```
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\usepackage[T1]{fontenc}
\usepackage{amsmath}
\usepackage{amsfonts}
\usepackage{amssymb}
\usepackage[version=4]{mhchem}
\usepackage{stmaryrd}
\usepackage{graphicx}
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\usepackage{hyperref}
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\urlstyle{same}

\title{Chapter 2
 The nature of money }

\author{}
\date{}


\begin{document}
\maketitle
\subsection{Introduction}
Fiat money is indispensable in modern economic systems, although it has been part of daily life as far bac
k as 9000 BC, when grain and cattle were used in Anatolia and Mesopotamia for exchange purposes. Wh
ile we take the usage of some form of money as granted, we need to understand why it exists, its function
s and properties. In particular, we need to solve the double coincidence of wants problem associated with
barter and to resolve the lack of trust between the payer and the payee in a transaction.

\subsection{Aims}
The aim of the chapter is to introduce the main ingredients of a monetary economy. We will introduce mo
ney for exchange purposes of goods and services. We will describe its functions, why money is useful in t
rade, its types and properties.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able
to:

\begin{itemize}
  \item discuss the nature and shortcomings of a barter economy

  \item list and describe the general functions performed by money

  \item describe the Wicksell problem and demonstrate how indirect barter can lead to the emergence of a
 commodity money

  \item describe the differences between transactions using credit cards and bank debit cards

  \item list and describe what the different types of money are.

\end{itemize}

\subsection{Reading advice}
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You will certainly find it easiest, and probably most useful, to read the appropriate sections on the nature of money in one or more of the basic textbooks before you move 2. The nature of money

on to the material in this chapter. [1] All textbooks on money and banking will have a section covering the material here. However the best, although most difficult, is that of Goodhart (1989a) Chapter 2 - see below under 'Essential reading'. You may find it helpful to read Goodhart (1989b) first - see below under 'Further reading'.

Essential reading

Goodhart, C.A.E. Money, Information and Uncertainty. (London: Macmillan, 1989a) Chapter 2.

Lewis, M.K. and P.D. Mizen Monetary Economics. (Oxford; New York: Oxford University Press, 2000) Chapters 1 and 2.

Further reading

Clower, R.W. 'Introduction' in Clower, R.W. (ed.) Monetary Theory: Selected readings. (Harmondsworth: Penguin, 1969).

Goodhart, C.A.E. 'The Development of Monetary Theory' in Llewellyn, D.T. (ed.) Reflections on Money. (Basingstoke: Macmillan, 1989b).

Harris, L. Monetary Theory. (New York; London: McGraw-Hill, 1985) Chapter 1.

Kiyotaki, N. and J.H. Moore 'Evil is the Root of all Money'. Clarendon Lecture series, Lecture 1 (2001).

Kiyotaki, N. and R. Wright 'Acceptability, Means of Payment, and Media of Exchange', Federal Reserve Bank of Minneapolis Quarterly Review, Summer 1992. Also in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

McCallum, B. Monetary Economics. (New York; Macmillan; London: Collier Macmillan, 1989).

Newlyn, W.T. and R.P. Bootle Theory of Money. (Oxford: Clarendon Press, 1978) Chapter 1.

What is money?

Money is defined by its function rather than the form in which it takes. In this sense, money is defined as 'anything which is in general use, and generally accepted, as a means of payment.' In the past, money has taken the form of corn, rice, cattle, shells, various precious metals and more recently, pieces of paper issued by governments. In all cases though, money was and is used as a means of payment in exchange. The payer in a transaction (he or she who is purchasing a good or service) hands over money to the value of the item bought and at this point the payee (the seller of the good or service)

[1] See for example, Harris, and Newlyn and Bootle. accepts that the payment is complete. The payee neither holds any further claims on the payer, nor on any third party who may have produced or issued the money.

In this chapter we will discuss and explain the important functions, properties and different types of money. We will also explain why we use money and compare the alternative trading strategies, both with and without money.

The functions of money

Below, we will discuss the three main functions of money, although, arguably, some authors will expand these into four. [2]

Money as a means of payment

As suggested above, the most important function of money is its use as a means of payment - money bei ng used to pay for items purchased or to settle any debts. A related role of money is that as a medium of exchange, which Wicksell defined as an object which is taken in exchange, not on its own account. . not to be consumed by the receiver or to be employed in technical production, but to be exchanged for something else within a longer or shorter period of time. [3] In this sense, a means of payment can also be a medium of exchange. A gold coin for example, used to buy a piece of land, will be a means of payment (the seller of the land will not hold a claim on the payee who has just handed over the gold) but it will also be a medium of exchange. The receiver of the gold coin will not use it for decorative purposes or to 'make' any other goods or services, but will use it as a medium of exchange when he or she wishes to purchase a good or service some time in the future.

However, the converse is not necessarily true. [4] That which may act as a medium of exchange may not act simultaneously as a means of payment. For example, if I wish to pay for a television set using a credit card, the seller may accept this as a medium of exchange.

Although the television shop's account may be credited effectively immediately, it may not be a means of payment since I, the payer, still have a debt outstanding, namely that to the credit card company. I have merely replaced a debt to the television shop with a debt to the credit card issuer. The purchase of the television set with the credit card is then not a means of payment. For this reason we will not include credit cards, trade credit between firms, or any other line of credit such as unused overdraft facilities, in our definition of money.

[2] The fourth function of money commonly quoted, is that of a standard for deferred payment. This simply means that if something is bought today although payment for it does not have to be made until some later date, then the amount due for deferred payment can be measured in terms of money.

[3] Wicksell, 1906, quoted by Kiyotaki and Wright, (1992).

[4] For an excellent analysis of the differences between 'means of payment' and 'medium of exchange' see Goodhart (1989a) Chapter 2 , Section 4.

The nature of money

Money as a unit of account

This function is also described as money acting as a measure of exchange value, money acting as a standard of value, or money as a numeraire. The essential point about this function is that money is acting as a common denominator, in terms of which the value in exchange of all goods and services can be expressed. Money is simply acting as a unit of measurement in the same way that metres measure length and kilograms measure weight. Money in this sense is being used to measure the value of goods, services and assets relative to other goods, services and assets. If it is convenient to trade all commodities in exchange for a single commodity, so it is convenient to measure the prices of all commodities in terms of a single unit, rather than record the relative price of every good in terms of every other good. If there is to be a single unit of account, it is again clearly convenient (though not necessary) that the unit of account be the medium of exchange, given that goods actually exchange against the medium of exchange.

A clear advantage of having a single unit of account is that it greatly reduces the number of exchange ratios between goods and services. With four goods $(\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D), in order to facilitate exchange, exchange ratios of each good in terms of all the others must be available (i.e. the six ratios $A: B, A: C, A: D, B: C, B: D$ and $C: D$ must be available). In fact with n goods, there are $n(n-1)=2$ relative prices. However, if we introduce a fifth good, 'money' that acts as a unit of account then there only need to be four prices. With n goods and money being the $n+1$-th commodity acting as a unit of account, we only need n prices. For example, with 1,000 goods and no unit of account, the economy needs 499,500 relative prices of one good in terms of another. Introducing money as a unit of account dramatically reduces this to only 1,000 . Thus, having money as a unit of account can encourage trade by making it easier for individuals to know how much one good is worth in terms of another.

Money as a store of value

The exchange attributes of money, in particular that it is durable and can readily be used in the purchase of goods, also mean that people may wish to hold it as an asset, that is as part of their stock of wealth. In this sense, money serves as a store of value: it is permitting the separation in time of the act of sale from the act of purchase. The existence of a means of payment enables a person to sell a good without simultaneously having to buy another good in exchange. Receiving a means of payment in exchange for the good sold allows the seller to hold on to it until such time as it is needed to be exchanged for the goods and services he or she requires.

Money is not unique as a store of value: any asset, such as equities, bonds, real estate, antiques and works of art can all act as stores of value. Money itself is sometimes a poor store of value. This will occur when the relative price of money falls as a result of the money prices of other goods and services rising, that is, during periods of inflation.

\subsection{Why do we have money?}
The use of money helps facilitate trade since in the absence of money, trade has to proceed through barter, that is the direct exchange of one good for another.

\section{Barter}
Barter tends to be associated with primitive economies in which individual households operate in an isolated manner, and in particular have no sophisticated information systems concerning what is going on in the rest of the economy. For various reasons households may wish to consume a different bundle of goods from those they produce, so that there are gains from trade. The problem is how to achieve these potential gains, given that trade is a voluntary activity and can proceed only when it is to the benefit of both parties.

In barter, there has to be what is known as a ’double coincidence of wants’. If I grow corn but want to consume apples, not only do I have to find someone willing to trade apples but they must also want what I have to offer, namely corn. In other words for trade to be mutually beneficial, it is necessary not only that trader A has what trader B wants but also that trader B has something to offer in exchange which trader A wants. It is quite possible that no trade will occur, especially in cases where the goods desired are so specialised that the probability of a double coincidence of wants occurring is so low that the cost of finding a match (for example in terms of advertising, transport, and so on) becomes very large and outweighs the increased utility derived from trade. Even if the goods offered for trade are readily available, it may still be the case that no trade occurs. This is the subject of the Wicksell problem in which it is impossible to secure gains from trade through bilateral exchange.

\section{The Wicksell problem}
Consider the situation in Table 2.1 and shown in Figure 2.1. There are three individuals who each only produce one good but derive more utility from the consumption of another. Avinash produces bread but wants wine, Nicole produces wine but wants apples and Edwin has apples but wants bread. In this situation, no bilateral exchange will result. Edwin will not trade with Nicole since he derives more utility from consuming his own apples than he does from Nicole’s wine. The utility maximising situation is shown in Figure 2.1 but under the assumption of no commitment, this will never arise. Avinash will not want to give his bread to Edwin, hoping to receive wine from Nicole since after receiving bread, Edwin has no incentive to give apples to Nicole who in turn will have no incentive to hand over her wine to Avinash. The system will collapse to autarky (in which all individuals become self-sufficient).

However, models of this type can lead to the emergence of indirect barter. In indirect barter a trader accepts a good not because she or he wants to consume it but because of the possibility that it may result in a future trade. In the above example, Avinash may trade his bread for apples, not because he wants to consume the apples but in the knowledge that when meeting Nicole, she will be willing to trade her wine for those goods. This may seem an obvious solution at first glance but if we assume traders meet at random (or do not know how long it will be before they meet a prospective trader) then considerable risk may be taken on in this transaction. The goods Avinash accepts in exchange for his bread may deteriorate before he meets Nicole, in which case Nicole will not want to trade.

Let us assume that the apples do not perish. The situation of indirect barter is shown in 2. The nature of money

\begin{center}
\begin{tabular}{|l|l|l|l|l|}
\hline
Individual & Produce & \multicolumn{3}{|c|}{Consumption utility} \\
\hline
 & & Bread & Apples & Wine \\
\hline
Avinash & Bread & 0.5 & 0 & 1 \\
\hline
Edwin & Apples & 1 & 0.5 & 0 \\
\hline
Nicole & Wine & 0 & 1 & 0.5 \\
\hline
\end{tabular}
\end{center}

Table 2.1:
\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-006}

Figure 2.1:

Figure 2.2 .

If apples are used in both stages of the trading process then each trader will gain. In indirect barter, commodities are acquired not for consumption but for the purposes of making future exchanges. They should therefore have appropriate physical attributes; in particular they should be durable, portable and homogeneous (or at least easily valued). In primitive societies money emerged from the process of indirect barter, and took the form initially of staple foodstuffs like corn or rice. With the development of technologies, which allowed the separation of metals, money increasingly took the form of precious metals, especially gold.

\section{Money and trust}
The solution of indirect barter in the Wicksell problem arises almost entirely from the lack of trust between traders. As shown in Figure 2.1, each trader could maximise utility by giving his or her goods to the next person in the triangle. However, Avinash could not trust Edwin to hand over his apples to Nicole once he gave him his bread and neither could he trust Nicole to hand over her wine once (or if) she received Edwin's 2.9. Why do we have money?

Step 1. Avinash trades apples for bread with Edwin

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-007}
\end{center}

Avinash loses 0.5 , Edwin gains 0.5

Step 2. Avinash trades apples for wine with Nicole

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-007(1)}
\end{center}

Avinash gains 1 , Nicole gains 0.5

In total, Avinash, Edwin and Nicole each gain 0.5

Figure 2.2:

apples.

Let us consider another situation. Suppose I visit my dentist and I pay for his services using a bank debit card.[5] The bank will debit my account and credit that of the dentist. I have effectively given an IOU[6] to the bank and the bank has given the dentist one of its IOUs. But why did I not just hand over one of my IOUs directly to the dentist? Perhaps it may be that my dentist does not trust me to repay once he tries to redeem my IOU. More realistically, he cannot use my IOU to make purchases with anyone else and the likelihood that he will ever want to redeem the IOU from me, by receiving an economics lesson, may be so small that he would never have agreed to spend his time fixing my teeth. If the dentist went to the grocers the following day trying to buy provisions with an IOU from an economics teacher, of whom the grocer had never heard, it is highly unlikely that the trade would be completed. On the other hand, if he tried to pay using the bank's IOU, the grocer should happily agree to the sale, purely because everyone knows and trusts the bank.

The difference between my IOU and that of the bank is that the bank's IOU is liquid

[5] This example is taken directly from 'Evil is the Root of all Money' by Kiyotaki and Moore (2001). The model presented in this paper is not necessary for this subject but the ideas discussed therein will greatly improve the reader's intuition as to what is money and why we hold it.

[6] 'IOU' is short for 'I owe you' and is merely a written statement acknowledging that a debt has been created, to be repaid sometime in the future.

The nature of money

and functionally equivalent to cash. My IOU, on the other hand, is not liquid and cannot flow around the economy facilitating trades. As we will discuss later in this chapter, it is 'credit' money but serves the same purpose as the commodity money derived in the indirect barter problem, without having to use or lock up any goods in the payment process.

Activity 2.1 Consider the problem in Figure 2.1. How could you introduce a bank and a system of IOUs to resolve the problem without having to use any commodity to act as a medium of exchange?

The idea of trust and uncertainty are important ones when explaining the existence of money.[7] If you do not know, or have any way of obtaining information as to the creditworthiness of, a prospective customer, there is a high risk that he or she may default on that debt. The risk is, however, minimised if the payer hands over an item of worth, such as commodity money or another store of value, or an IOU redeemable from a third party on which the payee has sufficient information, such as a bank. In this way it may be essential to hold money beforehand in order to make purchases. This is the whole idea behind 'cash in advance' (CIA) models that we will consider in a more macroeconomic setting in later chapters. People have to hold money in order to alleviate the problem of trust (or lack of trust) when transactions are made.

Types of money

Up to now we have considered only two types of money: commodity money in the solution to the Wicksell problem, and credit money as the solution to the lack of trust in the issuance of private IOUs. Traditionally, however, money is divided into three types:

- commodity money

\item fiat money

\item credit money.

\end{itemize}

\section{Commodity money}
Commodity money derives from the use of commodities as exchange intermediaries in indirect barter. Commodity money has taken many forms, such as cattle, corn, seashells and suchlike, but in most societies, it has evolved towards the precious metals, copper, silver and, above all, gold. Such commodities are accepted in exchange because of their intrinsic value, even though the trader intends to use them in further exchange rather than for their own consumption. Commodity money is inefficient, in part because the commodities being used may not have ideal properties as exchange intermediaries but more fundamentally because it is unnecessary to use goods which have intrinsic value for a purpose which does not make use of that value. For example, using gold as a commodity money does not make use of the fact that gold does have some alternative utility-yielding use and could be used in more satisfying ways, such as through jewellery or ornaments.

${ }^{7}$ On this issue, see Goodhart (1989a), Chapter 2, Section 2.

\section{Fiat money}
Fiat money, whose archetype is the government-issued bank note or metal coin, is money that has physical substance but no intrinsic value. It is used because its use is established by custom and practice. People accept fiat money that they know to be intrinsically worthless because they know others will accept it in payment for goods and services. The value of fiat money is reinforced in society by the attribute of being 'legal tender', that is making it illegal for anyone to refuse payment in fiat money in settlement of a debt. In fact it is this legal 'stamp' and recognised power of the issuer, usually governments, which gives fiat money its unquestionable acceptance as a means of payment.

Since fiat money is essentially worthless, there is a difference between the value of the goods such money can purchase and the smaller cost of printing this money. The difference is known as seigniorage and is effectively the profit made by the issuing authority when it produces currency.

\section{Credit money}
Whereas fiat money is not a claim on any commodity or individual, credit money is; it is the debt of a private person or institution. In the dentist example above, I gave an IOU to the bank, which in turn hands this debt over to the dentist. If he can pass this claim on to another individual, the grocer, in exchange for goods, then this debt is being used as a means of payment and so constitutes money. However, the asset that the grocer now has, the claim on the bank, is matched one-for-one with the bank's claim on me. What the bank owes to the grocer (the bank's liability) must be matched by what I owe the bank (the bank's asset) in order for the bank's balance sheet to balance. The limitation of credit money is that it can only function if both individuals, the dentist and me in the example, have complete confidence in the willingness and ability of the intermediary to honour the debt. If the bank fails then both my dentist and I would lose out since the dentist would still have a claim on me and I may not have the funds to honour that claim if the bank has been forced to close. Since there exists a private liability perfectly offsetting the asset acting as a means of payment (i.e. the money is generated inside the private system) credit money is also known as 'inside money'. Fiat money, which cannot be matched against private sector claims (and is generated outside the private sector) is in a similar fashion, known as 'outside money'. In the UK today, inside money is quantitatively much larger than outside money. At the time of writing, the nominal value of inside money is approximately 30 times the value of outside money, notes and coins.

There may appear to be a contradiction at this point when we compare the definition of credit money to the discussion in the section 'money as a means of payment' where we argued that all forms of credit should not be included in our definition of money. In the above example, although I have replaced a debt to my dentist with a debt to my bank, this is commonly accepted as a means of payment since a transfer of cred it to a current account has advantages such as safe-keeping and making use of book-keeping services at the bank. Only when all parties concerned are entirely confident the bank will not fail, will such transfers be a means of payment. Once the bank hears of my purchase at the dentist (via the swiping of the debit card), the bank debits my account and credits that 2. The nature of money

of the payee, the dentist. Nothing further needs to be done since the payment is complete. If a purchase is made by credit card, however, a debt is still outstanding and has to be repaid. Similarly, if I paid by my bank debit card but went overdrawn, a debt would remain outstanding which I would later have to pay off. For this reason, overdraft facilities are not a means of payment.

\subsection{Properties of money}
When discussing the appropriate properties of commodity money, we mentioned durability, portability and that it should be homogeneous. Other necessary properties are divisibility and recognisability. In fact all money should possess these properties: electronic transfers when purchases are made by bank debit card are themselves durable, if not in a physical sense, in that $\$ 1$ credited to your account does not deteriorate over time. Another property that is just as important is that of acceptability, the probability that it will be accepted as a means of payment. Kiyotaki and Wright (1992) emphasise this property and argue that acceptability is not a property of the money per se. An equilibrium can be reached where one individual uses one money purely because they believe everyone else will use it:

'When an object is more readily acceptable to other people in the economy, it is more likely that each individual will desire it and accept it as a medium of exchange. The implication is that the property of acceptability can have a self-reinforcing nature... (This) lead(s) to the conclusion that acceptability may not actually be a property of an object as much as it is a property of social convention.'

Kiyotaki and Wright, 1992, p.19.

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
\item discuss the nature and shortcomings of a barter economy

\item list and describe the general functions performed by money

\item describe the Wicksell problem and demonstrate how indirect barter can lead to the emergence of a commodity money

\item describe the differences between transactions using credit cards and bank debit cards

\item list and describe what the different types of money are.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
\item 'The evolution of commodity money from indirect barter, as the solution to the Wicksell problem, arises because of a lack of trust between the relevant parties.'
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{1}
  \item In a monetary economy, money usually performs the three roles of medium of exchange, unit of ac
count and store of value. Why are these three roles typically combined in one entity? Is it possible to conc
eive of an economy in which they are performed by three separate entities?

  \item What is money and why do we use it? Outline the three different types of money and explain what i
s different between them.

  \item ’Money should essentially be perceived as an instrument that allows an increasingly widespread an
d anonymous economic society to deal with the inevitable resulting shortcomings in information and trust
of each of the members on the others.’ (Goodhart) Explain and discuss.

  \item Explain why some economists argue that payment by cheque is the same as giving trade credit. If t
his is the case and considering that cheques are drawn on bank current accounts (as are payments by ba
nk debit cards), do cheque payments count as money?

\end{enumerate}

\subsection{Feedback to Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragra
ph.

\begin{enumerate}
  \item The statement is true. Suggested answers may include Figure 2.1 and an explanation as to why ea
ch person cannot trust the others to pass their goods along the chain. In the example of Figure 2.1, Avina
sh could not trust Edwin to hand over his apples to Nicole once he gave him his bread and neither could h
e trust Nicole to hand over her wine once (if) she received Edwin’s apples. By using a commodity money,
such as wine, the problem can be solved. Edwin trades his apples for Nicole’s wine, not because he want
s to consume the wine, but because he can trade it with Avinash for his bread. Wine is thus being used as
 a commodity money. 2. The nature of money
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{2}
  \item Money is anything which is in general use, and generally accepted, as a means of payment. Althou
gh money has a number of functions, including unit of account and store of value, it is its use as a means
of payment that defines it. The main reason why money is used is to avoid the problems associated with b
arter, namely the double coincidence of wants. You could discuss the origins of a commodity money here,
 from the Wicksell problem, and also explain how money solves the lack of trust problem in a world of larg
ely anonymous individuals. The three types of money are: commodity money, fiat money and credit mone
y. All these are described near the end of this chapter.
\end{enumerate}

\section{Chapter 3}
\section{The demand for money}
\subsection{Introduction}
In Chapter 2 we saw why there was a need for money:

\begin{itemize}
  \item to solve the double coincidence of wants problem associated with barter

\item to obviate the lack of trust between the payer and the payee in a transaction.

\end{itemize}

However, what determines the quantity of money that individuals and economies demand is a separate q uestion. It is the aim of this chapter to explain what determines the quantity of money we demand and als o to present a number of models (or theories) of the demand for money. The chapter is split into two main sections. The first part considers the demand for money from individuals or institutions/firms the microeco nomic determinants of money demand. The second part examines the demand for money at the macroec onomic level gives a brief history of money demand, focusing on the breakdown of the macroeconomic de mand for money function.

\subsection{Aims}
The aim of the chapter is to study the money demand as one of the building blocks of the money market e quilibrium.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
\item explain why it is important to study the demand for money

\item describe the four main microeconomic determinants of money demand

\item outline the inventory theoretic model of Baumol-Tobin and the portfolio selection model of Tobin

\item discuss why Tobin's model solves the 'plunger' problem of the demand for money model of Keynes

\item describe the general set-up of macroeconomic money demand equations 3. The demand for mone y

\item discuss empirical evidence on money demand functions, especially on income and interest elasticit ies

\item describe what happened and what is meant by 'the case of the missing money' and give reasons f or the breakdown of the estimated money demand equations.

\end{itemize}

\subsection{Reading advice}
Before embarking on this chapter, and before consulting any of the recommended reading, you should re view your understanding of 'the demand for money' from your studies in EC2065 Macroeconomics. A very useful text on the demand for money is Laidler (1993), which is both readable and comprehensive, and s hould be consulted on everything covered in this chapter. Goodhart (1989) is also essential reading and s hould be read while you work through the chapter.

\subsection{Essential reading}
Goodhart, C.A.E. Money, Information and Uncertainty. (London: Macmillan, 1989) Chapters 3 and 4.

Laidler, D.E.W. The demand for money: Theories, evidence and problems. (New York: Harper Collins, 19 93) Section II.

Lewis, M.K. and P.D. Mizen Monetary Economics. (Oxford; New York: Oxford University Press, 2000) Ch

apters 5, 6, 11 and 12.

\subsection{Further reading}
\section{Books}
Friedman, M. 'The quantity theory of money: a restatement', in Friedman, M. (ed.) Studies in the quantity t
heory of money. (University of Chicago Press, 1956).

Goldfeld, S.M. 'Demand for money: empirical studies', in Newman, P., M. Milgate and J. Eatwell (eds) Th
e New Palgrave Dictionary of Money and Finance. (London:

Macmillan, 1994).

Goldfeld, S.M. and D.E. Sichel 'The demand for money', in Friedman, B. and F. Hahn (eds) Handbook of
monetary economics. (Amsterdam: North-Holland, 1990).

Harris, L. Monetary Theory. (New York; London: McGraw-Hill, 1985) Chapters 9 and 10.

\section{Journal articles}
Baumol, W. 'The transactions demand for cash: an inventory theoretic approach', Journal of Econometric
s (1952) 66, November, pp.545-56. Judd, J. and J. Scadding 'The search for a stable money demand fun
ction: a survey of the post-1973 literature', Journal of Economic Literature 20(2) 1982, pp.993-1023.

Miller, M. and D. Orr 'A model of the demand for money by firms', Quarterly Journal of Economics 80(3) 1
966, pp.413-35.

Sprenkle, C. 'The uselessness of transactions demand models', Journal of Finance 24(5) 1969 , pp.835-4
7.

Tobin, J. 'The interest elasticity of transactions demand for cash', The Review of Economics and Statistics
38(3) 1956, pp.241-47.

Tobin, J. 'Liquidity preference as behaviour towards risk', Review of Economic Studies $25(1) 1958$, pp.
65-86.

\subsection{Microeconomic determinants of the demand for money}
There are essentially four main determinants of money demand at the individual level. These are

\begin{enumerate}
\item interest differentials: the difference between the yield on money, commonly assumed in the literatur
e to be zero, ${ }^{1}$ and the rate of return on other assets. The greater the rate of return on assets other
than money, the greater the opportunity cost of holding money, and so the fewer money balances will be
demanded.

\item cost of transfers: the costs associated with transferring between higher interest earning assets and
money, needed to purchase goods and services. If the cost of transfers, known as brokerage fees, are hig
h then it is unlikely that we will put our wealth into the higher interest earning assets as to do so will involv
e substantial costs. Demand for money will then be a positive function of these transfer costs.

\item price uncertainty of assets: there is inevitably risk involved in holding assets. Even though there exi
sts a risk in holding money as a store of value, since we do not know for certain how many goods a given
quantity of money can buy in the future due to inflation, the risk associated with holding other, interest ear
ning, assets is generally considered to be greater. If the price of assets is likely to vary over time then, by
the time we want to sell those assets in order to obtain money to undertake transactions, we may face co
nsiderable capital loss. If we are risk-averse then our demand for money will therefore be a positive functi
on of the riskiness or price uncertainty of alternative assets.

\item the expected pattern of expenditures and receipts: if individuals were paid their wages in lump sums weekly then average cash balances would be less than if wages were paid monthly. If the pattern of payments and receipts was uncertain then cash balances would be likely to be higher; it may be unwise to face the brokerage fees and transfer cash to bonds if there is a possibility that you will need to make a large cash payment in the near future.

\end{enumerate}

${ }^{1}$ In practice the interest paid on sight deposit accounts is positive. 3. The demand for money

These determinants are also related to Keynes' description of money demand motives. Keynes (1936) broke down the demand for money into three types: transactions, precautionary and speculative motives:

\begin{itemize}
\item The transaction demand for money is essentially that needed to buy goods and services where money is needed as a medium of exchange.

\item Precautionary money balances are simply holdings of money kept in case of emergencies (an unexpectedly large tax bill or hospital treatment for example).

\item Finally, the speculative demand for money considers money as an alternative to interest earning assets. Due to the capital loss involved with holding bonds when the interest rate increases ${ }^{2}$ if an individual expects the interest rate to rise, then he will expect to experience a capital loss on his bond holdings. Knowing that the bond price will fall, he will want to hold a larger quantity of money.

\end{itemize}

In fact, Keynes originally assumed that individuals held their expectations of interest rate movements with certainty. When the interest rate was below what they expected in the long run, R^{*} in Figure 3.1, then they would put all of their financial wealth in the form of money to avoid the capital loss associated with holding bonds. When the interest rate was above what was expected, then the expected interest rate fall would be associated with a capital gain from holding bonds. The individual would then hold as little money as possible, only covering the transactions and precautionary motives $(T$ representing minimum cash required to conduct transactions). The individual's demand for money, as a function of the interest rate, would then be a step function, shown in Figure 3.1 below.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-016}
\end{center}

Figure 3.1:

${ }^{2}$ This is due to the negative relationship between the price of a bond and the yield the bond earns. This is considered in more detail in Chapter 9 .

\subsection{Baumol-Tobin transactions demand for money}
It has become standard to model each component of the demand for money separately. ${ }^{3}$ The transaction demand for money is typically modelled by allowing money to be a function of determinants 1, 2 and 4 above (i.e. ignoring asset price uncertainty). By allowing more flexible assumptions on the expected pattern of expenditures and receipts, similar analysis can incorporate a precautionary motive for money holdings. 4 Analysis of the speculative demand for money, however, concentrates on determinant 3 , asset price uncertainty, while dropping one or more of the other factors for tractability.

\subsection{Baumol-Tobin transactions demand for money}

The two papers by Baumol (1952) and Tobin (1956) explicitly model the transactions demand for money in an inventory theoretic approach. Each assumes that a cash manager (individual or firm) has to manage an inventory of cash and other interest-earning assets. In the original Baumol model, the cash manager is paid in bonds and spends money (or makes transactions) at a constant, known rate. The objective is to choose the number of times she transfers between the stock of bonds and cash that maximises profits, or equivalently minimises costs. These costs come in two forms: brokerage fees and the interest foregone by holding money. By making a large number of transfers out of bonds and into cash, she will be able to earn more interest on the bonds that are kept for longer. However, such a strategy will involve large brokerage fees. On the other hand, by making only a few transfers, she will avoid paying frequent brokerage fees but will miss out on the interest the wealth could earn if kept in the form of bonds.

Let T equal the value of the cash manager’s expenditures, equal in this case to her income: i is the interest rate earned on bonds, b is the fixed cost of making a transfer between bonds and cash and Z is the value of money transferred each time, equal to the amount of bonds sold each time. If the money is spent at a constant, known rate then the average money balances will be equal to $M=Z / 2$. The cost minimisation problem can be written as:

$$
\min _{Z} C=\frac{b T}{Z}+\frac{i Z}{2}
$$

Costs, C, are made up of brokerage costs, equal to the cost per transfer, b, multiplied by the number of transfers, T / Z, plus the interest foregone by (opportunity cost from) holding money. The interest rate foregone will equal the interest rate, i, multiplied by the average money holdings, $Z / 2$. Differentiating the cost with respect to the choice variable, Z, the amount we transfer each time, gives:

$$
\frac{\mathrm{d} C}{\mathrm{~d} Z}=-\frac{b T}{Z^{2}}+\frac{i}{2}
$$

Setting this equal to zero and solving for $M=Z / 2$, gives:

$$
M=\frac{Z}{2}=\sqrt{\frac{b T}{2 i}}
$$

The average money demand, M, is then a positive function of both the brokerage cost, b, determinant number 2 , and income/receipts, T, determinant number 4 . It is also a

[3] See Goodhart (1989) Chapter 3, Sections 1 and 2 for a more complete discussion of the modelling techniques.

[4] See Miller and Orr (1966) 3. The demand for money

negative function of the interest rate earned on alternative assets, i, determinant number 1. One can also show that the interest and income elasticities of money demand are $-1 / 2$ and $1 / 2$, respectively. This is left as an exercise. So average money demand will increase by $1 / 2 \%$ if income increases by 1% and decreases by the same amount if the interest rate earned on alternative assets increases by 1%. The model generates a ’saw tooth’ pattern of money holdings as shown in Figure 3.2 below. For simplicity, assume that interest payments are made at the end and do not accrue during the period.

The diagram shows a situation where the optimal number of times to transfer between bonds and money is 4 . At time t_{0}, the cash manager receives T in bonds and transfers Z to cash immediately in order to buy goods and services. The amount of bonds left is therefore B_{1} (the difference between T and B_{1} being Z). Money balances and hence financial wealth decline gradually as the cash is sp ent. At time t_{1}, another transfer is made that reduces bond holding by a further Z to B_{2}. The process continues until all wealth is spent and a new income is received. Money holdings are therefore shown by the saw-tooth pattern, financial wealth is shown by the straight line from T and the level of bond holdings is shown by the dashed step function.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-018}
\end{center}

Figure 3.2:

Activity 3.1 A taxi driver takes $\pounds 15,000$ net over the course of a year, at an approximately constant daily rate. He spends 80% of his takings on consumption goods, also at an approximately constant daily rate, but saves the remainder to pay for a world cruise at the end of the year. He can hold his savings in a deposit account in a bank paying 4% per annum, with costless deposits and withdrawals, or he can purchase bonds paying a known yield of 7%. The brokerage fee in purchasing or selling bonds is $\pounds 5$ per transaction. Assume the taxi driver manages his finances optimally by making n transactions, $n-1$ of these being purchases of bonds spaced equally through the year, and the n-th transaction being the sale of bonds at the end of the year to pay for the world cruise.

(a) Draw the time profile of the taxi driver’s holdings of deposits and bonds.

(b) What is the optimal value of n ? (Note that n must be a whole number.)

(c) What is the taxi driver’s ’demand for money’, or average deposit balance?

(For Feedback, see the end of this chapter.)

Criticisms of the inventory theoretic model

Despite the intellectual appeal and simplicity of the inventory theoretic model, it faces a number of criticisms. One argument made against such models is that they make the assumption that the pattern of expenditure and receipts is known perfectly. This is clearly not true in reality. Adding uncertainty to these processes is exactly what Miller and Orr (1966) do but, even then, these inventory theoretic models face a huge uphill struggle when faced with empirical evidence. At the micro level, the percentage of actual cash balances held by firms that is explained by the Baumol model is tiny.

Tobin’s model of portfolio selection

One problem with Keynes’ original ideas of the micro level demand for money was that individuals held expectations of interest rate movements with certainty. This resulted in a step demand function given in Figure 3.1 where individuals either held no money and all long term bonds (or as little money as possible to cover transactions) or held all money and no bonds. Individuals were ’plungers’ - all or nothing. This did not explain the empirical regularity that people held their financial wealth in both forms of assets and could only provide a downward sloping aggregate money demand function by assuming that every individual had a different expectation of the equilibrium interest rate, also assuming that each of these expectations were held with certainty. This clearly is too strong an assumption.

To overcome this problem, Tobin (1958) considered the problem of how much money to hold as one where individuals maximise utility by choosing between assets in a portfolio. Money is assumed to have a zero rate of return and is considered riskless. The variance of the return on money is then also zero. Bonds, however, pay a positive rate of return, μ, but are risky due to the possibility of capital gains or losses if the bond is sold before maturity. Let the variance of the return on bonds be σ^{2}.

Let B be the proportion of your financial wealth held in bonds. $1-B$ is therefore the proportion held in the form of money if we assume a two-asset world. The distribution of money and bonds are given below:

$$
\begin{aligned}
& \text { Money } \sim(0,0) \\
& \text { Bonds } \sim\left(\mu, \sigma^{2}\right)
\end{aligned}
$$

3. The demand for money

A portfolio containing a share B of bonds and $1-B$ of money therefore has a distribution:

$$
\text { Portfolio } \sim\left(0 \cdot(1-B)+\mu \cdot B, 0 \cdot(1-B)^{2}+\sigma^{2} B^{2}\right)=\left(\mu B, \sigma^{2} B^{2}\right) .
$$

Let the mean return of this portfolio be μ_{p} and the variance be σ_{p}^{2}.

$$
\begin{aligned}
\mu_{p} & =\mu B \\
\sigma_{p}^{2} & =\sigma^{2} B^{2} .
\end{aligned}
$$

Writing B in terms of σ and σ_{p} from (3.8) and substituting into (3.7) gives a budget constraint relating the maximum return on a portfolio to the standard deviation, which we assume is a proxy for risk.

$$
\mu_{p}=\frac{\mu \sigma_{p}}{\sigma}
$$

Assuming that agents are risk-averse, the indifference curves will be convex and upward sloping, as shown in Figure 3.3.

The top part of the figure shows the budget constraint and indifference curves of the agent. Note that there is an upper bound on the return and risk of the portfolio, $\mu_{p}^{\prime}$ and $\sigma_{p}^{\prime}$, respectively, where the individual has put all of their financial wealth in bonds; $B=1$. At this point it is impossible to increase the return or risk of the portfolio by substituting between money and bonds. The bottom part of the figure simply shows the share of the portfolio held in bonds and the corresponding risk of the portfolio.

As can be seen in the figure, the individual maximises utility by being at the point where the indifference curve is tangential to the portfolio budget constraint, point E. The share of wealth held in bonds is B^{*} and the share held in money is $1-B^{*}$. By using portfolio analysis, Tobin showed how individuals can diversify their wealth into more than just one asset. By changing the return earned on bonds, μ, this will shift the lines in the figure, resulting in a new equilibrium and different bond/money allocation (see 'Feedback to the Activity' at the end of this chapter).

Macroeconomic determinants of money demand

As seen in the previous section and the models referenced there, money demand at the micro level is mo delled as a function of a number of variables: interest differentials, cost of transfers, asset price uncertainty and the pattern of expenditures and receipts. At the macro level, however, the demand for money is modelled under, arguably, a simpler framework. That is not to say that the analysis is simpler or there are fewer complications to overcome. Indeed, the econometric techniques used to model the aggregate demand for money are complex and have needed to be after the breakdown of the demand for money functions starting in the 1970s.

