ps4-5-sol

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(a)

$$
\begin{aligned}
\|B(V_1) - B(V_2)\|_\infty &= \|V_1' - V_2'\|_\infty \\
&= \max_{s \in S} |V_1'(s) - V_2'(s)| \\
&= \max_{s \in S} \left| \gamma \max_{a \in A} \sum_{s' \in S} P_{sa}(s')V_1(s') - \gamma \max_{a \in A} \sum_{s' \in S} P_{sa}(s')V_2(s') \right| \\
&= \gamma \max_{s \in S} \left| \max_{a \in A} \sum_{s' \in S} P_{sa}(s')V_1(s') - \max_{a \in A} \sum_{s' \in S} P_{sa}(s')V_2(s') \right| \\
&= \gamma \max_{s \in S} \left| \max_{a \in A} \sum_{s' \in S} P_{sa}(s')\big(V_1(s') - V_2(s')\big) \right| \\
&\leq \gamma \max_{s \in S} |V_1(s) - V_2(s)|
\end{aligned}
$$

(b)

Let V_1 be a fixed point, then

$$
\|V_1 - V_2'\|_\infty \leq \gamma \|V_1 - V_2\|_\infty
$$

Finally, we have

$$
\begin{aligned}
\|V_1 - V_2\|_\infty &= 0 \\
V_1 &= V_2
\end{aligned}
$$

It means there is only one fixed point V_1.