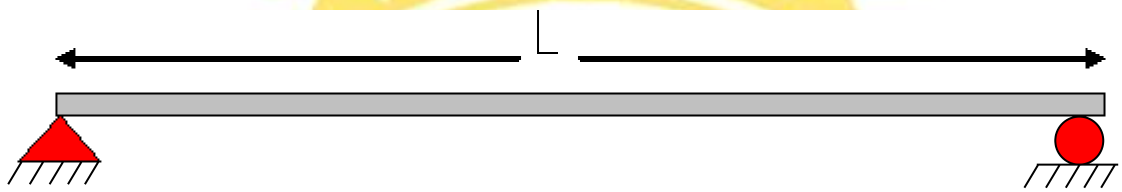
DEPARTMENT OF CIVIL ENGINEERING

IIT DELHI

VIRTUAL SMART STRUCTURES AND DYNAMICS LAB

EXPERIMENT 1 (SIMULATION)

Modes of Vibration of Simply Supported Beam



INTRODUCTION

This simulation based experiment aims to study the modes of vibration of a simply supported beam under flexure. The simply supported beam, a continuous system, is different from a discrete system. Where Stiffness, the Mass and Damping are modelled as discrete properties. The mathematical models for discrete systems are ordinary differential equations, which thereby render themselves quite conducive to numerical solution techniques. An alternative method of modelling physical system, which is considered for this beam, is based on the principle of distributed mass and stiffness characteristics. Such a system for which stiffness and mass are considered to be distributed properties (rather than discrete) is referred to as a distributed or continuous system.

Unlike a discrete system that possess a finite number of Degree Of Freedom (DOF), the distributed systems, which are considered to be composed of infinite number of infinitesimal mass particles, theoretically possesses an infinite number of Degrees Of Freedom (DOF). However, only the first few modes are significant. It is therefore not necessary to study all of them.

This computational model of Simply Supported Beam is based on distributed system. By using this online simulation, the student/user can easily determine the natural frequencies of beams and simulate the first five mode shapes. In addition, there is an exercise for user: The user can study and plot a graph between natural frequency and length of beams keeping all others factors constant. Similarly, relation between natural frequency and the Young's Modulus of Elasticity can be studied.

THEORY

General solution for displacement for beam is given by (Chopra, 2001)

$$y(X) = c_1 \sinh \beta x \; + c_2 \cosh \beta x + c_3 \sin \beta x + c_4 \cos \beta x \qquad (1)$$

After applying the boundary condition, we get

$$\sin \beta L = 0 \qquad (2)$$

Hence,

$$\beta L = n\pi \qquad (3)$$

and $$\beta_n^2 = \omega_n LC \quad (4)$$

The final solution for frequencies is

$$f_n = \frac{\pi n^2}{2L^2}\sqrt{\frac{EI}{\rho A}} \quad (5)$$

and for the nth mode shape, the solution is

$$y(x) = \sin\frac{n\pi x}{L} \quad (6)$$

Fig.1 shows the profile of nth mode of vibration of a simply supported beam.

Where L is the length of the beam, EI is the flexural rigidity (E = Young's modulus, I = Moment of inertia), A is the cross- sectional area, f_n is the natural frequency and C is constant.

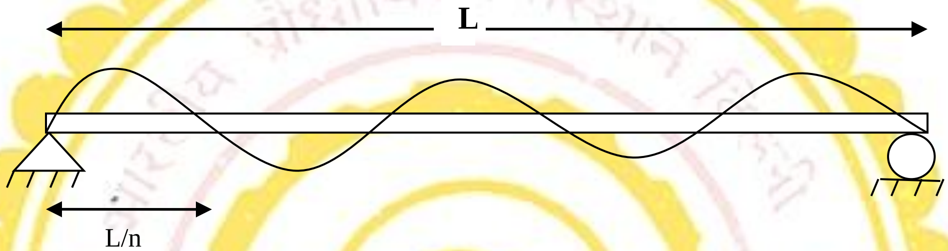


Figure 1 nth mode of vibration of a simply supported beam

REFERENCES

1. Chopra, A. (2001), Dynamics of Structures, Prentice Hall of India limited, New Delhi.

2. Paz, M. (2004), Structural Dynamics: Theory and Computations, 2nd ed., CBS Publishers and Distributors, New Delhi.