The demand for money is one of the most thoroughly researched topics in the field of economics but why has it attracted so much attention? We study the demand for something in order to be able to predict the consequences of changes in its supply, and this is as true of money as of anything else. Changes in the supply of any good, service or asset will alter at least one of the variables upon which the demand for that good, 3.10. Macroeconomic determinants of money demand

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-021}
\end{center}

Figure 3.3:

service or asset depends. For example, assume that the demand for apples depends primarily on the following:

\begin{enumerate}
  \item their price

  \item the price of pears

  \item the incomes of consumers, and

  \item the tastes and preferences of consumers.

\end{enumerate}

Then a change in the supply of apples will be expected to alter one of these factors influencing demand. In this case most probably the price of apples. In a similar fashion, changes in the supply of money can be expected to bring about a change in the value of one or more of the determinants of the demand for money. The possibility that these determinants may include interest rates, real income and the general level of prices themselves important macroeconomic variables - gives the study of the demand for money a particular importance.

A particular aggregate money demand function takes the form:

$$
M^{d}=f\left(Y, R_{i}, W\right)
$$

M^{d} is the demand for nominal money balances, Y is nominal income and R_{i} is the rate of return on asset i. Since the rate of return on a number of assets will determine the demand for money, including that on money itself, i can represent a number of assets. The R_{i} s represent the opportunity cost of holding money and Y acts as a proxy for the level of transactions undertaken. Wealth (W) is included as it forms the budget constraint on which the choice of money holdings depends but since wealth is capitalised current and future income, it is not independent of Y. For this reason, and 3. The demand for money

also because data on wealth levels of nations are very difficult to obtain, W is often dropped from the a nalysis. If money demand is homogenous of degree one in prices (i.e. a doubling of the price level leads to a doubling of the demand for nominal money balances), (3.10) can be re-written as:

$$
\left(\frac{M}{P}\right)^{d}=g\left(y, R_{i}\right)
$$

where y is real income. A common log linear form of this equation is:

$$
m_{t}-p_{t}=a y_{t}-b R_{t}
$$

where m_{t}, p_{t} and y_{t} are log values of the nominal money supply, price level and income at time t respectively and R_{t} is 'the' nominal interest rate at time t. Two important parameters of the money demand function are the elasticities with respect to income and the interest rate. For a summary of the empirical evidence on these estimates, see Lewis and Mizen, especially Chapter 11. This chapter also explains in detail the methods used to estimate such money demand functions.[5] The interest elasticity of money demand is important in the debate over whether monetary or fiscal policy is more powerful. A low value of b implies a relatively steep LM curve and, other things being equal, monetary policy has a larger effect on output than fiscal policy. Keynesians on the other hand argue the opposite: a high value of b and therefore a relatively shallow LM curve, implying a greater role for fiscal policy.

Of equal importance is whether or not the money demand equation is stable. If the monetary authorities decide to target the money supply then an unstable money demand function can lead to unexpected and adverse changes in nominal and possibly real factors in the economy. The stability of money demand can only be determined by statistical analysis of the relevant data - hence the enormous number of empirical studies relating to the demand for money.

The stability of the money demand function

Relatively simple functional forms of money demand were estimated until the early 1970s and these appeared to work reasonably well at explaining the demand for money. One such specification was used by Goldfeld (1973) and is similar to (3.12).

$$
\ln \left(\frac{M}{P}\right)_{t}=b_{0}+b_{1} y_{t}+b_{2} R_{t}+b_{3} \ln \left(\frac{M}{P}\right)_{t-1}+u_{t}
$$

where u_{t} is a random error term and a lagged dependent variable, $\ln (M / P)_{t-1}$, is also included. Goldfeld himself suggested:

'Perhaps most interesting is the apparent sturdiness of a quite conventional formulation of the money demand function, however scrutinised...(T)he conventional equation exhibits no marked instabilities, in either the short run or the long run.. [6]

[5] For a more technical review, see also Goldfeld and Sichel listed in the 'Further reading' section.

[6] Goldfeld, 1973, quoted by Goodhart, 1989. However, from 1974 Goldfeld's equation overpredicted real money balances, M1, in the US. This was known as 'the case of the missing money' (Goldfeld, 1976). Basically, for any given level of real income and interest rates, the above equation suggested that there should be more money in circulation than there actually was.[7] Such demand for money functions were breaking down, not only in the US but also elsewhere, such as the UK see Hacche (1974) who examined a broader measure of money, M3. Whereas Goldfeld's equation overpredicted the amount of money in the US, money demand equations for the broader M3 aggregate in the UK were underpredicting the amount of money.

\subsection{Reasons for the breakdown of the money demand functions}
A common reason quoted for why the money demand equations broke down in the 1970s was greater financial innovation. The oil shocks of the mid-1970s and the resulting high inflation caused interest rates to increase substantially. This meant that the opportunity cost of holding money increased and was eventually so large that it became worthwhile for cash managers to find more efficient ways of holding cash balances, allowing more wealth to be put into interest-earning assets. Hence for any level of income and interest rate, the demand for money would be reduced, explaining the 'missing money.'

Another possible explanation for the negative results of the 1970s is the fact that a single equation money demand function may be misspecified. An equation relating real money balances, income and interest rates may not represent a true money demand equation, but a reduced form equation; a mixture of both money demand and supply equations, especially if the money supply set by the authorities is dependent on conditions in the economy such as inflation and output. A changing policy stance by the authorities, as was the case from 1979 to 1982 in the $\mathrm{US},{ }^{8}$ will cause the reduced form relationship to alter, explaining the breakdown of the estimated 'demand' function. Overly simplified econometrics was also used up until the last couple of decades. Issues including stationarity, spurious regressions and co-integration need to be addressed before any meaningful interpretation can be taken from empirical results. See Lewis and Mizen (2000), Chapter 11.

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item explain why it is important to study the demand for money

  \item describe the four main microeconomic determinants of money demand

  \item outline the inventory theoretic model of Baumol-Tobin and the portfolio selection model of Tobin

\end{itemize}

${ }^{7}$ For a review of the stability of the demand for money function, see Judd and Scadding (1982).

${ }^{8}$ See Goodhart (1989), Chapter 10. 3. The demand for money

\begin{itemize}
  \item discuss why Tobin's model solves the 'plunger' problem of the demand for money model of Keynes

  \item describe the general set-up of macroeconomic money demand equations

  \item discuss empirical evidence on money demand functions, especially on the income and interest elasticities

  \item describe what happened and what is meant by 'the case of the missing money' and give reasons for the breakdown of the estimated money demand equations.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
\item 'Tobin's portfolio model implies that the demand for money will always fall when the interest rate rises.'
\end{enumerate}

\section{Section B}
\begin{enumerate}
\setcounter{enumi}{1}
\item According to some economists, Tobin's portfolio selection model suffers from the following deficiencies: (a) it explains the allocation of wealth between assets, but it does not explain the demand for money; (b) it implies that investors view investment in long-term assets as more risky than investment in short-term assets: this, however, is false. Discuss these shortcomings.

\item Somebody suggested that instead of introducing the Euro, the European Central Bank should have introduced a debit card that could be used for any kind of transaction; a 'Eurocard'. If this suggestion had been followed by the ECB, what would be the transaction demand for cash in the euro-area economy?

\item At the micro-level, the transactions elasticity of money demand has been found to be close to unity and the interest elasticity has been found to be close to zero. How do these estimates compare to those of the Baumol-Tobin model? Explain why they differ.

\item By definition, wealth forms the budget constraint that people face when deciding how much money to hold. Why then is wealth not present in many macroeconomic money demand functions?

\item Many economists have produced empirical, econometric studies purporting to have established a stable money demand function, which subsequently broke down. What went wrong?

\end{enumerate}

\subsection{Feedback to Activity 3.1}
If 80% of the taxi driver's income is spent, that implies non-spent income increases gradually from zero at the start of the year to $\pounds 3,000$ at the end of the year. As in the example, the problem amounts to minimising the costs of holding inventory associated with making transfers, buying and selling bonds, and the opportunity cost of not holding the higher interest-earning asset. This time, however, the problem involves choosing the number of times to make transfers. The time profile of deposits and bonds for an arbitrary number of transfers, n, is shown in Figure 3.2.

With n transfers in the year, the amount transferred each time from money/deposits to bonds is $3000 / n$ and so the average deposit balance, M, is $3000 / 2 n$. Average bond holdings, B, is then the difference between average non-spent income, $\pounds 1,500$, and M :

$$
B=1500-\frac{3000}{2 n}
$$

The problem then is to maximise profits, i.e.:

$$
\max _{n} 0.07\left(1500-\frac{3000}{2 n}\right)+0.04\left(\frac{3000}{2 n}\right)-5 n .
$$

The first term is the interest earned on bonds $(7 \%=0.07)$ multiplied by average bond holdings. The second term is the interest earned on deposits, 4%, multiplied by the average deposit balance, and the last term is the cost per transfer, $\pounds 5$, times the number of transfers. Simplifying the expression lea ds to:

$$
\max _{n} 105-\frac{45}{n}-5 n
$$

Differentiating with respect to n and setting equal to zero gives an optimal number of transfers, n, equal to 3 , and the average deposit balance, M, is given by:

$$
M=\frac{3000}{2 n}=500
$$

Feedback to Sample examination questions

Section A

1. The statement is false. Suggested answers may include Figure 3.3 and will explain what happens after an increase in the interest rate, namely an anticlockwise movement of the budget constraint. The new point of tangency can lie either to the left or right of point E, in which case the share of wealth held in money can be higher or lower than before. This is due to the offsetting income and substitution effects (to be shown in the diagram).

Section B

2. The suggested answer should show the cost minimisation problem in the Baumol-Tobin model and also show that the average money demand, M, is given 3. The demand for money

as in (3.3). The elasticities of money demand, with respect to T and i, denoted M_{T} and M_{i}, respectively, are given by

$$
\begin{aligned}
M_{T} & =\frac{\partial M}{\partial T} \cdot \frac{T}{M} \quad \text { and } \quad M_{i}=\frac{\partial M}{\partial i} \cdot \frac{i}{M} \\
M_{T} & =\frac{1}{2} \cdot T^{1 / 2} \sqrt{\frac{b}{2 i} \cdot \frac{T}{M}}=\frac{1}{2}
\end{aligned}
$$

and M_{i} can be shown to equal $-1 / 2$. The Baumol-Tobin model therefore suggests transactions and interest elasticities of money demand to be $1 / 2$ and $-1 / 2$ respectively. These are different to the empirical estimates of 1 and 0 . The main reason for the difference is that many people cannot afford to enter the bond markets due to the large brokerage fees. When income increases, agents increase their money holdings one-for-one. If the interest rate increases, agents do not convert any wealth to bonds since the high brokerage fees outweigh the benefit of higher interest received on bonds.

Chapter 4

The supply of money

Introduction

In Chapter 2 we discussed what constitutes money and in Chapter 3 we analysed the factors that determine how much money individuals, and economies as a whole, demand. In this chapter we will discuss how money, credit money in particular, is created and what determines the supply of money. As we shall see in later chapters, the supply of money is hugely important since a change in the money supply can lead to changes in the price level and inflation but also to changes in real variables such as output and unemployment. By affecting the money supply, the monetary authorities can 'control' or at least help limit the fluctuations of these variables. Later in this chapter we will see how the authorities can implement monetary policy to help control the money supply.

\subsection{Aims}
The aim of the chapter is to study the money supply as one of the building blocks of the money market equilibrium, the role of financial intermediaries in the credit supply process and how monetary policy makers can control money supply.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
\item describe and discuss the different roles of financial intermediaries

\item describe why maturity transformation is so important in financial markets

\item define on which side of a bank's balance sheet deposits and loans appear and explain why the balance sheet must indeed balance

\item explain how the monetary authorities can influence the total money supply by changing the monetary base or by introducing mandatory reserve ratios or other regulation. 4. The supply of money

\end{itemize}

\subsection{Reading advice}
Any monetary economics textbook will have a section on the supply of money but the book followed most closely in this chapter is Goodhart (1989). Chapter 5 covers the role of and need for financial intermediaries and Chapter 6 covers the ideas of the money multiplier and high-powered money. Unfortunately there is no good reference for the model of the banking sector at the end of the chapter so you are advised to work carefully through this section and through the Activities. [1]

\subsection{Essential reading}
Artis, M.J. and M.K. Lewis Money in Britain: Monetary policy, innovation and Europe. (New York; London: Philip Allan, 1991).

Goodhart, C.A.E. Money, Information and Uncertainty. (London: Macmillan, 1989) Chapters 5,6 and 10.

\subsection{Further reading}
\section{Books}
Brunner, K. 'High-powered money and the monetary base', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Goodhart, C.A.E. 'The monetary base', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Laidler, D.E.W. Taking money seriously and other essays. (New York; London: Philip Allan, 1990) Chapters 4 and 5.

McCallum, B. Monetary Economics. (New York; Macmillan; London: Collier Macmillan, 1989).

Papademos, L. and F. Modigliani 'The supply of money and the control of nominal income', in Friedman, B. and F. Hahn (eds) Handbook of monetary economics. (Amsterdam: North-Holland, 1990).

\section{Journal articles}
Goodhart, C.A.E. 'What should Central Banks do? What should be their macroeconomic objectives and operations?', Economic Journal 104(427) 1994, pp.1424-36.

${ }^{1}$ The survey article of Papademos and Modigliani is good but difficult. You should, however, be able to read and understand the basics of Section 3 of this article.

\subsection{Financial intermediaries}
Financial intermediaries, such as banks, are hugely important in activities such as the financing of investment projects and in the safekeeping of savings. At any point in time, there are agents who spend less than they earn, and so wish to save, and there are agents who spend more than they earn, and so need to borrow. The main service that a bank provides is the collection of funds from those who wish to save and the lending out of funds to those who wish to borrow. The reward for providing such services comes from the difference between the rate of interest paid on savings and that charged on loans. By charging a higher rate of interest rate on the funds they lend to firms and households than on the funds they accept from depositors, banks can make profits. This is also called credit spreads.

If there are agents who want to save and others who want to borrow, why do those with funds to spare not just lend directly to those who want to borrow (i.e. why do we need a bank or financial institution at all)? There are a number of reasons to explain the existence of financial intermediaries, which include:

\begin{enumerate}
  \item Economies of scale in transactions and information. In order to have a sufficiently diversified portfolio, agents with funds to save should hold a large number of different assets. If each agent only had limited funds available, this could only be done by 'clubbing' together. This is exactly the type of service unit trusts and pension funds provide. Also, if an agent wanted to borrow a large sum of money, to buy a house or factory for example, it is unlikely that they would find a single other agent willing to lend such a large sum. If a large number of agents deposited small quantities of savings at a bank, the bank could then lend a large amount to a single borrower. The financial intermediary may also be able to obtain better information about the creditworthiness of a prospective borrower than an individual agent can; and the intermediary may be more likely to retrieve assets from a borrower who defaulted on the loan after being made redundant.

  \item Insurance. Agents are, in general, risk-averse. If there are two states of the world, one where an agent received a high income and another where (s)he received a low income, the average of the utilities from both high and low incomes will be less than the utility of the average income. As such, the agent would be willing to pay a fraction of their income in order to smooth their income receipts and hence their consumption. This is exactly the service insurance firms provide. In a similar fashion, banks provide insurance services by guaranteeing a rate of return to depositors even if loans made to borrowers turn bad. Without the bank, the default risk would be faced entirely by the individual/depositor. By paying the bank, depositors accept a lower deposit rate than that they have to pay on loans, depositors can protect themselves from the default risk and obtain a higher utility. ${ }^{2}$

  \item Maturity transformation. Arguably the most important service provided by banks is that of issuing one form of debt that is illiquid (of long maturity), while taking on another which is of short maturity. Individual lenders generally want to lend (to the bank) while still having quick access to their money, in order to make transactions or for precautionary motives. The liabilities of the bank (the deposits

\end{enumerate}

${ }^{2}$ Unless, however, the bank fails as a result of too many borrowers defaulting, for example! 4. The supply of money

of savers) are then liquid and the bank will promise to convert the depositors' assets on demand. The ban ks assets, on the other hand, will tend to be illiquid since private borrowers tend to want to hold long maturity liabilities (the loans/assets of the bank). For example, if a firm wanted to borrow money to build a factory, they would want to borrow long term. This would allow the steady income earned from the investment to gradually pay off the loan.

Activity 4.1 If you had $\$ 1,000$ in cash and you deposited this at a bank, on what side of the bank's balance sheet would this appear? On what side of your own balance sheet would it appear? What could and should the bank do with this cash?

Since banks hold short maturity liabilities, which they promise to pay to the depositors on demand, and hold long maturity assets in the form of loans to borrowers, a paramount concern of banks is therefore to ensure that they can honour demands for withdrawals by customers. To do this they must always hold an adequate supply of liquid assets. The problem is that the most profitable assets of a bank are those that possess least liquidity. Indeed, if the banks held only the most liquid assets in their portfolios they would not even cover their operating costs! The highest profits are earned on long-term loans and investments but these are comparatively illiquid assets in a bank's balance sheet. Banks are therefore obliged to balance liquidity on the one hand against profitability on the other.

Activity 4.2 Why are liquidity and profitability sometimes referred to as conflicting objectives?

The money multiplier and base money

Base money, also known as high-powered money is the monetary liabilities of the Central Bank. This consists of notes and coins, and the deposits and reserves of banks with the Central Bank. If the monetary authorities increased the monetary base by printing money and giving it to private individuals, those individuals will, more likely than not, deposit a fraction of it with their bank. The bank will not want to keep all of these deposits in the form of liquid assets, as this strategy will not earn the bank the most money. Remember that liquid assets earn the lowest interest. Instead it will keep a fraction of it in liquid form but lend out the rest in order to earn a higher return. The funds given out as loans will be spent and subsequently deposited at the bank of whoever sold the goods to the initial borrower. Again the bank will lend out a fraction of these new deposits and the process continues. In equilibrium the total increase in the money stock, the liabilities of the government (notes and coins) and the liabilities of banks (deposits) will be a multiple of the increase in the high-powered money. This multiple will depend on the fraction of the money individuals wish to deposit with their bank, the currency-deposit ratio, and the fraction of the deposits the bank wishes to keep in cash reserves and not lend out, the reserve-deposit ratio. If these ratios are stable through time then by controlling the base money, the monetary authorities can control the total money supply in the economy.[3] Let the total money supply in the

[3] Controlling the money supply is not as simple as it sounds, however. See Goodhart (1989). economy, M, be made up of deposits, D, and the liabilities of the government, notes and coins, C. Therefore:

$$
M=D+C
$$

Also, let high-powered money, H, be made up of the notes and coins in the general public, C, and the remainder of the government's liabilities held by banks in the form of reserves, R :

$$
H=C+R
$$

Dividing (4.1) and (4.2) by D and then dividing one by the other, we can write:

$$

$$
M=H\left(\frac{1+(C / D)}{(C / D)+(R / D)}\right)
$$

The bracketed term is the money multiplier: the factor which, when multiplied by the base money, gives the total money supply in the economy. As can be seen, it is a function of the currency-deposit ratio (C / D), and the reserve-deposit ratio (R / D). The monetary authorities should be able to directly control the stock of high-powered money, H, but the determination of the total stock of money, M, ultimately depends on the banking sector and the preferences of private individuals. It is the workings of the banking sector to which we now turn.

A simple model of the banking sector

Assume a banking industry whose market is described by:

$$
\begin{aligned}
D & =d_{0}-d_{1}\left(i-i_{D}\right) \\
L & =d_{0}+l_{1}\left(i-i_{L}\right)
\end{aligned}
$$

where L is the (demand for) bank loans, D is the (supply of) bank deposits, i_{L} the rate charged on loans, i_{D} the interest paid on deposits and i the 'market' interest rate. Note that the supply of deposits is a positive function of the interest rate paid on deposits and a negative function of the market interest rate - the interest rate one could earn elsewhere. If i_{D} was large relative to i, that would encourage individuals to deposit more funds with the bank. Equivalently, the demand for loans is a negative function of the interest rate charged on loans. If the interest rate you had to pay on a loan was high relative to the market rate, you would borrow, not from the bank, but from elsewhere. Assume the banks have no operating costs and have no other assets or liabilities. Finally, d_{0} captures an arbitrary banking relationship constant. It represents the minimum an individual would deposit to the bank and the minimum a firm receives from the bank to hold a bank relationship.

Competitive equilibrium

The profits made by the bank will equal the quantity of loans, L, multiplied by the interest earned on those loans, i_{L} (which is the bank's revenues) [4] minus the costs faced

[4] Assume there are no borrowers defaulting on their debts.

by the bank. Costs will equal the interest paid to depositors in order to encourage them to hand over their funds to the bank. Costs then equal the quantity of deposits, D, times the interest rate paid on deposits, i_{D}. Denoting bank profits by Π and noting that in a competitive equilibrium, profits equal zero:

$$
\begin{aligned}
\Pi & =L \cdot i_{L}-D \cdot i_{D} \\
\Pi & =0 \text { (competitive equilibrium) } .
\end{aligned}
$$

When making the additional assumption that the bank holds no reserves, in order for the bank's balance sheet to balance, loans (assets) must equal deposits (liabilities), $L=D$. Substituting into (4.6) and using (4.7) implies that in equilibrium:

$$
i_{D}=i_{L}
$$

Substituting into (4.5) and equating L with D gives:

$$
d_{0}-d_{1}\left(i-i_{D}\right)=d_{0}+l_{1}\left(i-i_{L}\right)
$$

Solving for the interest rate on deposits, which equals the interest rate charged on loans from (4.8), gives:

$$
i_{D}=i_{L}=i
$$

This implies total deposits, D, which equals total loans, L, equals d_{0} from either (4.4) or (4.5). This is shown in Figure 4.1 below.

In equilibrium, the interest rate paid on deposits equals the interest rate charged on loans, which equals the market interest rate, i. The level of deposits, which in this model is equal to the level of the money stock, equals the amount of loans.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-032}
\end{center}

Figure 4.1:

Government regulation of the deposit rate

One way that the government could reduce or control the money supply is by setting a limit on the interest rate paid on deposits. If the maximum interest rate banks can pay on deposits is less than i in the above example, then the amount of funds savers are willing to deposit with the bank will fall, resulting in a lower money supply. Note that this will also be associated with a lower quantity of loans that, despite the lower money supply, may be harmful to the economy if it means some investment opportunities are not undertaken.

Consider the case where the government sets the interest rate on deposits equal to zero. From (4.4), the quantity of deposits is fixed at $D=d_{0}-d_{1} i$. Diagrammatically, this is shown in Figure 4.2.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-033}
\end{center}

Figure 4.2:

The level of deposits, and hence of the money supply, has fallen from d_{0} to $d_{0}-d_{1} i$ because of the government regulation. Since deposits must equal loans in order for the bank’s balance sheet to balance, the lower level of deposits means a lower level of loans, which is associated with a higher interest rate charged, $i_{L}>i$. Also, since there is a difference between the interest rate charged on loans and that paid on deposits (which has been set at zero), the government regulation has allowed the banks to make positive profits.

Activity 4.3

1. By equating deposits with loans, and noting that $i_{D}=0$, find an expression for i_{L} in terms of the market interest rate, i.

\item Using (4.6) find an expression for the profits made by the banks in terms of the market interest rate . 4. The supply of money

\end{enumerate}

\section{Reserves}
As discussed above, banks will try to keep a fraction of their assets in liquid form in order to meet the day-to-day needs of depositors who withdraw their funds. Assume that the government introduces a mandatory reserve ratio, r^{*} (i.e. total reserves of the bank, R, equals $r^{*} \times D$). The bank’s assets now comprise loans, as before, but now include these reserves. Its liabilities are just the deposits of its customers.

$$
\begin{aligned}
\text { Assets } & =\text { Liabilities } \\
\Rightarrow \quad r^{*} \cdot D+L & =D \\
\Rightarrow L & =\left(1-r^{*}\right) D .
\end{aligned}
$$

Substituting (4.11) into the zero profit condition, (4.6) and (4.7), and rearranging will give the interest rate charged on loans to be:

$$
i_{L}=\frac{i_{D}}{1-r^{*}}>i_{D}
$$

The interest rate charged on loans is now higher than the interest rate paid on deposits. This is in order to compensate the banks for holding reserves, on which it earns no interest. Total loans are now less than total deposits so each dollar lent out must earn a higher return than that paid on each dollar deposited with the bank. i_{L} then needs to be greater than i_{D}.

Activity 4.4 Substitute out L and D from (4.4) and (4.5) into (4.11). Then substituting out i_{L} from (4.12), show that the rate of interest paid on deposits is equal to:

$$
i_{D}=\left(1-\frac{l_{1} r^{*}}{l_{1}+d_{1}\left(1-r^{*}\right)^{2}}\right) i+\frac{r^{*}\left(1-r^{*}\right) d_{0}}{l_{1}+d_{1}\left(1-r^{*}\right)^{2}}
$$

Substitute this out into the supply of deposits equation, (4.4), and show that total deposits are given by:

$$
D=\frac{l_{1}+d_{1}\left(1-r^{*}\right)}{l_{1}+d_{1}\left(1-r^{*}\right)^{2}} d_{0}-\frac{d_{1} l_{1} r^{*}}{l_{1}+d_{1}\left(1-r^{*}\right)^{2}} i
$$

Although (4.13) and (4.14) appear complicated, they simply relate the deposit rate and total deposits (money supply) to the market interest rate and to the reserve ratio. By changing the mandatory reserve ratio, the government can change the total money supply through the activities of the banking sector. However, the way in which D varies with r^{*} is uncertain and depends on the parameters of the model. With no reserve requirements we equate L to D in order for the bank’s balance sheet to balance. In this situation we equate L to $\left(1-r^{*}\right) D$ from (4.11). Therefore the new intersection results in a lower value of L but a higher interest rate charged, i_{L}. Total revenues for the bank, $L \times i_{L}$, cou ld then increase or decrease depending on the elasticity of the demand for loans function. If the demand for loans is elastic then, if L falls, revenue will fall. Banks will be forced to cut the interest paid on deposits, so i_{D} falls, resulting in fewer deposits and hence causing the money supply to fall.

If the demand for loans is inelastic, a fall in L, caused by the reserve ratio, will increase revenues. The banks will increase i_{D} and so total deposits will increase.

Monetary base control

As seen above, the government can control the money supply by directly controlling the rate of interest paid on deposits and it can also affect it through imposing a mandatory reserve ratio. However, by forcing the banks to hold a certain level of reserves, not just a reserve ratio, it can directly control the monetary base. Note that high-powered money equals reserves plus cash held by the general public (see (4.2)) and the government directly controls the amount of cash in the economy since it is the monopoly supplier.

The government fixes the amount of bank reserves, R, at R^{*}. Together with the reserve ratio, r^{*}, this explicitly determines the amount of deposits in the economy since $R^{*}=r^{*} D$. Inverting (4.14) will give an expression for the market interest rate in terms of r^{*} and R^{*}.

$$
i=\frac{l_{1}+d_{1}\left(1-r^{*}\right)}{d_{1} l_{1} r^{*}} d_{0}-\frac{l_{1}+d_{1}\left(1-r^{*}\right)^{2}}{d_{1} l_{1} r^{*}} D
$$

where:

$$
D=\frac{R^{*}}{r^{*}}
$$

This is shown in Figure 4.3.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-035}
\end{center}

Figure 4.3:

The supply of money

The level of total deposits, and hence the supply of money, is determined by the mandatory requirement that the banks hold a fixed level of reserves, R^{*}, and have a fixed reserve ratio, r^{*}. Changing R^{*} will directly affect the money supply, shifting the vertical $D=R^{*} / r^{*}$ schedule left or right. However, doing so will necessitate a change in the market interest rate in order to achieve equilibrium in the banking sector.

Activity 4.5 Assume an economy in which the demand for bank loans is given by:

$$
L=1000+25\left(i-i_{L}\right)
$$

and in which the portfolio preferences of the public generate a supply of bank deposits given by:

$$
D=1000-50\left(i-i_{D}\right)
$$

where i_{L} is the interest rate charged on bank loans, i_{D} the interest rate paid on bank deposits and i the ’market’ interest rate (and interest rates are measured in percentage points so that a rate of two per cent is given as 2 , not 0.02).

The banks hold no reserves and have no other assets and liabilities and incur operating costs of $\pounds 60$ a year for every $\pounds 1,000$ of deposits.

1. Assuming the banking system is competitive, solve for the loan rate and the deposit rate as functions of the market interest rate and determine the stock of money in equilibrium.

2. Imagine that the banks were now to collude and reach an agreement to set loan and deposit rates which maximise the profits of the industry. What loan and deposit rates will they set, what will be the profits of the industry and what will be the effects of this cartel on the stock of money?

3. Assume the banking industry is described as in the first part of the question (i.e. competitive and unregulated). The government can influence the ’market’ interest rate through its dealings in the gilt-edged money markets. What is the effect of a change in the market interest rate on deposit and loan rates and on the stock of money? Explain. Is monetary policy impotent in this economy?

(For Feedback, see the end of this chapter.)

A reminder of your learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- describe and discuss the different roles of financial intermediaries

- describe why maturity transformation is so important in financial markets

- define on which side of a bank’s balance sheet deposits and loans appear and explain why the balance sheet must indeed balance - explain how the monetary authorities can control the total money supply by changing the monetary base or by introducing mandatory reserve ratios or other regulation.

Sample examination questions

Section A

Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

1. ’From the money multiplier analysis, the stock of money is a multiple of the quantity of high-powered money. Since this is set by the monetary authorities, the authorities can directly set the quantity of money in the economy.’

Section B

2. Why should banks be required to satisfy restrictions placed by the regulatory authorities in the form

of required reserve ratios?

\item Do banks perform a role in the financial system that is qualitatively different from the role of other 'non-bank' financial intermediaries (NBFIs)? Should all financial institutions be regulated by the authorities in the same way?

\item It is sometimes argued that regulation of the banking system (by minimum reserve requirements, for example) acts like a tax on banks, and leads to the growth of non-bank financial intermediation, which escapes the regulation. Therefore, it is argued that bank regulation is self-defeating and undesirable. Discuss.

\item What is the effect of an increase in the 'market' interest rate on the equilibrium value of bank deposits (and hence on the money stock) on the assumptions that:

\end{enumerate}

(a) the banking system is competitive and unregulated, but the banks require for commercial reasons to hold some proportion of their deposits in the form of cash

(b) the banking system is regulated by a rule preventing banks from paying interest on deposits?

Is the effect bigger under assumption (a) or (b)? Explain.

\subsection{Feedback to Activity 4.5}
\begin{enumerate}
\item With no reserves, it must be the case that loans equal deposits, as in the first part of the example in Chapter 3 ((4.4) onwards). The profits of the bank are given by revenues, $L \times i_{L}$, minus costs. Costs include the interest on deposits, $D \times i_{D}$, but 4. The supply of money
\end{enumerate}

also the operating costs of $\pounds 60$ per $\pounds 1,000$ deposited, i.e. 6% of deposits. Profits, Π, are then given by:

$$
\begin{aligned}
& \Pi=L \cdot i_{L}-D \cdot i_{D}-6 D \\
& \Pi=0 .
\end{aligned}
$$

Noting that $L=D$, the zero profit condition can be solved to give:

$$
i_{L}=i_{D}+6
$$

Substituting (4.19) into the loan demand equation and equating this to the supply of deposits equation gives:

$$
1000+25\left(i-\left(i_{D}+6\right)\right)=1000-50\left(i-i_{D}\right)
$$

Solving for i_{D} gives:

$$
i_{D}=i-2
$$

which implies:

$$
i_{L}=i+4
$$

From either the loan demand or supply of deposits equation, we find that the stock of money in equilibrium is:

$$
D=L=1000+25(i-(i+4))=900
$$

This is shown in Figure 4.4.

2. Equating loans with deposits gives a relationship between i_{L} and i_{D} of:

$$
i_{L}=3 i-2 i_{D}
$$

Using (4.24) and the loan demand and supply of deposits equations we can use (4.17) to obtain an expression for profits.

$$
\Pi=\left(1000-50\left(i-i_{D}\right)\right)\left(3 i-2 i_{D}-i_{D}-6\right) .
$$

Differentiating this with respect to i_{D}, the bank’s choice variable, results in an optimal deposit rate of:

$$
i_{D}=i-11
$$

Using (4.24) and the supply of deposits equation, this results in:

$$
\begin{aligned}
i_{L} & =i+22 \\
D & =1000-50(i-(i-11))=450 .
\end{aligned}
$$

3. From ’1.’, $D=900$ (i.e. deposits are a constant in equilibrium and are not a function of the market interest rate). Changing i will not change D and hence will not change the stock of money. The gover

nment bids for funds by increasing i so banks have to compete by increasing their own rates. In total, nothing happens to
\end{enumerate}

\subsection{Feedback to Sample examination questions}
D or L since they both only depend on interest differentials. From (4.21) and (4.22), $\mathrm{d} i_{L} / \mathrm{d} i=\mathrm{d} i_{D} / \mathrm{d} i=1$. The effect of changing the market interest rate is shown in Figure 4.4 below.

Even though the money stock has not changed, an increase in the nominal interest rate will increase the real interest rate if prices (inflation) do not change. Therefore, investment and consumption will be affected (reduced) and so monetary policy may not be impotent.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-039}
\end{center}

Figure 4.4:

\subsection{Feedback to Sample examination questions}
\section{Section A}
\begin{enumerate}
  \item The statement is uncertain. The suggested answer should show the relationships between M (money stock) and H (high-powered money) from (4.1) to (4.3). The money multiplier will be shown to be a function of two ratios (currency-deposit and reserve-deposit). Only if these ratios are stable will the monetary authorities be able to control the stock of money directly.
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{4}
  \item For the first part, where the banking system is competitive and unregulated, the effects of an increase in the market rate of interest will be to increase deposit and loan rates one-for-one but to leave the money stock unaffected. See the outline answer to '3.' of Activity 4.5.
\end{enumerate}

In (b), where banks do not pay interest on deposits, consider Figure 4.2 and (4.4). Deposits are fixed at $D=d_{0}-d_{1} i$ and therefore an increase in the market rate will 4. The supply of money

cause the money stock to fall. It is therefore likely that the effect of the increase in market rates is greater in the second case. The money stock falls and the increase in the market rate can have real effects, as described above.

\section{Chapter 5
Classical theory}
\subsection{Introduction}
The classical model is hugely important for the analysis of monetary theory. If one commodity has been introduced into the economy to serve the functions of money and has no other use or value, what determines the value of such a commodity, or the prices of other goods and services in relation to that money commodity? These are essentially the questions we will be answering in this chapter. We will analyse the general equilibrium framework of Walras and use it to explain what is meant by the 'Classical dichotomy', namely the separation of the real side of the economy from the monetary side. Finally, we will give an explicit example of Walras' law by introducing money in a general equilibrium framework; a set-up we will use extensively in later chapters.

\subsection{Aims}
In this chapter we aim to study a general equilibrium environment with money.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item define and explain the term ’neutrality of money’

  \item describe the essential features and predictions of the quantity theory of money

  \item describe the main features of the classical system and the classical dichotomy

  \item discuss the implications of Walras’ law in the determination of general equilibrium

  \item state the nature and importance of Patinkin’s real balance effect for the internal consistency of the classical dichotomy.

\end{itemize}

\subsection{Reading advice}
You are recommended to read the appropriate chapters or sections in one or more of the basic monetary economics textbooks before tackling the material presented here. All will discuss, to greater or lesser exte
nts, classical theory and issues surrounding

\section{Classical theory}
monetary neutrality. The book followed most closely here is Lewis and Mizen, Chapters 3 and 4 , but see also Harris, Chapters 4 and 5.

You are also strongly advised to read the entries on ’Neutrality of money’ and ’Quantity theory of money’ i
n The New Palgrave Dictionary of Money and Finance. The latter chapter is quite long but covers material
 relevant not only for this, but for other chapters in the guide.

\subsection{Essential reading}
Friedman, M. ’The quantity theory of money’, in Newman, P., M. Milgate and J. Eatwell (eds) The New Pa
lgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Harris, L. Monetary Theory. (New York; London: McGraw-Hill, 1985) Chapters 4 and 5.

Lewis, M.K. and P.D. Mizen Monetary Economics. (Oxford; New York: Oxford University Press, 2000) Ch
apters 3 and 4.

Patinkin, D., ’Neutrality of money’, in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dicti
onary of Money and Finance. (London: Macmillan, 1994).

\subsection{Further reading}
Cagan, P. ’Monetarism’, in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Laidler, D. ’The quantity theory is always and everywhere controversial - why?’, Economic Record 67(199
) 1991, p.289.

Patinkin, D. Money, interest and prices: an integration of monetary and value theory. (New York: Harper a
nd Row, 1965).

The quantity theory of money

The value of money can be established through the well-known 'equation of exchange', which is at the centre of the classical quantity theory of money. The equation of exchange,

$$
M V \equiv P T
$$

is an identity since the two sides of the equation are simply different ways of measuring the same thing, namely the total value of all monetary transactions in the economy. M is the stock of money, the number of units of the money commodity, V is the velocity of circulation, the number of times each unit of money is used in transactions per period, T is the number of transactions in a period and P is the average monetary value of each transaction. [1] Essentially, the value of the money paid out, $M V$, must be equal to the value of goods and services it buys, $P T$.

