ps3-1-solve

May 24, 2021

1 (a)

Our update function for $w_{1,2}^{[1]}$:

$$w_{1,2}^{[1]} := w_{1,2}^{[1]} - \alpha \frac{\partial l}{\partial w_{1,2}^{[1]}}$$

For $\frac{\partial l}{\partial w_{1,2}^{[1]}}$, we have:

$$\begin{aligned}
\frac{\partial l}{\partial w_{1,2}^{[1]}} &= \frac{\partial l}{\partial o}\frac{\partial o}{\partial z^{[2]}}\frac{\partial z^{[2]}}{\partial a_2^{[1]}}\frac{\partial a_2^{[1]}}{\partial z_2^{[1]}}\frac{\partial z_2^{[1]}}{\partial w_{1,2}^{[1]}} \\
&= \frac{2}{m}\sum_{i=1}^{m}(o^{(i)} - y^{(i)})\sigma(z^{[2](i)})(1 - \sigma(z^{[2](i)}))w_2^{[2]}\sigma(z_2^{[1](i)})(1 - \sigma(z_2^{[1](i)}))a_1^{[0]} \\
z^{[2](i)} &= w_0^{[2]} + \Sigma_{k=1}^{3} w_k^{[2]}\sigma(w_{0,k}^{[1]} + \Sigma_{j=1}^{2} w_{j,k}^{[1]} x_j^{(i)}) \\
z_2^{[1](i)} &= w_{0,2}^{[1]} + \Sigma_{j=1}^{2} w_{j,2}^{[1]} x_j^{(i)} \\
a_1^{[0]} &= x_1^{(i)} \\
\sigma(z) &= (1 + \exp(-z))^{-1}
\end{aligned}$$

2 (b)

It's possible.

We can find that for any $x^{(i)} \in X$, if

$$\begin{aligned}
x_1^{(i)} &< 0.5 \text{ or} \\
x_2^{(i)} &< 0.5 \text{ or} \\
x_1^{(i)} + x_2^{(i)} &> 4.0
\end{aligned}$$

then $y^{(i)} = 1$

In hidden layer, we have three neurons, so we can simply implement one condition in one neuron.

Then we get

$$
\begin{aligned}
w_{0,1}^{[1]} &= -0.5, w_{1,1}^{[1]} = 1, w_{2,1}^{[1]} = 0 \\
w_{0,2}^{[1]} &= -0.5, w_{1,2}^{[1]} = 1, w_{2,2}^{[1]} = 0 \\
w_{0,3}^{[1]} &= -4.0, w_{1,3}^{[1]} = 1, w_{2,3}^{[1]} = 1 \\
w_{0}^{[2]} = -0.5, w_{1}^{[2]} = 1, w_{2}^{[2]} &= 1, w_{3}^{[2]} = 1
\end{aligned}
$$

3 (c)

It's impossible.

If our activation functions be the linear function $f(x) = x$, then for $z^{[2]}$, we have:

$$
\begin{aligned}
z^{[2]} &= w_0^{[2]} + \Sigma_{k=1}^{3} w_k^{[2]} a_k^{[1]} \\
&= w_0^{[2]} + \Sigma_{k=1}^{3} w_k^{[2]} f(w_{0,k}^{[1]} + \Sigma_{j=1}^{2} w_{j,k}^{[1]} x_j) \\
&= w_0^{[2]} + \Sigma_{k=1}^{3} w_k^{[2]} (w_{0,k}^{[1]} + \Sigma_{j=1}^{2} w_{j,k}^{[1]} x_j) \\
&= \Sigma_{k=1}^{3} w_k^{[2]} \Sigma_{j=1}^{2} w_{j,k}^{[1]} x_j + (w_0^{[2]} + \Sigma_{k=1}^{3} w_k^{[2]} w_{0,k}^{[1]}) \\
&= \Sigma_{i=1}^{2} w_i x_i + C
\end{aligned}
$$

This shows that in this case, our neural network becomes a linear model.

And obviously, the linear model can't segment the data displayed in the figure.

[ ]: