Importing Libraries

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import mltools as ml
        %matplotlib inline
```

Problem 1

Problem 1 - Part 1

```
In [2]: data = np.genfromtxt("data/curve80.txt", delimiter = None)
        X = data[:, 0]          # Scalar feature value
        X = np.atleast_2d(X).T # Converts the array into a transpose of the origina array
        Y = data[:, 1]          # target value y for each example
        Xtr, Xte, Ytr, Yte = ml.splitData(X,Y, 0.75) # split data set 75/25
```

Printing the shapes of the objects

```
In [3]: print("Shape of Training Data\n------------------------", "\nNumber of points: ", Xtr.shape[0], "\nN
        umber of features: ", Xtr.shape[1], "\n")
        print("Shape of Test Data\n------------------------", "\nNumber of points: ", Xte.shape[0], "\nNumbe
        r of features: ", Xte.shape[1], "\n")
        print("Shape of Training Target\n------------------------", "\nNumber of points: ", Ytr.shape[0], "
        \n")
        print("Shape of Test Target\n------------------------", "\nNumber of points: ", Yte.shape[0])
```

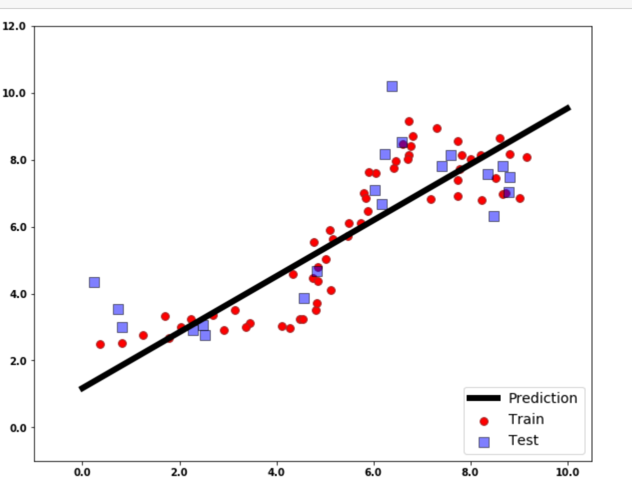
```
Shape of Training Data
------------------------
Number of points:  60
Number of features:  1

Shape of Test Data
------------------------
Number of points:  20
Number of features:  1

Shape of Training Target
------------------------
Number of points:  60

Shape of Test Target
------------------------
Number of points:  20
```

Problem 1 - Part 2

```
In [4]: lr = ml.linear.linearRegress( Xtr, Ytr ) # create and train model
        xs = np.linspace(0, 10, 200) # densely sample possible x-values
        xs = xs[:, np.newaxis] # force "xs" to be an Mx1 matrix (expected by our code)
        ys = lr.predict(xs) # make prediction at xs
```

Problem 1 - Part 2 - Part a

```
In [5]: fig, ax = plt.subplots(1, 1, figsize = (10, 8))
        ax.scatter(Xtr, Ytr, s = 60, color = "red", edgecolors = "brown", label = "Train")
        ax.scatter(Xte, Yte, s = 90, color = "blue", edgecolors = "black", label = "Test", marker = "s", alp
        ha = 0.5)
        ax.plot(xs, ys, color = "black", lw = 6, label = "Prediction")
        ax.set_xlim(-1.0, 10.5)  # Sets the limit for the X-axis
        ax.set_ylim(-5.0, 8.0 )  # Sets the limit for the Y-axis
        ax.set_xticklabels(ax.get_xticks(), fontdict = {"fontsize": 10, "fontweight": "bold"}) # Changes the
        styles of the X-axis numbers
        ax.set_yticklabels(ax.get_xticks(), fontdict = {"fontsize": 10, "fontweight": "bold"}) # Changes the
        styles of the Y-axis numbers
        plt.legend(fontsize = 14, loc = 4)
        plt.show()
```



Problem 1 - Part 2 - Part b