[1] Note that the 'stock' of any variable refers to the total accumulated value of a variable over years, whereas 'flow' refers to a change in the variable within a given time period. The quantity theory determines the price level on the basis of the assumptions made about the variables in the equation. V is taken as exogenous, being determined by various institutional features of the economy, such as the frequency with which workers get paid. T, the number of transactions, is determined by real variables such as preferences, endowments and technology, and M is, in a fiat money economy, determined by the government. [2] On these assumptions the price level, P, is proportional to the stock of money, M, in the sense that changes in the stock of money lead to equi-proportional changes in the price level. Taking logs of (5.1) gives:

$$
\ln M_{t}+\ln V_{t}=\ln P_{t}+\ln T_{t}
$$

where we have introduced a time subscript to all variables. Differentiating this with respect to time gives:

$$
\frac{1}{M_{t}} \cdot \frac{\mathrm{d} M_{t}}{\mathrm{~d} t}+\frac{1}{V_{t}} \cdot \frac{\mathrm{d} V_{t}}{\mathrm{~d} t} \equiv \frac{1}{P_{t}} \cdot \frac{\mathrm{d} P_{t}}{\mathrm{~d} t}+\frac{1}{T_{t}} \cdot \frac{\mathrm{d} T_{t}}{\mathrm{~d} t}
$$

If velocity and transactions are considered exogenous, meaning these are not changing through time, then their derivatives with respect to time are simply zero. For the remaining terms, noting that $\left(1 / X_{t}\right) \cdot\left(\mathrm{d} X_{t} / \mathrm{d} t\right)$ is simply the growth rate of X_{t}, then (5.3) implies that the growth rate of the money supply is equal to the growth rate of the price level (i.e. inflation). In this sense, inflation cannot occur without a commensurate increase in the supply of money.

The Cambridge view of the quantity theory

Whereas the equation of exchange is a flow equation, relating the number of transactions made per period to the flow of money per period handed over to facilitate these transactions, the Cambridge school, under Marshall, transformed it to a stock equation. As such, the Cambridge school transformed the quantity theory to a demand function of the form: [3]

$$
M=k P Y
$$

In its simplest form, velocity was assumed constant, in which case $k=1 / V$ and the amount of money on e held was equal to a proportion of the number of transactions, or alternatively, income, Y.

\section{The velocity of money}
The velocity of money measures how many times a unit of money is used to purchase goods and services per period. It is also a measure of the stability of the money demand. Consider now a situation where the velocity of money doubles (i.e. every individual spends their money twice as fast as before). What is the implication of this on the equation of exchange? Assume initially that Joanna gets paid £800 a month and spends a quarter of this at the beginning of each week. Her money balances are then $\pounds 600$ during the first week, $\pounds 400$ during the second week, $\pounds 200$ during the third and $\pounds 0$ during the final week of the month when she is waiting to receive her next salary. This is depicted in Figure 5.1a below.

2 'Transactions', T, are often proxied by income, Y, to give a quantity theory of $M V=P Y$.

${ }^{3}$ Notice the similarity between this equation and the macroeconomic money demand equations in Chapter 2 of the subject guide. 5. Classical theory
\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-044}

Figure 5.1:

Joanna's average money holdings over the month are then:

$$
\pounds 600 \cdot \frac{1}{4}+\pounds 400 \cdot \frac{1}{4}+\pounds 200 \cdot \frac{1}{4}+\pounds 0 \cdot \frac{1}{4}=\pounds 300
$$

Now consider the case where Joanna gets paid twice per month, receiving $\pounds 400$ at the beginning of week 1 and $\pounds 400$ at the beginning of week 3 . Again she spends a quarter of her income in each quarter of the payment period, so that her money balances are $\pounds 300$ in the first half of week $1, \pounds 200$ in the second half, $\pounds 100$ in the first half of the second week and zero in the last half of that week. Similarly, for the second half of the month. Notice that her transactions remain unchanged at $\pounds 800$ per month but the velocity of money has doubled; any money balance is spent twice as quickly. Her money holdings are now shown in Figure 5.1b. Joanna's average money holdings over the month are therefore given by:

$$
\left(\pounds 300 \cdot \frac{1}{8}+\pounds 200 \cdot \frac{1}{8}+\pounds 100 \cdot \frac{1}{8}+\pounds 0 \cdot \frac{1}{8}\right) \times 2=\pounds 150
$$

As a result of the velocity of money doubling, caused by a change in the way people are paid, the average money holdings over the period have halved from $\pounds 300$ to $\pounds 150$. Other determinants of the velocity of money include individual habits and spending patterns, social conditions, the efficiency of the payments system and possibly also the interest rate. As discussed in Chapter 2 , if the interest rate remains high for some time, this may cause individuals to try to find more efficient ways of holding their wealth in order to avoid the high opportunity cost of holding money. One consequence of high interest rates may then be that any money balances are spent more quickly, implying an increase in velocity.

\section{The classical dichotomy and monetary neutrality}
The classical economists assumed that, although money was essential for the efficient functioning of the economy, the quantity of money units had no impact on real variables. According to the 'Classical Dichotomy' only real variables (preferences, endowments and technology) determined real outcomes (quantities and relative prices). ${ }^{4}$ The quantity of money, on the other hand, determined the absolute price level or the value of goods in terms of monetary units, from the equation of exchange. According to the classi

cal economists, money was neutral, in the sense that the quantity of it has no effect on any real variable in the economy. In order to explain these ideas more fully, we first consider the notions of Say's law and Walras' law.

Say's law and Walras' law

Say's law essentially states that supply creates its own demand. In a barter economy in which there are $n-1$ goods, each supplier has an endowment of some good. They each exchange these goods for those they desire and it is the level of their endowment, that determines how much they can buy. Supply funds demand. It must therefore be true that the sum of expenditures over all goods $i=1, \ldots, n-1$ must equal the sum of the supplies of all goods, denoted S_{i}.

$$
\sum_{i=1}^{n-1} p_{i} D_{i} \equiv \sum_{i=1}^{n-1} p_{i} S_{i}
$$

where p_{i} is the relative price of one good in terms of another. Remember we are considering a barter economy here, which therefore has no monetary prices. Alternatively, defining the excess demand for good $i, E D_{i}$, as $D_{i}-S_{i}$, then:

$$
\sum_{i=1}^{n-1} p_{i} E D_{i} \equiv 0
$$

In Say's view, market laws imply that there cannot be a 'general glut'. If there exists an excess demand for one good then there must be an excess supply of another. However, there cannot be a general excess demand or general excess supply at the aggregate level (in a closed economy). Each household has endowments of one or more goods, and a utility function defined over all goods, from which we can derive the demand of each household for each good as a function of relative prices. Adding up the demand for each good across households, and given the total endowment of that good, we can write down a market clearing equation for each good as a function of relative prices.

Since all households must balance their budgets, the sum of all market clearing equations must add up to zero. Therefore in a market for $n-1$ goods, if there is equilibrium in $n-2$ goods, there must be equilibrium in the final market. This is Walras' law. The $n-1$ market clearing equations are not independent; if all markets except one clear, the last market must clear also. Only $n-2$ equations are independent, but this is sufficient to solve for $n-2$ relative prices, for example the prices of all goods in terms of good 1. To establish the equilibrium of an economy of $n-1$ goods we may solve for $n-2$ relative prices and this will in turn determine the demand from each household for each good.

[4] Relative prices being the exchange ratios of one good for another, for example one kilogram of tomatoes equals two loaves of bread.

Money in general equilibrium

Now consider a monetary economy (i.e. we introduce fiat money as the n-th good). The price of this good is p_{n}, usually normalised to unity for simplicity. From Walras' law:

$$
\sum_{i=1}^{n} p_{i} D_{i} \equiv \sum_{i=1}^{n} p_{i} S_{i}
$$

and separating out good n, money:

$$
\sum_{i=1}^{n-1} p_{i} D_{i}+p_{n} D_{n} \equiv \sum_{i=1}^{n-1} p_{i} S_{i}+p_{n} S_{n} .
$$

D_{n} and S_{n} are the demand and supply of nominal money balances, respectively, and from Walras' law we can see that if there is equilibrium in the $n-1$ goods markets then there must be equilibrium in the money market. However, Say's law may not hold. There can exist a general excess supply in the $n-1$ goods markets; (5.7) may not hold, but only if this is offset by excess demand in the money market. The demand for good i will depend on all relative prices and income, implying excess demand for good $i, E D_{i}$, of:[5]

$$
E D_{i}=f_{i}\left(\frac{P_{1}}{P_{n}}, \frac{P_{2}}{P_{n}}, \ldots, \frac{P_{n-1}}{P_{n}}, Y\right)-S_{i}^{*}
$$

where Y is the total output. However, the absolute price level, the price of money, p_{n}, will have no effect on excess demand. If the price of money, p_{n}, doubles, caused by a doubling of the money supply from the equation of exchange, since p_{n} is an average of all other prices, then $p_{1}, \ldots, p_{n-1}$ must all double also. The relative prices, $p_{1} / p_{n}, \ldots, p_{n-1} / p_{n}$, will not change, resulting in no change in (excess) demands for any goods. In this way, a changing of the money supply will have no repercussions in the real economy. Hence money is neutral.

Patinkin and the real balance effect

There were found to be a number of criticisms of the dichotomy the classical economists had proposed.[6] One problem was that the model was internally inconsistent. On the one hand, the excess demand for each commodity, $i=1, \ldots, n-1$, was only dependent on relative prices, not the absolute price level, p_{n}, and from Walras' law, the excess demand for money is then determined. The excess demand for money is then only a function of relative prices. However, from the quantity theory, which is needed to solve the entire system of equations, the demand for money explicitly depended on the absolute price level. On one side, money market equilibrium depends only on relative prices while on the other side, it depends only on the absolute price level.

An attempt to resolve this problem was made by Patinkin who included the value of real money balances, S_{n} / p_{n}, as a determinant of the demand for each good; in other words the excess demand for each good i is given by:

$$
E D_{i}=f_{i}\left(\frac{P_{1}}{P_{n}}, \frac{P_{2}}{P_{n}}, \ldots, \frac{P_{n-1}}{P_{n}}, Y, \frac{S_{n}}{P_{n}}\right)-S_{i}^{*}
$$

[5] Assume for simplicity that the supply is fixed at S_{i}^{*}.

[6] See Lewis and Mizen (2000) Chapter 4 for an excellent description of these criticisms. Real money balances are the ratio between nominal money balances and the price level. A change in either of these nominal variables will change the value of the real variable and thereby change the demand for commodities. Thus, starting from a position of equilibrium in all markets including the money market, assume that there is an increase in the supply of money. This increases real money balances which in turn increases the demand for commodities, even though there has been no change in relative prices. But the increase in demand, having started from a position of equilibrium, must mean that there is now excess demand, a situation which denies Say's law. The increased demand for commodities will then bring about an increase in the general level of prices that will reduce real balances. Eventually real balances will return to their equilibrium level, as will the demand for commodities. Real balances provide a bridge between the real and monetary sectors of the classical system and dispose of the classical dichotomy, that is factors influencing nominal variables are separated from factors influencing real variables, while retaining the neutrality of money, that is monetary changes do not change real variables.

A simple general equilibrium framework

Whereas in the previous analysis, individuals' utility was defined over goods, we now assume it depends also on the level of real money balances. The justification, according to Patinkin, is that even if households plan to balance their budgets so that planned purchases are equal in value to planned sales, it may be convenient to buy and sell goods at different times. The more money they hold, the greater the extent to which they can purchase goods ahead of making sales. Money holdings stand as a proxy for the more convenient sequence of transactions they make possible. Thus, money is in the utility function.

Assume a household's utility depends on the quantity of goods consumed, X, and on real money balances, M / P. Let the household have initial endowments X_{0} of goods and M_{0} of nominal money balances. The budget constraint faced by the household is then, in nominal terms:

$$
P X+M \leq P X_{0}+M_{0}
$$

So that the nominal expenditure on goods, $P X$, plus the holdings of nominal money balances, M, must not be greater than the nominal value of the endowments of goods and money. Writing the budget constraint in real terms (dividing by the price level) gives:

$$
X+\frac{M}{P} \leq X_{0}+\frac{M_{0}}{P}
$$

The household's utility function takes the specific form $U=X^{1 / 2}(M / P)^{1 / 2}$. In order to determine the demands for goods and real money balances, we maximise the utility function subject to the budget constraint. To do this we form the Lagrangian: [7]

$$
L=X^{1 / 2}\left(\frac{M}{P}\right)^{1 / 2}+\lambda\left(X_{0}+\frac{M_{0}}{P}-X-\frac{M}{P}\right) .
$$

[7] We also impose an equality in the budget constraint as this implies no wastage of goods or money.

Classical theory

Differentiating with respect to the two choice variables, X and M / P, gives the first order conditions of:

$$
\begin{aligned}
\frac{\partial L}{\partial X} & =\frac{1}{2} X^{-1 / 2}\left(\frac{M}{P}\right)^{1 / 2}-\lambda=0 \\
\frac{\partial L}{\partial(M / P)} & =\frac{1}{2} X^{1 / 2}\left(\frac{M}{P}\right)^{-1 / 2}-\lambda=0,
\end{aligned}
$$

from which we obtain

$$
X=\frac{M}{P}
$$

Substituting into the budget constraint will give solutions for the demands for goods and nominal money balances of:

$$

X=\frac{X_{0}+M_{0} / P}{2} \quad \text { and } \quad M=\frac{P X_{0}+M_{0}}{2}
$$

Assume now that the economy consists of n households each identical to the one described above. The market clearing condition in the goods market then becomes:

$$
n\left(\frac{X_{0}+M_{0} / P}{2}\right)=n X_{0}
$$

In other words, total demand equals total supply. Solving for the price level gives:

$$
P=\frac{M_{0}}{X_{0}}
$$

Alternatively, we can write down the market clearing condition for the money market:

$$
n\left(\frac{P X_{0}+M_{0}}{2}\right)=n M_{0}
$$

If we solve for the price level here, we obtain:

$$
P=\frac{M_{0}}{X_{0}}
$$

In this economy, money is neutral. Real output per household is fixed at X_{0} as it depends on endowments. From the solution of the price level, a change in the money supply will only lead to a proportional increase in prices. Real money balances and ’production’ of goods do not change. An increase in money, M_{0}, will shift the demand function for good X outwards in Figure 5.2 but this simply causes the price level to increase.

Activity 5.1 Why is the solution for the price level the same when we solve for the goods market equilibrium as for when we solve for the money market equilibrium? (Hint: Consider Walras’ law and the fact we are considering only two markets: those for goods and money!)

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-049}
\end{center}

Figure 5.2:

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item define and explain the term ’neutrality of money’

  \item describe the essential features and predictions of the quantity theory of money

  \item describe the main features of the classical system and the classical dichotomy

\item discuss the implications of Walras' law in the determination of general equilibrium

\item state the nature and importance of Patinkin's real balance effect for the internal consistency of the classical dichotomy.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragra
ph.

\begin{enumerate}
\item 'A doubling of the velocity of circulation results in the price level doubling.' 5. Classical theory
\end{enumerate}

\section{Section B}
\begin{enumerate}
\setcounter{enumi}{1}
\item To what do economists refer by the phrases 'neutrality of money' and 'the classical dichotomy'? W
hat problem did Patinkin have with these ideas and how did he solve it?

\item Which factors did the Cambridge quantity theorists (A. Marshall, A. Pigou) stress as significant in e
xplaining the transactions demand for money?

\item 'A given increase in the stock of money has the same long-run effect on the price level whether it r
esults from a temporary budget deficit, or from open market operations, or is dropped from a helicopter.'
Discuss.

\item Walras' law states that the sum of excess demands in all markets must be zero. How does this diff
er from the following statement? If there are n goods, the markets for $n-1$ goods must always be in e
quilibrium.

\end{enumerate}

\subsection{Feedback to Sample examination questions}
\section{Section A}
\begin{enumerate}
\item The statement is false. Suggested answers will introduce the quantity theory, $M V=P T$, explainin
g that V depends on institutional factors such as the frequency with which people are paid, spending pa
tterns, etc. You should also show that a doubling of V results in a halving of M, the demand for mone
y. This could be done using the example of (5.5) and (5.6).
\end{enumerate}

\section{Section B}
\begin{enumerate}
\setcounter{enumi}{1}
\item According to the classical dichotomy, only real variables (preferences, endowments and technolog
y) determine real outcomes (quantities and relative prices of one good in terms of another). Money, on the
other hand, determines the absolute price level, or the value of goods in terms of monetary units, from th
e equation of exchange, $M V=P T$. Real variables determine real outcomes, while monetary variables d
etermine monetary outcomes, hence the dichotomy between real and monetary sectors. Since the quantit
y of money has no real effect on the economy, money is therefore said to be neutral. This is what econom
ists refer to as 'the neutrality of money'. The problem that Patinkin had with these ideas was that the syste
m was not internally consistent. On the one hand, the excess demand for each commodity, $i=1, \ldots, n-

1$ was only dependent on relative prices, not the absolute price level, p_{n}, and from Walras' law, the excess demand for money is then determined. The excess demand for money is then only a function of relative prices. However, from the quantity theory, which is needed to solve the entire system of equations, the demand for money explicitly depended on the absolute price level. On one side, money market equilibrium depends only on relative prices while on the other side, it depends only on the absolute price level. In order to solve the problem, Patinkin argued that the demand for each good was a function of real money balances. See the discussion in the section 'Patinkin and the real balance effect'. 5. Walras' law can be derived from Say's law, which essentially states that supply creates its own demand. If the sum of excess demands in all markets must add up to zero, then if the sum of excess demands in markets 1 to $n-1$ are zero, it must be the case that excess demand in the n-th market is also zero. However, with n goods, the markets for $n-1$ goods can only be in equilibrium if the market for the n-th good is in equilibrium. If there is positive excess demand in the n-th market, there must be positive excess supply in the $n-1$-th market, implying disequilibrium in these $n-1$ markets. 5. Classical theory
\end{enumerate}

\section{Chapter 6 Stylised facts}
\subsection{Introduction}
Macro-monetary models aim to explain the macroeconomic phenomena that we observe in the data. In order to do so we need to have a set of stylised facts that we want to explain. Bear in mind that a model can only be judged by its usefulness to explain those stylised facts. Collecting data from statistical agencies and policy institutions (such as national statistical offices, OECD, IMF/IFS, central banks), about production, consumption, investment, inflation, employment, unemployment and so on, is the first step to start analysing those facts we want to evaluate statistically and provide an internally consistent narrative to explain these facts. However, statistical (time series) data often need to be adjusted for establishing these stylised facts. In this chapter we will introduce several concepts of macroeconomics for which we need to look into the data and adjust these so that they become our statistical benchmarks.

\subsection{Aims}
This chapter aims to introduce key macroeconomic stylised facts which need to be explained by theoretical modelling. To this end, some simple statistical tools such as means, variances, correlations will be utilised.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item describe trends versus business cycles

  \item calculate the means, volatility, cyclicality and persistence of business cycle components of macro-variables

  \item list key business cycle stylised facts

  \item discuss the relevance of monetary aggregates and short-term interest rates as potential policy instruments. 6. Stylised facts

\end{itemize}

\subsection{Reading advice}
Facts about macroeconomics are constantly changing with the arrival of new and better data and the debate about the actual stylised facts are seamlessly going on with the data and new statistical techniques. For a general discussion on measurement issues Williamson (2010) is a good starting point. Calderon and Fuertes (2014) provide an excellent analysis of the recent business cycle stylised facts. For the relevance of monetary aggregates and short-term interest rates in explaining variations in income and prices you sh

ould read Friedman and Kuttner (1992) and Aksoy and Piskorski (2006). For an analysis of the evolution of the Phillips curve in a small sample of countries you should read Kuttner and Robinson (2010).

\subsection{Essential reading}
Aksoy, Y. and T. Piskorski ’US domestic money, inflation and output’, Journal of Monetary Economics 53, 2006, pp.183-97.

Calderon, C. and R. Fuertes ’Have business cycles changed over the last two decades? An empirical investigation’ Journal of Development Economics 109, July 2014, pp. $98-123$.

de Grauwe, P. and Polan, M. (2005) ’Is inflation always and everywhere a monetary phenomenon?’ Scandinavian Journal of Economics, 107 (2), 2005, pp.239-59.

Friedman, B. and K. N. Kuttner ’Money, income, prices and interest rates’, American Economic Review 82, 1992, pp.472-92.

\subsection{Further reading}
\section{Books}
Carlin,W. and D. Soskice Macroeconomics: Imperfections, Institutions and Policies. (Oxford: Oxford University Press, 2006) Chapter 6.

Williamson, S.D. Macroeconomics. (Prentice Hall; fourth edition 2010) Chapters 1,2,3.

\section{Journal articles}
Aksoy, Y. and G. Melina ’US fiscal indicators, inflation and output’, North American Journal of Economics and Finance 22 2011, pp.221-36.

Alvarez, L. , E. Dhyne , M. Hoeberichts , C. Kwapil , H. Le Bihan , P. L’unnemann , F. Martins, R. Sabbatini, H. Stahl, P. Vermeulen and Jouko Vilmunen, ’Sticky Prices in the Euro Area: A Summary of New Micro-Evidence’, Journal of the European Economic Association 4 2006, pp.575-584.

Bils, M and P.J. Klenow, ’Some Evidence on the Importance of Sticky Prices’, Journal of Political Economy, 112 2004, pp.947-985. Cotis, J.P. and J. Coppel ’Business cycle dynamics in OECD countries: Evidence, causes and policy implications’, OECD, \href{http://www.oecd.org/economy/growth/35125435.pdf}{http://www.oecd.org/economy/growth/35125435.pdf}.

Kuttner, K.N. and T. Robinson ’Understanding the flattening of Phillips curve’, North American Journal of Economics and Finance, 21 2010, pp.110-125.

\subsection{Trends and business cycles}
Most macroeconomic time series can be decomposed into its long-term (trend) and short-term (business cycle) components. This distinction between short-term and long-term is important for there is a consensus among economists that monetary policy makers can only influence short-term fluctuations in output, and their constituent parts such as consumption and investment. Most economists believe that the long-term (trend) component in such time series is mainly driven by factors that are not easily controllable by monetary or fiscal authorities, such as demographic changes and long-term technological progress.

Figure 6.1 displays a stylised example of real GDP time series. Business cycles are fluctuations of real GDP around its trend component. The lowest point of the business cycle characterises the trough, whereas the highest point characterises the peak of the business cycle. If the business cycle is below the trend we will refer to a negative output gap, and if it is above the trend we will refer to a positive output gap.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-055}
\end{center}

Figure 6.1:

What do business cycles and trends look like in time series data? In Figure 6.2, we plot seasonally adjusted real GDP levels in Germany, Turkey and the US for the period of 1960 Q1 - 2013 Q2 as reported by the OECD. 6. Stylised facts
\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-056}

Figure 6.2:

As can be seen from Figure 6.2, all series are trending upward with some cyclical variations. We would like to decompose the data into its trend and business cycle component and then analyse volatility, persistence and cyclicality of the business cycle components. This cyclical component of the real GDP levels data is sometimes labelled as the output gap.

Before we discuss how to extract the business cycle component from a level series, it is useful to transform the data by taking its natural logarithms. There are various good reasons for doing so. First, it rescales the time series from large numbers to more reasonable numbers without affecting the time series properties significantly. Second, the difference between the natural logarithms of two numbers is approximately the percentage change in the series across time periods as long as the difference is rather small. Third, graphically you can interpret the changes in the time series, since the slope of the log of a series is approximately the growth rate. ${ }^{1}$

There are several ways of removing the time trend from the log level data; the simplest being to remove a linear trend. If the trend in the data series is characterised by a constant growth rate, removing the linear trend would be a good way to extract the business cycle component. However, this is rarely the case and may be misleading. A popular and better way to remove the trend from the level data is the application of the Hodrick-Prescott (HP) filter. ${ }^{2}$ This filter removes a time varying trend by minimising the sum of squared errors (remember the OLS regression) but incorporates a penalty term λ to having the slope changing too rapidly. The minimisation problem is given by:

$$
\left.\min _{\left(x_{t}^{G}\right)_{t=1}^{T}} \sum_{t=1}^{T}\left(x_{t}-x_{t}^{G}\right)^{2}+\lambda \sum_{t=2}^{T-1}\left[\left(x_{t+1}^{G}\right)-x_{t}^{G}\right)-\left(x_{t}^{G}-x_{t-1}^{G}\right)\right]^{2}
$$

where λ denotes the penalty parameter imposed on changing the slope of the smoothed series, $\left(x_{t}\right)_{t=1}^{T}$ is the original real output data and $\left(x_{t}^{G}\right)_{t=1}^{T}$ is the smoothed real output series. When $\lambda=0$, the HP filter will set $x_{t}^{G}=x_{t}$ for all t. There will be no smoothing. When $\lambda \rightarrow \infty$, the resulting trend will be linear. For quarterly data, typically, a λ value of 1600 is assigned.

When we apply the HP filter to real GDP, consumption and investment data, we obtain the following trend (Figure 6.4) and cyclical components (Figure 6.5) of the real output series for the US.

\subsection{Moments}
After removing the growth trend from the data we would like to understand its business cycle behaviour. As we have already taken natural logarithms, the business cycle component is interpretable as a percent deviation of the variable from its growth trend. As a next step we want to study their mean, volatility, cyclicality and persistence. Mean is simply the average of the business cycle component of a variable. Volatility measures how the variable of our concern fluctuates in time. Cyclicality shows how the business cycle component of a particular variable evolves in relation to the business cycle component of the real GDP. Persistence provides us a clue about the likelihood of a variable to stay, say, above trend next period, when it is above trend in the current period.

Means

The first moment of any time series is the sample average or the sample mean $(\bar{x})$. To calculate $(\bar{x})$ of the business cycle component $\left(x_{t}^{b c}\right)$ we use the following formula:

$$
\bar{x}=\frac{1}{T} \sum_{t=1}^{T} x_{t}^{b c}
$$

${ }^{1}$ This implies that the constant growth is a straight line with $\log$ GDP.

${ }^{2}$ Note that in Excel you can extract the business cycle component with the help of a macro file $($ HPFilter.xla) 6. Stylised facts

\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-058}

Figure 6.3:

Given that the business cycle fluctuates around a trend component, by construction, the sample mean is zero.

Standard deviation

The second moment of any time series is the variance around its mean. A sample standard deviation $\left(\sigma_{x}\right)$ is simply the square root of the variance of the series.

$$
\sigma_{x}=\sqrt{\frac{1}{T-1} \sum_{t=1}^{T}\left(x_{t}^{b c}-\bar{x}\right)^{2}} .
$$

The larger the deviations of a time series from its mean, the larger the sum will be on average. In other words, a more volatile series will have a larger standard deviation. Volatility measures how a particular variable fluctuates around over time. The standard deviation of the HP filtered series serves as a measure for volatility.

In Table 6.1, we display standard deviations of the business cycle component of real GDP, consumption and investment for three countries, Germany, Turkey and the US. Table 6.1 shows two results we would like to take home. First, relative standard deviations of the cyclical component of consumption to real output ratio are less than one in Germany and the US. This is often interpreted as smoother consumption than income over time. Note that for the same sample period cyclical consumption in Turkey exhibits more volatility than income. Thus, consumption smoothing may not be such a general rule, however, most macro-monetary models aim to explain less volatile consumption relative to income. Second, in all these three countries, cyclical component of investment is more volatile than income, roughly in the order of 3.

\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-059}

Figure 6.4:

Cyclicality

Another second moment about means is the covariance. It measures whether two business cycle components, $x^{b c}$ and $y^{b c}$, move in tandem. The sample covariance $\left(\sigma_{x y}\right)$ 6. Stylised facts

$\left(\sigma_{x}\right)$	Germany	Turkey	US
$\sigma_{x}^{b c}$	0.0169	0.0327	0.0135
$\sigma_{c}^{b c}$	0.0138	0.0389	0.0106

$\sigma_{i}^{b c}$ & 0.0483 & 0.0977 & 0.0392 \\
\hline
$\sigma_{c}^{b c} / \sigma_{y}^{b c}$ & 0.82 & 1.19 & 0.78 \\
$\sigma_{i}^{b c} / \sigma_{y}^{b c}$ & 2.86 & 2.99 & 2.90 \\
\end{tabular}
\end{center}

Table 6.1: Volatilities.

between two variables is calculated as:

$$
\sigma_{x y}=\frac{1}{T-1} \sum_{t=1}^{T}\left(x_{t}^{b c}-\bar{x}\right)\left(y_{t}^{b c}-\bar{y}\right)
$$

Terms in the sum may be clearly positive or negative. If two series tend to 'comove' in the same direction, then whenever a term in series x is above its mean, then the corresponding term in series y tends to be above its mean as well and whenever a term in series x is below its mean, then the corresponding term in series y tends to be below its mean as well. Therefore, a sample covariance will have mostly positive numbers which yield a positive sum. The opposite is true, when two series co-move in the opposite direction. A large covariance may make life somewhat difficult for us. It may be due to high correlation between these two variables or both series may have large variances that lead to large terms in the sum. In order to deal with this, we divide the covariance by the two standard deviations. Resulting ratio is labelled as the 'correlation coefficient'.

$$
\rho_{x y}=\frac{\sigma_{x y}}{\sigma_{x} \sigma_{y}}
$$

The correlation coefficient will be between -1 and 1 . Highly correlated series will have a correlation coefficient close to 1 in absolute value. If y represents the real GDP, a high positive correlation coefficient with a series x indicates that the series x is 'procyclical'. Similarly, a high negative correlation coefficient with a series x indicates that the series x is 'countercyclical'. If the correlation coefficient is near zero, we label the series as being 'acyclical'.

Table 6.2 displays correlation coefficients of consumption, investment and real output in the US, Germany and Turkey.

First, we observe that for each country, consumption and investment are positively correlated with their real output, hence both consumption and investment are procyclical variables. Second, although investment constitutes a smaller fraction in the calculations of real GDP than consumption, the correlation coefficients between investment and real GDP are somewhat higher than the correlation coefficients between consumption and real GDP. Third, while the US business cycle to some extent seems to be positively correlated with business cycles in Germany and Turkey, German and Turkish business cycles appear to be slightly negatively correlated with each other.
\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-061}

Figure 6.5:

\section{Persistence}
To measure persistence in a variable we can use autocorrelations, that is the correlation coefficient $\left(\rho_{x}\right)$ of a variable with the lagged value of itself. It is calculated as:

$$
\rho_{x}=\frac{a_{x}}{\sigma_{x}^{2}}

with

$$
a_{x}=\frac{1}{T-2} \sum_{t=2}^{T}\left(x_{t}^{b c}-\bar{x}\right)\left(x_{t-1}^{b c}-\bar{x}\right) .
$$

A persistent series is more likely to stay above its mean in the next period when it is currently above its mean. A highly persistent series will display an autocorrelation coefficient closer to 1 .

Table 6.3 displays autocorrelation coefficients for our quarterly series between 1984 Q1 and 2007 Q2 and these are large, implying high persistence in these time series. 6. Stylised facts

$\left(\rho_{x y}\right)$	$c^{U S}$	$i^{U S}$	$y^{U S}$	$c^{G E R}$	$i^{G E R}$	$y^{G E R}$	$c^{T U R}$	$i^{T U R}$	$y^{T U R}$
$c^{U S}$	1	0.69	0.83	0.27	0.23	0.24	0.23	0.16	0.28
$i^{U S}$		1	0.85	0.03	0.25	0.12	0.17	0.10	0.17
$y^{U S}$			1	0.23	0.20	0.21	0.20	0.16	0.26
$c^{G E R}$				1	0.50	0.71	0.10	-0.07	0.02
$i^{G E R}$					1	0.77	0.04	-0.21	-0.11
$y^{G E R}$						1	0.02	-0.11	-0.07
$c^{T U R}$							1	0.35	0.73
$i^{T U R}$								1	0.77
$y^{T U R}$									1

Table 6.2: Correlation coefficients.

	$\left(\rho_{x i}\right)$
$c^{U S}$	0.84
$i^{U S}$	0.92
$y^{U S}$	0.89
$c^{G E R}$	0.70
$i^{G E R}$	0.76
$y^{G E R}$	0.82
$c^{T U R}$	0.66
$i^{T U R}$	0.70
$y^{T U R}$	0.77

Table 6.3: Autocorrelations.

Inclusion of more recent data-points after the financial crisis leads in most cases to a slight decline in their persistence.

Autocorrelation coefficients for all series are quite large. When they are above (below) the trend in the current period, they tend to remain above (below) the trend next period. We conclude that series in our samp le display strong persistence.

Cross country business cycle evidence

In an excellent recent paper, Calderon and Fuertes (2014) study business cycles of 71 countries (23 industrial countries and 48 emerging market economies, or EMEs), from 1970 Q1 to 2012 Q4. They report that

- Recessions are costlier for emerging market economies (vis-à-vis industrial economies). While recessions in emerging market economies are, on average, as long as those in industrial countries, they are deeper and steeper.
- Recoveries in emerging markets are slower and more volatile.
- Recessions are on average shorter in duration, smaller in amplitude and less costly during the globalisation period (1990-2007) when compared to the pre-globalisation period $(1970-89)$. - Recessions in East Asia are deeper than in Latin America and Eastern Europe, however East Asian economies experience the fastest recoveries.
- The behaviour of consumption and investment around recessions resembles that of real GDP. Consumption fluctuates less than output for both industrial economies and emerging markets. The opposite is true for investment.
- Although both industrial and emerging markets have experienced deep recessions during the recent global financial crisis, the emerging markets have recovered faster.

Activity 6.1 Calculate the business cycle component and its statistical properties for the country you are living in by using excel.

Key macroeconomic relationships

We are also interested how real macroeconomic variables are related to nominal variables, such as monetary aggregates, and prices in general. Such statistical information is important. A theoretical macro-monetary models usefulness can only be judged by its ability to fit the historical data and perhaps more importantly by its ability to forecast the future. Next sections looks at some of these existing statistical relationships found in the data.

Relationship between monetary aggregates and inflation

Inflation is always and everywhere a monetary phenomenon. M. Friedman and A. Schwartz in 'A Monetary History of the United States, 1867-1960', (Princeton University Press, 1963).

In the long run

Ever since the work by Friedman and Schwartz on the determinants of U.S. inflation, there has been a strong consensus among economists that variations in monetary aggregates are closely associated with variations in inflation over a reasonably long time period. There is also broad agreement about the direction of causation: variations in money lead to variations in inflation. This consensus has some theoretical support from the so called Quantity Theory of Money (QTM) (See Chapter 5). In its simplest form, the QTM argues that variations in some measure of money supply are followed by the inflation rate. We can write the QTM directly in logarithmic differences (lowercase letters) so that we can refer to changes in the variables concerned.

$$
m+v=p+y
$$

where m represents the variations in money supply, p the inflation rate, y variations in real income and v variations in the velocity of money supply. If we solve for the inflation rate, p, we obtain:

$$
p=m-y+v
$$

Stylised facts

When we assume that variations in the money supply, m, do not impact the long run real income, y, or velocity, v, money supply variations and inflation will be proportional. In other words, a one per cent increase in the money supply will lead to a one per cent increase in the inflation rate. To test the hypothesis De Grauwe and Polan (2005) study 159 countries for the period of 1969-1999. First, they find a strong positive relation between the long run growth rate of money and inflation. However, this relation is not one to one. Second, they argue that this strong link between inflation and money growth is mainly due to the presence of high inflation countries in their sample; there seems to be no clear long run link between money growth and inflation in low inflation economies. Third, they confirm the QTM assumption of no relationship between real income and money growth in the long run. In other words money is long run neutral, i.e. does not have permanent effects on output.

In the short run (Phillips Curve)

Phillips curve is an empirical inverse relationship between the rate of unemployment and the rate of inflation in the short run. [3] We will see later different versions of Phillips curve in theoretical models as it provides a justification for the existence and relevance of monetary policy in the short run. Here, we will simply state whether such relationship is visible in the time series data. In Figure 6.6 we plot the annual unemployment and the rate of inflation in the UK and the US together for the period 1971-2008. A simple correlation analysis shows that the contemporaneous correlation between the UK inflation and unemployment for the whole sample period is in the order of -59 per cent, whereas in the US the correlation is only about -5 per cent.

We also note that the contemporaneous correlations between inflation and unemployment are changing over time. In the UK, there was a long lasting inverse relationship between inflation and unemployment up to the global financial crisis. Note that the negative relationship was already disappearing before the onset of the crisis. In the US, there is a slight positive contemporaneous correlation up to the global financial crisis. (Kuttner and Robinson (2010) provide an excellent discussion on why such relationships may be changing over time; students are urged to read their paper). Of course, these are simple correlations; we need to evaluate these relationships with the use of advanced econometric techniques. We conclude that, while there is no clear empirical support for a negative (positive) relationship between unemployment (output gap) in the short-term, most theoretical business cycle models start with the premise that the inflation-unemployment trade-off does exist in the short-term.

Relationship between monetary instruments and macroeconomy

Standard macro-monetary analysis considers some measure of monetary aggregates or short-term interest rates as potential monetary policy instruments to influence macroeconomic variables. Economic research long understood the fact that a policy

[3] Similarly, many researchers are interested in studying the relationship between the business cycle component of real GDP (output gap) and the rate of inflation.

\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-065}

Figure 6.6:

instrument can only be useful in the monetary policy implementation to the extent that variations in the policy instrument over time regularly and reliably associated with variations in income, prices, unemployment or any other variable the policy maker is seeking to influence. The question for a policy maker is then cho osing either a monetary aggregate or a short-term rate as the policy instrument.

To answer the question, Friedman and Kuttner (1992) analysed the 'information content' of variations in a
range of US monetary aggregates and short-term interest rates to explain variations in US real/nominal G
DP and inflation. They conclude that while monetary aggregates were useful in explaining US real output
and inflation up until the early 1980s, thereafter these lost their informational value, whereas short-term int
erest rates or the spread between the commercial paper rate and Treasury bill rate retained their informati
on content in explaining movements in real income. Hence, not surprisingly, starting from the early 1990s
onwards, nearly all major central banks focused on short-term interest rates as their policy variable. Curre
ntly, monetary policy announcements of central banks are almost always about short-term interest rates.

\section{Stylised facts}
More recently, Aksoy and Piskorski (2006) argued that the loss of information content in US monetary agg
regates has actually something to do with the measurement of monetary aggregates. In particular, they ar
gue that as the US currency enjoys a reserve currency position in the global financial and trade system, fo
reign holdings of the US dollar are sizeable. Moreover, the foreign demand for the US dollar is changing o
ver time affecting the time series properties of the monetary aggregates. What matters for the US domesti
c economic conditions is not the aggregate level of monetary aggregates, but monetary aggregates correc
ted for the foreign holdings (domestic money). They show that variations in domestic money contain valua
ble information in explaining variations in not only US real income but also variations in US inflation. Thus,
any analysis, that does not take the international currency feature of US dollar into account, may be misle
ading. In other words, money continues to be useful as an information variable and a potential policy instr
ument.

\subsection{Price level stickiness}
Observed aggregate and good specific prices are rather sticky i.e. there is persistence in price levels. Pric
es of goods and services do not adjust immediately in response to changing demand and supply condition
s. Similarly, inflation displays persistence. Using individual price data, Alvarez et al. (2006) study frequenc
y and size of price adjustments in the euro area. They report that:

\begin{itemize}
  \item Prices change infrequently. In a given month 15.1 per cent of prices are changed and the average
duration of a price spell ranges from 4 to 5 quarters. These figures mean that price adjustment in the euro
 area is considerably less frequent than in the US (Bils and Klenow (2004)).

  \item Frequency of price changes across products differ. Price changes are very frequent for energy (oil
products) and unprocessed food, while they are relatively infrequent for non-energy industrial goods and s
ervices.

  \item Differences across countries appear to be less important than cross-sector differences.

  \item On average, 40 per cent of the price changes are price reductions.

  \item Price changes, either increases or decreases, are sizeable compared to the inflation rate prevailing
 in each country.

\end{itemize}

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the essential reading and activities, you should be able t
o:

\begin{itemize}
  \item describe trends vs. business cycles

\item calculate the means, volatility, cyclicality and persistence of business cycle components of macroeconomic variables - list key business cycle stylised facts

\item discuss the relevance of monetary aggregates and short-term interest rates as potential policy instruments.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
\item 'Business cycles refer to fluctuations in the entrepreneurial activities.'
\end{enumerate}

\section{Section B}
\begin{enumerate}
\setcounter{enumi}{1}
\item What is an HP Filter? Why is it important to extract the business cycle component from the level data to analyse policy issues?

\item What are the key stylised facts in business cycle analysis that a theoretical model should try to explain? Are there any differences between industrialised economies and emerging market economies?

\item 'Inflation is always and everywhere a monetary phenomenon.' (Friedman and Schwartz). Explain and critically discuss.

\item Is there empirical evidence in favour of Phillips curve? In the light of the discussion in Kuttner and Robinson, provide an explanation for recent developments in in the statistical relationship between output gap and inflation.

\end{enumerate}

\subsection{Feedback to Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
\item The statement is false. Suggested answers should include an exact definition of the business cycle. A complementary stylised business cycle figure is recommended.
\end{enumerate}

\section{Section B}
\begin{enumerate}
\setcounter{enumi}{3}
\item While there is some relationship between variations in money and inflation in the long run, the relationship appears to be different between high and low inflation economies. (De Grauwe and Polan (2005)). In the short run the relationship appears to be weaker (Friedman and Kuttner (1992)). For the US one needs to take
\end{enumerate}

\section{Stylised facts}

international currency feature of US dollar into account and only then apply the relevant statistical analysis (Aksoy and Piskorski (2006)).

\section{Chapter 7 Money, inflation and welfare}
\subsection{Introduction}
In Chapter 5 , we saw that in classical theory money was neutral, not having any effects on real variables such as consumption or output. The welfare effects of money and inflation, however, were not considered. Even if a change in the money supply does not cause output or consumption to change, does it cause the utility or welfare of individuals to be affected? This chapter will discuss such issues, noting the difference between the neutrality and superneutrality of money. Finally, issues in hyperinflation will be considered.

\subsection{Aims}
This chapter aims to study money, inflation and welfare in a flexible price environment. It will introduce such concepts as the real and nominal interest rates, the superneutrality of money, inflation tax and hyperinflations.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item describe the relationship between real and nominal interest rates and why the relationship holds

  \item explain what superneutrality means and give examples of when monetary policy can be neutral but not superneutral

  \item define a monetary policy which maximises welfare and quantify the welfare cost associated with a given level of inflation

  \item describe the concepts of seigniorage and inflation tax

  \item discuss the merits of using inflation as a source of government revenue rather than other forms of tax

  \item define hyperinflation, and explain why it is undesirable and how it forms.

\end{itemize}

\subsection{Reading advice}
Before embarking on this chapter and before looking at any of the recommended reading you should revise your understanding of inflation from your EC2065

Macroeconomics course. You should also have re-read Chapter 5 of the subject guide on classical theory and monetary neutrality and the references therein. The model of hyperinflation and that of high but stable inflation are based on the papers by Cagan (1956) and Dornbusch (1992), respectively, and should be read after completing this chapter.

\subsection{Essential reading}
Cagan, P. 'The monetary dynamics of hyperinflation', in Friedman, M. (ed.) Studies in the Quantity Theory of Money. (Chicago: University of Chicago Press, 1956).

Dornbusch, R. 'Lessons from experience with high inflation', The World Bank Economic Review (6) 1992, pp.13-31.

Lewis, M.K. and P.D. Mizen Monetary Economics. (Oxford; New York: Oxford University Press, 2000) Ch

apter 7.

McCallum, B. Monetary Economics. (New York; Macmillan; London: Collier Macmillan, 1989) Chapters 6 and 7.

Further reading

Books

Cagan, P. 'Hyperinflation', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Danthine, J.P. 'Superneutrality', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Friedman, M. 'The quantity theory of money', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Harris, L. Monetary Theory. (New York; London: McGraw-Hill, 1985) Chapter 19. Mankiw, N.G. Macroeconomics. (New York: Worth Publishers, 2002).

Journal articles

Diamond, P.A. 'Search, sticky prices and inflation', Review of Economic Studies 59(4) 1993, pp.53-68.

Sargent, T. and N.Wallace 'Some unpleasant monetarist arithmetic', Federal Reserve Bank of Minneapolis Quarterly Review 531 (1981), Fall.

Neutrality

In the classical model, which we assume here, prices are perfectly flexible and the economy is then one in which output is always at (full employment) equilibrium. We will also not consider other sources of non-neutrality such as contracts or debts denominated in nominal terms. In such an economy, as established in Chapter 5, money is neutral in the sense that in static equilibrium all real variables in the economy are independent of the quantity of nominal money and the price level is proportional to the quantity of money.

It might appear that an immediate implication of neutrality is that the rate of growth of the quantity of money would also be neutral in the sense of affecting the rate of growth of the price level (inflation) but no real variables. This is known as 'superneutrality'. Then inflation would indeed be a monetary phenomenon and at the same time would have no real effects. However, it turns out that even in a flexible price economy this implication is not correct, the reason being that expectations of the future growth rate of the quantity of money create expectations of inflation and expectations of inflation are not neutral. The starting point is the distinction between real and nominal interest rates, and the relationship between them; the 'Fisher equation'.

Real and nominal interest rates

If people expect there to be inflation over some period, quoted interest rates no longer represent the rate at which goods today can be traded for goods in the future. Imagine a deposit paying a fixed interest rate, $R_{t} \%$ per annum on balances at the beginning of period t, paid at the beginning of period $t+1$. Then $\pounds 100$ today becomes $\pounds 100\left(1+R_{t}\right)$ in a year's time. However, if the price level at the beginning of the year were P_{t}, and prices were to rise during the year at a rate π_{t+1} (π_{t+1} being the inflation rate between periods t and $t+1$), the quantity of goods that can actually be purchased at the end of the year is $\pounds 100\left(1+R_{t}\right) / P_{t}\left(1+\pi_{t+1}\right)$. Thus not consuming $\pounds 100 / P_{t}$ goods at the beginning of the year enables one to consume $\pounds 100\left(1+R_{t}\right) / P_{t}\left(1+\pi_{t+1}\right)$ more goods at the end of the year. For small values of R_{t} and p_{t+1}, this expression can be approximated by $\pounds 100\left(1+R_{t}-\pi_{t+1}\right) / P_{t}$, and the term $\left(R_{t}-\pi_{t+1}\right)$ is known as the 'real' rate of interest, r_{t}. The relationship between the real and nominal rate of interest, which is known as the Fisher equation

, is therefore by definition:

$$
r_{t}=R_{t}-\pi_{t+1}
$$

While a borrower or financial institution can commit to a particular nominal interest rate at the beginning of the period, it is evident that the actual rate of inflation over that same period, between t and $t+1$, will not be known until date $t+1$. Hence at the beginning of the period, borrowers and lenders can only form expectations of the real interest rate based on their expectations of inflation over the period.

- The ex-ante real interest rate, r_{t}^{A}, measures the quantity of goods at the end of the period which people expect to be able to buy as a proportion of the quantity they could buy today with the money they deposit or lend. Then:

$$
r_{t}^{A}=R_{t}-\pi_{t+1}^{e}
$$

Money, inflation and welfare

where π_{t+1}^{e} is the expected rate of inflation between dates t and $t+1$, formed at date t

- In the same way the ex-post real interest rate, r_{t}^{P}, measures the actual real interest rate during some past period, which is the quantity of goods at the end of the period that a depositor was actually able to buy relative to what could have been bought at the beginning. Evidently,

$$
r_{t}^{P}=R_{t}-\pi_{t+1}
$$

In measuring ex-post real interest rates it should be noted that current quoted nominal interest rates relate to the future whereas quoted inflation rates refer to the past, hence the different time subscripts. If one wanted to know the ex-post real interest rate for 2002 it would be necessary to subtract the 2002 inflation rate (as recorded at the end of the year/beginning of 2003) from the nominal rate quoted for one-year money at the beginning of the year.

If everyone correctly forecasts the rate at which prices will rise over some period and acts on the basis of these expectations, inflation is said to be fully anticipated. In this case of course, $\pi_{t+1}^{e}=\pi_{t+1}$, and the ex-ante and ex-post real interest rates are therefore equal.

Activity 7.1 How can borrowers commit to paying the real rate of return on any sums lent to them?

Superneutrality

Superneutrality may be defined as the proposition that, where inflation is fully anticipated, the real rate of interest $\left(r_{t}\right)$, is independent of the fully anticipated inflation rate, π_{t+1}. The real rate of interest measures the relative price of goods in the future as against goods today so it is clearly a 'real' variable relevant to intertemporal decisions (saving and investment). If it were to change in response to a change in the inflation rate, then inflation would not be neutral in that it would affect savings and investment decisions. However, if the real interest rate does not change when the inflation rate changes, then from the Fisher equation, it follows that the nominal rate, R_{t}, must adjust one-for-one with changes in the inflation rate.

In the standard macro model, the real interest rate affects the goods market through its effects on saving and investment decisions. From the resource constraint; output, Y, is split between consumption, C, investment, I, and government spending, G, we can write:

$$
Y=C(\stackrel{+}{Y}-\tau, \bar{r})+I(\stackrel{+}{Y}, \bar{r})+G
$$

τ are taxes so that consumption depends positively on disposable income and negatively on the real interest rate. A high real interest rate will encourage saving (decrease current consumption) as one unit of consumption goods today can yield a larger number of consumption goods tomorrow. There is then a negative relationship between the real rate and output.

$$
r=H(\bar{Y})
$$

From the Fisher equation:

$$
R=H(\bar{Y})+\pi
$$

Equation (7.6) represents an IS curve, showing the combinations of output, Y, and nominal interest rate, R, that clear the goods market.[1] Therefore, in the standard IS-LM model with the nominal interest rate on the vertical axis, an increase in the inflation rate will shift the IS curve upwards but leave the position of the LM curve unchanged. There must therefore be a one-off increase in the price level to restore equilibrium in the money market at the given level of equilibrium output, see Figure 7.1. The increase in the nominal interest rate reduces the demand for real money balances and the jump in the price level reduces the supply.

Figure 7.2 shows (by way of concrete example) the case of a static economy, where initially the stock of money and hence the price level are constant, and where at time t_{0} a new policy is introduced such that the money stock grows at $g \%$ per year. The growth in the money stock will cause prices to increase, also at $g \%$ per year, and the anticipation of this inflation at time t_{0} will cause the nominal interest rate to jump at t_{0} by g percentage points. This in turn will cause the price level to jump at time t_{0} by an amount equal to g multiplied by the interest elasticity of the demand for money. In this standard model, changes in the fully anticipated inflation rate leave the real interest rate unchanged but affect not only the nominal interest rate but also the stock of real money balances, and it is this which leads to the welfare costs of fully anticipated inflation (see below).

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-073}
\end{center}

Figure 7.1: Superneutrality of money.

The standard model is somewhat simplified in that it has no wealth effects in the IS curve. A full and general model would include such effects, and in such a model a fall in real money balances would constitute a fall in wealth, which would tend to raise savings. A rise in savings would shift the IS curve inwards and thus lower the equilibrium real interest rate and increase investment. This is known as the Mundell-Tobin effect. It says that in inflationary times people wish to hold less wealth in the form of real money balances and therefore attempt to acquire other forms of wealth including real capital.

${ }^{1}$ While IS-LM model used to analyse economies with short term price stickiness, remember that there are no such rigidities in this model. 7. Money, inflation and welfare

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-074}
\end{center}

Figure 7.2: Nominal money balances and the price level.

In practice, however, in most advanced economies fiat money balances constitute so tiny a proportion of people's wealth that this effect can be ignored.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-074(1)}
\end{center}

Figure 7.3: Non-superneutrality of money.

The case of non-superneutrality, caused by wealth effects in the consumption function, is shown in Figure 7.3. The IS curve is derived from an amended version of (7.4).

$$
Y=C\left(\stackrel{+}{Y}-\tau, \bar{r}, \frac{\stackrel{+}{M}}{P}\right)+I(\stackrel{+}{Y}, \bar{r})+G
$$

Alternatively, this can be written in the form:

$$
r=H\left(\bar{Y}, \frac{\stackrel{+}{M}}{P}\right)
$$

For any given output level, a fall in real money balances necessitates a fall in real interest rates in order to offset the decrease in aggregate demand. In goods market equilibrium there is then a positive relationship between M / P and r. Following the increase in inflation, caused by the positive growth rate of nominal money balances, the IS curve shifts out as in Figure 7.1 but is partially offset because of the fall in wealth (fall in M / P) which reduces consumption. The nominal interest rate only increases to $R_{1}^{\prime}\left(<R_{1}\right.$ in Figure 7.1) and since inflation is the same in both Figure 7.1 and Figure 7.3 , the real interest rate, $r_{1}^{\prime}$, will be less than r_{0}, (i.e. the real rate has fallen).

\section{The welfare costs of inflation and the optimal quantity of money}
The key insight, which follows from Friedman's price-theoretic approach to modelling the demand for money, is that the area under the demand curve measures the utility derived by individuals from holding money balances. The opportunity cost to individuals of holding money balances is measured by the nominal rate of interest, R. The consumers' surplus enjoyed by individuals from holding money balances, that is, the excess of the utility they derive over the opportunity cost they incur, is thus measured by the area A in Figure 7.4.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-075(1)}
\end{center}

Figure 7.4: Seigniorage and inflation tax.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-075}

\end{center}

Figure 7.5: ... with positive inflation.

Suppose that, initially, there is no inflation, so that the nominal interest rate in Figure 7.4 is equal to the real interest rate $\left(R_{0}=r\right)$. Given this interest rate, the total stock of

\section{Money, inflation and welfare}
money balances, which individuals wish to hold, is denoted $(M / P)_{0}$. The area B measures the annual income stream the government as issuer of money is able to acquire by the once and for all purchase of productive assets with the money it issues. (In a static economy with no inflation, the stock of money outstanding is constant, but the government was able to acquire real assets with it at the time it was introduced.) But if the money is fiat money, the cost of production is approximately zero, so the government is essentially able to acquire real assets 'for free' by virtue of its monopoly position on the money issue. This is known as 'seigniorage', and it represents a transfer of resources from money holders (households and firms) to the government.

Seigniorage is essentially a form of taxation, and the area C in Figure 7.4 is the excess burden or deadweight loss associated with this tax. Holdings by individuals of money balances in excess of $(M / P)_{0}$ yield positive utility and the additional money costs nothing to produce. Therefore, there would be economic benefit from expanding the quantity of real money balances held from $(M / P)_{0}$ up to the point of 'full liquidity', L. However, this cannot be done through an expansion of the nominal money stock, because with an interest rate of R_{0} the desired holding of money balances is $(M / P)_{0}$, and any additional nominal money will lead to an increase in the price level to restore the desired quantity of real balances.

The social utility of money in Figure 7.4 is measured by the area $\mathrm{A}+\mathrm{B}$, the sum of consumers' surplus and the revenues obtained by the government from seigniorage. Given the demand curve, the maximum possible social utility that could be achieved is when the holdings of money balances are at L (full liquidity), which is Friedman's 'optimal quantity of money'. To induce individuals to hold the optimal quantity of money it is necessary that the nominal interest rate be zero. Only in this case will the demand for money be equal to L. Given the Fisher equation, the nominal interest rate will be zero when the rate of price inflation is equal to minus the real interest rate (i.e. $R=0$ when $p=-r)$.

The welfare cost of fully anticipated inflation arises because inflation raises the nominal interest rate, reducing the demand for real money balances and therefore reducing the utility obtained from the use of money. In Figure 7.5 we consider an economy with fully anticipated inflation at a rate of p_{1}. The nominal interest rate is now $R_{1}\left(=r+p_{1}\right)$, and the demand for money is $(M / P)_{1}$. The consumers' surplus is again measured by the area under the demand curve and above the opportunity cost of holding money, and is therefore the area A. The area $\mathrm{B}+\mathrm{D}$ represents the revenue the government now derives from money issue, of which B is generally known as the 'inflation tax', and as before, D is seigniorage, real interest rate times real money balances. The inflation tax refers to the resources obtained by the government by the continuing process of printing money during times of inflation. (If the inflation rate is p_{1}, and the volume of real money balances $(M / P)_{1}$, the amount of nominal money the government will need to print per period to maintain the real stock of money constant is $p_{1}(M / P)_{1}$.)

Comparing Figure 7.4 with Figure 7.5 , the welfare cost of inflation may be measured as the loss of consumers' surplus, $B+C$, plus the loss of seigniorage, E, less the inflation tax revenue, B.

$$
\text { Welfare cost }=\mathrm{B}+\mathrm{C}+\mathrm{E}-\mathrm{B}=\mathrm{C}+\mathrm{E} \text {. }
$$

This is equal to the reduction in the quantity of real money balances demanded multiplied by the nominal i nterest rate averaged over that range. The welfare cost of fully anticipated inflation arises because individuals choose to incur the cost and inconvenience of economising on money balances in times of inflation. Inflation raises the private cost of holding money balances though the social cost of creating money remains negligible.

Inflation as taxation

If inflation is like a tax it not only reduces demand and has welfare costs, but also raises revenue for the government. Even though inflation has welfare costs, these costs may be lower than those of other forms of taxation. Returning to Figure 7.5, the net increase in government revenue from inflation is the inflation tax, area B, less the reduced yield from seigniorage, area E. If one measures the efficiency of a tax as the ratio of the welfare cost to the revenue raised, then the efficiency of inflation as a form of taxation is given by:

$$
\text { Efficiency }=\frac{\mathrm{C}+\mathrm{E}}{\mathrm{B}-\mathrm{E}}
$$

The most efficient tax scores zero on this scale (no welfare cost) and the least efficient scores infinity (welfare cost but no revenue yield), or negative if the tax actually lowers total revenue. The relationship between the welfare cost and the revenue raised by the inflation tax depends on the interest elasticity of demand for money. The more inelastic the demand for money, the more efficient is inflation as a form of taxation.

Activity 7.2 What other factors, other than efficiency, should determine the extent to which the government uses inflation as a form of taxation?

Hyperinflation

The popular image of hyperinflation is of very rapid, and in particular very rapidly accelerating, inflation, leading to astronomical prices and to a collapse of the monetary system. The most famous hyperinflation of this type in a major industrial country was Germany in 1922-23. Prices increased by 500,000 times within the space of a few months in the autumn of 1923, the prices of staple commodities like a loaf of bread or a newspaper ran into hundreds of billions of marks. Notwithstanding this, money remained the medium of exchange, although the attempt to spend money as soon as it was received itself generated faster inflation. The greatest cost of the hyperinflation, however, was the impoverishment of households with monetary assets whose value was destroyed.

Hyperinflations of this type result from political instability such that the government is unable to finance its expenditure except through money creation. The acceleration of inflation results in part from deteriorating public finances and in part from the 'flight of money' caused by expectations of faster inflation. In the twentieth century, there were hyperinflations after the First World War in Austria, Germany, Hungary, Poland and Russia, and after the Second World War in Bulgaria, Greece, Hungary, Poland and Romania. More recently, there have been hyperinflations associated with the collapse of the former Soviet Union and with the effects of war, in particular in parts of former Yugoslavia (e.g. Serbia).

Money, inflation and welfare

Activity 7.3 What has been the highest rate of inflation in your country?

These rapidly accelerating hyperinflations are unsustainable, and are usually terminated by a financial reconstruction involving a new government, a new currency whose value is sometimes secured by a fixed exchange rate or currency board, and a fiscal reconstruction that may involve a repudiation of government debt or other obligations. To demonstrate how hyperinflations can lead to a collapse in the demand for money, consider the model of money demand by Cagan (1956). Take a standard money demand equation of the form:

$$
\ln \left(\frac{M}{P}\right)_{t}=a y_{t}-b R_{t}
$$

Replacing R_{t} with $r_{t}+p_{t+1}^{e}$ from the Fisher equation, this can be written as:

$$
\ln M_{t}=\ln P_{t}+a y_{t}-b r_{t}-b \pi_{t+1}^{e}
$$

Differentiate this with respect to time and assume that output growth and real interest rate changes are small relative to changes in nominal quantities, as is likely in periods of hyperinflations, which is what the Cagan model tries to study.

$$
\frac{\mathrm{d}\left(\ln M_{t}\right)}{\mathrm{d} t}=\frac{\mathrm{d}\left(\ln P_{t}\right)}{\mathrm{d} t}-b \frac{\mathrm{d}\left(\pi_{t+1}^{e}\right)}{\mathrm{d} t}
$$

$\mathrm{d}\left(\ln M_{t}\right) / \mathrm{d} t$ is the growth rate of the money supply, m_{t}, and $\mathrm{d}\left(\ln P_{t}\right) / \mathrm{d} t$ is the growth rate of the price level, or inflation, π_{t}. Rearranging gives:

$$
\pi_{t}=m_{t}+b \frac{\mathrm{d}\left(\pi_{t+1}^{e}\right)}{\mathrm{d} t}
$$

If inflation is expected to increase in the future (i.e. $\mathrm{d}\left(\pi_{t+1}^{e}\right) / \mathrm{dt}$ is positive), then these expectations can cause inflation to increase to a level above m_{t}. Increases in inflation will cause individuals to increase their expectations of inflation, which in turn causes p_{t} to increase even further from (7.14). The collapse in the demand for money is then partly caused by expectations of faster inflation next to growth rate of the money supply $\left(m_{t}\right)$, which are, ultimately, self-fulfilling.

Very rapid, continuing and non-explosive inflation

A more recent phenomenon has been a widespread experience of very rapid but none the less continuing and non-explosive inflation. For example, there have been inflation rates averaging over 100 per cent a year lasting for 10 years or more in many South American countries such as Argentina, Brazil or Peru during the 1980s. Such very rapid but stable inflations can be explained in terms of a continuing but non-deteriorating government deficit. This can be demonstrated in the form of a very simple model due to Dornbusch.

Assume a government whose expenditure falls short of the money it can raise from taxation by some amount, which is a fixed proportion, g, of nominal income. The government has a poor credit rating and is unable to borrow from the capital markets so this deficit can be financed only through the creation of money. The deficit per unit time period is given by:

$$
\text { Deficit }=g P y
$$

where P is the price level and y is real GDP. This deficit is financed by an increase in the money stock, so that:

$$
\frac{\mathrm{d} M}{\mathrm{~d} t}=g P y
$$

$$

In the simplest version of the model the monetary sector is described by the quantity theory equation:

$$
M V=P_{y}
$$

Dividing (7.16) by (7.17) gives:

$$
\frac{1}{M} \cdot \frac{\mathrm{dM}}{\mathrm{d} t}=g V
$$

But $(1 / M) \cdot(\mathrm{d} M / \mathrm{d} t)$ is equal to the growth rate of the money supply, which is equal to the rate of inflation, p. This leads to the very simple expression for the inflation rate under these assumptions of:

$$
\pi=g V
$$

For large values of g and/or V the inflation rate can be quite rapid, but in the simplest versions of this model there is no reason why it should accelerate. A straightforward modification of the model is to allow the velocity of circulation to be increasing in the inflation rate, $V=V(\pi)$ with $V^{\prime}>0$, which may lead to a deteriorating trade-off between the deficit and inflation and to the possibility of instability. See Figure 7.6.

If we are initially at point $\mathrm{E}^{\prime}$ and inflation increases, this will cause velocity to increase since $V=V(\pi)$. Higher velocity will cause inflation to rise $(\pi=g V)$ and the process continues with ever increasing inflation. Point E' is therefore an unstable equilibrium.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-079}
\end{center}

Figure 7.6:

Activity 7.4 Explain, using Figure 7.6, why point E is a stable equilibrium. (Hint: start by examining what happens when inflation increases slightly from point E.) 7. Money, inflation and welfare

\section{Activity 7.5}
\begin{enumerate}
  \item In the economy of Mythica, prices are perfectly flexible and output is always at its full employment level, Y. The velocity of circulation of money is constant, and the stock of money, M, grows at a constant rate, m. Aggregate demand, the IS curve, is given by $Y^{d}=A-b\left(R-\pi^{e}\right)$, where R is the nominal rate of interest and π^{e} is the expected rate of inflation.
\end{enumerate}

(a) What is the inflation rate, π, in Mythica?

(b) If the inhabitants of Mythica have perfect foresight, what is the effect of increasing the growth rate of the money supply on real and nominal interest rates?

(c) Is the stock of real money balances affected by the rate of growth of the money supply?

Imagine now that due to an increase in financial sophistication in Mythica the demand for money becomes sensitive to the interest rate so that in place of the quantity theory equation, we have $(M / P)^{d}=H-k R$. How are your answers to parts (a) to (c) above affected by this development? Is money i. neutral, and ii. superneutral in Mythica?

\begin{enumerate}
\setcounter{enumi}{1}
\item An econometrician makes the following estimates of the key economic parameters of the Mythican economy. $Y=1000$, and in the IS equation $A=1400$ and $b=100$. In the money demand equation $H=1200$ and $k=50$. The government of Mythica is committed to a policy of price stability and hence the rate of growth of the money supply, $\mu=0$, and the stock of nominal money balances is 2000 . If interest rates are measured in percentage points, so for example a 7% interest rate means $R=7$, then:
\end{enumerate}

(a) Calculate the nominal interest rate, the price level and the stock of real money balances in Mythica.

(b) What value of seigniorage does the government derive from its monopoly of the money issue?

If the government were instead to allow the money stock to grow at a rate of 10% per annum, $\mu=10$, what will happen to the inflation rate, the nominal interest rate and the stock of real money balances in Mythica? What resources does the government now derive from the combination of inflation and seigniorage? (For feedback, see the end of this chapter.)

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
\item describe the relationship between real and nominal interest rates and why the relationship holds

\item explain what superneutrality means and give examples of when monetary policy 7.9. Sample examination questions

\end{itemize}

can be neutral but not superneutral

\begin{itemize}
\item define a monetary policy which maximises welfare and quantify the welfare cost associated with a given level of inflation

\item describe the concepts of seigniorage and inflation tax

\item discuss the merits of using inflation as a source of government revenue rather than other forms of tax

\item define hyperinflation, and explain why it is undesirable and how it forms.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
  \item 'The government of Zeroland acquires a fixed quantity, G, of real goods and services per capita each year, which it purchases through printing fiat money. It has no other income. If there is no economic growth in Zeroland, and velocity of circulation, V, of money is constant, the inflation will be independent of G.'
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{1}
  \item What is meant by the 'superneutrality' of money? How does it relate to the Fisher equation? If money is superneutral, does it follow that inflation has no welfare costs?

  \item Assume that the government can control the rate of inflation exactly. What rate should it set? Outline arguments in favour of positive, zero and negative inflation rates.

  \item What causes hyperinflation? How can hyperinflations be stopped?

  \item Consider the economy of Mythica in Activity 7.5, noting the values of the variables in part 2.

\end{enumerate}

(a) If monetary policy affects social welfare only through its effect on the consumer surplus obtained from the use of money, what rate of monetary growth maximises social welfare in Mythica? What is the nominal rate of interest in this optimal regime?

(b) If the government of Mythica can finance its expenditure only through money creation, what is the relationship between government expenditure as a proportion of GDP (g) and the growth rate of the money stock? Is there a limit to the expenditure, which can be financed in this way?

\subsection{Feedback to Activity 7.5}
\begin{enumerate}
  \item First part.
\end{enumerate}

(a) Take logs of the quantity theory and differentiate with respect to time. Use (7.2) and (7.3) to show that inflation is equal to m.

(b) Perfect foresight implies inflation is equal to m at all times. From the IS equation and the fact that Y is constant, $r(=R-\pi)$ must therefore be constant. The real interest rate is constant and the nominal interest rate moves one-for-one with the inflation rate.

(c) From the quantity theory, $M V=P Y, M / P$ is constant and so not affected by the rate of growth of the nominal money supply.

Second part.

(a) Differentiate the money demand equation with respect to time, noting that $\mathrm{d} R / \mathrm{d} t=0$ in equilibrium.

$$
\frac{\mathrm{d} M}{\mathrm{~d} t} \cdot \frac{1}{P}-\frac{\mathrm{d} P}{\mathrm{~d} t} \cdot \frac{M}{P^{2}}=0
$$

From which we can see that the growth rate of money equals the growth rate of prices. $\pi=\mu$.

(b) Same as in the first part.

(c) From the money demand equation, it is clear that an increase in R, caused by the increase in μ, will cause the demand for real money balances to fall. The price level jumps in order for the real money supply to equal the lower real money demand.

Money is neutral in Mythica; increasing the money supply just increases the price level with no change in any real variables.

Money is, however, not superneutral. Increasing the growth rate of the money supply causes a real variable, M / P, to fall. The real interest rate, however, remains unchanged.

\begin{enumerate}
  \setcounter{enumi}{1}
  \item First part.
\end{enumerate}

(a) $R=4 \%$ from the IS equation. Substituting into the money demand equation gives $P=2$ and hence $M / P=1000$.

(b) Seigniorage equals the real interest rate multiplied by real money balances. Therefore seigniorage equals 4000 .

Second part.

(a) $M=10 \%$ therefore $\pi=10 \%$. Since $R=4 \%, R$ must equal 14% and from the money demand equation, $M / P=500$.

(b) Seigniorage equals 2000 and the inflation tax, inflation rate multiplied by real money balances, equals 5000 . Total revenue therefore equals 7000 .

\subsection{Feedback to Sample examination questions}
\section{Section A}
\begin{enumerate}
  \item The statement is false. Suggested answer will use and explain (7.15) to (7.18) to show that inflation is equal to $G V$.
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{4}
  \item (a) The money demand equation is $(M / P)^{d}=1200-50(r+\pi)$. If social welfare is maximised by maximising the consumer surplus under the money demand curve, then welfare is maximised when $i=0$ (Friedman rule). If $r=4 \%$ and $i=r+\pi$, then welfare is maximised when $\pi=-4 \%$. Hence, in order to maximise welfare, the money supply should contract by 4% per year, $\mu=-4 \%$.
\end{enumerate}

(b) Using the model of Dornbusch, government spending is financed by money creation. (Ignore seigniorage here; government spending is only financed from the inflation $\operatorname{tax})$.

$$
\frac{\mathrm{d} M}{\mathrm{~d} t}=g P Y

Dividing by P and noting that $(\mathrm{d} M / \mathrm{d} t) \cdot 1 / M=\mu$, the growth rate of money, gives:

$$
\mu \cdot \frac{M}{P}=g Y
$$

Substituting in for $M / P(=1200-50 i)$ and using $i=r+\pi(=4+\pi)$ and $\pi-\mu$, together with our value for $Y(=1000)$:

$$
\mu(1200-50(4+\mu))=1000 g
$$

Rearranging gives

$$
g=\mu-0.05 \mu^{2}
$$

This is the relationship between the money growth rate, μ, and the proportion of GDP that the government can spend, financed only through money creation. Differentiating this with respect to μ and setting equal to zero will give a maximum value of g of 5%. This is associated with a money growth rate of 10%. 7. Money, inflation and welfare

Chapter 8
Classical models and monetary policy

Introduction

In Chapter 5 we introduced the classical model where money was neutral. Real outcomes such as investment decisions and output were determined by real factors; tastes and technology, and the money supply only determined the price level. In this chapter we will examine classical models in more detail, exploring the possibilities that monetary policy can have real effects, at least in the short run. However, in order for us to do this, we must first consider the benchmark case of the classical model and we do this by considering a model based on 'reduced form' macro equations. [1]

A 'simple' model is introduced and solved in the appendix but you are not expected to develop, or be able to solve, these models in the examination. They are purely left for the more interested reader who wishes to further understand how RBC theory works.

Aims

This chapter introduces first major macro model without nominal 'frictions' that will serve as a benchmark model. We then study cases with real frictions in the classical model (in the form of asymmetric information, the requirement to hold cash before you could buy, and limited participation in financial markets) all lead to money having real effects.

Learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

■ describe how, in the classical economy, output is determined by the factor markets

- describe and discuss the effects of monetary policy, with reference to the effects of money on nomi nal variables such as prices and nominal wages, and on real variables such as employment

\item describe what business cycle models are and what they try to do

\item list and explain the workings behind, the flexible price models where money has real effects.

\end{itemize}

${ }^{1}$ I.e. equations we assume hold in this particular instance. 8. Classical models and monetary policy

\subsection{Reading advice}
Before working through this chapter, it is vital that you have a thorough understanding of the macroeconomic classical models. Background reading can be found in the textbooks of Mankiw (2002) and Branson (1989) among others.

In this chapter, we discuss the Classical model in more detail, analysing the effects of monetary policy.

Once you have worked through the chapter, you can read the articles in the reading list; those of Long and Plosser (1983) and Plosser (1989) are probably the most readable. The chapters in Hoover and in Hargreaves Heap are also very useful.

\subsection{Essential reading}
Hargreaves Heap, S.P. The New Keynesian Macroeconomics: Time, Belief and Social Independence. (Aldershot: Edward Elgar Publishing, 1992) Chapter 4.

Hoover, K.D. The New Classical Macroeconomics. (Oxford: Blackwell, 1988) Chapter 3. Long, J. and C. Plosser 'Real business cycles', Journal of Political Economy 91(1) 1983, pp.39-69.

Plosser, C. 'Understanding real business cycles', Journal of Economic Perspectives 3(3) $1989, \mathrm{pp} .51-77$.

\subsection{Further reading}
\section{Books}
Branson, W.H. Macroeconomic Theory and Policy. (New York; London: Harper and Row, 1989).

Mankiw, N.G. Macroeconomics. (New York: Worth Publishers, 2002).

Walsh, C.E. Monetary Theory and Policy. (Cambridge, Mass.: MIT Press, 2003) Chapter 1.

\section{Journal articles}
King, R.G. and C. Plosser 'Money, credit and prices in a real business cycle', American Economic Review 74(3) 1984, pp.363-80.

Kydland, F.E. and E.C. Prescott 'Business cycles: real facts and a monetary myth', Federal Reserve Bank of Minneapolis Quarterly Review 14(2) 1990, p.3.

Lucas, R.E. Jr. 'Some international evidence on output-inflation trade-offs', American Economic Review 66(5) 1976, p.985.

Lucas, R.E. Jr. 'Nobel lecture: monetary neutrality', Journal of Political Economy $104(3) 1996$, pp.661-82.

\subsection{The classical model revisited}
As should be clear by now, in the classical model all markets do clear instantaneously such that output is

at its natural rate, resulting in involuntary unemployment being equal to zero. Note that the labour market variable is hours worked, not the level of employment, thus when individuals decide to work they always find work at a given wage rate. Prices are perfectly flexible and move instantly to clear both goods markets and factor markets (the prices in the labour and capital factor markets are wages and real rates, respectively). We shall assume that output in the economy is determined only by labour and capital.

$$
y=f(k, l)
$$

where k is the amount of capital, which for the time being we will assume to be fixed, and l is the amount of labour. Also, f_{1} (the derivative of the production function with respect to the first element) $>0, f_{2}>0, f_{11}<0, f_{22}<0$ and $f_{12}>0$. That is, factors are complementary in production and they have positive but decreasing marginal productivities. The demand for labour by firms will depend negatively on the real wage that they have to pay, W / P. If labour is more expensive, firms will demand less of it. Firms demand labour up to the point where the marginal cost of production, the real wage, equals the marginal benefit from production, the marginal product of labour. Due to the properties of the production function, namely diminishing marginal returns, a high level of labour is associated with a low marginal product so firms are only willing to pay a low wage at that employment level. The supply of labour is positively related to the real wage since a high wage encourages individuals to work more hours and causes those who initially chose not to join the labour force, to participate in work.

Labour market equilibrium is shown in Figure 8.1. If capital is fixed at k^{*}, then output is determined as $y=f\left(k^{*}, l^{\prime}\right)$.

The effect of monetary policy

The effect of monetary policy can be best shown using the set of diagrams in Figure 8.2, below.

