ps4-1-sol

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Suppose the angle between $x^{(i)}$ and u is θ, then

$$\alpha|u| = |x^{(i)}|\cos(\theta)$$
$$\alpha|u| = \frac{|x^{(i)}||u|\cos(\theta)}{|u|}$$
$$\alpha|u| = \frac{x^{(i)} \cdot u}{|u|}$$
$$\alpha = \frac{x^{(i)} \cdot u}{|u|^2}$$
$$\alpha = x^{(i)} \cdot u$$
$$\alpha u = (x^{(i)} \cdot u)u$$
$$v = (x^{(i)} \cdot u)u$$

So,

$$\begin{aligned}
&\arg\min_{u:u^Tu=1} \sum_{i=1}^{m} \|x^{(i)} - f_u(x^{(i)})\|_2^2 \\
= &\arg\min_{u:u^Tu=1} \sum_{i=1}^{m} \|x^{(i)} - (x^{(i)} \cdot u)u\|_2^2 \\
= &\arg\min_{u:u^Tu=1} \sum_{i=1}^{m} (x^{(i)} - (x^{(i)} \cdot u)^T(x^{(i)} - (x^{(i)} \cdot u) \\
= &\arg\min_{u:u^Tu=1} \sum_{i=1}^{m} (x^{(i)})^T x^{(i)} - 2(x^{(i)} \cdot u)u^T x^{(i)} + (x^{(i)} \cdot u)^2 u^T u \\
= &\arg\min_{u:u^Tu=1} \sum_{i=1}^{m} (x^{(i)})^T x^{(i)} - 2u^T x^{(i)}(x^{(i)})^T u + u^T x^{(i)}(x^{(i)})^T uI \\
= &\arg\max_{u:u^Tu=1} \sum_{i=1}^{m} u^T x^{(i)}(x^{(i)})^T u \\
= &\arg\max_{u:u^Tu=1} u^T(\sum_{i=1}^{m} x^{(i)}(x^{(i)})^T)u
\end{aligned}$$