ps4-5-sol

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(a)

$$
\begin{aligned}
&\|B(V_1) - B(V_2)\|_\infty \\
&= \|V_1' - V_2'\|_\infty \\
&= \max_{s\in S} |V_1'(s) - V_2'(s)| \\
&= \max_{s\in S} \left|\gamma \max_{a\in A} \sum_{s'\in S} P_{sa}(s')V_1(s') - \gamma \max_{a\in A} \sum_{s'\in S} P_{sa}(s')V_2(s')\right| \\
&= \gamma \max_{s\in S} \left| \max_{a\in A} \sum_{s'\in S} P_{sa}(s')V_1(s') - \max_{a\in A} \sum_{s'\in S} P_{sa}(s')V_2(s')\right| \\
&= \gamma \max_{s\in S} \left| \max_{a\in A} \sum_{s'\in S} P_{sa}(s')\big(V_1(s') - V_2(s')\big)\right| \\
&\le \gamma \max_{s\in S} |V_1(s) - V_2(s)| \\
&= \gamma \|V_1(s) - V_2(s)\|_\infty
\end{aligned}
$$

(b)

Let V_1 be a fixed point, then

$$\|V_1 - V_2'\|_\infty \le \gamma \|V_1 - V_2\|_\infty$$

Finally, we have

$$
\begin{aligned}
\|V_1 - V_2\|_\infty &= 0 \\
V_1 &= V_2
\end{aligned}
$$

It means there is only one fixed point V_1.