The middle left diagram shows the labour market equilibrium. This is the same as Figure 8.1 only we have the nominal, as opposed to the real, wage on the vertical axis. This is because we are examining the effects of changing nominal variables, namely the nominal money supply.

The bottom left diagram is the production function for a given level of capital, k^{*}, and shows the decreasing marginal returns to labour.

The top right panel shows the standard IS-LM analysis and the middle right panel shows the aggregate demand and supply schedules. Note that the aggregate supply schedule is vertical in the classical model since markets always clear which allows output always to equal its natural/full employment rate, $y^{\prime}$.

Suppose the famous 'helicopter drop' analogy of monetary policy of Friedman. Imagine an economy, where authorities conduct monetary policy by sending off helicopters to drop cash over the country. Such an increase in the money supply, caused by a 8. Classical models and monetary policy

\begin{center}
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\end{center}

Figure 8.1: Labour market equilibrium.

'helicopter drop' of newly printed money or by open market operations, whereby the monetary authorities buy government bonds in exchange for money, leads to an outward shift of the aggregate demand curve to $\mathrm{AD}_{1} \cdot{ }^{2}$ The monetary expansion also causes the LM curve to initially shift out to LM_{1}. However, the increase in demand has no effect on output since the AS schedule is vertical at $y^{\prime}$, and simply causes the price level to increase from P_{0} to P_{1}. The increas e in the price level reduces real money balances, causing the LM curve to shift back to its initial position, LM_{0}. Note that although the outward shift of the LM curve, caused by the initial monetary expansion, suggests at least a temporary increase in output, the price level increases immediately so that the LM curve effectively never changes position.

The increase in the price level causes the labour demand schedule to shift out. Note that we have the nominal wage on the vertical axis. For any given nominal wage, a higher price implies a lower real wage and hence an increase in labour demand. The price rise also causes the labour supply schedule to shift to the left. Again, for any given nominal wage, the price rise implies a fall in the real wage so individuals supply less labour. In equilibrium the nominal wage increases to the point where the real wage remains at the initial level. Real wages and real money balances are left unchanged in equilibrium; and output, labour, and so on are all unaffected. All the change in the money supply does is to change other nominal variables, nominal wages and prices, one-for-one. Money, as expected, is neutral.

[2] Why a change in nominal money balances affects the AD schedule is documented in all macroeconomic texts. See for example Mankiw (2002).

\begin{center}
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\end{center}

Figure 8.2: 8. Classical models and monetary policy

Real business cycle theory

Real business cycle (RBC) theory extends the classical model in that it derives the ad hoc macroeconomic equations, represented in Figure 8.2, from the optimising behaviour of agents. Money still turns out to be neutral in the basic set-up but business cycles are argued to be the natural and efficient responses of the economy to changes in the available production technology. It distinguishes between nominal shocks (shocks to money supply or money demand) which only affect the LM curve and real shocks (shocks to the production function, real government spending or to savings and consumption decisions) which affect only the IS or AS curves.

According to RBC theory, real shocks to the economy are the primary cause of business cycles. The theory focuses on shocks to the production function: supply shocks or productivity shocks. Such shocks can be the result of:

- the development of new products or production methods
- changes in the quality of labour or capital
- changes in the availability of raw materials
- unusually good or bad weather and
- changes in government regulations affecting production.

Economic booms result from beneficial productivity shocks and recessions are caused by adverse productivity shocks. Consider a temporary adverse supply shock. The marginal product of labour falls as labour becomes less productive, which reduces the demand for labour. Both the real wage and the equilibrium level of employment fall. The latter causes a fall in output (a recession).

Business cycle facts and RBC theory

RBC theory does a good job at explaining a number of the stylised facts of the macroeconomy and of the business cycle. RBC models explain the procyclical nature of employment, labour productivity and the real wage and also explain why investment is more volatile than consumption.[3] This last point is explained by the fact that agents, being riskaverse, prefer to smooth consumption streams over time. Any fluctuations in income due to productivity shocks will be absorbed into the saving/investment decisions of individuals, resulting in smooth consumption and volatile investment. However, despite explaining these features of the business cycle, it is, in reality, very difficult to identify the productivity shocks that cause the cyclical fluctuations. Also, in order to explain the large cyclical fluctuations in hours worked and the small cyclical fluctuations in the real wage observed in the real world, RBC theory needs a flatter aggregate labour supply curve than has been found by most studies. RBC theory also predicts that the price level will be countercyclical when the empirical evidence for real output and inflation is, as we saw in the 'Stylised facts' chapter, not conclusive. Also,

[3] We define a variable as being procyclical (countercyclical) if it moves positively (negatively) with output. RBC theory, being an extension of the classical model, suggests that money is neutral. However, the problem with claiming that money is neutral is that the money stock has been found to be a leading, procyclical variable. RBC theorists argue that the apparent relationship between money and output is one of reverse causality; anticipated changes in output lead to changes in the money supply in the same direction. Historical research as discussed in Chapter 7, however, suggests that money is not short-run neutral.

Classical models with real effects of money

In the standard classical model analysed above, money has no real effects. As mentioned before, this is a pitfall of standard classical theory; money has been found to have real effects at least in the short run. However, with some modifications to the reference model, money can have real effects, even in models with perfectly flexible prices. These are discussed below.

Lucas 'misperceptions' model

In the standard classical model every individual knows what the money supply is and there is full and symmetric information about the state of the economy and about the state of each market. Relaxing this assumption will allow money to have real effects. Suppose an individual who produces goods in one market sees the price of his good increasing due to an increase in demand. If that was caused by a relative demand shock for his good (i.e. preferences changed so that the demand for his product increased relative to some other goods) then it will be optimal for him to increase production, essentially moving along his upward sloping marginal cost/supply curve. If, on the other hand, the price rise was caused by an economy-wide increase in prices as a result of the money supply increasing, then since nothing 'real' has changed, the demand for his good has not altered relative to the demand for other goods, then it will be optimal for him to leave output unchanged.

In reality, it will be unlikely that he knows the cause of the increase in price and so he will opt for an intermediate strategy whereby he increases output by a little (not as much as if he knew the price rise was caused by a relative demand shock but more than if it was due to a monetary expansion, that being a zero output change). The Lucas aggregate supply function is then given by (8.2).

$$
y_{t}=y^{*}+d\left(P_{t}-\mathrm{E}_{t-1}\left[P_{t}\right]\right)
$$

where y_{t} is aggregate supply, y^{*} is the full employment/market clearing level of output and P_{t} is the price level at date t.

At date $t-1$, individuals make an expectation of the price level in date t, denoted $\mathrm{E}_{t-1}\left[P_{t}\right]$. If the price level in date t is greater than expected, $P_{t}-\mathrm{E}_{t-1}\left[P_{t}\right]>0$, agents believe this is caused by a relative demand shock in date t, or by an increase in the money supply that was unexpected. As a consequence, each individual/firm produces more, which causes aggre gate supply to increase above y^{*}. Hence d is positive and the aggregate supply schedule is upward sloping rather than vertical. Also, an unexpected increase in the money supply can have real effects even though prices are perfectly flexible. People effectively 'misperceive' an increase in the price level as having been

caused by a relative demand shock, rather than an increase in the money supply. If the monetary expansion was perfectly anticipated rather than unexpected, then everyone would allow for this when making their expectation of P_{t} at date $t-1$ and the monetary expansion would have no real effects.[4]

Even though monetary policy has real effects in this set-up, the effects only persist in the short run. In the long run, when people realise that the price rise is due to a monetary expansion, output will revert back to its full employment level. This is true not only of the Lucas misperceptions model but of all classical models that allow real effects of monetary policy.

Cash in advance models

A problem with the money in utility (MIU) model is that it assumes people obtain utility from something that is intrinsically useless. Adding money to the utility function is then an odd way of introducing money. The cash in advance (CIA) model, however, uses cleaner micro-foundations to explain the existence of money. It effectively tries to model the trading frictions, such as the lack of trust between payer and payee discussed in Chapter 2 that causes the trading process to necessitate a medium of exchange cash. People need cash in order to buy goods and sellers of goods will accept nothing but cash when they trade. The trading round is assumed to be split into two periods.

In the first period individuals receive an income and decide how much of their wealth to hold in cash and how much to hold in other forms such as bonds or capital that cannot be used as a medium of exchange.

In the second period individuals buy and consume goods, facing the constraint that the nominal value of their consumption must not be more than the nominal money holdings they brought forward from the first period. Therefore, in addition to the budget constraint, individuals face a 'cash in advance' constraint:

$$
P_{t} C_{t} \leq M_{t-1}
$$

Dividing by P_{t-1} and noting that $P_{t} / P_{t-1}=1+\pi_{t}$, this can be written as:

$$
\left(1+\pi_{t}\right) C_{t} \leq m_{t-1}
$$

where $m_{t-1}=M_{t-1} / P_{t-1}$, and is equal to real money balances at date $t-1$. (8.4) implies that an increase in π_{t} acts like an inflation tax; the consumers cannot purchase in real terms the same amount of goods. Since their utility depends on consumption and leisure, they will substitute (the now relatively more expensive) consumption goods for leisure. That is, they will decrease their supply of labour due to the increase in inflation. Thus employment, and therefore output, will decrease as a result of the increase in inflation.

Limited participation models

Monetary policy can have real effects if there are a limited number of agents participating in financial markets meaning some consumers may be restricted in their access to the banking system and therefore cannot save via interest bearing assets. If the monetary authorities decided to increase the money stock, such policy actions would be made through banks and other financial intermediaries with whom the open market operations were conducted. Now faced with a glut of liquid assets, banks wis

[4] See Chapter 9 and the section on the policy ineffectiveness proposition.

h to lend out some of these in order to maintain their desired reserve ratio. The increase in the supply of loans will cause the interest rate charged on these loans to fall (the liquidity effect) and since firms borrow from financial intermediaries to finance investment projects, this makes investment cheaper. Cheaper investment causes investment to increase, which therefore causes output, and employment if labour and capital are complementary inputs, to increase.

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

■ describe how, in the classical economy, output is determined by the factor markets

\begin{itemize}
  \item describe and discuss the effects of monetary policy, with reference to the effects of money on nominal variables such as prices and nominal wages, and on real variables such as employment

  \item describe what business cycle models are and what they try to do

  \item list and explain the workings behind, the flexible price models where money has real effects.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
  \item 'According to the stylised facts of the business cycle, real wages, employment and government spending are acyclical.'
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{1}
  \item Explain why money might be neutral in the long run but not in the short run.

  \item Suppose the supply curve is given as in (8.2) and that aggregate demand is given by: $y=1000+3(M / P)$. Suppose there has been no shock in the economy for some time and no changes in policy are expected in the near future. If $M=600$, find y and P in terms of y^{*}. What happens when the monetary authorities announce (in advance) that M will increase to 650 ? What happens if this increase is entirely unexpected? In this final part, you do not need to solve for P and y. 8. Classical models and monetary policy

  \item Suppose the economy of Myland is completely described by a cash-in-advance model. However, for the population of Myland, leisure and consumption goods are not substitutes in any way. The Governor of Myland decides to increase the money supply in an attempt to expand the economy. Explain why this leaves output in Myland unchanged.

\end{enumerate}

\subsection{Feedback to Sample examination questions}
\section{Section A}
\begin{enumerate}
  \item The statement is false. Suggested answers should say that the real wage and employment are bot

h procyclical. A productivity shock, for example, will increase labour demand, pushing up the real wage and increasing employment. With this higher level of employment, more output can be produced, implying real wages and employment are procyclical.

Section B

3. If the economy has not experienced any shocks for some time then $P_{t}=\mathrm{E}_{t-1} P_{t}$ and hence, from (8.2), $y_{t}=y^{*}$. If $M=600$, then from the aggregate demand equation, one can solve for P to obtain

$$
P_{t}=\frac{1800}{y^{*}-1000}
$$

If the monetary authorities announce that the money supply at date t will be $M=650$, and if this announcement was made at or before date $t-1$ (i.e. when expectations of P_{t} were formed), then rational agents will change the prices they charge on their goods in order to take this new money supply into account.

Expectations of P_{t} made at $t-1$ will increase and again $P_{t}=\mathrm{E}_{t-1}\left[P_{t}\right]$. Again, from (8.2), $y_{t}=y^{*}$ (i.e. the monetary expansion has no effect on output). From the aggregate demand equation, we can solve for the price.

$$
P_{t}=\frac{1950}{y^{*}-1000}
$$

Note that the growth rate of the price level is equal to the growth rate of the money supply. If the monetary expansion is entirely unexpected, then $\mathrm{E}_{t-1}\left[P_{t}\right]$ will not have changed, and will be equal to the original $1800 /\left(y^{*}-1000\right)$. The monetary expansion will increase aggregate demand and this outward shift of the demand curve along the supply curve of (8.2) will cause output to increase, assuming $\delta \neq 0$. Output increases and so does the price level, P_{t}. However, the growth of the price level is less than that of the money supply, at least initially.

4. From the cash-in-advance model we know $P_{t} C_{t} \leq M_{t-1}$ and as shown in (8.4), an increase in inflation acts like a tax on consumption. The greater the inflation rate, the lower the level of consumption is permitted for any level of real money balances. Ordinarily, an increase in inflation that makes consumption more expensive, would cause people to switch to more leisure, which is now relatively cheap. However, if leisure and consumption are not substitutes in any way, then an increase in inflation, which still causes consumption goods to become more expensive, will have no real effect on the work/leisure decision and so output in the economy will not be affected.

Appendix to Chapter 8: a 'simple' RBC model

Please note that the material presented in this appendix is not examinable. Those who do not wish to study this can move directly on to Chapter 9. For those who do wish to follow and understand the model presented below, you may find it useful to consult the following references.

Cooley, T. and G. Hansen 'Money and the business cycle', in Cooley, T. (ed.) Frontiers of Business Cycle Research. (Princeton, N.J.; Chichester: Princeton University Press, 1995) [ISBN 9780691043234].

Kydland, F.E. and E.C. Prescott 'Time to build and aggregate fluctuations', Econometrica 50(6) 1982, pp.1345-370.

Lucas, R.E. Jr and N. Stokey 'Money and interest in a cash-in-advance economy', Econometrica 55(3) 1987, pp.491-513.

A 'simple' RBC model with money in the utility

The model presented below is a simple RBC model. It is based on micro-foundations/ first principles whereby individuals, in a deterministic setting used here, choose today a time path for consumption and other variables under their control in order to maximise utility. We make the assumption that we live in a deterministic world (i.e. an economy with no uncertainty) so that we do not need to worry about expectations of future variables. This simplifies the notation and also means that we do not need to concern ourselves with how expectations are formed. As such, the model presented is not strictly in the true spirit of RBC theory but gives the reader a basic introduction to modelling techniques so commonly used today.

Similar to the model of money in general equilibrium introduced in Chapter 5 , the representative agent derives utility from consumption goods, C_{t}, and real money balances, M_{t} / P_{t}, but also from leisure, defined as $1-l_{t}$ where l_{t} is labour supplied. Utility is obtained from these variables not only today but into the future, discounted at rate b. Assume a utility function of the form:

$$
U=\sum_{t=0}^{\infty} \beta^{t}\left(\ln C_{t}+\zeta \ln \frac{M_{t}}{P_{t}}+\gamma \ln \left(1-l_{t}\right)\right)
$$

where ζ and γ are constant preference parameters. The individual maximises utility subject to a budget constraint given in real terms in (8.6).

$$
C_{t}+k_{t}+\frac{M_{t}}{P_{t}}+\frac{B_{t+1} Q_{t+1}}{P_{t}}=\frac{W_{t}}{P_{t}} l_{t}+\left(1+r_{t-1}\right) k_{t-1}+\frac{M_{t-1}}{P_{t}}+\frac{B_{t}}{P_{t}}
$$

The constraint simply states that your consumption and asset holdings, left-hand side, equal your income and assets brought over from last period, right-hand side. The

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left-hand side comprises consumption, C_{t}, investment in capital, I_{t} (assume that the capital stock you hold equals your investment made so $k_{t}=I_{t}$, i.e. there is 100% depreciation), M_{t} / P_{t} is real money balances and $B_{t+1} Q_{t+1} / P_{t}$ is savings, not in a productive technology/capital, but in 'discount' bonds. B_{t+1} is the number of bonds you bought at date t that mature at date $t+1$, paying one unit of money, one pound or one dollar, for example. Since they are pure discount bonds, they pay no interest but are bought at a price, Q_{t+1}, that is below 1 , the maturity value. For example, a bond could be bought today for $\pounds 90$ that pays $\pounds 100$ in one year's time, effectively earning you an interest rate of $11.1 \%(=(100-90) / 90 \times 100) .^{5} B_{t+1} Q_{t+1}$ is therefore the nominal value of the bonds you purchase at date t. The budget constraint is in real terms so we divide this quantity by the price level, P_{t}.

The right-hand side is your income and assets brought forward from last period. $\left(W_{t} / P_{t}\right) l_{t}$ is the real income from your labour services. k_{t-1} was the capital invested at date $t-1$, which earns a total return of $1+r_{t-1}, r_{t-1}$ being the real interest rate. M_{t-1} was the amount of money you held at the end of date $t-1$, divided by P_{t} in the budget constraint to put it in real terms at date t. Finally, B_{t} is the number of bonds bought that mature at date t, paying $\pounds 1$ or $\$ 1$, etc. per bond. Hence we do not multiply by Q since, by definition, the bond price at maturity is unity. Again, to put this in real terms we divide by P_{t} in the budget constraint.

Agents then maximise (8.5) subject to (8.6) so we can form the Lagrangian, L. Note that the budget constraint must hold in every time period so there is a Lagrange multiplier for each time period, λ_{t}.

$$
\begin{aligned}
L= & \sum_{t=0}^{\infty} \beta^{t}\left(\ln C_{t}+\zeta \ln \frac{M_{t}}{P_{t}}+\gamma \ln \left(1-l_{t}\right)\right) \\
& +\sum_{t=0}^{\infty} \lambda_{t}\left(C_{t}+k_{t}+\frac{M_{t}}{P_{t}}+\frac{B_{t+1} Q_{t+1}}{P_{t}}-\frac{W_{t}}{P_{t}} l_{t}-\left(1+r_{t-1}\right) k_{t-1}-\frac{M_{t-1}}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t}}-\frac{B_{t}}{P_{t}}\right) .
\end{aligned}
$$

Notice in the budget constraint we replace M_{t-1} / P_{t} by $\left(M_{t-1} / P_{t-1}\right) \cdot\left(P_{t-1} / P_{t}\right)$ so that the numerator and denominator of real money balances are dated at the same time. The individual's choice variables are $C_{t}, C_{t+1}, M_{t} / P_{t}, l_{t}, l_{t+1}, B_{t+1}$ and k_{t}. Table 8.1 lists the choice variables and the first order conditions; derivatives of the Lagrangian with respect to the choice variables set equal to zero.

[5] See Chapter 9 for more discussion of bonds and interest rates.

Choice variable	First order condition	
C_{t}	$\frac{\beta^{t}}{C_{t}}+\lambda_{t}=0$	$(8.8 \mathrm{a})$
C_{t+1}	$\frac{\beta^{t+1}}{C_{t+1}}+\lambda_{t+1}=0$	$(8.8 \mathrm{~b})$
$\frac{M_{t}}{P_{t}}$	$\zeta \beta^{t} \frac{P_{t}}{M_{t}}+\lambda_{t}-\lambda_{t+1} \frac{P_{t}}{P_{t+1}}=0$	$(8.8 c)$
l_{t}	$-\frac{\gamma \beta^{t}}{1-l_{t}}-\lambda_{t} \frac{W_{t}}{P_{t}}=0$	$(8.8 \mathrm{~d})$
l_{t+1}	$-\frac{\gamma \beta^{t+1}}{1-l_{t+1}}-\lambda_{t+1} \frac{W_{t+1}}{P_{t+1}}=0$	$(8.8 \mathrm{e})$
B_{t+1}	$\lambda_{t} \frac{Q_{t+1}}{P_{t}}-\lambda_{t+1} \frac{1}{P_{t+1}}=0$	$(8.8 f)$
k_{t}	$\lambda_{t}-\left(1+r_{t}\right) \lambda_{t+1}=0$	$(8.8 \mathrm{~g})$

Table 8.1

Fisher equation

From $(8.8 \mathrm{~g}), \lambda_{t+1} / \lambda_{t}=1 /\left(1+r_{t}\right)$. Also, (8.8f) implies $\lambda_{t+1} / \lambda_{t}=Q_{t+1} P_{t+1} / P_{t}$. If Q_{t+1} is the price of a bond at date t that pays 1 unit of money at date $t+1$, then $Q_{t+1}=1 /\left(1+R_{t}\right)$ where R_{t} is the nominal interest rate between t and $t+1$. If inflation, π_{t+1}, is defined as $\left(P_{t+1}-P_{t}\right) / P_{t}$ then $P_{t+1} / P_{t}=1+\pi_{t+1}$. Combining the results from $(8.8 \mathrm{f})$ and $(8.8 \mathrm{~g})$ gives:

$$
\begin{array}{lr}
1+r_{t}=\frac{1+R_{t}}{1+\pi_{t+1}} \approx 1+r_{t}-\pi_{t+1} & \text { for small } R_{t} \text { and } \pi_{t+1} \\
\Rightarrow r_{t}=R_{t}-\pi_{t+1} & \text { (the Fisher equation) }
\end{array}
$$

Consumption decisions

Dividing (8.8a) by $(8.8 \mathrm{~b})$ results in

$$
\frac{C_{t+1}}{C_{t}}=\beta \frac{\gamma_{t}}{\gamma_{t+1}}=\beta\left(1+r_{t}\right)
$$

(8.9) states that the growth rate of consumption is positively related to the real interest rate and the extent to which we value future consumption shown by the discount factor, β. If the real interest rate is high, we can forego one unit of consumption today to gain 8. Classical models and monetary policy

a lot more consumption tomorrow. We will then postpone some of our consumption until later, implying a high consumption growth rate.

Labour decisions

Dividing (8.8d) by (8.8a) gives:

$$
\frac{W_{t}}{P_{t}}=\frac{g C_{t}}{1-l_{t}}
$$

This is the labour supply curve used in the first part of this chapter. It essentially states that the marginal rate of substitution between consumption and leisure, the ratio of marginal utilities, is equal to the real wage. If the real wage is high, we will supply more labour, l_{t}. Dividing $(8.8 \mathrm{e})$ by $(8.8 \mathrm{~d})$ and using $(8.8 \mathrm{~g})$ implies:

$$
\frac{1-l_{t+1}}{1-l_{t}}=\beta\left(1+r_{t}\right) \frac{W_{t} / P_{t}}{W_{t+1} / P_{t+1}}
$$

A high real wage today compared to tomorrow will cause individuals to supply more labour today and less tomorrow. Knowing that you can earn more for any amount of labour supplied now, you will forego some leisure time in order to work, the proceeds of which can be used to buy even more consumption goods, increasing utility.

Activity 8.1 If a temporary productivity shock increases today's real wage only, this causes labour supply to increase today relative to tomorrow. What happens to intertemporal labour supply decisions if there is a permanent productivity shock that causes the real wage to increase today and in every future period?

Money holdings

Dividing (8.8c) by $(8.8 \mathrm{a})$ gives:

$$
\zeta \frac{P_{t} C_{t}}{M_{t}}=1-\frac{\lambda_{t+1}}{\lambda_{t}} \cdot \frac{P_{t}}{P_{t+1}}
$$

Using the fact that $\lambda_{t+1} / \lambda_{t}=\left(1+\pi_{t+1}\right) /\left(1+R_{t}\right)$ and $P_{t+1} /$

$P_{t}=1+\pi_{t+1}$, we can rearrange (8.12) to get a money demand equation of the form:

$$
\frac{M_{t}}{P_{t}}=\zeta C_{1}\left(1+\frac{1}{R_{t}}\right)
$$

The demand for real money balances are positively related to consumption (transaction demand) and the preference parameter, j, showing our preference for money compared to consumption. Real money balances are also negatively related to the nominal interest rate, R_{t}, as expected. Multiplying (8.12) by $M_{t} / \zeta P_{t} C_{t}$ gives:

$$
1=\frac{M_{t}}{\zeta P_{t} C_{t}}-\frac{\lambda_{t+1}}{\lambda_{t}} \cdot \frac{P_{t}}{P_{t+1}} \cdot \frac{M_{t}}{\zeta P_{t} C_{t}}
$$

Noting that $\lambda_{t+1} / \lambda_{t}=\beta C_{t} / C_{t+1}$ from (8.9) and cancelling terms results in:

$$
1=\frac{M_{t}}{\zeta P_{t} C_{t}}-b \frac{M_{t+1}}{\zeta P_{t+1} C_{t+1}} \cdot \frac{M_{t}}{M_{t+1}}
$$

Let $z_{t}=M_{t} / \zeta P_{t} C_{t}$ and $X_{t+1}=M_{t+1} / M_{t}$, (i.e. X_{t+1} is the growth factor of the money supply $=1+m_{t+1}$ where m_{t+1} is the growth rate). For simplicity, assume that the growth rate of money is fixed at m, implying $X_{t}=X_{t+1}=X$.

$$
z_{t}=1+\frac{\beta}{X} z_{t+1}=1+\frac{\beta}{X}\left(1+\frac{\beta}{X} z_{t+2}\right)
$$

Substituting for future values of z and assuming z_{t+n} does not explode for large n will allow us to solve for z_{t} as:

$$
\begin{aligned}
z_{t} & =1+\frac{\beta}{X}+\frac{\beta^{2}}{X^{2}}+\cdots \\
\Rightarrow z_{t} & =\sum_{n=0}^{\infty} \beta^{n}\left(\frac{1}{X^{n}}\right)
\end{aligned}
$$

With X constant, this simplifies to $z_{t}=1 /(1-\beta / X)$. Real money balances are then negatively related to the growth rate of the nominal money supply, and so negatively related to the inflation rate.

Activity 8.2 Using $X_{t+1}=1+\mu_{t+1}=M_{t+1} / M_{t}$ and the fact that

$P_{t+1} / P_{t}=1+\pi_{t+1}$, prove that in the steady state, when real money balances are constant through time, the inflation rate, π, is equal to the growth rate of money, μ.

Production

Assume a Cobb-Douglas production function of the form $y_{t}=A_{t} k_{t-1}^{\alpha} l_{t}^{1-\alpha}$. A_{t} is a time varying productivity parameter and the source of the productivity shocks that cause the business cycle. Notice that yesterday's capital stock, k_{t-1}, determines today's output (i.e. we invest in one period and it only pays off in the next). This is consistent with the way capital enters the budget constraint, (8.6). Firms are assumed to maximise real profits, which are real revenues, output, minus the real costs of employing labour and capital (i.e. firms face the problem):

$$
\begin{aligned}
\max \Pi_{t} & =y_{t}-\frac{W_{t}}{P_{t}} l_{t}-r_{t-1} k_{t-1} \\
\text { s.t. } y_{t} & =A_{t} k_{t-1}^{\alpha} l_{t}^{1-\alpha} \\
\Rightarrow \max \Pi_{t} & =A_{t} k_{t-1}^{\alpha} l_{t}^{1-\alpha}-\frac{W_{t}}{P_{t}} l_{t}-r_{t-1} k_{t-1} .
\end{aligned}
$$

Differentiating this with respect to the two choice variables, l_{t} and k_{t-1}, shows us that the marginal cost of capital, r_{t-1}, will equal its marginal product. The marginal cost of labour, W_{t} / P_{t}, will similarly equal its marginal product.

$$
\begin{aligned}
r_{t-1} & =\alpha A_{t} k_{t-1}^{\alpha-1} l_{t}^{1-\alpha} \\
\frac{W_{t}}{P_{t}} & =(1-\alpha) A_{t} k_{t-1}^{\alpha} l_{t}^{-\alpha}
\end{aligned}
$$

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Model solution

In order for us to solve this $R B C$ model, deriving solutions for C_{t}, k_{t}, l_{t} and y_{t}, we first have to impose the resource constraint that output equals consumption plus investment. This basically states that what is produced is either consumed or put back into the economy to produce goods next period. Since we assume 100% depreciation, investment, I_{t}, must equal the capital stock, k_{t}, that is used in the production of tomorrow's goods.

$$
y_{t}=C_{t}+k_{t}
$$

We also have to note that individuals face two forms of saving devices, capital and bonds, the real returns from which must be equal in order to avoid infinite arbitrage opportunities that are clearly inconsistent with the real world. One unit of capital at date t will yield r_{t} units at date $t+1$. On the other hand, one bond costing Q_{t+1} units of money at date t will pay 1 unit of money at date $t+1$. We can convert this into real terms by dividing each date by its respective price level so that $Q_{t+1} P_{t+1} / P_{t}$ units of goods saved in bonds at date t will yield 1 unit of consumption good at date $t+1$. If the price level rose between dates t and $t+1$ then the real return on bonds will be lower since more consumption goods have to be put into bonds at date t in order to get one unit at date $t+1$. This is shown more clearly in Table 8.2.

	Date t	Date $t+1$
Capital	1	r_{t}
Bonds	$Q_{t+1} P_{t+1} / P_{t+1}$	1

Table 8.2

This implies that there is a relationship between the returns to capital and bonds that satisfies the following:

$$
\frac{Q_{t+1} P_{t+1}}{P_{t}} r_{t}=1
$$

Using the fact that $Q_{t+1} P_{t+1} / P_{t}=l_{t+1} / l_{t}=\beta C_{t} / C_{t+1}$ and substituting r_{t} from (??) will give us:

$$
\frac{\beta C_{t}}{C_{t+1}} \alpha A_{t+1} k_{t}^{\alpha-1} l_{t+1}^{1-\alpha}=1
$$

Using (8.23), the resource constraint, (8.24), and labour market clearing, (8.25), we will be able to solve for current and future values of C, k, l and y if we impose an initial condition for k_{0}.

$$
\begin{aligned}
C_{t}+k_{t} & =A_{t} k_{t-1}^{\alpha} l_{t}^{1-\alpha} \\
(1-\alpha) A_{t} k_{t-1}^{\alpha} l_{t}^{-\alpha} & =\frac{\gamma C_{t}}{1-l_{t}} .
\end{aligned}
$$

Notice that in (8.23) to (8.25) there are no nominal quantities, Q, W, M or P. Therefore a change in M, for example, will not have any effect on any of the real variables, C, k, l or y. Money is neutral. In the sense that (8.23) to (8.25) do not depend on the growth rate of money, money can be argued to be superneutral. However, real money balances do depend on the growth rate of money as demonstrated in (8.17) so, strictly speaking, money is not superneutral. Since utility depends positively on real money balances, an increase in the growth rate of money that reduces real money holdings will have detrimental effects on welfare.

Activity 8.3 For the more interested student, show that the solution to (8.23) to (8.25) is:

$$
C_{t}=\frac{1-\alpha \beta}{\alpha \beta} k_{t}, \quad k_{t}=\alpha \beta A_{t} k_{t-1}^{\alpha} l_{t}^{1-\alpha}, \quad l_{t}=\left(1+\frac{g(1-\alpha \beta)}{1-\alpha}\right)^{-1}
$$

Outline of answer

From (8.23):

$$
1=\alpha \beta \frac{C_{t} / k_{t}}{C_{t+1} / k_{t+1}} \cdot \frac{A_{t+1} k_{t}^{\alpha} l_{t+1}^{1-\alpha}}{k_{t+1}}=\alpha \beta \frac{C_{t} / k_{t}}{C_{t+1} / k_{t+1}} \cdot \frac{C_{t+1}+k_{t+1}}{k_{t+1}} .
$$

Using the resource constraint, this implies:

$$
\frac{k_{t}}{C_{t}}=\alpha \beta\left(\frac{k_{t+1}}{C_{t+1}}+1\right)
$$

Solving forward for k_{t+1} / C_{t+1}, noting that $|\alpha \beta|<1$ and assuming that k_{t+n} / C_{t+n} does not explode will give $k_{t} / C_{t}=\alpha \beta /(1-\alpha \beta)$, from which we obtain the solution for C_{t}. Substitute this into the resource constraint and solving for k_{t} gives us the second part of the solution that tells us how k_{t} evolves over time. Using the resource constraint to substitute out $A_{t} k_{t-1}^{\alpha} l_{t}^{1-\alpha}$ in (8.25) and substituting C_{t} from the first part of the solution will give us the solution for l_{t}.

Chapter 9
Keynesian models with money supply as a policy instrument

Introduction

In the last chapter we examined the effects of monetary policy and saw that frictions in the classical model (in the form of asymmetric information, the requirement to hold cash before you could buy, and limited participation in financial markets) all led to money having real effects. Here, we will examine one particular type of friction, namely the friction to prices. If prices, of goods or of labour, are sticky then changes in the money supply will cause changes to real money balances and/or to the real wage. A change in the stock of nominal money will then have consequences in the real economy. Despite some of these models being thought up and refined in the latter years of the 20th century, we put such models of nominal rigidities under the umbrella of 'Keynesian models', the term that is usually used when referring to them.

Aims

This chapter aims to introduce models with nominal (price) frictions where the monetary policy instrument is a monetary aggregate. We find short-term real effects of monetary policy under certain conditions.

Learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- explain the importance of nominal rigidities and why such sticky prices occur in the real world
- examine the effects of monetary policy in the short and long run when there are nominal rigidities
- describe the workings of the Phillips curve and augmented Phillips curve, noting the differences between the two
- build and work through models of sticky prices, examining the effects of monetary policy
- explain why multi-period pricing can lead to real and persistent effects of monetary policy
- discuss the merits of various sticky price models, noting, in particular, the different short-run and long-run effects of monetary policy.

Reading advice

The content of this chapter, especially the latter half, is based primarily on McCallum (1989), Chapters 9 and 10, and it is recommended that you read these chapters while working through this section. However, before doing so you should refresh your memory with the Keynesian macroeconomic models, found in all macro textbooks. (see for example Mankiw (2002) or Branson (1989)).

The review article by Phelps in the New Palgrave Dictionary of Money and Finance is also very good and should be read after completing this chapter. The article by Blanchard is thorough but difficult and should only be read when you feel comfortable with the material here and in the other readings. For empirical evi dence of nominal rigidities, see Gordon.

Essential reading

Gordon, R.J. 'A century of evidence on wage and price stickiness in the US, the UK and Japan', in Tobin, J. (ed.) Macroeconomics, Prices and Quantities. (Oxford: Blackwell, 1983).

Hargreaves Heap, S.P. The New Keynesian Macroeconomics: Time, Belief and Social Independence. (Edward Elgar Publishing, 1992) Chapters 5 and 6.

McCallum, B. Monetary Economics. (New York; Macmillan; London: Collier

Macmillan, 1989) Chapters 9 and 10.

Phelps, E.S. 'Phillips curve', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Further reading

Books

Blanchard, O.J. 'Why does money affect output? A survey', in Friedman, B. and F. Hahn (eds) Handbook of Monetary Economics. (Amsterdam: North-Holland, 1990).

Branson, W.H. Macroeconomic Theory and Policy. (New York; London: Harper and Row, 1989).

Mankiw, N.G. Macroeconomics. (New York: Worth Publishers, 2002).

Phelps, E.S. Microeconomic Foundations of Employment and Inflation Theory. (New York: Norton, 1970).
Rotemberg, J.J. and M.Woodford 'Dynamic general equilibrium models with imperfectly competitive markets', in Cooley, T. (ed.) Frontiers of Business Cycle Research. (Princeton, N.J.; Chichester: Princeton University Press, 1995).

Journal articles

Friedman, M. 'The role of monetary policy', American Economic Review 58(1) 1968, pp. 1-17.

Phillips, A.W. 'The relation between unemployment and the rate of change of money wage rates in the United Kingdom', Economica 25(100) 1958, pp.283-99.

Keynesian aggregate supply function

An early model of nominal rigidities was that of sticky nominal wages. Labour unions were argued to negotiate wages on a yearly basis, which would not allow nominal wages to adjust instantaneously to changes in market conditions. As in Chapter 8, output is produced from capital and labour, and firms employ workers up to the point where the marginal product of labour is equal to the marginal cost, the real wage. However, unlike in classical models, labour employed, and hence output produced, only depends on labour demand (not on the intersection of demand and supply!). Firms are assumed to have the 'right to manage', whereby firms can choose how much labour to employ, even if this is more than the amount workers wish to supply at the going real wage. This could happen, for example, by firms asking, or forcing, labour to work overtime. Similarly, if the demand for labour by firms was below that which workers wish to supply, this will result in unemployment. [1]

The aggregate supply curve in the economy is then going to be upward sloping. If the nominal wage is fixed at W^{*}, an increase in the price level will cause the real wage, W^{*} / P, to fall, resulting in higher labour demand. More labour employed (if firms have the right to manage) will lead to more output supplied, implying an upward-sloping aggregate supply schedule. However, in the long run, wages will be renegotiated to the market clearing level, at which point employment, output and other real variables are determined by tastes and technology, not the price level. The long-run aggregate supply schedule is then vertical as in Chapter 7.

\section{The effect of monetary policy}
The effect of monetary policy can best be shown by using a similar set of diagrams to those of Figure 8.2. This is shown, for sticky nominal wages, in Figure 9.1. The middle left panel shows the labour market when the nominal wage is fixed at W^{*} and the bottom left panel shows the standard production function with diminishing marginal returns to labour. The top right diagram shows the IS-LM curves and the middle right panel shows aggregate demand and supply. Note that aggregate supply is upward sloping as explained above.

${ }^{1}$ Note that the 'right to manage' is not needed where the real wage is above the market-clearing wage rate.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-106}
\end{center}

Figure 9.1: Expansionary monetary policy causes the aggregate demand schedule to shift out and also causes the LM curve to shift to the right. Since the AD shift causes a price rise, this will tend to reduce real money balances, causing a partial offsetting of the outward LM shift. As in the analysis in Chapter 8, a monetary expansion causes the labour demand curve to shift out. The increase in the price level reduces the real wage for any given nominal wage, leading to an increase in labour demand. However, since the nominal wage is fixed, and given the 'right to manage' assumption, firms employ more labour so that employment increases to $l^{\prime \prime}$ and output increases to $y^{\prime \prime}$. A monetary expansion therefore has real effects caused by nominal wages being sticky. In the long run, however, the nominal wage will be bid up as workers renegotiate their contracts to counter the fall in their real wage. In the long run, employment will remain at $l^{\prime}$, output will remain at $y^{\prime}$ and the monetary expansion simply causes a one-for-one movement in prices and nominal wages. ^{2}

\section{The Phillips curve}
The assumption of sticky nominal wages can easily explain the short-run real effects of monetary policy. However, this implies that the real wage is strongly countercyclical; clearly inconsistent with empirical findings. Also, despite the ability of the above model to explain the existence of unemployment, it sheds very little light on to the mechanism by which wages are determined. If wages are predetermined in the current period, then changes in the economy must be reflected in wages in the next period. This is not explicitly modelled above, but was the focus of research made by Phillips (1958). Phillips overcame the problem of assuming exogenously fixed wages by assuming that the nominal wage depends on recent values of unemployment. This assumption was based on an empirical regularity between unemployment and nominal wage inflation in the UK from 1861 to 1957 . This is shown in Figure 9.2.

Intuitively, if unemployment was high, trade unions, and labour in general, could not negotiate larger pay increases since firms would have a large pool of unemployed with which to fill its vacancies. When unemployment is high, labour tends to be in a weak bargaining position. The specification Phillips gave for the relationship was:

$$
\begin{aligned}
\ln W_{t} & =\zeta\left(u_{t-1}\right)+\ln W_{t-1} \\
\Rightarrow \Delta w_{t} & =\zeta\left(u_{t-1}\right) \quad \text { with } \zeta^{\prime}<0
\end{aligned}
$$

where w_{t} is the log of the nominal wage, $=\ln W_{t}$, and Δ denotes the first difference operator. Firms that maximise profits will set the marginal product of labour equal to the real wage, W_{t} / P_{t}, which implies that an increase in the nominal wage will be associated with an increase in the price level. Δw_{t} and Δp_{t} will be highly correlated, with the result that (9.1) can be written as:

$$
\Delta p_{t}=\zeta\left(u_{t-1}\right) \quad \text { with } \zeta^{\prime}<0
$$

The relationship states that there is a permanent trade-off between inflation and unemployment. It appeared that all policy makers had to do to lower unemployment

${ }^{2}$ The increase in the price level from P_{1} to the long-run equilibrium of P_{2} should cause the labour demand and supply schedules to shift up from l_{1}^{d} and l_{1}^{s} but this is ignored here as it would simply complicate the diagram.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-108}
\end{center}

Unemployment rate

Figure 9.2:

and increase output was to allow inflation to rise. However, in the 1970s the Phillips curve relationship broke down and this was explained, and indeed predicted, by Friedman (1968) and Phelps (1970) who emphasised the importance of inflation expectations, which had been ignored thus far.

Augmented Phillips curve

Friedman and Phelps claimed that agents cared, not about their nominal wage, but about how many goods such a wage could buy (i.e. what really mattered was their real wage). By augmenting (9.1) with inflation, Δp_{t}, unemployment in period $t-1$ would determine changes in real wages. This is shown in (9.3).

$$
\Delta w_{t}-\Delta p_{t}=\zeta\left(u_{t-1}\right)
$$

However, since there is no current information about Δp_{t} (inflation is only realised after nominal wages have been negotiated), this has to be anticipated. If Δp_{t}^{e} is the expectation formed at date $t-1$ of inflation at date t, then the expectations-augmented Phillips curve can be written as:

$$
\Delta w_{t}=\zeta\left(u_{t-1}\right)+\Delta p_{t}^{e} \quad \text { with } \zeta^{\prime}<0
$$

The expectations augmented Phillips curve explained the breakdown of the simple version that occurred in the 1970s. There were argued to be a number of short-run Phillips curves, one for each level of expected inflation. Unexpected inflation would move you along a given short-run Phillips curve but in the long run there would be no trade-off between unemployment and inflation. As people's expectations of inflation increased to meet actual inflation we would move to another short-run Phillips curve. In equilibrium, when inflation was equal to expected inflation, unemployment would be constant at its natural rate. In the long run, any attempt to reduce unemployment to below its natural rate would simply be inflationary. The reason the Phillips curve broke down was because of the persistent and high inflation of the 1970s, caused partly by policy makers trying to exploit the Phillips curve to reduce unemployment and partly by the supply side shocks in the form of large oil price rises in 1974. The high inflation caused expectations of inflation to increase, causing the existing 'stable' Phillips curve to shift. In the period 1861 to 1957, although there were periods of notable price rises and falls, inflation, and therefore expected inflation, was on the whole rather stable.

Okun's law

As can be seen in Figure 9.1, there is somewhat negative relationship between unemployment and departures of output from potential, y^{*}. This is known as Okun's law, which can be applied to the Phillips curve to transform (7.4) into:

$$
\begin{aligned}
\Delta p_{t} & =\gamma\left(y_{t}-y^{*}\right)+\Delta p_{t}^{e} \quad \text { with } \gamma^{\prime}>0 \\
\Rightarrow y_{t} & =y^{*}+\frac{1}{\gamma}\left(\Delta p_{t}-\Delta p_{t}^{e}\right) .
\end{aligned}
$$

This is the same as the Lucas aggregate supply curve, where $d=1 / g$ and we have replaced unanticipated prices by unanticipated inflation. Note, however, that even though both Lucas model and Okun's law have similar predictions (real effects of unanticipated monetary policy caused by an upward-sloping aggregate supply curve), they have different microfoundations. The Lucas supply curve is based on imperfect information on the sources of good specific price changes that affects real output and employment fluctuations are voluntary, whereas the Phillips curve assumes nominal rigidities that leads to involuntary employment fluctuations.

Menu costs and sticky prices

We now outline a model of sticky prices in the goods market. Prices are set at the beginning of the period and cannot change to accommodate shocks or other developments in the economy. As employment was demand determined at the start of this chapter in the case of sticky wages, output is demand determined in this situation of sticky goods prices. Firms produce as much as is demanded at the price that was set at the beginning of the period, even if this means supplying more than they would ordinarily wish to produce given those market conditions.

Activity 9.1 Why would a firm want to produce at a level that is demand determined? (Hint: think of the relationship between customers and producers!)

How can we justify the assumption of sticky prices? If, after setting a price, market conditions change, why do firms not change their prices to accommodate such developments and thus maximise profits? There are a number of theories to rationalise sticky prices. Two common arguments are the existence of menu costs and the attempts by firms not to disturb the loyal customer base with frequent price changes. If prices do

Keynesian models with money supply as a policy instrument

change frequently, a number of repeat customers may look elsewhere in the market. Prices may then be kept constant in order to keep these valuable customers happy. Menu costs, on the other hand, refer to the costs that firms face when they change their list prices. Such costs arise due to the need to issue updated catalogues, for example. It should then not be that unreasonable to assume that prices are, to a greater or lesser extent, inflexible.

Activity 9.2 What industries can you think of that have very sticky prices? Which industries have very flexible prices?

A model of sticky prices

This model is taken from McCallum (1989), Chapter 10, and you are strongly recommended to read this chapter while working through what follows. First consider aggregate demand. By combining the IS and LM equations, we can derive an AD expression of the form:

$$
y_{t}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}\right)+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}\right]+v_{t}

y_{t}, m_{t} and p_{t} are the logs of real output, nominal money balances and the price level, respectively, at date t. v_{t} is a random demand shock with zero mean (i.e. an element of aggregate demand that is not picked up by real money balances or expected inflation). β_{0}, β_{1} and β_{2} are positive parameters and E_{t-1} is the rational expectations operator with the $t-1$ subscript denoting the date the expectation was formed. In rational expectations, people not only take all available information into account when they form their expectations, but their expectations are also consistent with the way in which the variables actually evolve. Rational expectations are sometimes also known as 'model consistent' expectations. [3] The equation states that aggregate demand depends positively on real money balances and positively on expected inflation; for any given nominal interest rate, higher inflation implies a lower real interest rate, making investment cheaper.

Aggregate supply is a little more difficult to specify because of the assumption that prices are set at the beginning of the period, with supply being demand determined. Denote the price that clears the market at date t as p_{t}^{*}. Where is the price stickiness in the model coming from? We assume that the prices the firms set at date $t-1$, to be operational in the market at date t, p_{t}, are the prices they expect to clear the market at date t, i.e.:

$$
p_{t}=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]
$$

As in time $t-1$, shocks are unknown, these cannot be taken into account when setting the prices. Also, denote the output level that clears the market at date t as y_{t}^{*}. Then, if the price equals that which allows markets to clear, by definition markets must clear and so y_{t}^{*} must equal the demand when $p_{t}=p_{t}^{*}$.

$$
y_{t}^{*}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}^{*}\right)+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}^{*}\right]+v_{t} .
$$

[3] You should refer for a refresher on rational expectations to work with expectations operators. Also, see Branson (1989), Chapter 11. Rearranging (7.8) to obtain p_{t}^{*} on the left-hand side gives:

$$
p_{t}^{*}=\frac{\beta_{0}-y_{t}^{*}+\beta_{1} m_{t}+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}\right]+v_{t}}{\beta_{1}+\beta_{2}}
$$

noting that $\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$ equals p_{t} from (9.7) and that in the situation of market clearing, $p_{t}=p_{t}^{*}$. We now need an expression telling us how the market clearing/full employment level of output, y_{t}^{*}, evolves over time. Here we will generalise the process used in McCallum (1989) so that y_{t}^{*} increases gradually through time but also depends positively on last period's full employment output level, which allows persistence of full employment output. If full employment output is high today, it is likely to be high tomorrow.

$$
y_{t}^{*}=\delta_{0}+\delta_{1} t+\delta_{2} y_{t-1}^{*}+u_{t}
$$

u_{t} is a zero mean, random supply, shock, which could include the discovery of oil reserves or other raw materials or a sudden technological breakthrough that can increase the productive capacity of the economy.

Activity 9.3 Consider expressions for y_{t}^{*} of the form, i. $y_{t}^{*}=\delta_{0}+\delta_{1} t+u_{t}$ and ii. $y_{t}^{*}=y_{t-1}^{*}+\delta_{0}+u_{t}$. What are the differences between these two processes? (Hint: what is the persistence of the shock, u_{t}, in both equations; how long does the shock last?)

(9.7), (9.9) and (9.10) together can be regarded as constituting aggregate supply. Along with aggregate demand, (9.6), we can solve for the output level, y_{t}, in terms of deviations from y_{t}^{*}, and also for the price level, p_{t}. The solution method is given in McCallum, Chapter 10, and you should read the relevant section in order to see how the solution is derived. However, a brief outline is given below.

Take expectations of (9.10) conditional on information available at date $t-1$:

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[y_{t}^{*}\right] & =\delta_{0}+\delta_{1} t+\delta_{2} y_{t-1}^{*} \\
\Rightarrow \mathrm{E}_{t-1}\left[y_{t}^{*}\right] & =y_{t}^{*}-u_{t} .
\end{aligned}
$$

Do the same for (9.8), noting that $\mathrm{E}_{t-1}\left[v_{t}\right]=0$:

$$
\mathrm{E}_{t-1}\left[y_{t}^{*}\right]=\beta_{0}+\beta_{1} \mathrm{E}_{t-1}\left[m_{t}-p_{t}^{*}\right]+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}^{*}\right]
$$

Equating (9.11) and (9.12), noting that $\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$ in (9.12) is equal to p_{t} (from (9.7)) gives:

$$
y_{t}^{*}=\beta_{0}+\beta_{1}\left(\mathrm{E}_{t-1}\left[m_{t}\right]-p_{t}\right)+\beta_{2}\left(\mathrm{E}_{t-1}\left[p_{t+1}\right]-p_{t}\right)+u_{t}
$$

If we subtract (9.13) from (9.6), we will have an expression for $y_{t}-y_{t}^{*}$ (i.e. deviations of output from the market clearing level).

$$
y_{t}-y_{t}^{*}=\beta_{1}\left(m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]\right)+v_{t}-u_{t}
$$

9. Keynesian models with money supply as a policy instrument

Monetary policy

Imagine that the money supply set by the authorities changes according to:

$$
m_{t}=\mu_{0}+\mu_{1} m_{t-1}+e_{t}
$$

where e_{t} is a zero mean, random error representing monetary policy shocks. Taking expectations of (9.15) conditional on information at date $t-1$ will give us an expression for the unexpected money supply at date $t, m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]$, which simply equals e_{t}.

Substituting this into (9.14) will give us a solution for the output deviation, $y_{t}-y_{t}^{*}$.

$$
y_{t}-y_{t}^{*}=\beta_{1} e_{t}+v_{t}-u_{t}
$$

Notice that the systematic component of monetary policy $\left(\mu_{0}+\mu_{1} m_{t-1}\right)$ does not have any real effects here. This is because at date $t-1$, when prices for date t are set, firms take into consideration what they expect the monetary authorities will do. If they expect the money supply to increase, knowing that money should have no real effects, they will increase their prices for date t accordingly. Only the random component of monetary policy, the monetary policy shock e_{t}, will have real effects since this is realised after the prices have been set. This result, that the systematic component of monetary policy has no real effect, will be discussed in more detail in the next section 'policy ineffectiveness proposition'.

Activity 9.4 What happens if the monetary authorities react to last period's output deviation, in other words, if we add $\mu_{2}\left(y_{t-1}-y_{t-1}^{*}\right)$ to the monetary policy rule (9.15)? Explain your results.

Why would the monetary authorities want to respond to lagged deviations in the first place?

The fact that unanticipated monetary policy, in the form of the shock, et, has real effects, is essentially because prices are fixed for one period. Prices are set at date $t-1$ for the market at date t. Any event occurring, relevant for the market at date t, after prices have been set will naturally be reflected in real variables such as output and employment. Now consider what happens when prices are set for two periods. The model of multiperiod pricing, again from McCallum, Chapter 10, will be used to analyse the effects of monetary policy in this setting.

\section{Multi-period pricing}
Let there be two types of firms, ' A ' firms and 'B' firms. Both set prices for two periods, but at different times. At date $t-2$ (based on the information available at that date) 'A' firms set, possibly different, prices for the market at dates $t-1$ and t. 'B' firms also set prices for two periods but do so a period later (i.e. at date $t-1$ they set prices for the market at dates t and $t+1)$. This is shown more clearly in Figure 9.3.

At date $t-2$, ' A ' firms set their prices (for the market at $t-1$ and t) at the level they expect will clear the market. ' B ' firms set their prices at date $t-1$ for t and $t+1$ similarly. Therefore, at date t, the price level will be the average of the prices set by 'A'

\section{'A' firms}
\begin{center}
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\end{center}

'B' firms

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-113(1)}
\end{center}

Figure 9.3:

firms and by ' B ' firms. This is just the average of prices set by ' A ' and ' B ', conditional on the information available at the time they were set.

$$
p_{t}=\frac{\mathrm{E}_{t-1}\left[p_{t}^{*}\right]+\mathrm{E}_{t-2}\left[p_{t}^{*}\right]}{2}
$$

To solve the model, we simplify the aggregate demand function by setting $\beta_{2}=0$, i.e. $y_{t}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}\right)+v_{t}$. Also, assume that the market clearing output level, y_{t}^{*}, follows the process given by $y_{t}^{*}=d_{0}+d_{1} t$, i.e. y_{t}^{*} is purely deterministic, subject to no shocks. Using the methods outlined above, aggregate demand will equal the market clearing level of output when the price is equal to that which clears the market, $p_{t}=p_{t}^{*}$.

$$
y_{t}^{*}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}^{*}\right)+v_{t}
$$

Taking expectations of (9.18) conditional on information available at $t-1$ and $t-2$ will give:

$$
\begin{aligned}
& \mathrm{E}_{t-1}\left[y_{t}^{*}\right]=\beta_{0}+\beta_{1} \mathrm{E}_{t-1}\left[m_{t}-p_{t}^{*}\right] \\
& \mathrm{E}_{t-2}\left[y_{t}^{*}\right]=\beta_{0}+\beta_{1} \mathrm{E}_{t-2}\left[m_{t}-p_{t}^{*}\right]
\end{aligned}
$$

Averaging these two expressions, noting that $\mathrm{E}_{t-1}\left[y_{t}^{*}\right]=\mathrm{E}_{t-2}\left[y_{t}^{*}\right]=y_{t}^{*}$ from the fact $y_{t}^{*}=\delta_{0}+\delta_{1} t$, gives us:

$$
y_{t}^{*}=\beta_{0}+\beta_{1} \cdot \frac{1}{2}\left(\mathrm{E}_{t-1}\left[m_{t}\right]+\mathrm{E}_{t-2}\left[m_{t}\right]\right)-\beta_{1} \cdot \frac{1}{2} \underbrace{\left(\mathrm{E}_{t-1}\left[p_{t}^{*}\right]+\mathrm{E}_{t-2}\left[p_{t}^{*}\right]\right)}_{p_{t}} .
$$

Subtracting (7.20) from the expression for aggregate demand results in:

$$
y_{t}-y_{t}^{*}=\beta_{1} \cdot \frac{1}{2}\left(m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]\right)+\beta_{1} \cdot \frac{1}{2}\left(m_{t}-\mathrm{E}_{t-2}\left[m_{t}\right]\right)+v_{t}
$$

Let monetary policy be the same as before:

$$
m_{t}=\mu_{0}+\mu_{1} m_{t-1}+e_{t}=\mu_{0}+\mu_{1}\left(\mu_{0}+\mu_{1} m_{t-2}+e_{t-1}\right)+e_{t} .
$$

Keynesian models with money supply as a policy instrument

In order to find an expression for $\mathrm{E}_{t-2}\left[m_{t}\right]$, for us to solve (7.), we need to write m_{t} in terms of variables that we know at date $t-2$. Hence we replace m_{t-1} by backward substitution. As before, we take expectations of m_{t} conditional on information available to us at $t-1$ and $t-2$ and we can find that:

$$
\begin{aligned}
$$

$$
\begin{aligned}
m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right] & =e_{t} \\
m_{t}-\mathrm{E}_{t-2}\left[m_{t}\right] & =\mu_{1} e_{t-1}+e_{t} .
\end{aligned}
$$

Substituting these expressions into (9.21) will give us our solution for $y_{t}-y_{t-1}^{*}$.

$$
y_{t}-y_{t}^{*}=\beta_{1} e_{t}+\frac{1}{2} \beta_{1} \mu_{1} e_{t-1}+v_{t}
$$

This is the main result of the assumption of multi (two)-period pricing. A shock to the supply of money, e_{t-1}, not only has real effects at date $t-1$, but also in the following period, t. The reason why it affects output at $t-1$ should be clear; prices for the market at date $t-1$ were set at $t-2$ ('A' firms) and $t-3$ ('B' firms) so a shock at date $t-1$ will cause output to change at $t-1$. However, once the shock has been realised, only 'B' firms can take this into account when they set period t prices (see Figure 7.3). 'A' firms cannot take e_{t-1} into consideration since their period t prices were set at date $t-2$. Since not all firms can change strategies after the realisation of new market conditions, the shock at date $t-1$ will have real effects at date t. With multi-period pricing, monetary shocks can have real and, more importantly, persistent effects. The more periods for which a firm sets its prices, the longer and more persistent any monetary shocks will be. Not only that, but also the output deviation, $y_{t}-y_{t}^{*}$, now depends on the parameters of the monetary policy rule, m_{1}. The systematic component of monetary policy now has real effects (by determining the persistence of any shocks) although the choice of the m parameters will not cause permanent deviations of output from y^{*}. Money, in the long run, is still neutral.

Predictable and unpredictable components of the money supply

The monetary authorities react to events in the economy; if inflation is high in one period, the authorities may reduce the money supply in an attempt to reduce the undesired inflation. Alternatively, the current money stock may be dependent on past output, unemployment, lagged money supplies, and so on. One can call the part of the money stock that depends on these variables the predictable component of monetary policy, since the monetary authorities change the stock of money 'systematically' with the observed economic variables known to them when they make their policy choice. The remainder of the money supply can be considered as a monetary shock, either originating from the authorities themselves or in the financial markets, if for example there was a change in the public's desired currency-deposit ratio that affects the money multiplier (see Chapter 4).

The money supply in the last chapter (Lucas misperceptions model) followed a process of the form:

$$
m_{t}=\underbrace{\mu_{0}+\mu_{1} m_{t-1}}_{\text {predictable component }}+\underbrace{e_{t}}_{\text { monetary shock }}
$$

Predictable and unpredictable components of the money supply

We assume that the monetary shock is purely random/white noise and so our best guess of it before it is realised, $\mathrm{E}_{t-1}\left[e_{t}\right]$, is its unconditional mean, zero. Under the assumption of rational expectations, the expectation of m_{t}, based on information available at date $t-1$, will be given by:

$$
\mathrm{E}_{t-1}\left[m_{t}\right]=\mu_{0}+\mu_{1} m_{t-1}
$$

For simple models, where the monetary authorities and the public share the same information sets, the expected money supply will simply be the predictable component and the unpredictable component of the m oney supply will be the monetary shock, e_{t}.

Policy ineffectiveness proposition (PIP)

As we saw in the previous chapter, monetary shocks have real effects in the economy while changes in the predictable component of monetary policy do not change output at all. This is because people can change their prices in anticipation of the change in policy. Whereas the model in the previous chapter assumed sticky prices in order for monetary shocks to have real (short-run) effects, one can easily use the Lucas supply curve model to obtain a similar result for flexible price economies.

Activity 9.5 Assume aggregate demand is given by $y_{t}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}\right)+v_{t}$ and aggregate supply is given by $y_{t}=y^{*}+\alpha\left(p_{t}-\mathrm{E}_{t-1}\left[p_{t}\right]\right)$. If monetary policy follows the rule of (9.25), show that the predictable component of monetary policy has no real effects.

(Hint: to answer this, equate AD and AS and bring all terms in p_{t} only over to the left hand side. Take expectations of this conditional on date $t-1$ information and hence obtain an expression for $p_{t}-\mathrm{E}_{t-1}\left[p_{t}\right]$ in terms of $m_{t}, \mathrm{E}_{t-1}\left[m_{t}\right]$, etc. Substitute this expression into the AS function and use (9.26) to solve for $y_{t}-y^{*}$.)

So even when prices are perfectly flexible, monetary shocks can have real effects, caused by the asymmetric information assumption of the Lucas model. The predictable component of monetary policy, showing how the authorities change the money supply depending on the state of the economy, has no effect at all on real variables. This is the policy ineffectiveness proposition (PIP). A policy of the form $m_{t}=\mu_{0}+e_{t}$ will have the same effect on the economy as when money depends on lagged money, inflation, output, and so on.

Rational expectations models and PIP

When proving that PIP holds in the sticky price and Lucas models, we assumed that agents had rational expectations when forming estimates of future variables. Hence the $m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]$ term which enters the output equation can be replaced by e_{t}, the monetary shock. It may appear that the only reason that PIP holds is because of the assumption of rational expectations. However, PIP does not hold in all rational expectations, RE, models. Consider the aggregate demand function in (9.27).

$$
y_{t}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}\right)+\beta_{2} \mathrm{E}_{t}\left[p_{t+1}-p_{t}\right]+v_{t}
$$

Keynesian models with money supply as a policy instrument

This is exactly the same as (9.6) in the sticky price model, only the expectation of inflation is made at date t, not $t-1$. When making similar assumptions as before, $p_{t}=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$ and $y_{t}^{*}=d_{0}+d_{1} t$, we can show that y_{t}^{*} can be given by:

$$
y_{t}^{*}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}^{*}\right)+\beta_{2} \mathrm{E}_{t}\left[p_{t+1}-p_{t}^{*}\right]+v_{t}
$$

Taking expectations of (9.28), conditioning on $t-1$ information, noting that $\mathrm{E}_{t-1}\left[y_{t}^{*}\right]=y_{t}^{*}$ and that $\mathrm{E}_{t-1}\left[\mathrm{E}_{t}\left[p_{t}^{*}\right]\right]=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$, from the law of iterative expectations (see for example, Branson (1989) Chapter 11), which equals p_{t}, then we obtain:

$$
y_{t}^{*}=\beta_{0}+\beta_{1}\left(\mathrm{E}_{t-1}\left[m_{t}\right]-p_{t}\right)+\beta_{2}\left(\mathrm{E}_{t-1}\left[p_{t+1}\right]-p_{t}\right) .
$$

Subtracting (9.29) from the aggregate demand equation, (9.27), will give us an expression for the output deviation, $y_{t}-y_{t}^{*}$.

$$
y_{t}-y_{t}^{*}=\beta_{1}\left(m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]\right)+\beta_{2}\left(\mathrm{E}_{t}\left[p_{t+1}\right]-\mathrm{E}_{t-1}\left[p_{t+1}\right]\right)+v_{t}
$$

The monetary shock term, $m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]$, enters as before, but a change in the predictable component of monetary policy could indeed alter people's expectations of tomorrow's price level. If a change in the predictable component of monetary policy causes a change in expectations (between dates $t-1$ and t) of p_{t+1}, then PIP no longer holds even though rational expectations are assumed. Monetary policy can then have real effects, but again only in the short run. Prices are set at date $t-1$ for date t and so can only contain information available up until date $t-1$. Aggregate demand, on the other hand, depends on the expectation of inflation made at date t (relevant for investment decisions through the Fisher equation). Firms, setting their prices at the level they expect to clear the market, have to make an expectation of this term but do so at date $t-1$. The fact that expected inflation can rise with new information means aggregate demand can be greater than firms initially anticipated, causing output to increase.

Lucas critique

The Lucas critique refers to the instability of reduced-form expressions used for policy making or policy appraisal. In the sticky price model of the previous section, the structural equations were given by the aggregate demand equation, the price equation, $p_{t}=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$, the equation showing how y_{t}^{*} evolves and the monetary policy reaction function showing how m_{t} depends on the state of the economy. When we solve for $y_{t}-y_{t}^{*}$, the equation we derive is a reduced-form equation; a mixture of aggregate demand, aggregate supply and the authorities' reaction function. For example, consider the sticky price model and the reduced-form expression, (9.16), $y_{t}-y_{t}^{*}=\beta_{1} e_{t}+v_{t}-u_{t}$. Instead of e_{t}, write $m_{t}=\mathrm{E}_{t-1}\left[m_{t}\right]=m_{t}-\mu_{0}-\mu_{1} m_{t-1}$.

$$
y_{t}-y_{t}^{*}=-\beta_{1} \mu_{0}+\beta_{1} m_{t}-\beta_{1} \mu_{1} m_{t-1}+\left(v_{t}-u_{t}\right)
$$

If this equation were given to an econometrician, he or she would run a regression of the form:

$$
y_{t}-y_{t}^{*}=\gamma_{0}+\gamma_{1} m_{t}+\gamma_{2} m_{t-1}+\eta_{1}
$$

and would find a positive coefficient for γ_{1} since we know $\gamma_{1}=\beta_{1}$ (only if the economy was accurately represented by the structural equations given above!). Since $\gamma_{1}>0$, we may then think that an increase in the money supply should cause an increase in output above y_{t}^{*}. For example if γ_{1} was found to equal 0.5 , then increasing the money supply by 2% should cause a 1% increase in output. However, as we saw in the earlier section, the predictable component of monetary policy had no real effects. Even though we may think an increase in the money supply will increase output (from the reduced form equation, (9.32)), in reality the change in people's expectations associated with this policy change will cause the reduced form to break down. If the authorities increased the money supply by increasing μ_{0}, indeed this will have a positive effect on output, via γ_{1}, but it will also have a negative effect on output since $\gamma_{0}=-\beta_{1} \mu_{0}$. The effect of an expansionary monetary policy will be purely inflationary. Quoting Romer (2001) p.251, 'If policy makers attempt to take advantage of statistical relationships, effects operating through expectations may cause the relationships to break down. This is the famous Lucas critique.'

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item explain the importance of nominal rigidities and why such sticky prices occur in the real world

  \item examine the effects of monetary policy in the short and long run when there are nominal rigidities

  \item describe the workings of the Phillips curve and augmented Phillips curve, noting the differences between the two

  \item build and work through models of sticky prices, examining the effects of monetary policy

  \item explain why multi-period pricing can lead to real and persistent effects of monetary policy

  \item discuss the merits of various sticky price models, noting, in particular, the different short-run and long-run effects of monetary policy.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
  \item 'The assumption of rational expectations implies that agents know that prices are flexible.' 9. Keynesian models with money supply as a policy instrument
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{1}
  \item If prices are sticky, what happens to the short-run Phillips curve after an increase in expected inflation?

  \item Why, in Taylor's relative price theory, can changes in money induce permanent deviations of output from its flexible price level?

  \item Suppose that recent economic indicators suggest that although output is growing at 3.5% per year, inflation is at 4% per year, up from 3% last year. If someone asks you to forecast the effect of this new inflation data, what would be your response? (Hint: make sure you specify what model you are using, what variables you are considering and what kind of real effects you forecast. Also mention if you expect any of the stylised facts of the business cycle to occur.)

  \item Suppose that you know that the full employment level of output in England is $y=100$. You observed that in 2001, inflation and output in England were 5% and $y=100$. In 2002, they were 3% and $y=96$. According to the theory of the Phillips curve, what can you say about expected inflation in 2001 and in 2002 ?

  \item Suppose a country with 400 people is set up so that everybody supplies 10 units of labour per peri od no matter how small or large the real wage. In this economy there are also 1000 competitive firms, eac h maximising their profit function: $\Pi=Y P-W N$, where Y is output, P is the price of output, W is t he nominal wage and N is the number of units of labour. Suppose each firm is identical in every respect and each has a production function: $Y=\sqrt{N}$.

\end{enumerate}

(a) Derive the demand for labour of one firm, its output and the aggregate demand for labour in the econo my.

(b) Draw on a graph the aggregate demand and supply of labour. Compute algebraically the equilibrium w age $(W / P)^{*}$ and equilibrium level of labour supply N^{*}.

(c) Suppose the aggregate demand for goods in the economy is given by $Y=\frac{1}{2}(M / P)$ and the money supply is equal to 100 . What is the aggregate supply function for goods in the economy? Calculat e Y^{*} and P^{*}.

(d) Suppose that the government increases once and for all the money supply. Does this improve the posi tion of the workers in the economy?

(e) Would your answer to part (d) change if we also assumed that nominal wages were rigid in the econo my? What would be your answer if prices were rigid? What does the AS curve look like in the case of rigid wages?

\subsection{Feedback to Sample examination questions}
\section{Section A}
\begin{enumerate}
  \item The statement is false. Suggested answers should define what rational expectations are. In particul ar, in rational expectations, people not only take all available information into account when they form thei r expectations, but their expectations are also consistent with the way in which the variables actually evolv e. Rational expectations are sometimes also known as 'model consistent' expectations. If prices are sticky , agents then know that prices are sticky.
\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{1}
  \item The augmented Phillips curve is shown in (9.3). Replacing actual inflation with expected inflation, si nce agents have to make expectations of this unknown quantity, gives (9.4). Since ζ is a negative f unction of unemployment, a rise in expected inflation will increase unemployment for any given wage gro wth. This implies that the short-run Phillips curve shifts out. If prices are sticky, then an increase in expect ed inflation will cause unemployment to rise above its natural (full employment) level, at least temporarily. In the long run, prices will adjust, shown by a higher inflation rate, and unemployment will move back to th e full employment level.

  \item (a) Substituting the production function into the profit equation gives us a choice problem for the fir m of

\end{enumerate}

$$
\max _{N} \Pi=\sqrt{N} P-W N
$$

Differentiating this with respect to N, setting equal to zero and solving, gives a value of $N=P^{2} /\left(4

W^{2}\right)$ per firm and since $Y=\sqrt{N}$, then output per firm is given by $Y=P / 2 W$. With 1000 identical firms, aggregate demand for labour in the economy, $N^{A D}=250 P^{2} / W^{2}$.

(b) If the 400 people in the economy each supply 10 units of labour, irrespective of the real wage, the labour supply is fixed at 4000. N^{*} therefore equals 4000 . Equating N^{*} with $N^{A D}$ and solving for the real wage gives $(W / P)^{*}=1 / 4$.

(c) Since labour supply is fixed at 4000, output produced in the economy will not be dependent on demand at all. With 1000 firms, this implies $N=4$ per firm and hence $Y=\sqrt{4}=2$. Aggregate supply is then fixed at $Y^{*}=2000$, (i.e. 1000 firms $\times 2)$. Equating aggregate demand with supply will allow us to calculate price, giving $P^{*}=0.025$.

(d) An increase in the money supply will shift the aggregate demand curve out and since the aggregate supply curve is fixed at $Y^{*}=2000$, output in the economy will not change. Equating Y^{*} with aggregate demand, we can see that the money supply and the price level will move one-for-one. Equating N^{*} $(=4000)$ with $N^{A D}$ shows us that W and P increase one-for-one so the change in money supply has no effect on real wages. The position of the workers is therefore not improved in the economy.

(e) With rigid nominal wages, the increase in the money supply will still shift the aggregate demand curve out, along the vertical aggregate supply curve. Output is not affected and again, prices move one-for-one with the money supply. With an increase in the price level but rigid nominal wages, workers are worse off since real wages, W / P, are now lower. With rigid prices, aggregate demand still shifts out but the price level does not change. One may think that output necessarily increases but since output is not

demand-determined in this economy, but is fixed at $Y^{*}=2000$, then neither price nor output change. Nothing happens in the labour market either and the aggregate supply curve is always vertical.

Chapter 10
New Keynesian models of monetary policy

Introduction

In the previous chapter, we analysed the effects of monetary policy which utilises monetary aggregates as the monetary policy instrument. However, modern macroeconomic research and actual monetary policy implementation suggest that the current policy instrument is the short-term interest rate controlled by the central bank via open market operations. Pure classical models suggest that money has no real effects at any horizon. If we introduce nominal and real frictions or relax some other key assumptions of the classical model, such as rational expectations, then monetary policy can have real effects in the short run. In this chapter we will examine the efficacy of a monetary instrument, the short-term interest rate, using a small scale new Keynesian macro model.

Aims

This chapter aims to introduce models with nominal (price) frictions where the monetary policy instrument are short-term interest rates. We find short-term real effects of monetary policy under certain conditions.

Learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- explain key ingredients of the New Keynesian macro-model
- describe the monetary policy reaction function or the Taylor rule
- analyse the impact of demand shocks on macroeconomic outcomes

\item analyse the impact of supply shocks on macroeconomic outcomes

\item discuss the relevance of financial frictions in such models. 10. New Keynesian models of monetary policy

\end{itemize}

\subsection{Reading advice}
This chapter follows on directly from Chapter 8 and Chapters 3 and 5 of Carlin and Soskice (2006) so you should familiarise yourself with the material presented in these chapters. You should read these while wo rking through this section.

The articles by Carlin and Soskice (2005) and by Taylor (1998) are also essential reading. Bernanke et al. (1999) discusses the role of financial accelerator in new Keynesian models.

\subsection{Essential reading}
\section{Books}
Carlin,W. and D. Soskice Macroeconomics: Imperfections, Institutions and Policies. (Oxford: Oxford Unive rsity Press, 2006) Chapters 3, 5 and 15.

\section{Journal articles}
Aksoy, Y. H.S. Basso, J. Coto-Martinez 'Lending relationships and monetary policy', Economic Inquiry 51 2013, pp.368-393.

Bernanke, B., M. Gertler and S. Gilchrist,'The financial accelerator in a quantitative business cycle framew ork', Chapter 21, in 'Handbook of Macroeconomics', Volume 1, Edited by J.B. Taylor and M. Woodford, 19 99, Elsevier.

Carlin, W. and D. Soskice 'The 3-equation New Keynesian model - A graphical exposition', BE Journals in Macroeconomics: Contributions, 5(1) 2005, pp.1-38.

Clarida, R., J. Gali and M. Gertler 'The science of monetary policy: A new Keynesian perspective', Journal of Economic Literature 37 1999, pp.1661-1707.

Clarida, R., J. Gali and M. Gertler 'Monetary policy rules and macroeconomic stability: evidence and some theory', Quarterly Journal of Economics 115(1) 2000, pp.147-80.

Kiyotaki, N. and J. Moore 'Credit cycles', Journal of Political Economy, 105 1997, $211-248$.

Kuttner, K. and T. Robinson 'Understanding the flattening Phillips curve', North American Journal of Econ omics and Finance, 21 2010, pp.110-125.

Taylor, J.B. 'Discretion versus policy rules in practice', Carnegie-Rochester Conference Series on Public Policy, 39 1993, pp.195-214.

Taylor, J.B. 'An historical analysis of monetary policy rules', National Bureau of Economic Research worki ng paper, w6768, (1998).

\subsection{Further reading}
Romer, D. Advanced Macroeconomics. (Boston; London: McGraw Hill, 2012, fourth edition), Chapter 7.

\subsection{IS-PC-MR model of new Keynesian economics}
Up to end of 1980's macro-monetary analysis was dominated by IS/LM models starting with a graphical a nalysis. Although standard Keynesian models were under constant attack from rational expectations econ omists led by Robert Lucas and Edward Prescott, among others. They argued that the theoretical foundati

ons of Keynesian models were weak even absent. It took a long time to bring in some microeconomic fou
ndations into Keynesian economics. In the last 20 years or so the focus turned into the Dynamic Stochasti
c General Equilibrium (DSGE) models with optimising behaviour of households, firms and sometimes poli
cy makers. This literature uses methodological foundations of the Real Business Cycle (RBC) research. A
t the same time it incorporates:

\begin{itemize}
  \item imperfect competition in the goods market

  \item nominal rigidities, that is, firms cannot adjust their prices continuously in the face of shocks and/or
nominal wage contracts cannot be changed continuously in the face of the shocks hitting the economy.

