# ps4-3-sol

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Suppose the angle between $x^{(i)}$ and $u$ is $\theta$, then

$$\begin{aligned}
\alpha|u| &= |x^{(i)}|\cos(\theta) \\
\alpha|u| &= \frac{|x^{(i)}||u|\cos(\theta)}{|u|} \\
\alpha|u| &= \frac{x^{(i)} \cdot u}{|u|} \\
\alpha &= \frac{x^{(i)} \cdot u}{|u|^2} \\
\alpha &= x^{(i)} \cdot u \\
\alpha u &= (x^{(i)} \cdot u)u \\
v &= (x^{(i)} \cdot u)u
\end{aligned}$$

So,

$$\begin{aligned}
& \arg\min_{u:u^Tu=1} \sum_{i=1}^{m} \|x^{(i)} - f_u(x^{(i)})\|_2^2 \\
= & \arg\min_{u:u^Tu=1} \sum_{i=1}^{m} \|x^{(i)} - (x^{(i)} \cdot u)u\|_2^2 \\
= & \arg\min_{u:u^Tu=1} \sum_{i=1}^{m} (x^{(i)} - (x^{(i)} \cdot u)^T(x^{(i)} - (x^{(i)} \cdot u) \\
= & \arg\min_{u:u^Tu=1} \sum_{i=1}^{m} (x^{(i)})^T x^{(i)} - 2(x^{(i)} \cdot u)u^T x^{(i)} + (x^{(i)} \cdot u)^2 u^T u \\
= & \arg\min_{u:u^Tu=1} \sum_{i=1}^{m} (x^{(i)})^T x^{(i)} - 2u^T x^{(i)}(x^{(i)})^T u + u^T x^{(i)}(x^{(i)})^T uI \\
= & \arg\max_{u:u^Tu=1} \sum_{i=1}^{m} u^T x^{(i)}(x^{(i)})^T u \\
= & \arg\max_{u:u^Tu=1} u^T(\sum_{i=1}^{m} x^{(i)}(x^{(i)})^T)u
\end{aligned}$$