Project 1: Advanced Euler Solver in 1D

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One dimensional Euler equation is being solved using first order upwind with approximate Riemann solver. Roe Solver is used with SuperBee limiter for the implementation of TVD scheme. The density values of low order scheme and high order scheme is compared.

Equations:

$$u_{,t} + F_{,x} = 0$$

$$u = \begin{bmatrix} \rho \\ \rho v \\ \rho e \end{bmatrix}$$

$$F = \begin{bmatrix} \rho v \\ \rho v^2 + p \\ v(\rho e + p) \end{bmatrix}$$

$$p = (\gamma - 1)\left[\rho e - 0.5 \rho v^2\right]$$

Domain:

$$0 \le x \le 100$$

Initial conditions:

$$\gamma = 1.4$$

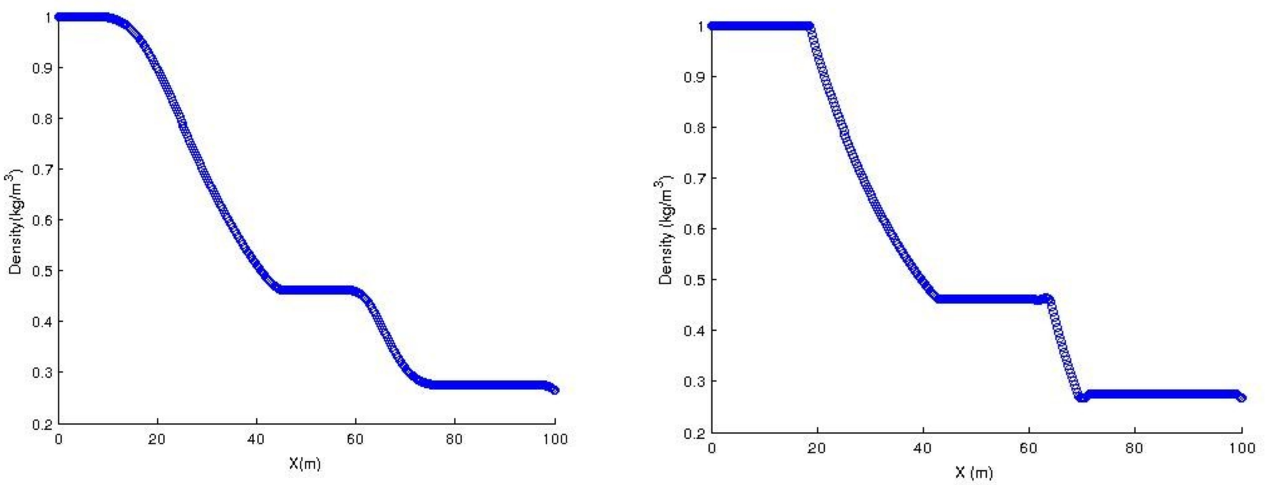
$$x < 50: \quad u = (1.0, 0.0, 2.5)$$

$$x > 50: \quad u = (0.1, 0.0, 0.25)$$

Approach:

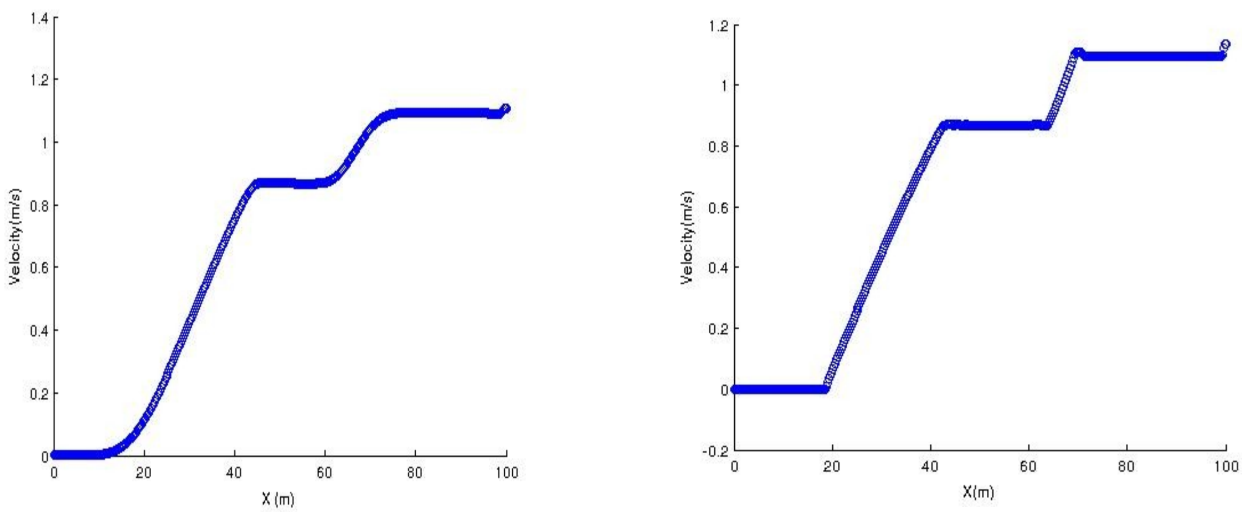
The algorithm has been implemented using C++ programming language. There are two types of unknowns here: the flow unknowns density, density velocity and density energy. Along with that there are Godunov unknows i.e. density, velocity and pressure.

At first the initial values are assigned to the gridpoints. Then a time loop is being run. In the time loop, first the boundary values are updated in each time step. Then the value of dt is calculated with maintaining CFL condition for stability. Afterwards, the Riemann problem is solved for all internal nodes using the Roe solver. In the Roe solver, the left and right state along the diaphragm is calculated. In the TVD case the slopes are maintained using SuperBee limiter. In the non TVD case no limiting is being performed. First order upwind method of discritization is chosen. The length of the domain is set to 100 units and total 400 grid points are being used. For the case of TVD, the left and right state conditions are calculated following the equations shown in the class. From the Roe solver the Godunov variables are first updated followed by the conservative flow variables. The result for density values are shown in Figure 1. From the figure it can be seen that the TVD scheme is monotonically preserving than and non TVD case. In Figure 2 the velocity vs. distance plot is presented and in Figure 3 pressure vs. distance results are shown.



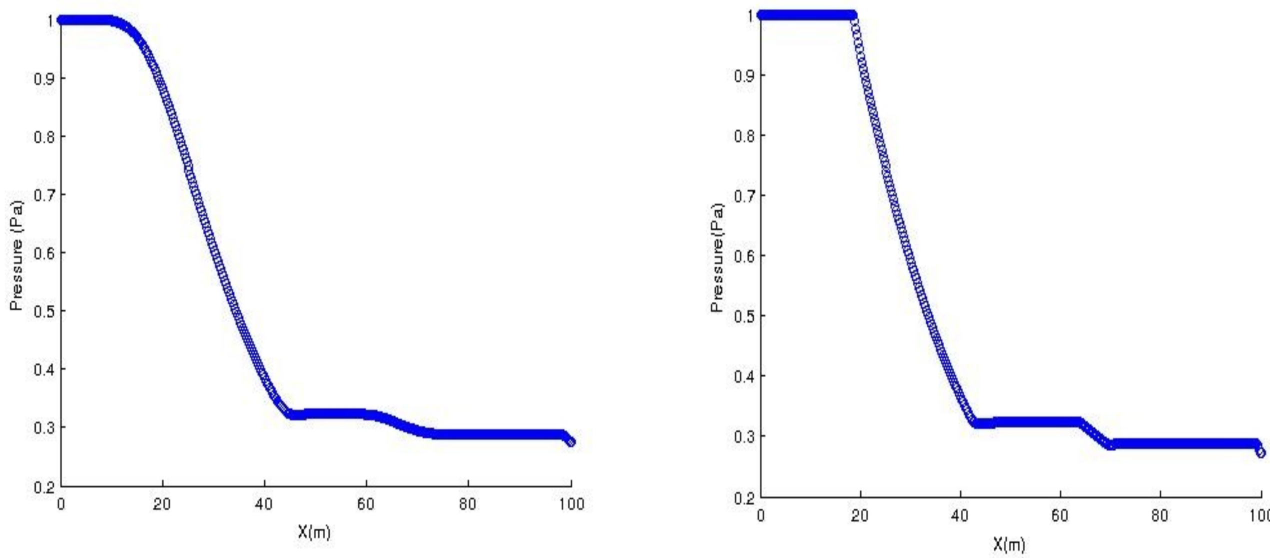
(a) 1st order updwind (b) 1st order upwind with SuperBee limiter

Figure 1. Density vs. distance plot for 1D Euler equation



(a) 1st order updwind (b) 1st order upwind with SuperBee limiter

Figure 2. Velocity vs. distance plot for 1D Euler equation



(a) 1st order updwind

(b) 1st order upwind with SuperBee limiter

Figure 3. Pressure vs. distance plot for 1D Euler equation