\end{itemize}

This literature, now labelled as the new Keynesian Macroeconomics, is still under construction. The model
 implies that the monetary policy conducted by means of short-term rates can be effective to stabilise infla
tion and real output in the short run. It is important to note that the policy is neutral long run. Thus classica
l dichotomy between real and nominal variables are still valid under new Keynesian assumptions.

In its simplest form these models have three building blocks:

\begin{itemize}
  \item an IS curve (IS)

  \item a New Keynesian Phillips curve (PC)

  \item an optimal monetary policy reaction function or a simple monetary policy rule (also known as the T
aylor rule) (MR).

\end{itemize}

In this chapter we will follow closely Carlin and Soskice $(2005,2006)$ when describing the key mechanis
ms of the model. Typically, the model develops a framework of forward looking households and firms and
therefore both IS and Phillips curves are forward looking, that is, expectations about future real output and
 inflation matter when setting optimal consumption and production. For the sake of simplicity, Carlin and S
oskice present a case with a backward looking IS and Phillips curves (that is, looking at what has happen
ed in the previous period when decisions are made). This is of course an important simplification and dep
arture from the standard analysis of New Keynesian models. Other than its usefulness for simplicity, back
ward looking IS and Phillips curves can be to some extent justified by the presence of consumers who for
m habits in their consumption decisions. That is consumers care about how much they consumed in the p
revious periods when they decide about their current consumption.

However, the policy maker, in other words, the central bank, is forward looking when deciding the level of
short-term interest rates by taking the Phillips and IS curves as constraints. Key assumptions are the pres
ence of nominal rigidities (wages and/or prices do not adjust instantaneously to shocks hitting the econom
y) and imperfect competition. 10. New Keynesian models of monetary policy

The IS curve is given as:

$$
y_{1}=A-a r_{0}
$$

where y_{1} is the actual real output in period $1, A$ is an autonomous expenditure variable and r_{0} is real interest rate set in period 0 .

The simplified Phillips curve is given as:

$$
\pi_{1}=\pi_{0}+\alpha\left(y_{1}-y_{e}\right)
$$

where π_{1} is the rate of inflation in period 1 , where π_{0} is the rate of inflation in the previous period. y_{e} describes the 'trend' or long-term output and is the level of output associated with constant rate of inflation. y_{e} reflects the level of real output associated with the structural features of the economy such as the degree of competition in the goods market and the nature of the labour markets. The presence of π_{0} is ad hoc and based on empirical observations of inflation persistence. Inflation persistence may be a reflection of backward looking expectations, lags in wage/price setting behaviour, consumption habits and other types of real/informational imperfections not discussed here.

The central bank minimises a loss function such that next period inflation $\left(\pi_{1}\right)$ is close to a target inflation level $\left(\pi^{T}\right)$ and next period real output $\left(y_{1}\right)$ is close to the 'trend' output $\left(y_{e}\right)$ subject to IS and PC equations.

$$
L=\beta\left(\pi_{1}-\pi^{T}\right)^{2}+\left(y_{1}-y_{e}\right)^{2}
$$

Note that the parameter β describes the importance of inflation against output gap for the central bank. The assumption that the central bank can control real rates ensures that the central bank uses forecasts of the future inflation rate when setting the policy instrument. We show the precise timing in the 3-equation model in Figure 10.1 (Carlin and Soskice $(2005,2006))$

To derive the optimal interest rate rule (or the monetary reaction function), Carlin and Soskice introduce a variable called the short-term interest rate relative to the natural real rate of interest r_{s} that the central bank chooses when the rate of inflation deviates from its target or the real output deviates from its trend output. Let us define the real rate of interest, the interest rate that would prevail when the economy functions at the full employment level and compatible with trend (or long-term) output as:

$$
y_{e}=A-a r_{s}
$$

Subtracting this expression from the IS curve, gives us the IS curve defined in terms of output gap, i.e. $y_{1}-y_{e}$.

$$
y_{1}-y_{e}=-a\left(r_{0}-r_{s}\right)
$$

The central bank minimises the loss by choosing the $\left(r_{0}-r_{s}\right)$. After substituting the IS and PC equations into the central bank loss function, the loss function becomes:

$$
L=\beta\left(\pi_{0}+\alpha\left(-a\left(r_{0}-r_{s}\right)\right)-\pi^{T}\right)^{2}+\left(-a\left(r_{0}-r_{s}\right)\right)^{2} .
$$

The central bank chooses $\left(r_{0}-r_{s}\right)$ optimally. Therefore first order condition with respect to $\left(r_{0}-r_{s}\right)$ is given by:

$$
\frac{\partial L}{\partial\left(r_{0}-r_{s}\right)}=2 \beta\left(\pi_{0}-a \alpha\left(r_{0}-r_{s}\right)-\pi^{T}\right) \cdot(\alpha(-a))+2\left(-a\left(r_{0}-r_{s}\right)(-a)\right)=0
$$

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-125}
\end{center}

Figure 10.1: Timing: Source (Carlin and Soskice (2005)).

Solving for $\left(r_{0}-r_{s}\right)$ yields:

$$
\left(r_{0}-r_{s}\right)=\frac{1}{a\left(\alpha+\frac{1}{\alpha \beta}\right)}\left(\pi_{0}-\pi^{T}\right)
$$

This is the interest rule (IR) as a response to deviations from target inflation and output gap. For $a=\alpha=\beta=1$ the rule becomes simply:

$$
\left(r_{0}-r_{s}\right)=0.5\left(\pi_{0}-\pi^{T}\right)
$$

Essentially, by setting the interest rates, the central bank needs to forecast inflation and output gap next period. Even though the central bank observes the shock in period 0 (we will define in the following sections), due to lagged effect of interest rates on aggregate demand it cannot fully offset the impact of the shock in the current period. According to the 'rule' interest rates need to increase whenever actual inflation exceeds target inflation and vice versa. In the following we present a graphical exposition how the demand-side stabilisation works with respect to demand or supply shocks (Carlin and Soskice, (2005)).

\subsection{Analysing demand shocks}
Following Carlin and Soskice (2005, 2006) Figure 10.2 exhibits the vertical (long-run) Phillips curve at the equilibrium output level, y_{e}, under imperfectly competitive product and labour markets so that the equilibrium output level is where both wageand price-setters make no attempt to change the prevailing real wage or relative prices and each Phillips curve is indexed by the pre-existing or the past rate of inflation, 10. New Keynesian models of monetary policy

$\pi^{I}=\pi_{1}$. The economy is in a constant inflation equilibrium at the output level of y_{e}; inflation is constant at the target rate of π^{T}. In the upper panel of Figure 10.2 is the goods market equilibrium characterised by the IS equation: the stabilising interest rate, r_{s}, will produce a level of aggregate demand equal to equilibrium (trend) output, y_{e}.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-126}
\end{center}

Figure 10.2: IS and Phillips curves: Source (Carlin and Soskice (2005)).

Figure 10.3 shows the case when the economy is subject to an aggregate demand (IS shock) shock when the target inflation $\left(\pi^{T}\right)$ is 2 per cent (Carlin and Soskice 2005)). A positive shock to demand will lead to a positive output gap above the trend output, which in turn leads to an increase in price inflation above the target level. This is shown in the lower panel of Figure 10.3. It shows that given the prevailing short-term rate, r_{s}, there will be a positive output gap $\left(y>y_{e}\right)$ and associated inflatio n rate will be $\pi_{0}>\pi^{T}$ (Point A). The central bank conducts the monetary policy by manipulating the interest rate r_{0}. As in Carlin and Soskice (2005), it does so by forecasting the Phillips curve for next period. Due to the past inflation, its forecast π^{I} will be 4 percent as shown by the dashed line in the PC diagram. Note that, all points on the forecast Phillips curve with inflation below 4 per cent are associated with a negative output gap $\left(y_{1}<y_{e}\right)$. It is clear that conducting disinflationary policies is costly in terms of real output output.[1]

How is the policy maker going to choose the r_{0} along the new (forecast) Phillips curve? This depends on the stabilisation preferences of the central bank. If the central bank cares more about inflation stabilisation relative the output gap, the higher the preference parameter, β, will be in its loss function. Given that the interest rate $\left(r_{0}-r_{s}\right)$ determines output gap which in turn determines the rate of inflation, the more inflation averse the central bank is, the more it is willing to sacrifice output (below y_{e}) to achieve its inflation objective. In Figure 10.3 the central bank will choose point B where the central bank indifference curve (loss function) and the forecast PC will be tangent. Therefore the new desired output level y_{1} is the aggregate demand target according to the optimal monetary policy rule. The monetary reaction function connects point B and the zero loss point at Z. The next step is to forecast the IS curve (IS') associated with desired output level y_{1}. The dashed line in the upper panel shows the central bank interest rate r_{0} in agreement with the new desired output to stabilise the inflation. Clearly, interest rate needs to increase.

In the absence of further IS shocks, the increase in the interest rate leads to a fall in output to y_{1} and inflation starts to decrease. The central bank repeats the steps above to achieve the target inflation. Essentially, both preferences of the central bank and the Phillips curve represent the inflation output trade-off the central bank faces. Given the policy instrument, the short-term rate, the central bank stabilises the economy through aggregate demand management following an IS (demand) shock.

\section{Temporary or permanent demand shocks}
According to our example the central bank needs to decide whether the demand shock is a temporary or permanent one. If the central bank believes that the shock is permanent (would persist for another period), central bank policy rate $r_{0}^{\prime}$ should be higher than the new stabilising interest rate, $r_{s}^{\prime}$. If the central bank's forecasts are such that the output will revert to its initial level, then it will increase the interest rate by less since the stabilising interest rate would have remained equal to r_{s} (i.e. its chosen interest rate would have been on the IS pre-shock curve in Figure 10.3 rather than on the IS curve).

\subsection{Analysing supply shocks}
Structural changes in the economy may lead to a shift in the trend output therefore a shift in the vertical (long-run) Phillips curve. Prominent examples of such structural changes are changes in the wage- or price-setting behaviour (curbing labour union power for instance), a change in tax regime or benefits or changes in the nature of the goods market competition such that price mark-up changes.

Suppose that the degree of competition intensifies which leads to a shift in the vertical Phillips curve to the right. Equilibrium output shifts from y_{e} to y_{e} associated with a

[1] It is worth mentioning that in other versions of the New Keynesian models demand shocks do not lead to a trade-off between output and inflation. The central bank can stabilise inflation by stabilising the output gap. (See, for instance, Clarida et al. (1999). 10. New Keynesian models of monetary policy

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-128}
\end{center}

Figure 10.3: New Keynesian model and demand shocks: Source (Carlin and Soskice $(2005))$.

stabilising interest rate r_{s} decline at Z'. Short-run Phillips curve corresponding to inflation equal to th e target (shown by the Phillips curve $\left.\left(\pi^{I}=2, y_{e}\right)\right)$ shifts to the right as well. Some observations are in order: first inflation falls from target inflation level of 2 percent to zero percent as the economy moves from equilibrium point A to B. Second, monetary policy maker should forecast the Phillips curve constraint (Phillips curve $\left.\left(\pi^{I}=0, y_{e}\right)\right)$ for the next period. Corresponding optimal level of output is shown by point C. To raise output to this level, it is necessary to cut the interest rate in period zero to r as shown in the IS diagram. The economy is then guided along the MR-AD curve to the new equilibrium at Z. The positive supply shock is associated initially with a fall in inflation and a rise in output - in contrast to the initial rise in both output and inflation in response to the aggregate demand shock. In other words, according to the model, a supply shock leads to countercyclical inflation, whereas an aggregate demand shock as in the previous section leads to a procyclical inflation.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-129}
\end{center}

Figure 10.4: New Keynesian model and supply shocks: Source (Carlin and Soskice (2005)).

\subsection{Financial accelerator models}
This section briefly discusses issues that may arise between banks and real economic activity. Banks are important for economic activity as they often provide much needed external funding for entrepreneurial activity. It has also been long recognised that credit-market conditions may themselves be the source factor depressing economic activity. Bernanke et al. (1999) study a new Keynesian model with a modification for financial intermediation process that captures credit market imperfections. There are several ways of doing this within the general equilibrium setting. One is to assume collateral constraints as in Kiyotaki and Moore (1997), or the existence of lending relationships as in Aksoy, Basso, Coto-Martinez (2013). Bernanke et al. assume asymmetric information. They formulate a model where asymmetries of information play a key role in borrower-lender relationships. In this model, financial contracts reflect the costs of gathering information about the quality of borrower's investment 10. New Keynesian models of monetary policy

project. Several problems can occur in credit markets that may lead to a worsening of informational asymmetries and increases in the associated agency costs and thereby lead to fewer projects get external financing. These then will have widespread real effects. [2]

In their model, there are three types of agents, called households, entrepreneurs, and retailers. Entrepreneurs play a key role in their model. These individuals are assumed to be risk-neutral and have finite horizons: Specifically, they assume that each entrepreneur has a constant probability of surviving to the next period (implying an expected lifetime of $1 /(1-\gamma))$. In each period t entrepreneurs acquire physical capital that is financed by either wealth (profits plus labour income) and borrowing that is subject to external finance premium. If the wealth or net worth is high, this allows for internal finance therefore entrepreneurs will be able to finance their investment rather cheaply, or equivalently if they seek external finance they can post these as collateral, therefore financing the project is less risky. As the external finance needs should be met via some form of lending contract and there are informational problems about the quality of the entrepreneurial project (agency problem - uncertain returns on capital that are subject to both aggregate and idiosyncratic risk), uncollateralised external finance should be more expensive than collateralised one.

$$
B_{t+1}^{j}=Q_{t} K_{t+1}^{j}-N_{t+1}^{j} .
$$

The expression above formalises the need to borrow from some credit institution. At time t, the entrepreneur who manages firm j purchases capital for use at $t+1$. The quantity of capital purchased is denoted K_{t+1}^{j}, with the subscript denoting the period in which the capital is actually used, and the superscript j denoting the firm. The price paid per unit of capital in period t is Q_{t}. Capital is homogeneous, and so it does not matter whether the capital the entrepreneur purchases is newly produced within th e period or is old, depreciated capital. At the end of period t (going into period $t+1$) entrepreneur j has an available net worth, N_{t+1}^{j}. To finance the difference between his expenditures on capital goods and his net worth he must borrow an amount B_{t+1}^{j}. The entrepreneur borrows from a financial intermediary that obtains its funds from households. The financial intermediary faces an opportunity cost of funds between t and $t+1$ equal to the economy's riskless gross rate of return, R_{t+1}. The riskless rate is the relevant opportunity cost because in the equilibrium, the intermediary holds a perfectly safe portfolio (it perfectly diversifies the idiosyncratic risk involved in lending). Because entrepreneurs are risk-neutral and households are riskaverse, the loan contract the intermediary signs has entrepreneurs absorb any aggregate risk. Lenders have to pay a fee (auditing, accounting, legal and so on) to verify the state of the entrepreneur, so they incur costs by lending. In equilibrium, Bernanke et al. show that for an entrepreneur who is not fully self-financed, the return to capital will be equated to the marginal cost of external finance, i.e.

$$
\mathrm{E}_{t}\left[R_{t+1}^{k}\right]=s\left(\frac{N_{t+1}^{j}}{Q_{t} K_{t+1}^{j}}\right) R_{t+1} .
$$

The ratio $s(\cdot)$ of the cost of external finance to the safe rate - which is the discounted return to capital but may be equally well interpreted as the external finance premium -

[2] Remember that Modigliani-Miller theorem implicitly states that the source of funding of a particular project does not matter for the outcome, be it by firm's internal resources using, for instance, equity or by external resources, for instance, using bank loans. When credit markets are characterised by asymmetric information, the Modigliani-Miller irrelevance theorem is violated. depends inversely on the share of the firm's capital investment that is financed by the entrepreneur's own net worth. It shows that capital expenditures by each firm are proportional to the net worth of the owner/entrepreneur, with a proportionality factor that is increasing in the expected discounted return to capital $s\left(\frac{N_{t+1}^{j}}{Q_{t}^{j} K_{t+1}^{j}}\right)$. A high s reduces default probability and entrepreneurs can take on more debt and expand the size of the firm, thus economy expands, and vice versa. In other words, financial (or equivalently credit) constraints can lead to stronger downturns than when these constraints are absent; hence it works as a financial accelerator.

Simple monetary policy rules

We conclude this chapter by a brief discussion of the simple policy rules, that is where monetary authorities implementing policy according to a rule or formula that is chosen to be applicable over a large number of periods. These are simple rules thus by definition not reflecting optimal choice of the monetary policy instrument given the state of the economy as done in the previous sections. Despite reducing the inflation bias which we will discuss in the chapter, possibly to zero, depending on the rule, such policy will not be able to counter the productivity shocks that hit the economy. If the economy is prone to productivity shocks so its variance of was large, it may be desirable for the monetary authorities to lean against the wind using expansionary monetary policy in a time of negative productivity shocks, for example. However, to do so will require discretion on the part of the authorities, but this will naturally lead to the inflation bias. In the argument of rules versus discretion we have to weigh up the cost of using discretion (the inflation bias) with the cost of using a rule (inability to counter productivity shocks). [3] Two possible rule scenarios are discussed next.

Taylor rules

Taylor rules are simple monetary policy rules that prescribe how a central bank should adjust its interest rate policy instrument in a systematic manner in response to developments in inflation and macroeconomic activity. Following a rule is transparent. By committing to follow a rule, policy makers can communicate and explain their policy actions easily and should at least in principle enhance the accountability and credibility of the central bank.

In Taylor (1998) an interest rate rule of the form:

$$
R_{t}=\pi_{t}+g\left(y_{t}-y^{*}\right)+h\left(\pi_{t}-\pi^{*}\right)+r^{*}
$$

was argued to be a valid representation of how nominal interest rates respond to economic variables for a number of different monetary regimes. R_{t} is the nominal interest rate, π_{t} is inflation, π^{*} is the target level of inflation, $y_{t}-y^{*}$ is output deviations and r^{*} is the estimate of the real interest rate. (10.12) can be rewritten, by collecting terms together to obtain:

$$
R_{t}=\left(r^{*}-h \pi^{*}\right)+g\left(y_{t}-y^{*}\right)+(1+h) \pi_{t}
$$

[3] Note that next chapter discusses inflation bias problem in detail. 10. New Keynesian models of monetary policy

When fitting this model to US data from 1987, quarter 1, to 1997, quarter 3 , the g coefficient was found to be 0.765 and the coefficient on inflation was found to be 1.533 . The fact that these are positive, and for the case of the coefficient on inflation, greater than unity, is important for the stability of the economy.

Activity 10.1 Consider (10.12). If inflation was equal to the desired/target rate of inflation and output deviations were zero $\left(y_{t}-y^{*}\right)$, to what does the equation collapse?

Consider a fall in demand that causes output to fall below y^{*} with no immediate impact on inflation. A positive g coefficient implies that nominal interest rates set by the monetary authorities should fall, since $y_{t}-y^{*}$ is now negative. With no change in inflation, the fall in nominal rates will decrease real rates and so encourage investment spending and aggregate demand. Output should then return to the full employment level. Now consider the case where inflation has increased above the target rate. If the coefficient on inflation in (11.1) is greater than unity, then the increase in nominal interest rates will be greater than the increase in inflation. This causes real interest rates to increase, leading to reduced investment spending and so reduces the

inflationary pressure in the economy. With positive coefficients for g and h (causing $1+h$, the coefficient on inflation in (11.1), to be greater than unity) we should see stability in the economy, with output tending to be at its full employment level and inflation staying close to the target rate. If the coefficient on inflation in (11.1) was found to be less than one $(h<0)$, then an increase in inflation will be met by a less than one-for-one rise in nominal rates. This will cause real rates to fall, encouraging investment and aggregate demand that will cause further inflationary pressure. The economic system in this case would be unstable.

Friedman's rule of constant money growth

Due to the long and variable lags associated with the formulation and implementation of monetary policy, Milton Friedman suggested that trying to control the economy through changing monetary variables, would purely lead to greater instability. Consider Figure 10.5. At date t_{0} a shock occurs that causes output to start to fall. The first recorded data of such an event may become available at date t_{1}. However, the authorities need more than just one data observation before changing their policy. After all, the data reading could be a blip or involve considerable measurement error. At date t_{2} enough data are available for the authorities to determine that a downturn is indeed happening and the decision is made to implement an expansionary monetary policy. Since it takes some time for the monetary policy decision to take effect, output is only affected at date t_{3}, but this is the point when the economy is starting to recover. The monetary expansion at a time when the economy is naturally starting to accelerate could lead to a more volatile path for output. Hence, Friedman suggested monetary policy should not be used to combat output fluctuations, not because of neutrality arguments but because of the long and variable lags involved with such an activist policy.

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-133}
\end{center}

Figure 10.5:

\subsection{A reminder of your learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item explain key ingredients of the New Keynesian macro-model

  \item describe the monetary policy reaction function or the Taylor rule

  \item analyse the impact of demand shocks on macroeconomic outcomes

  \item analyse the impact of supply shocks on macroeconomic outcomes

  \item discuss the relevance of financial frictions in such models. 10. New Keynesian models of monetary policy

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
  \item 'Monetary policy is effective in a benchmark New Keynesian model.'

  \item 'Monetary policy is still effective in a New Keynesian model with financial accelerators.'

\end{enumerate}

\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{2}
  \item Consider a New Keynesian macro model with backward looking inflation in the Phillips curve. Derive the optimal reaction function of the policy maker that aims to stabilise inflation. a). Suppose the economy is subject to a temporary demand shock. How should the policy maker stabilise the economy? Discuss by means of diagrams. b). Do your results change when the economy is subject to a supply shock?
\end{enumerate}

\subsection{Feedback to Sample examination questions}
\section{Section A}
\begin{enumerate}
  \item The statement is true in the short run. Suggested answers should give examples of effective monetary policymaking in the presence of nominal and real rigidities. The model outlined in this chapter shows that the persistence of demand/supply shocks matters in the determination of central bank forecasts about future Philips curve and following optimal policy choice.
\end{enumerate}

\section{Section B}

\begin{enumerate}
\setcounter{enumi}{1}
\item Students should discuss the effectiveness of monetary policy in a macroeconomic environment as discussed in the subject guide and in Carlin and Soskice (2005).
\end{enumerate}

\section{Chapter 11 Time inconsistency and inflation bias}
\subsection{Introduction}
As discussed in Chapters 5 and 6 there is widespread belief among economists that inflation is largely a monetary phenomenon. If monetary variations lead to inflation variations, and if there is a positive statistical relationship between inflation and real output (or employment) in the short run, governments or politically motivated monetary policy makers may be tempted to exploit these short run relationships in order to achieve different objectives. These may range from achieving temporary high real output, reducing unemployment, financing budget deficits and even attaining balance of payments objectives by generating surprise inflation. While such outcomes may be feasible in the short-term, inflationary consequences distort economic agents incentives in the medium to long run. What is more, economic agents learn the type of policy makers over time such that they can not be easily fooled. Once consumers, firms and other agents in the economy understand the incentives of policy makers to create surprise inflation, they adjust their inflation expectations accordingly such that even in the short run there can be no output gains to achieve. Thus, we will discuss the so called inflation bias and ways reduce this bias. A simple game theoretic framework will allow us to analyse the importance of central banks credibility in the combat of inflation.

\subsection{Aims}
This chapter aims to discuss strategic interactions between policy makers and economic agents which may differ in their incentives with ultimate welfare deterioration for the society. We also study how to resolve these conflicts of interests.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
\item describe the causes of 'inflation bias' in an economy

\item distinguish between 'wet' and 'hard nosed' central banks

\item outline institutional and behavioural ways inflation bias can be reduced or avoided

\item describe the workings of the Taylor rule and give conditions needed for stability. 11. Time inconsistency and inflation bias

\end{itemize}

\subsection{Reading advice}
There is no single textbook that covers the material in this chapter but the most appropriate are those by Carlin and Soskice (2006) and Goodhart (1989) and you should read these while working through this section.

The articles by Carlin and Soskice (2005), Barro and Gordon (1983), Kydland and Prescott (1977) are also essential reading. The Goodhart article gives a more historical account of monetary policy, especially in the UK and US and the article by Orphanides (2007) provides a neat overview of Taylor rules.

\subsection{Essential reading}
Barro, R.J. and D.B. Gordon 'A positive theory of monetary policy in a natural rate model', Journal of Political Economy 91(3) 1983, pp.589-610.

Carlin, W. and D. Soskice 'The 3-equation New Keynesian model - A graphical exposition', BE Journals in Macroeconomics: Contributions, 5(1) 2005, pp.1-38.

Goodhart, C.A.E. Money, Information and Uncertainty. (London: Macmillan, 1989) Chapters 14 and 15.

Kydland, F.E. and E.C. Prescott 'Rules rather than discretion: the inconsistency of optimal plans', Journal of Political Economy 85(3)1977, pp.473-492.

Orphanides, A. 'Taylor rules', Finance and Economics Discussion Series: Divisions of Research Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C., $200718,(2007)$.

(\href{http://www.federalreserve.gov/Pubs/FEDS/2007/200718/200718pap.pdf}{http://www.federalreserve.gov/Pubs/FEDS/2007/200718/200718pap.pdf})

Taylor, J.B. 'An historical analysis of monetary policy rules', National Bureau of Economic Research working paper, w6768, (1998).(\href{http://papers.nber.org/papers/W6768}{http://papers.nber.org/papers/W6768})

\subsection{Further reading}
\section{Books}
Carlin,W. and D. Soskice Macroeconomics: Imperfections, Institutions and Policies. (Oxford: Oxford University Press, 2006) Chapter 6.

\section{Journal articles}
Clarida, R., J. Gali and M. Gertler 'Monetary policy rules and macroeconomic stability: evidence and some theory', Quarterly Journal of Economics 115(1) 2000, pp.147-80.

Goodhart, C.A.E. 'The conduct of monetary policy', Economic Journal 99(396) 1989, pp.293-346. Rogoff, K. 'The optimal degree of commitment to an intermediate monetary target', Quarterly Journal of Economics 100(4) 1985, pp.1169-1189.

Sargent, T.J. and N. Wallace 'Rational expectations, the optimal monetary instrument and the optimal money supply rule', Journal of Political Economy 83(1) 1975, pp. $241-54$.

Walsh, C.E. 'Is New Zealand's Reserve Bank Act of 1989 an optimal Central Bank contract?', Journal of Money, Credit and Banking 27(4, Part 1) 1995, pp.1179-191.

\subsection{Time inconsistency and inflation bias}
Throughout this subject guide we have stressed that money should be neutral in the long run but, using models such as the expectations-augmented Phillips curve or the Lucas supply curve, money can have real effects in the short run. Here, we will use such a framework to show that in equilibrium, a positive rate of inflation may be present even though a zero rate may be optimal. If there are no institutional or behaviourial restrictions, the monetary authorities have an incentive to increase inflation (when inflation is zero) in an attempt to move along the Phillips curve to increase output. An 'inflation bias' may then result in the economy. Let us call such policy makers as being 'wet' and those policy makers who always aim for target inflation as being 'hard nosed'.

Consider for example, the aggregate supply curve in (11.1). This is the same as an expectations-augmented Phillips curve or a Lucas supply curve, only we use inflation, π_{t}, instead of the price level.

$$
y_{t}=y^{*}+\alpha\left(\pi_{t}-\mathrm{E}_{t-1}\left[\pi_{t}\right]\right)+\varepsilon_{t}
$$

y^{*} is the market clearing level of output and ε_{t} is a productivity shock term where its mean is set equal to zero. Suppose the monetary authorities can choose the inflation rate directly and try to minimise the following loss function:

$$
L=\pi_{t}^{2}+\lambda\left(y_{t}-k y^{*}\right)^{2} \quad \text { where } k>1
$$

λ is a preference parameter showing the authorities' preference for output fluctuations relative to inflation fluctuations. We assume that $k>1$ so that a level of output greater than the market clearing level is desired reflecting political biases towards higher output. If output is at the market clearing level, a social loss is still incurred, $\left(y^{*}-k y^{*}\right)^{2}>0$, since the socially optimum level of output may be greater than the market clearing level. This may be due to the fact y^{*} is too low because of the monopoly powers of firms, for example, which causes output levels to be sub-optimal and hence unemployment to be too high. This is shown in Figure 11.1.

Initially, we are at the market clearing level of output and have zero inflation, point A. If output equals its market clearing level in the long term, this point is the best (welfare-maximising) that can be achieved. However, the monetary authorities know that by increasing inflation unexpectedly, we can move along the Phillips curve to point B, obtaining a higher level of welfare. At point B the inflation rate is higher than expected since $\mathrm{E}[\pi]=0$. Expected inflation therefore increases and the Phillips curve shifts up. The equilibrium compatible with expected inflation, point E, is the point where the Phillips curve is tangential to the indifference curve. At this point the 11. Time inconsistency and inflation bias

\begin{center}
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\end{center}

Figure 11.1:

authorities have no incentive to increase inflation since to do so will result in lower social welfare. Note that at point E, we have a positive rate of inflation, the inflation bias, and we are on a lower level of social welfare than at point A, our starting point.

To calculate the inflation bias, substitute the Phillips curve into the loss function, (11.2).

$$
L=\pi_{t}^{2}+\lambda\left((1-k) y^{*}+\alpha\left(\pi_{t}-\mathrm{E}_{t-1}\left[\pi_{t}\right]\right)+\varepsilon_{t}\right)^{2}
$$

Differentiating this with respect to the choice variable, π, taking expectations of inflation as given, setting equal to zero and solving for π_{t} will give us:

$$
\left(1+\alpha^{2} \lambda\right) \pi_{t}=\alpha^{2} \lambda \mathrm{E}_{t-1}\left[\pi_{t}\right]+\alpha \lambda\left((k-1) y^{*}-\varepsilon_{t}\right)
$$

Taking expectations of (11.4) conditional on information at date $t-1$, noting that $\mathrm{E}_{t-1}\left[\varepsilon_{t}\right]=0$, will give a solution for $\mathrm{E}_{t-1}\left[\pi_{t}\right]$. Substituting back into (11.1) will give a solution for inflation.

$$

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[\pi_{t}\right] & =\alpha \lambda(k-1)=y^{*} \\
\pi_{t} & =\alpha \lambda(k-1) y^{*}-\frac{\alpha \lambda \varepsilon_{t}}{1+\alpha^{2} \lambda}
\end{aligned}
$$

The inflation bias is given by $\mathrm{E}_{t-1}\left[\pi_{t}\right]=\alpha \lambda(k-1) y^{*}$ and represents the vertical distance AE in Figure 11.1. Note that when parameter $k=0$, there is no inflation bias as we have seen in the previous chapter.

Inflation aversion, steepness of the Phillips curve and interest rates

There are three elements that determine the level of inflation bias.

- the political bias towards higher output parameter $(k>1)$
- the degree of central bank inflation aversion in the loss function (λ)
- the responsiveness of inflation to output in the Phillips curve (α).

The parameter λ in the central bank's loss function (equation (11.2) governs the inflation aversion of the central bank. If λ is equal to 1 , indifference curves in Figure 11.2a. are circles and the central bank assigns equal weight to inflation and real output in the loss function. In alternative scenarios where λ is not equal to 1 , indifference curves are elliptical. If λ is less than 1 , the central bank is inflation averse (hard nosed). Finally, if λ is more than 1 , the central bank is real output (or unemployment) averse (wet). These scenarios are depicted in Figure 11.2.

Figures $11.2 \mathrm{~b}$ and $11.2 \mathrm{c}$ show differences between central bank responses. Given a Phillips curve, Figure 11.2b shows that when current inflation deviates from a target inflation level, the central bank will respond more aggressively to stabilise inflation back to its target level and Figure 11.2c shows that the central bank's response will be muted when inflation deviates from its target level since the central bank is more concerned with real output.

\begin{center}
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\end{center}

(a) Balanced $\beta=1$

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\end{center}

(b) Inflation averse $\beta>1$

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-139(1)}
\end{center}

(c) Unemployment averse $\beta<1$

Figure 11.2:

The parameter α measures the slope of the Phillips curve or alternatively, the responsiveness of in flation to changes in output. The higher the parameter α, the more responsive is inflation to changes in output. Given the central bank preferences, a steeper Phillips curve implies less aggressive policy response when the inflation deviates from the target level. A steeper Phillips curve allows the central bank to do less in response to an inflation shock since inflation responds strongly to a fall in output 11. Time inconsistency and inflation bias

associated with a tight monetary policy. For more discussion on the matter, see Carlin and Soskice (2005).

We will now outline a number of ways in which this inflation bias can be reduced/negated.

Ways inflation bias can be reduced

Delegation of monetary policy to a conservative central bank: central bank independence

If the role of monetary policy was delegated to such a conservative central bank, which is less concerned with output variations than the society, in other words, has a loss function of the form in (11.2) where $\mu<\lambda$:

$$
L^{C B}=\pi_{t}^{2}+\mu\left(y_{t}-k y^{*}\right)^{2}
$$

then we will still be faced with an inflation bias but the bias will be of the form $\alpha \mu(k-1) y^{*}$ and since $\mu<\lambda$, the inflation bias will be lower.

One way to formalise a conservative central bank is to establish an independent central bank with a formal price stability mandate. The popularity of independent central banks during the 1990 s can be seen as an attempt to separate monetary policymaking from political interference. Establishing an independent central bank, of course, is not sufficient to guarantee price stability if price stability is not part of their mandate. In order to ensure their commitment to price stability, most central banks in advanced economies recently adopted the so called 'inflation targeting policy'. This typically takes the form of a targeting policy with an explicit inflation target around two percent to tie down inflation expectations. While in some advanced economies, notably in the case of the US Federal Reserve, there is no formal central bank independence, historical practice suggests that there is no political pressure. Also, there were regular announcements by Fed officials about the desired level of inflation. In 2012, the than Chairman of the Federal Reserve, Ben Bernanke announced an inflation target of 2 percent departing from the non-committal stance of the Federal Reserve. Interestingly, in the aftermath of the global financial crisis in September 2008 there has been serious questioning of such inflation targeting policies in particular at times of high unemployment and weak output performance. A renewed scrutiny of the role of monetary policy other than price stability (such as unemployment and real output) in an economy has started and the debate is as yet unsettled. (See for instance writings of Paul Krugman in popular press.) One possible alternative to inflation targeting is the so called 'nominal income targeting' as such policy captures both inflation and real output at the same time.

Inflation contract for the central bank

Suppose the central bank was penalised, by way of reduced salary, for allowing any inflation above the socially desired level, set in this case at zero. The loss function that would be minimised would then be:

$$
L^{\text {Contract }}=\pi_{t}^{2}+\lambda\left(y_{t}-k y^{*}\right)^{2}+\Psi \pi_{t}
$$

So that the central bank's loss depends on the level of inflation through the Ψ parameter as well as squared inflation and squared output deviations. If we substitute the Phillips curve into (11.7) and minimise the loss function by choosing π_{t} (using the same method as above), it can be shown that the inflation bias, $\mathrm{E}_{t-1}\left[\pi_{t}\right]$, is given by:

$$
\mathrm{E}_{t-1}\left[\pi_{t}\right]=\alpha \lambda(k-1) y^{*}-\frac{\Psi}{2}
$$

Therefore, if the contract was written for the central bank such that $\Psi=2 \alpha \lambda(k-1) y^{*}$, then the inflation bias would be zero and one would achieve the point of highest attainable welfare, point A in Figure 11.1, as a time-consistent equilibrium.

Reputation

Policy credibility is at the core of monetary policymaking. A 'hard nosed' central bank cares only about inflation, whereas a 'wet' central bank attempts to exploit short-term relationships. However, even a 'wet' central bank can avoid inflation bias problem by establishing a reputation that it is serious about inflation. Such reputation can only be achieved in a multi period game between sceptical wage setters and the policy maker. Carlin and Soskice (2006) illustrate the point by using a two period game. Let us suppose that in period 1 , the society chooses an expected inflation $\left(\pi_{1}^{E}\right)$ with the knowledge that there is a certain probability, p, that the central bank is serious about inflation. Equipped with this knowledge, the central bank chooses output at, say, y_{1}. In period 2 the society chooses $\left(\pi_{2}^{E}\right)$ knowing how the central bank behaved in period 1 (that is they observe $\left.y_{1}\right)$; the central bank chooses y_{2} knowing $\left(\pi_{2}^{E}\right)$, and so on. In this scenario, even a 'wet' central bank will behave in period 1 as if it is 'hard nosed' about inflation. This is because by behaving tough on inflation in period 1, inflation will be on target and output on equilibrium. In period 2 (terminal period), however, the central can set output above the equilibrium by creating surprise inflation. When the reasoning extended to many periods, it pays off for the central bank to build a reputation of being tough on inflation. The 'wet' central bank would behave as if it is a 'hard nosed' central bank in all periods except the very last one of the game. The model provides a justification for the incentives to build a reputation of inflation toughness when the society is sceptical about the 'nature' of the central bank.

A reminder of your learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- describe the causes of 'inflation bias' in an economy
- distinguish between 'wet' and 'hard nosed' central banks
- outline institutional and behavioral ways inflation bias can be reduced or avoided
- describe the workings of the Taylor rule and give conditions needed for stability. 11. Time inconsistency and inflation bias

Sample examination questions

Section A

Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

1. 'Monetary policy makers only care about inflation.'

Section B

2. Discuss the main arguments in the debate rules versus discretion.

3. What is the Taylor rule and why are the size and sign of its parameters important for economic stability?

4. Country A's monetary policy rule has been estimated to be:

$$
R_{t}=0.6+1.6\left(\pi_{t}-\pi^{*}\right)+0.4\left(y_{t}-y^{*}\right)
$$

and country B's monetary policy rule is described by:

$$
R_{t}-0.6+0.5\left(\pi_{t}-\pi^{*}\right)+0.4\left(y_{t}-y^{*}\right)
$$

where π is inflation, y is output, π^{*} is the inflation target and y^{*} is equilibrium output. The standard deviation of the output gap in country A is 0.48 and in country B it is 1.05 .