```
In [6]: theta = lr.theta.ravel()
        print("Linear regression coefficients theta is: ", theta)
        print("Theta 0: -2.82765049")
        print("Theta 1: 0.83606916")
```

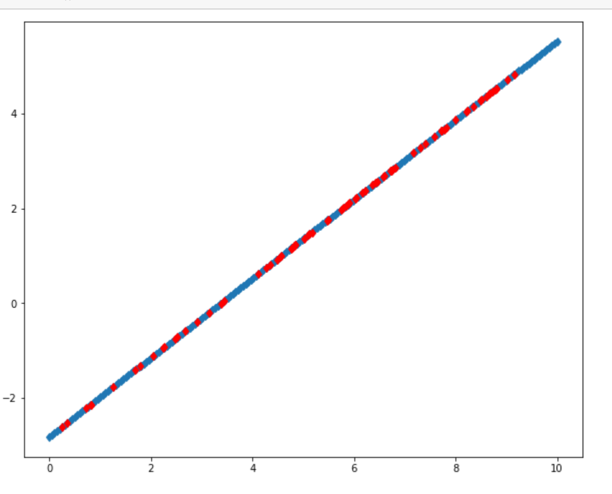
```
Linear regression coefficients theta is:  [-2.82765049  0.83606916]
Theta 0: -2.82765049
Theta 1: 0.83606916
```

Problem 1 - Part 2 - Part c

--> Theta 0 is the y-intercept and theta 1 is the slope if the line. Putting everything together we should have

yHat(x) = theta 1 * X + theta 0

```
In [7]: fig, ax = plt.subplots(1, 1, figsize = (10, 8))
        ax.plot(xs, ys, X, theta[1] * X + theta[0], "r.", marker = "d", lw = 4, dash_joinstyle = "bevel", fi
        llstyle = "full")
        plt.show()
```



```
In [8]: print("Mean squared error of training data is: " , lr.mse(Xtr,Ytr))
        print("Mean squared error of testing data is: " , lr.mse(Xte,Yte))
```

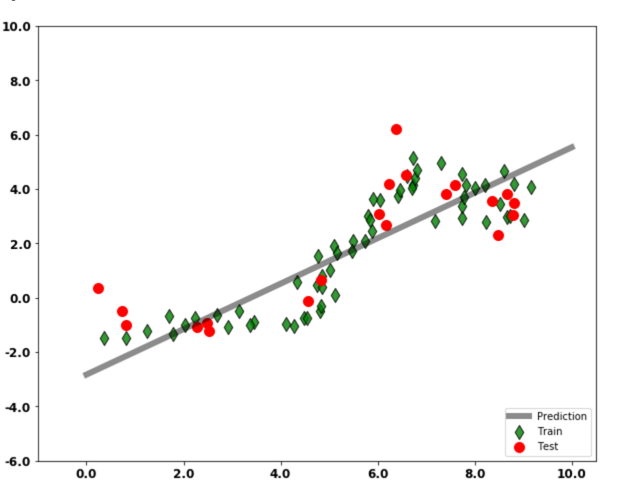
```
Mean squared error of training data is:  1.127711955609391
Mean squared error of testing data is:  2.2423492030101246
```

Problem 1 - Part 3

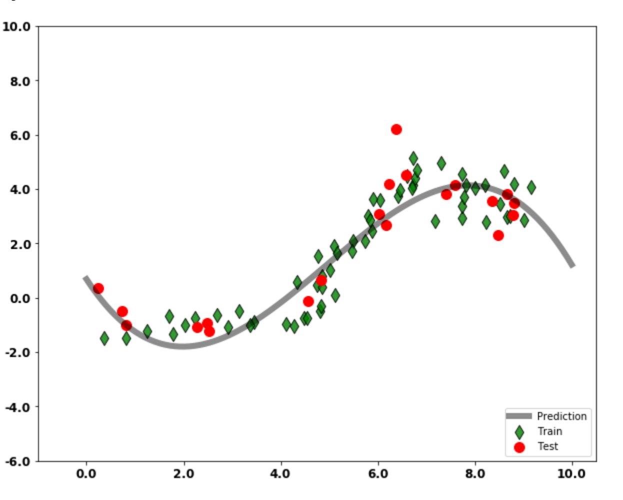
Problem 1 - Part 3 - Part a

```
In [9]: Xtr2 = np.zeros( (Xtr.shape[0], 2)) # Create Mx2 array to store features
        Xtr2[:, 0] = Xtr[:, 0]          # place original "x" feature as X1
        Xtr2[:, 1] = Xtr[:, 0] ** 2     # place "x^2" feature as X2
        # Now Xtr2 has two features about each data point: "x" and "x^2"

        d = [1, 3, 5, 7, 10, 15, 18]  # polynomial regression models of degrees
        for i, degree in enumerate(d):
            XtrP = ml.transforms.fpoly(Xtr, degree, bias = False)
            XtrP,params = ml.transforms.rescale(XtrP)
            lr = ml.linear.linearRegress(XtrP, Ytr)
            # Create axes for single subplot
            fig, ax = plt.subplots(1, 1, figsize = (10, 8))
            xs = np.linspace(0, 10, 100)
            xs = xs[:, np.newaxis]
            xsP = ml.transforms.fpoly(xs, degree, False)
            xsP,_ = ml.transforms.rescale(xsP, params)
            ys = lr.predict(xsP)
            XteP,_ = ml.transforms.rescale( ml.transforms.fpoly(Xte,degree,False), params)

            ax.scatter(Xtr, Ytr, s = 100, color = "green", edgecolors = "black", marker = "d", alpha = 0.8,
        label = "Train")
            ax.scatter(Xte, Yte, s = 400, marker = ".", color = "red", label = "Test")
            #Also plotting the regression line
            # Plot polynomial regression of desired degree
            ax.plot(xs, ys, lw = 6, color = "black", alpha = 0.45, label = "Prediction")
             # Set the minimum and maximum limits
            ax.set_xlim(-1.0, 10.5)
            ax.set_ylim(-6.0, 10.0)
            # Changes the styles of the X-axis numbers
            ax.set_xticklabels(ax.get_xticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
            # Changes the styles of the Y-axis numbers
            ax.set_yticklabels(ax.get_yticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
            ax.legend(fontsize = 10, loc = 4)
            print("Degree = " + str(degree))
            plt.show()
```

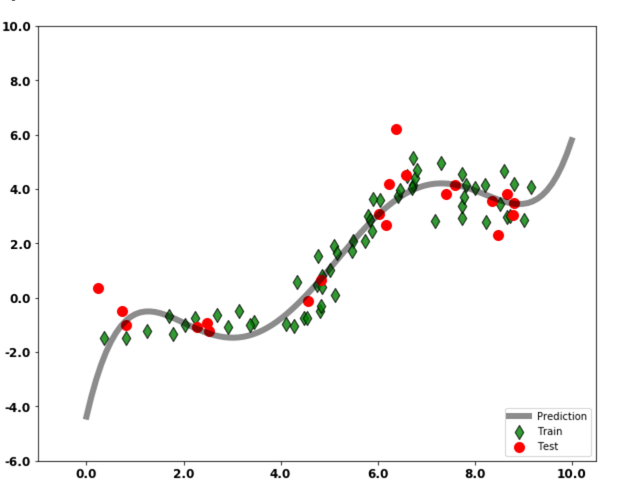
```
Degree = 1
```



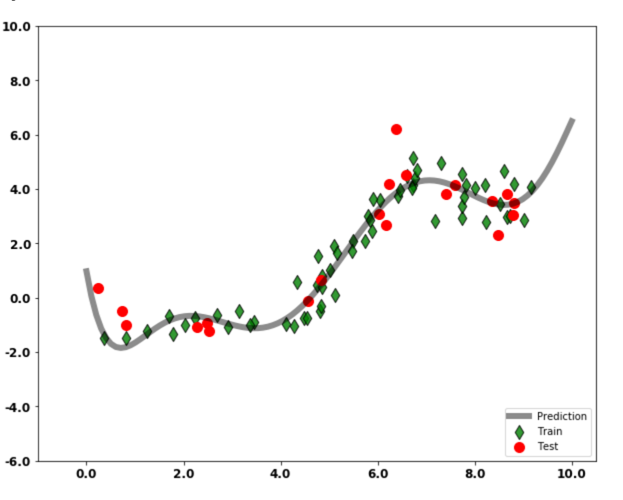
```
Degree = 3
```



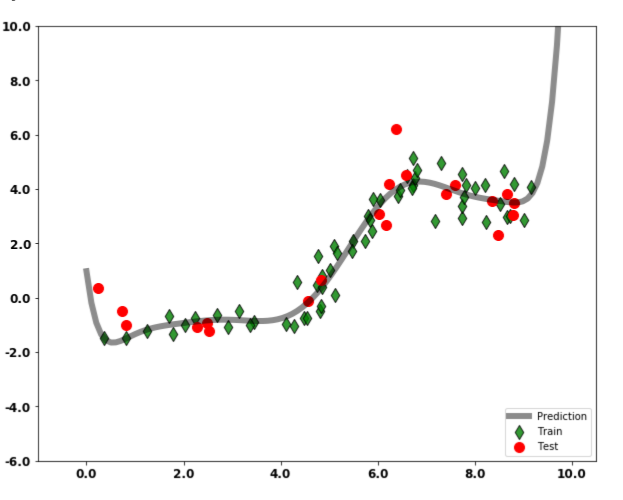
```
Degree = 5
```



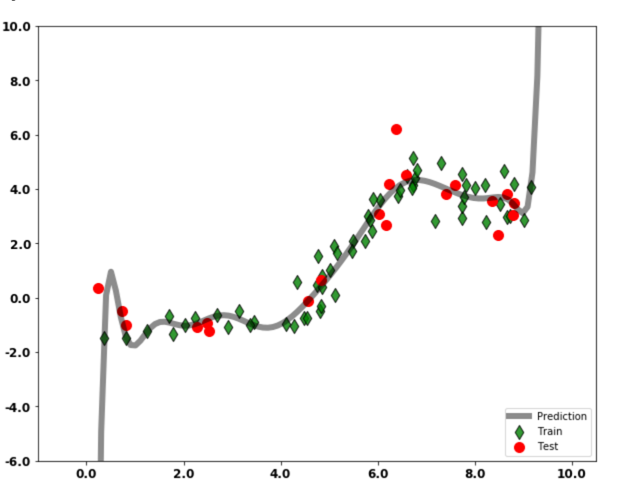
```
Degree = 7
```



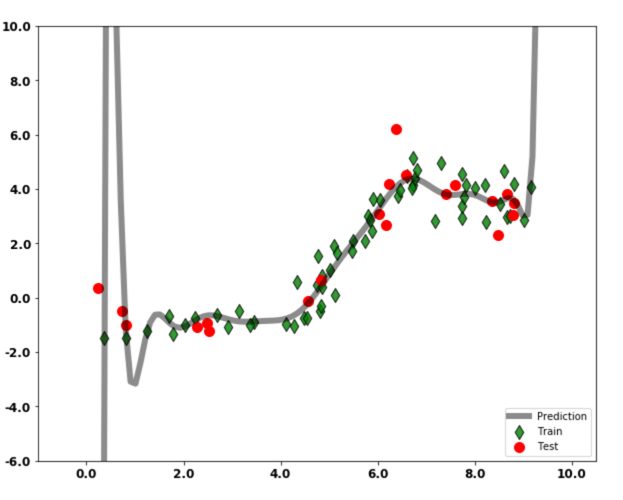
```
Degree = 10
```



```
Degree = 15
```

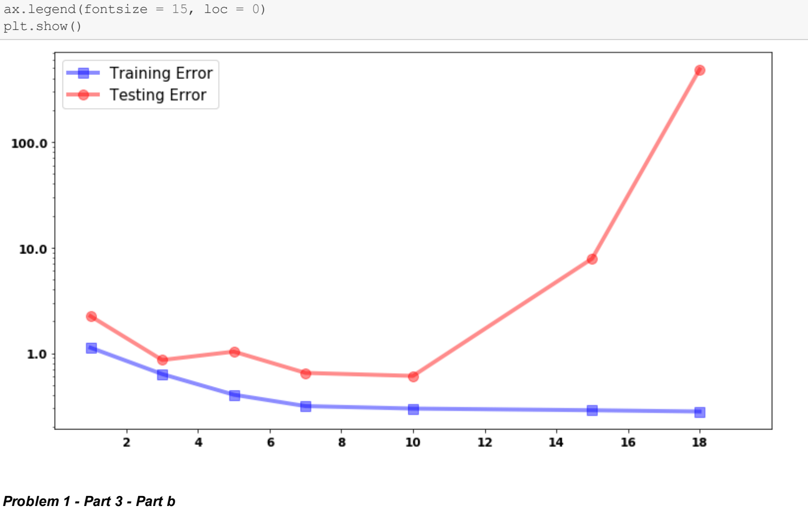


```
Degree = 18
```



Problem 1 - Part 3 - Part b

```
In [10]: train_error = [0] * 7
         test_error  = [0] * 7
         for i, degree in enumerate(d):
             XtrP = ml.transforms.fpoly(Xtr, degree, bias = False)
             XtrP, params = ml.transforms.rescale(XtrP)
             lr = ml.linear.linearRegress(XtrP, Ytr)
             XteP,_ = ml.transforms.rescale(ml.transforms.fpoly(Xte, degree, False), params)
             train_error[i] = lr.mse(XtrP, Ytr)
             test_error[i] = lr.mse(XteP, Yte)

         fig, ax = plt.subplots(1, 1, figsize = (13, 7))
         ax.semilogy(d, train_error, lw = 4, marker = "s", color = "blue", markersize = 10, alpha = 0.45, lab
         el = "Training Error")
         ax.semilogy(d, test_error, lw = 4, marker = "o", color = "red", markersize = 10, alpha = 0.45, label
         = "Testing Error")
         ax.set_xlim(0.0, 20)
         ax.set_xticks(np.arange(2, 20, 2))
         ax.set_xticklabels(ax.get_xticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
         ax.set_yticklabels(ax.get_yticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
         ax.legend(fontsize = 15, loc = 0)
         plt.show()
```



Problem 1 - Part 3 - Part b

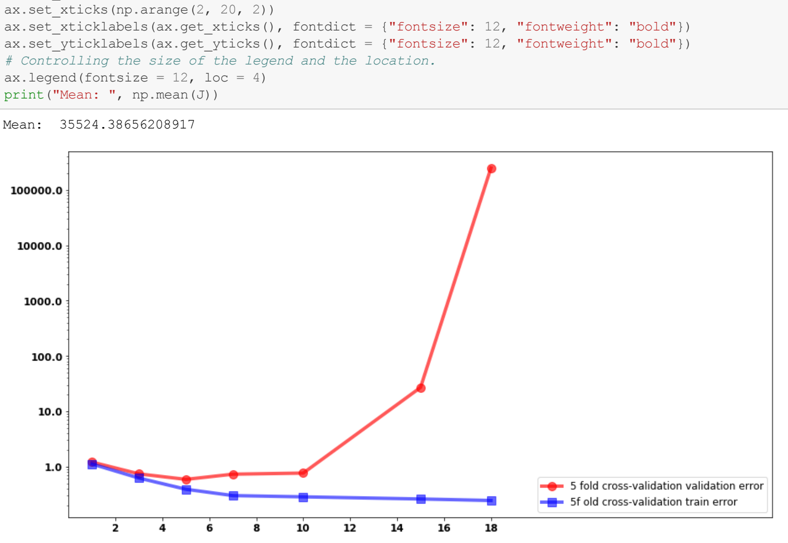
As shown in the figure above, error improves in some areas and breaks sharply at some other points. as degree increases the error increases. therefore degree 10 works the best due to sharp breaks at some points and improvements at some points

Problem 2

Problem 2 - Part 1

```
In [18]: nFolds = 5;
         d = [1, 3, 5, 7, 10, 15, 18]
         J = np.zeros((len(d), nFolds))
         training_J = np.zeros((len(d), nFolds))
         for t, degree in enumerate(d):
             for iFold in range(nFolds):
                 Xti, Xvi, Yti, Yvi = ml.crossValidate(Xtr, Ytr, nFolds, iFold) # use ith block as validation
                 XtiP = ml.transforms.fpoly(Xti, degree, bias = False)
                 XtiP, params = ml.transforms.rescale(XtiP)
                 lr = ml.linear.linearRegress(XtiP, Yti) # TODO: train on Xti, Yti, the data for this fold
                 XviP,_ = ml.transforms.rescale(ml.transforms.fpoly(Xvi, degree, False), params)
                 J[t,iFold] = lr.mse(XviP, Yvi)
                 training_J[t,iFold] = lr.mse(XtiP, Yti)

         mean_validation = [0]*7
         mean_training = [0]*7
         for k in range(7):
             mean_validation[k] = np.mean(J[k])
             mean_training[k] = np.mean(training_J[k])

