

We thank the reviewers for their time and thoughtful feedback. Taking their helpful comments into account, we have sought to clarify the presentation of our work and multiple tasks. Besides clarifying our current result for multiple tasks, we have also extended our result on variance reduction, in response to the main criticism by R1 and R2 (L11-17).

**Predicting a particular task using MTL? [R2]** As R2 pointed out, we focus on a situation where we have a target task for which we only have limited labeled data and a source task. We study when training the tasks together can benefit the target task. While this setting is different from traditional MTL that studies the avg performance of all tasks, it is also a common setting for MTL in practice. For example, for predicting a rare event or classifying an Xray-scan, collecting large amounts of labeled data is either not possible or very expensive, but auxiliary labeled data are often easier to obtain. Traditional MTL theory that studies the average performance of all tasks does not predict whether using MTL can benefit the target task. Our work applies to this setting and takes a step towards filling the gap.

**What can we say for multiple tasks? [R1, R2]** We have focused on two tasks in the submission to provide insight, since this is the simplest setting. We understand that having multiple tasks is more general, therefore, we have *extended our result on bias-variance tradeoff to multiple tasks*. (1) We can now show that *as long as the output dimension of the shared layer  $B$  is smaller than the total number of tasks, the variance of the MTL estimator for the target task is always smaller than the variance of the STL estimator but the bias is always larger*. We have included this result in the draft. (2) For multi-label settings where all tasks have the same features, i.e.  $X_i = X$  for any  $i$ , using Thm 3.6 *all of our insight for two tasks still applies except covariate shift* (covariate shift doesn't apply since tasks have the same features).

**Writing: [R2, R3]** We have corrected the typos that R2 pointed out and clarified the issues that R3 raised. (1) L112-118: we use  $t$  to denote the number of tasks hence for two tasks  $t = 2$ . (2) Validation set size: we only need it to be larger than the size of the hidden layer times the number of tasks, which can be much smaller compared to the size of the training set (cf. L108). (3) Def. of the prediction loss L113: the expectation is over a test sample  $x$  whose label is  $x^\top \beta_t$ . Taking expectation over  $\varepsilon$  gives the bias-variance decomposition, following standard linear regression literature [17,18].

**R1: (♦)** We thank R1 for suggesting looking at qualitative predictions of Thm 3.6 for multiple tasks, which we have added in the draft. (1) For task similarity, the more similar tasks are, the more variance reduces ( $\|v_t\|$  closer to 1), which leads to positive transfer as in Prop 3.3. (2) For sample ratio, the more dissimilar tasks are, the more bias increases w/ more source samples, which leads to negative transfer as in Prop 3.4. (♦) R1 asks how does our method compares to loss reweighing. Our method is equivalent to increasing task weight until performance drops. Our method is preferable since we only compute over a subset of samples whereas loss reweighing uses the full set. (♦) We thank R1 for pointing out the vague use of "similar performance" in experiments, which we replaced w/ (comparable) acc. numbers.

(♦) R1 asks how does our theory compares to previous work. The closest work to ours is [15] and that work uses standard concentration bounds to show that when two tasks are similar enough, MTL guarantees positive transfer. Our work uses advanced tools from random matrix theory and Thm 3.2 does not make such an assumption. This also allows us to study the *more challenging setting of varying sample size and covariate shift, both of which cannot be studied using standard concentration bounds*. (♦) R1 asks whether we can compute similarity via distance between classifier parameters, which we tried for predicting whether MTL outperforms STL but the result is worse than Table 1. We suspect the distance mainly captures difference between the trained model but does not capture properties of task data.

**R2: (♦)** We thank R2 for bringing up the confusion of which sample size regime does our theory/algorithm apply, which we have clarified in the updated draft. (1) R2 is correct that "our theory applies when the sample sizes are tens or hundreds of feature dimension". We think this is a reasonable regime to consider; for example, in our sentiment analysis experiment, the feature dimension of a sentence is 300 and the training set size ranges from 3 to 10 thousand. (2) R2 mentions "having imbalanced sample size btw source/target task": Our incremental training scheme does not assume that the tasks have imbalanced sample size; for example, in our sentiment analysis experiment, we have observed that our method can be effective even when the source task is smaller than the target. As shown in our theory, the transition threshold between positive/negative transfer provably depends on task similarity and can be less than one (Prop. 3.4).

(♦) We thank R2 for pointing out the connection between our incremental training procedure and curriculum learning. We are not aware of any previous work that proposes such an idea in MTL while having a strong theoretical basis. Adding more context, there is an ongoing discussion of how much data from each task the model should be trained on (cf. Google T5 and refs therein). We have focused on evaluating training efficiency as a further validation of our theory. It's conceivable that by combining our procedure w/ other ideas one might get better final performance of the target task. It is an interesting research question to further investigate the idea in future work.

**R3:** We thank R3 for commenting on our work. **L108:** We thought we have clearly stated in L108 that "a validation set that's larger than  $r \cdot t \leq t^2$  suffices" but we will clarify more ( $p^{0.99}$  can be replaced w/  $p^{0.5}$ ). **L113:**  $t$  always means the number of tasks so we're not where is the duplicate notation. **L117:** The sample covariance of task 1 is  $X_1^\top X_1$  not  $\Sigma_1$ . **L187:**  $\gamma$  is a free parameter and by varying it one can recover the entire precision-recall curve. **L220:** Our theory provides a theoretical basis for the algorithm. For two tasks, the algorithm can provably find the optimal sample ratio. As shown in Fig 1b, the performance curve, which is a quadratic function, has a single peak and our algorithm stops at the peak. The fact that the curve is quadratic is shown in our proof and we have added the connection to the draft.