         fig, ax = plt.subplots(1, 1, figsize = (15, 8))
         ax.semilogy(d, mean_validation, lw = 4, marker = "o", color = "red", markersize = 10, alpha = 0.65,
         label = "5 fold cross-validation validation error")
         ax.semilogy(d, mean_training, lw = 4, marker = "s", color = "blue", markersize = 10, alpha = 0.60, l
         abel = "5f old cross-validation train error")
         ax.set_xlim(0.0, 30)
         ax.set_xticks(np.arange(2, 20, 2))
         ax.set_xticklabels(ax.get_xticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
         ax.set_yticklabels(ax.get_yticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
         # Controlling the size of the legend and the location.
         ax.legend(fontsize = 12, loc = 4)
         print("Mean: ", np.mean(J))
```

```
Mean:  35524.38656208917
```



Problem 2 - Part 2

The same error happend in the figure as the one in problem 1 bur when MSE was used the error increased by higher difference compared to when cross validation was used and in that regards the error increases as the degree increases

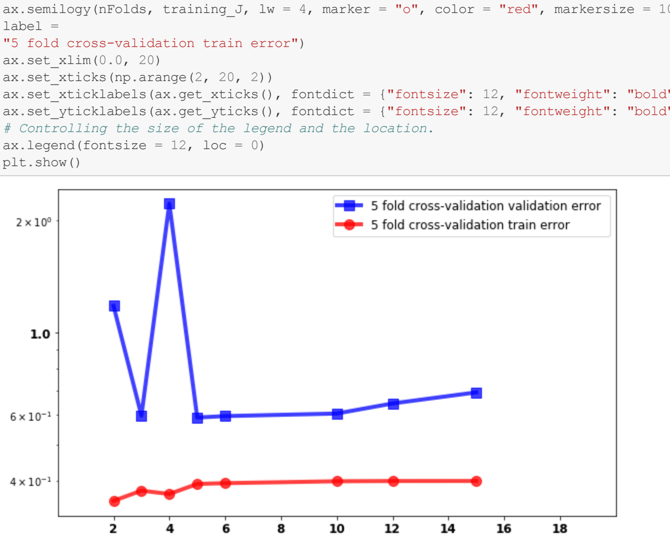
Problem 2 - Part 3

```
In [17]: print("Polynomial degree of: ", d[np.argmin(mean_validation)])
```

```
Polynomial degree of:  5
```

Problem 2 - Part 4

```
In [305]: nFolds = [2, 3, 4, 5, 6, 10, 12, 15]
          degree = 5

          J = np.zeros(len(nFolds))
          training_J = np.zeros(len(nFolds))
          for j, nFold in enumerate(nFolds):
              for iFold in range(nFold):
                  Xti, Xvi, Yti, Yvi = ml.crossValidate(Xtr,Ytr,nFold,iFold) # use ith block as validation
                  XtiP = ml.transforms.fpoly(Xti, degree, bias = False)
                  XtiP, params = ml.transforms.rescale(XtiP)
                  lr = ml.linear.linearRegress(XtiP, Yti) # TODO: train on Xti, Yti, the data for this fold
                  XviP,_ = ml.transforms.rescale(ml.transforms.fpoly(Xvi, degree, False), params)
                  J[j] += lr.mse(XviP, Yvi)
                  training_J[j] += lr.mse(XtiP, Yti)
              J[j] = J[j] / nFold
              training_J[j] = training_J[j] / nFold

          fig, ax = plt.subplots(1, 1, figsize = (10, 6))
          ax.semilogy(nFolds, J, lw = 4, marker = "s", color = "blue", markersize = 10, alpha = 0.75, label =
          "5 fold cross-validation validation error ")
          ax.semilogy(nFolds, training_J, lw = 4, marker = "o", color = "red", markersize = 10, alpha = 0.75,
          label =
          "5 fold cross-validation train error")
          ax.set_xlim(0.0, 20)
          ax.set_xticks(np.arange(2, 20, 2))
          ax.set_xticklabels(ax.get_xticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
          ax.set_yticklabels(ax.get_yticks(), fontdict = {"fontsize": 12, "fontweight": "bold"})
          # Controlling the size of the legend and the location.
          ax.legend(fontsize = 12, loc = 0)
          plt.show()
```



Problem 3

Reading the lecture notes and getting help from discussion section and piazza were very helpful.

It helped me to complete this homework.