Explain why, although the coefficient on the output gap is the same in the two countries, the variance of output is lower in country A.

5. Suppose the central bank was penalised, by way of reduced salary, for allowing any inflation above the socially desired level, set in this case at zero. The loss function will then be:

$$
L^{\text {Contract }}=\pi_{t}^{2}+\lambda\left(y_{t}-k y^{*}\right)^{2}+\Psi \pi_{t}
$$

Prove that under this loss function, the inflation bias will be:

$$
\mathrm{E}_{t-1}\left[\pi_{t}\right]=\alpha \lambda(k-1) y^{*}-\frac{\Psi}{2}
$$

Feedback to Sample examination questions

Section A

1. False. Students should answer this by providing a brief discussion on inflation bias (the political bias towards a higher output) and how to resolve the problem.

Section B

2. To answer this question we can follow the procedure used earlier in this chapter. Equate aggregate demand and aggregate supply and collect terms in p_{t} to obtain

$$
\left(\alpha+\beta_{1}\right) p_{t}=\beta_{0}+\beta_{1} m_{t}+v_{t}-y^{*}+\alpha \mathrm{E}_{t-1}\left[p_{t}\right]
$$

Take expectations conditional on date $t-1$ information, noting that $\mathrm{E}_{t-1}\left[v_{t}\right] \neq 0$. Subtract this from the above expression to obtain an expression for $p_{t}-\mathrm{E}_{t-1}\left[p_{t}\right]$. Substituting this into the Lucas supply curve and noting that $m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]=e_{t}$ and $v_{t}-\mathrm{E}_{t-1}\left[v_{t}\right]=\varepsilon_{t}$, gives us:

$$
y_{t}=y^{*}+\frac{\alpha}{\alpha+\beta_{1}}\left(\beta_{1} e_{t}+\varepsilon_{t}\right)
$$

The systematic component of monetary policy does not enter this expression and therefore, even though the monetary authorities react to a series of shocks in the economy, monetary policy is still ineffective. The expression for the output gap, $y_{t}-y^{*}$ can be written as:

$$
y_{t}=y^{*}+\frac{\alpha}{\alpha+\beta_{1}}\left(\beta_{1}\left(m_{t}-\mu_{0}-\mu_{1} v_{t-1}\right)+\varepsilon_{t}\right)
$$

simply by replacing e_{t}. This can also be written as

$$
y_{t}=y^{*}+\gamma_{0}+\gamma_{1} m_{t}+\eta_{t}
$$

From this equation, we may think that there is a positive relationship between money and output since the coefficient on mt is positive. This is not true, however. See the section on the Lucas critique in this chapter.

6. The variance of output in country A is lower than that in country B because the coefficient on inflation in the Taylor rule is greater than unity, while in country B it is less than 1. In country A, if inflation increases by 1%, due to an overheating economy for example, the nominal interest rate increases by 1.6%, so causing real rates to rise by 0.6%. This reduces the inflationary pressure in the economy by dampening demand. In country B, a similar rise in inflation leads to a rise in nominal rates of only 0.5%. This implies real rates fall, so increasing demand in the economy when it is already overheating. If output increases when it is already too high, this implies output is not stable, explaining the greater variance of output in country B. 11. Time inconsistency and inflation bias

Chapter 12
Monetary policy and data/parameter uncertainties

Introduction

Throughout the Guide we assumed that:

- the central bank is assumed to know the true model of the economy,

\item the central bank observes accurately all relevant variables timely and accurately,

\item the central bank knows sources and properties of economic disturbances.

\end{itemize}

For instance, in Chapter 9, we discussed New Keynesian model of monetary economics. Depending on th e nature of the shock (permanent or transitory, demand or supply) the central bank can change the intere st rates as soon as the shock is observed. There is no role for caution in this setting.

In practice, conducting monetary policy is very difficult. There is tremendous uncertainty about the true str ucture of the economy, the impact policy actions have on the economy, and even about the state of the ec onomy. Following quote from a prominent former central banker, Alan Blinder (1995), who was speaking a t a meeting in Minnesota best describes the challenges a central banker faces:

'Unfortunately actually to use such a strategy in practice you have to use forecasts knowing that they may be wrong. You have to base your thinking on some kind of monetary theory even though that theory migh t be wrong. And you have to attach numbers to that theory knowing that your numbers might be wrong. W e at the Fed have all these fallible tools and no choice but to use them.' Blinder continues to suggest the f ollowing 'What can you do to try to guard against failure? First of all be cautious. Dont oversteer the ship. I f you yank the steering wheel really hard a year later you may find yourself on the rocks.'

Blinder suggests to exercise caution in monetary policymaking. This effectively translates into a smooth a djustment of interest rates. The reasons for smooth adjustment of interest rates can be due to:

\begin{itemize}
\item data uncertainty (alternatively, additive uncertainty),

\item parameter uncertainty (alternatively, multiplicative uncertainty). 12. Monetary policy and data/para meter uncertainties

\end{itemize}

\subsection{Aims}
This chapter aims to introduce two major challenges, data and parameter uncertainty, that policy makers will face in deciding the monetary policy.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able t o:

\begin{itemize}
\item Discuss the importance of additive uncertainty

\item Distinguish between news and noise in data revisions

\end{itemize}

■ Discuss the choice of monetary policy instruments under additive uncertainty

\begin{itemize}
\item Discuss the importance of parameter uncertainty

\item Explain the concept of certainty equivalence.

\item Explain optimal policy conduct when the economy subject to parameter uncertainty.

\end{itemize}

\subsection{Reading advice}
This chapter follows the work by Aruoba (2008) for discussions of data uncertainty and Brainard (1967) fo
r discussions of parameter uncertainty. For empirical evidence see Aruoba (2008) and Sack (2000). For o
ptimal policy instrument choice under data uncertainty you should read Poole (1970).

\subsection{Essential reading}
Aruoba, B. 'Data revisions are not well-behaved', Journal of Money, Credit and Banking, 40(2-3) 2008, pp.
319-340.

Brainard, W. 'Uncertainty and effectiveness of policy', American Economic Review (papers and proceedin
gs), 57 (2) 1967, pp.411-425.

Poole, W. 'Optimal choice of monetary policy instrument in a simple stochastic macro model', Quarterly Jo
urnal of Economics, 84 (2) 1970, pp.197-216.

\subsection{Further reading}
Croushore, D. and T. Stark, 'A real time dataset for macroeconomists: does the data vintage matter?' Revi
ew of Economics and Statistics, 2003 pp.605-17. Sack, B. 'Does the Fed act gradually? A VAR analysis' J
ournal of Monetary Economics, 462000, pp.229-256.

\subsection{Data uncertainty}
Economic agents (policy makers, financial agents, firms, households) possess information and form their f
orecast in real time. As it turns out, most macroeconomic data is subject to continuous revisions. We can
define data revisions in the following way.

$$
X_{t}^{f}=X_{t}^{p}+r_{t}^{f}
$$

where X_{t}^{p} denotes the statistical agency's initial announcement of a variable that was realised at t
ime t, X_{t}^{f} denotes the final or true value of the same variable and r_{t}^{f} denotes the final revis
ion which may potentially be never observed.

These data revisions, r_{t}^{f}, can be due to:

\begin{itemize}
  \item short run revisions based on additional source data or

  \item benchmark revisions based on structural changes or updating base year.

\end{itemize}

If those revisions are done because of new data arrivals that are not forecastable at the time of the foreca
st (news), we can refer to 'well behaved' revisions since there is nothing economic agents can do about th
ese. If, however, future data revisions are forecastable at the time of the forecast these are not well behav
ed revisions (noise). By not making use of the forecastablity of future revisions, economic agents would vi
olate one of the main assumption of modern macroeconomics, that is rationality. Well behaved revisions h
ave three properties.

\begin{itemize}
  \item Revisions should have a mean zero. i.e. initial announcement of the statistical agency should be an

unbiased estimate of the final value of the data.

\item Final revision should be unpredictable given the information set at the time of the initial announcement and

\item The variance of the final revision should be small compared to the variance of the final value of the data.

\end{itemize}

Aruoba (2008) shows vast empirical evidence that none of these properties of well behaved revisions are full-filled in the US data. The measurement problem is more important for output series than it is for inflation or employment/unemployment series. We first look at a case where we assume that the structure of the economy is known with certainty. However, we also assume that the economy may be subject to additive shocks; this creates data uncertainty.

\section{Data (additive) uncertainty in Poole's model}
Poole (1970) analysed the implication of adding such news shocks to both the IS and LM schedules. Such shocks could come about from for instance changes in consumer tastes or government expenditures shocks on the IS side and stock market crashes and

financial crises such as the collapse of LTCM in 1998 or a change in the central bank behaviour on the LM side. Poole's model assumes that the parameters and structure of the model are known with certainty, which is an unrealistic assumption, but allows the IS and LM schedules to be subject to zero-mean random errors, again the so called news shocks. Define IS and LM schdules as:

$$
\begin{aligned}
Y & =a-b R+\varepsilon \\
M & =c-d R+e Y+\eta .
\end{aligned}
$$

ε and η are additive IS and LM shocks whose variances are σ_{ε}^{2} and σ_{η}^{2} respectively, and for simplicity we ignore the price level in the LM curve. Alternatively, assume the monetary authorities can control real money balances, denoted by M. The authorities can either set the money supply, M, or the interest rate, R, but not both. With a downward-sloping money demand schedule, either R or M can be set and the other variable will have to change to allow the markets to clear. If the authorities set the interest rate, then from the IS schedule, the expected value of output, Y, given R, denoted $\mathrm{E}[Y \mid R]$ will be:

$$
\mathrm{E}[Y \mid R]=a-b R
$$

Assume that the goal of the policy makers is to minimise the variance of output. From (12.2) and (12.3), $Y-\mathrm{E}[Y \mid R]$ will simply be ε and so the variance of output given that the authorities set the interest rate will be $\mathrm{E}[Y-\mathrm{E}[Y \mid R]]^{2}=\sigma_{\varepsilon}^{2}$. Alternatively, the monetary authorities could set the money supply, M. In order to calculate the variance of output in this scenario, we need to calculate Y as a function of M only. From the LM schedule, we can calculate R as a function of M, Y and η and then substitute this into the IS curve. Solving for Y will give:

$$
Y=\frac{a d-b c}{d+b e}+\frac{b M}{d+b e}+\frac{d \varepsilon-b \eta}{d+b e}
$$

$$

We can then find $\mathrm{E}[Y \mid M]$ and the variance of output given that the authorities directly control the money stock. This is given in 12.5 .

$$
\mathrm{E}[Y-\mathrm{E}[Y \mid M]]^{2}=\left(\frac{1}{d+b e}\right)^{2}\left(d^{2} \sigma_{\varepsilon}^{2}+b^{2} \sigma_{\eta}^{2}\right)
$$

We can now examine which policy instrument, when set by the authorities, will result in a lower output variance. Consider the case where there are no IS shocks, σ_{ε}^{2}. The variance of output under both interest rate and money targeting regimes is given in the top line of Table 12.1. It is clear that setting the interest rate and allowing money to change to clear the market is the optimal strategy. However, if there are no LM shocks, σ_{η}^{2}, then output variance is smaller under a policy of fixed money supply, see the bottom line of Table 12.1. Therefore, if an economy is prone to IS shocks, the authorities should keep the money supply constant. If the economy is prone to money market, LM, shocks, the interest rate should be the instrument of choice.

This is also shown in Figures 12.1a and 12.1b. Figure 12.1a shows the case with IS shocks only. The output variation when the interest rate is fixed at R^{*} is shown by $\left.\right|_{R}$, in which case the money supply has to change to clear the money markets causing the LM curve to shift so that equilibrium is at points A or B. If the money supply was kept

$$
\begin{array}{ll}
\text { Only LM shocks, } \sigma_{\varepsilon}^{2}=0 & \operatorname{Var}[Y \mid R]=0<\operatorname{Var}[Y \mid M]=\left(\frac{b}{d+b e}\right)^{2} \sigma_{\eta}^{2} \\
\text { Only IS shocks, } \sigma_{\eta}^{2}=0 & \operatorname{Var}[Y \mid R]=\sigma_{\varepsilon}^{2}>\operatorname{Var}[Y \mid M]=\left(\frac{d}{d+b e}\right)^{2} \sigma_{\varepsilon}^{2}
\end{array}
$$

Table 12.1:

fixed then output deviations are shown by $\left.\right|_{M}$. Therefore, with IS shocks, in order to keep output variance at a minimum, it is best to keep the money supply fixed.

In Figure 12.1b, by keeping the interest rate fixed after LM shocks, equilibrium will be unchanged at point E and output variance will therefore be zero. By keeping the money supply fixed, however, LM shocks will cause equilibrium to move between points A and B and output variance will be positive, equal to $\left.\Delta Y\right|_{M}$. So, depending on the main source of economic shocks, whether they originate from the goods or money markets, will determine which monetary instrument the authorities should target.

IS shocks (keep M fixed)

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-149(1)}
\end{center}

LM shocks (keep R fixed)

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-149}

\end{center}

Figure 12.1: Poole's analysis.

Parameter (or multiplicative) uncertainty

Whereas Poole considered the case where shocks were additive in nature, Brainard (1967) examined the case where the values of the parameters in the model were not known with certainty. This is, arguably, more realistic since any model must be estimated from data. An estimated model will not only give point estimates of the parameters but will also give standard errors since there will always be measurement error, model mis-specification and other problems that cause us not to know the exact structure of the economic model. Suppose output, y, depends on a policy mix, X, a vector containing fiscal and monetary instruments that the government can control. The relationship between y and X is given by:

$$
y=g X
$$

For simplicity, assume X is a single policy instrument so that g is a scalar parameter estimate with mean $\widehat{g}$ and variance σ_{g}^{2}. The aim of the authorities is to minimise the variance of y around some target level, y^{*}, full employment level of output for example, subject to the constraint (8.13), i.e.

$$
\min \mathrm{E}\left[y-y^{*}\right]^{2} \quad \text { s.t. } \quad y=g X
$$

If $y=g X$, taking averages will give $\widehat{y}=\widehat{g} X$. The problem can then be written as:

$$
\begin{aligned}
& \min \mathrm{E}\left[(y-\widehat{y}) X-\left(y^{*}-\widehat{y}\right)\right]^{2} \quad \text { s.t. } \quad y=g X \\
\Rightarrow & \min \mathrm{E}\left[(g-\widehat{g}) X-\left(y^{*}-\widehat{g} X\right)\right]^{2} \\
\Rightarrow & \min \mathrm{E}\left[(g-\widehat{g})^{2} X^{2}+\left(y^{*}-\widehat{g} X\right)^{2}-2(g-\widehat{g}) X\left(y^{*}-\widehat{g} X\right)\right]
\end{aligned}
$$

Noting that $\mathrm{E}[g-\widehat{g}]^{2}=\sigma_{g}^{2}$ and that $\mathrm{E}[g-\widehat{g}]=\widehat{g}-\widehat{g}=0$, the problem then becomes:

$$
\min _{X} X^{2} \sigma_{g}^{2}+\left(y^{*}-\widehat{g} X\right)^{2}
$$

Differentiating this with respect to X, the choice variable, setting equal to zero and solving will give:

$$
X=\frac{\widehat{g} y^{*}}{\sigma_{g}^{2}+\widehat{g}^{2}} \quad \Rightarrow \quad \widehat{y}=\widehat{g} X=\frac{\widehat{g}^{2} y^{*}}{\sigma_{g}^{2}+\widehat{g}^{2}}<y^{*}
$$

The implication of the model is that because of the presence of uncertainty, $\sigma_{g}^{2}>0$, the authorities will never push aggressively enough to make average output equal to the target level, y^{*}. To do so will simply cause output variation to increase to an intolerable level. The policy maker would rather have a stable level of output below the full employment level than very volatile output whose average was y^{*}. This is shown in Figure 12.2 .

From $y=g X$ and $\widehat{y}=\widehat{g} X$, it follows that $\sigma_{y}^{2}=\sigma_{g}^{2} X^{2}$ which implies $X=\sigma_{y} / \sigma_{g}$ Substituting into $y=g X$ will give the linear policy constraint in Figure 12.2. The authorities try to reach the indifference curve closest to y^{*}, the target level, subject to the policy constraint and as can be seen in Figure 12.2, the presence of uncertainty, σ_{g}^{2}, causes the authorities to opt for a less aggressive policy stance, causing equilibrium output to be below y^{*}.

Activity 12.1 What happens in Figure 12.2 when the uncertainty, shown by σ_{g}^{2}, increases? Do the policy makers become more or less aggressive in reaching the target level of output, y^{*} ? Explain. 12.8. Parameter (or multiplicative) uncertainty

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-151}
\end{center}

Figure 12.2: Poole's analysis.

\subsubsection{The New Keynesian model and parameter uncertainty}
In this section we will apply Brainard's ideas to a simple macroeconomic model in the New Keynesian spirit. We will show that if there is no parameter uncertainty, and the economy is subject to additive shocks with mean zero and constant variance, the policy maker should behave in a certainty equivalent manner, that is if the economic shocks did not occur. If, however, the economy is subject to structural changes as exemplified in parameter uncertainty, than Brainard conservatism applies. Now, suppose that the economy is characterised by:

$$
\pi_{t}=y_{t}+a \pi_{t-1}
$$

and

$$
y_{t}=-b i_{t}+\varepsilon_{t}
$$

with

$$
\varepsilon \sim\left(0, \sigma_{\varepsilon}^{2}\right)
$$

where π stands for inflation, y for the business cycle component of real output (or income), i for the short term rate that the policy maker can control and ε for the stochastic demand shocks hitting the economy. We assume that the mean value of these shocks are equal to zero and their variance is given by σ_{ε}^{2} and these are known by the policy maker. Parameters a and b are constants. By substituting (12.12) into (12.11) we obtain an expression for current inflation as a function of past inflation, the policy rate and shocks hitting the economy.

$$

\pi_{t}=a \pi_{t-1}-b i_{t}+\varepsilon_{t} .
$$

\begin{enumerate}
  \setcounter{enumi}{11}
  \item Monetary policy and data/parameter uncertainties
\end{enumerate}

The policy maker cares about inflation stabilisation. Suppose that the quadratic loss function of the central bank takes the following form:

$$
L=\left(\pi_{t}-\pi^{*}\right)^{2}
$$

where π^{*} represents the target inflation. In other words, whenever the current inflation deviates form the target inflation, the policy maker has an incentive to bring back the inflation to its target level by manipulating the policy rate, i.

\section{The case with additive uncertainty}
The only source of uncertainty is the presence of stochastic shocks. Given that the policy maker knows the nature of the shocks with a mean of zero $(\mathrm{E}(\varepsilon)=0)$ and a constant variance $\left(\mathrm{E}\left(\varepsilon^{2}\right)=\sigma_{\varepsilon}^{2}\right)$, it will form an expectation about these. Substitute the perceived structure of the economy into the objective function of the central bank:

$$
L^{e}=\mathrm{E}\left(a \pi_{t-1}-b i_{t}+\varepsilon_{t}-\pi^{*}\right)^{2}
$$

or

$$
\begin{aligned}
L^{e}= & a^{2} \pi_{t-1}^{2}+b^{2} i_{t}^{2}+\underbrace{\mathrm{E}\left(\varepsilon^{2}\right)}_{\sigma_{\varepsilon}^{2}}+\pi^{* 2}-2 a \pi_{t-1} b i_{t}+2 a \pi_{t-1} \underbrace{\mathrm{E}\left(\varepsilon_{t}\right)}_{0} \\
& -2 a \pi_{t-1} \pi^{*}-2 b i_{t} \underbrace{\mathrm{E}\left(\varepsilon_{t}\right)}_{0}+2 b i_{t} \pi^{*}-2 \underbrace{\mathrm{E}\left(\varepsilon_{t}\right)}_{0} \pi^{*} .
\end{aligned}
$$

Note that, by setting $\mathrm{E}(\varepsilon)=0$ and $\mathrm{E}\left(\varepsilon^{2}\right)=\sigma_{\varepsilon}^{2}$ we have inserted policy makers expectations about the shocks. The policy maker's job is to minimise the loss with the use of the monetary policy instrument i_{t}.

$$
\begin{aligned}
\frac{\partial L^{e}}{\partial i_{t}} & =2 b^{2} i_{t}+2 b \pi^{*}-2 a \pi_{t-1} b=0 \\
i_{t} & =\frac{-\pi^{*}+a \pi_{t-1}}{b} .
\end{aligned}
$$

It is important to notice that policy rate set by the policy maker is the same as the one that it would set if there were no shocks hitting the economy. This is the certainty equivalence result. That means additive shocks do not affect the way monetary policy is conducted. The best the policy maker can do is simply to igno re them.

The case with parameter uncertainty

Now, we can include a bit of complication. The economic environment is the same as in the previous section except that the parameter b of the real income equation is allowed to vary over time. We capture this by adding a time subscript to parameter b.

Specifically:

$$
y_{t}=-b_{t} i_{t}+\varepsilon_{t}
$$

with

$$
\varepsilon \sim\left(0, \sigma_{\varepsilon}^{2}\right), \quad b \sim\left(\widehat{b}, \sigma_{b}^{2}\right)
$$

where the shocks are still additive, however we also see that the parameter b has a certain distribution such that its mean is $\widehat{b}$ and its variance is σ_{b}^{2}. It is important to note that this information is available to the policy maker such that it can form expectations about the value of the parameter b.

By substituting (12.16) into (12.11) we obtain again an expression for current inflation as a function of past inflation, the policy rate, the shocks hitting the economy and the novel element of time varying parameter b.

$$
\pi_{t}=a \pi_{t-1}-b_{t} i_{t}+\varepsilon_{t}
$$

The problem of the central bank now becomes:

$$
\begin{aligned}
L^{e}= & \mathrm{E}\left(a \pi_{t-1}-b_{t} i_{t}+\varepsilon_{t}-\pi^{*}\right)^{2} \\
= & a^{2} \pi_{t-1}^{2}+\mathrm{E}\left(b_{t}^{2}\right) i_{t}^{2}+\underbrace{\mathrm{E}\left(\varepsilon^{2}\right)}_{\sigma_{\varepsilon}^{2}}+\pi^{* 2}-2 a \pi_{t-1} \mathrm{E}\left(b_{t}\right) i_{t}+2 a \pi_{t-1} \underbrace{\mathrm{E}\left(\varepsilon_{t}\right)}_{0} \\
& +2 a \pi_{t-1} \pi^{*}+2 \mathrm{E}\left(b_{t}\right) i_{t} \underbrace{\mathrm{E}\left(\varepsilon_{t}\right)}_{0}-2 \mathrm{E}\left(b_{t}\right) i_{t} \pi^{*}-2 \underbrace{\mathrm{E}\left(\varepsilon_{t}\right)}_{0} \pi^{*} .
\end{aligned}
$$

Remember that you can write the variance of b as $\sigma_{b}^{2}=\mathrm{E}\left(b_{t}-\widehat{b}\right)^{2}=\mathrm{E}\left(b_{t}^{2}-2 b_{t} \widehat{b}+\widehat{b}^{2}\right)$. Given that $\mathrm{E}\left(b_{t}\right)=\widehat{b}$ that is equal to $\sigma_{b}^{2}=\mathrm{E}\left(b_{t}^{2}\right)-\mathrm{E}\left(\widehat{b}^{2}\right)$. That allows us to rewrite the L^{e} as:

$$
\begin{aligned}
L^{e} & =a^{2} \pi_{t-1}^{2}+\mathrm{E}\left(b_{t}^{2}\right) i_{t}^{2}+\sigma_{\varepsilon}^{2}+\pi^{* 2}-2 a \pi_{t-1} \mathrm{E}\left(b_{t}\right) i_{t}+2 a \pi_{t-1} \pi^{*}+2 \mathrm{E}\left(b_{t}\right) i_{t} \pi^{*} \\
\end{aligned}
$$

$$
\begin{aligned}
& =a^{2} \pi_{t-1}^{2}+(\operatorname{sigma}_{b}^{2}+\underbrace{\mathrm{E}\left(\widehat{b}^{2}\right)}_{\widehat{b}^{2}}) i_{t}^{2}+\sigma_{\varepsilon}^{2}+\pi^{* 2}-2 a \pi_{t-1} \underbrace{\mathrm{E}\left(b_{t}\right)}_{\widehat{b}} i_{t}+2 a \pi_{t-1} \pi^{*}+2 \underbrace{\mathrm{E}\left(b_{t}\right)}_{\widehat{b}} i_{t} \pi^{*} \\
& =a^{2} \pi_{t-1}^{2}+\sigma_{b}^{2} i_{t}^{2}+\sigma_{\varepsilon}^{2}-2 a \pi_{t-1} \widehat{b} i_{t}+2 a \pi_{t-1} \pi^{*}+\widehat{b}^{2} i_{t}^{2}+\pi^{* 2}+2 \widehat{b} i_{t} \pi^{*} .
\end{aligned}
$$

We can now solve the policy maker's optimisation problem that is:

$$
\begin{aligned}
\frac{\partial L^{e}}{\partial i_{t}} & =2 \sigma_{b}^{2} i_{t}-2 a \pi_{t-1} \widehat{b}+2\left(\widehat{b} i_{t}+\pi^{*}\right) \widehat{b}=0 \\
i_{t} & =\frac{\left(a \pi_{t-1}-\pi^{*}\right) \widehat{b}}{\left(\sigma_{b}^{2}+\widehat{b}^{2}\right)} \\
& =\frac{\left(a \pi_{t-1}-\pi^{*}\right)}{\left(\sigma_{b}^{2}+\widehat{b}^{2}\right) / \widehat{b}} .
\end{aligned}
$$

The expression $\sigma_{b} \widehat{b}$ refers to the coefficient of variation. It represents the trade-off between returning inflation to target and increasing uncertainty about inflation depends on the variance of the parameter relative to its mean level. A large coefficient of variation means for a small reduction in the inflation bias the central bank induces a large variance into future inflation. Once parameter uncertainty is taken into account inflation variance depends on the interest rate reactions. Policy maker's decisions affect uncertainty of future inflation. Hence, rather than strong reaction or 'cold turkey', gradualism (sustained policy reaction) is preferable (Brainard conservatism). 12. Monetary policy and data/parameter uncertainties

Graphical exposition

Finally, we can show the effect of data and parameter uncertainties on policymaking in a simple graphical setting. Consider Figure 12.3 following Sack (2000).

Suppose that there is a negative relationship between real output (horizontal axis) and policy rates (vertical axis, which characterises the IS curve. Upper panel shows the case without any uncertainty. As is clear whenever the actual output deviates from desired output, the policy maker can adjust policy rates to reach the desired level. Middle panel shows the case with additive uncertainty (only news shocks are considered here). Here, the policy maker is not sure about the quality of the data it receives (as shown by parallel dashed lines). Nevertheless, it acts as if it knows the data with certainty, as there is nothing it can do about the shocks (certainty equivalence). Finally, lower panel shows the case with parameter uncertainty. Since, the nature of the relationship between output and policy rate is time-varying, the slope is changing over time. If the policy makers acts with aggression, that is trying to reach the desired output by increasing the policy rate a lot, the economy may find itself at an equilibrium that is even less desirable than before. What can the policy maker do?
\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-154}

Figure 12.3: Poole's analysis. In Figure 12.4 Sack (2000) demonstrates a possible alternative to the bad outcome in line with the analytical discussion in the previous section. The policy maker moves in steps (interest smoothing). First, it adjusts the policy rates in the right direction. When the (relatively benign) state of the economy is revealed, it moves again in the same direction. The policy maker thereby exploits arrival of new information and can reduce the size of potential errors it can commit.
\includegraphics[max width=\textwidth, center]{2023_06_01_01744bff3191fbf776f3g-155}

Figure 12.4: Poole's analysis.

Interest rate smoothing

As mentioned in the introduction central banks tend to change interest rates i) in small steps and ii) often in the same direction for consecutive periods. Figure 12.5 shows this case for the three major central banks. The Federal Reserve, the European Central Bank and the Bank of England. Policy makers are usually not only uncertain about the state of the economy (data uncertainty) but also on the structural parameters of the economy therefore interest rates are set in a smooth fashion. 12. Monetary policy and data/parameter uncertainties

A reminder of your learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- discuss the importance of additive uncertainty.
- distinguish between news and noise in data revisions.
- choice of monetary policy instruments under additive uncertainty.
- discuss the importance of parameter uncertainty.
- explain the concept of certainty equivalence.
- explain optimal policy conduct when the economy subject to parameter uncertainty.

Sample examination questions

Section A

Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

1. 'When the economy is subject to multiplicative uncertainty the best a policy maker do is to ignore the uncertainty'.

Section B

2. What is the certainty equivalence? Under which conditions can a policy maker act as if shocks do not happen?
3. Consider a policy maker aiming to minimise quadratic loss associated with actual inflation over a target inflation. Suppose also that the economic environment includes forward looking expectations in the Phillips curve such that:

$$
\pi_{t}=a \mathrm{E}_{t}\left(\pi_{t+1}\right)-b_{t} i_{t}+\varepsilon_{t}
$$

and the parameter b is allowed to vary over time. Note that distribution of the disturbance term and of the parameter b have following statistical properties:

$$
\varepsilon \sim\left(0, \sigma_{\varepsilon}^{2}\right), \quad b \sim\left(\widehat{b}, \sigma_{b}^{2}\right)
$$

Calculate optimal interest rate reaction function of the policy maker. Provide an intuition to your answer.

\begin{enumerate}
  \setcounter{enumi}{3}
  \item Suppose the economy is subject to i.i.d. mean zero IS and LM shocks. Discuss in a simple model t
heir implications on the policy instrument choice.
\end{enumerate}

\subsection{Feedback to Sample examination questions}
\section{Section B}
\begin{enumerate}
  \setcounter{enumi}{3}
  \item The question is about selecting the policy instrument that minimises the variance of output. Student
s should show analytically and graphically the impact of IS or LM shocks. See Poole (1970). 12. Monetary
 policy and data/parameter uncertainties
\end{enumerate}

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-158(2)}
\end{center}

(a) ECB Base Rates

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-158}
\end{center}

(b) BoE Base Rates

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-158(1)}
\end{center}

(c) US Federal Reserve Base Rates

Figure 12.5: Poole's analysis.

\section{Chapter 13
The term structure of interest rates}
\subsection{Introduction}
With the exceptions of Chapters 11 and 12, throughout this guide we have only ever considered 'the' inter
est rate, that is the one set by the monetary authorities, or that is allowed to change to clear the money m
arkets if the money supply is the instrument of choice. Even in Chapters 11 and 12, there was only ever o
ne market interest rate. In reality, there are a large number of interest rates, from those on debt that matur
e overnight to interest rates on debt that mature up to 30 years in the future. This chapter will examine in
more detail the links between short-term and long-term interest rates, explaining why such links are import
ant and providing theories that explain the short-termlong-term interest rate relationship.

\subsection{Aims}
This chapter will introduce the relevance of a rich variety of interest rates in financial and macroeconomic

decisions. We will also discuss different term structure theories.

\subsection{Learning outcomes}
By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\begin{itemize}
  \item explain what a yield curve is and why it is important for policy makers

  \item list and discuss the different empirical regularities of the yield curve

  \item list and describe the three different theories of the term structure outlined in this chapter, noting any differences and similarities and stating which theories can explain which empirical facts

  \item describe the relationship between rates of return and bond prices and why this is so important for the absence of arbitrage conditions on which the expectations hypothesis focuses. 13. The term structure of interest rates

\end{itemize}

\subsection{Reading advice}
The main readings for this section are the chapters in the textbooks by Goodhart (1989), Harris (1985) and especially Mishkin (2003). Chapter 7 of Mishkin should be read before starting this part of the subject as it starts by giving a general introduction to the ideas behind the term structure and then develops the relevant theory. The two entries in the New Palgrave Dictionary by Malkiel and Mishkin are very useful but the chapter by Shiller in the Handbook of Monetary Economics is difficult and should only be read if you feel comfortable with the material.

\subsection{Essential reading}
Goodhart, C.A.E. Money, Information and Uncertainty. (London: Macmillan, 1989) Chapter 11.

Harris, L. Monetary Theory. (New York; London: McGraw-Hill, 1985) Chapter 17.

Malkiel, B.G. 'The term structure of interest rates', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

Mishkin, F.S. The Economics of Money, Banking and Financial Markets. (Boston, Mass.; London: Addison Wesley, 2003) Chapter 7.

Mishkin, F.S. 'The yield curve', in Newman, P., M. Milgate and J. Eatwell (eds) The New Palgrave Dictionary of Money and Finance. (London: Macmillan, 1994).

\subsection{Further reading}
\section{Books}
Shiller, R.J. 'The term structure of interest rates', in Friedman, B. and F. Hahn (eds) Handbook of Monetary Economics. (Amsterdam: North-Holland, 1990).

Walsh, C.E. Monetary Theory and Policy. (Cambridge, Mass.: MIT Press, 2003) Chapter 10 .

\section{Journal articles}
Aksoy, Y. and H. Basso, 'Liquidity, term spreads and monetary policy', Economic Journal, 124, 2014, 1234-1278.

Mankiw, N.G. and L.H. Summers 'Do long-term interest rates overreact to short-term interest rates?', Brookings Papers on Economic Activity (1984) 1, pp.223-247.

McCallum, B.T. 'Monetary policy and the term structure of interest rates', National Bureau of Economic Research working paper, w4938, (1994).

Shiller, R.J. 'The volatility of long-term interest rates and expectations models of the term structure', Journal of Political Economy 87(5, Part 2) 1979, 1190-1219.

\subsection{The yield curve}
Governments issue a number of differently dated bonds, from short-term paper through to very long-dated bonds that mature in a number of years in the future. Each bond will pay a different rate of return, also known as yield to maturity. If we plot on a graph the time to maturity of government debt on the horizontal axis and the yield to maturity on the vertical axis, this will give us what is known as the yield curve. An example of such a curve is shown in Figure 13.1.

Figure 13.1 shows that, for this example, government debt that matures in one month's time pays an (annualised) interest rate of 4%. One-year paper pays 6% and debt that matures in 10 years from now pays an annual rate of return of 9%.

Activity 13.1 What does the yield curve look like in your country?

\begin{center}
\includegraphics[max width=\textwidth]{2023_06_01_01744bff3191fbf776f3g-161}
\end{center}

Figure 13.1:

\subsection{Why is the yield curve of importance to policy-makers?}
Central Banks, when setting interest rates, only set one interest rate, normally at the very short end of the spectrum. In the US, the Federal Reserve sets overnight rates. However, investment decisions and aggregate demand in the economy will tend to depend on long-term interest rates since firms will compare the rate of return on investment projects that accrue over the entire life of the project (15 years plus), to the 13. The term structure of interest rates

alternative of buying equivalent dated bonds maturing in 15 years' time or so. If aggregate demand depends on long-term interest rates and the monetary authorities set short-term rates but wish to affect aggregate demand, they will need to know the relationship between short-term and long-term rates of return (i.e. they need to know the shape of the yield curve).

But why then do Central Banks not just set long-term rates of return and allow the market to determine the rates of return along the rest of the yield curve? If the authorities set long-term rates, they may be faced with large capital losses due to fluctuations of bond prices in the market. To avoid the possibility of facing large price variations, the authorities instead set the rate of return on short-dated paper, on which the possibility of capital gains/ losses is very small or non-existent. However, not only is the yield curve important for analysing the effects of monetary policy on the economy, it is also important because it gives information on expected future inflation, as will be explained below.

\subsection{Bond prices and the rate of return}
There are, in general, two types of bonds a government can issue. These are:

\begin{itemize}
\item Coupon bonds. Typically, coupon bonds promise to pay a fixed sum of money, the coupon, every period, whether it is every month, quarter or year, and also promise to pay the holder of the bond its face value on maturity. For example, a ten-year bond may be bought from the government for $\pounds 100$ that promises to pay $\pounds 5$ (the coupon payment) every year for the next ten years. In the final year, both the last $\pounds 5$ coupon and the maturity value of the bond, $\pounds 100$, are paid.

- Discount bonds. Discount bonds do not offer any interest or coupon payments. Instead, they are sold at a 'discount' and pay a larger amount on maturity. For example, a ten-year discount bond may be sold today for $\pounds 60$ that pays no coupons at all but pays $\pounds 100$ on maturity, ten years from now.

Consider a coupon bond that pays C every year from now to infinity. Such bonds, without a maturity date, are called perpetuities or Consols (the UK term). The price of this bond, P, will be the future cash flow, discounted back to today by the interest rate, R, assumed constant here for convenience.

$$
P=\frac{C}{1+R}+\frac{C}{(1+R)^{2}}+\frac{C}{(1+R)^{3}}+\cdots=\frac{C}{1+R}\left(\frac{1}{1-1 /(1+R)}\right)=\frac{C}{R} .
$$

There is then a negative relationship between the price of a bond and the interest rate. This was discussed in Chapter 2 when examining Keynes individual money demand function. (For bonds that do have a maturity date, the relationship between bond prices and the interest rate, although still negative, is not as simple as (13.1). Intuitively, if the interest rate increased, people would be willing to pay less for a bond that pays a fixed coupon payment each period. If a perpetuity bond that paid a coupon of $\pounds 5$ had a price of $\pounds 100$, this implies the market interest rate is 5%. If the market interest rate doubled to 10%, no investor would be willing to pay more than $\pounds 50$ for the same bond since the $\pounds 5$ coupon on a $\pounds 50$ bond is 10%. Hence the bond price and interest rate are negatively related.

Empirical regularities of the term structure

There are three features of the term structure that any theory of the yield curve should be able to explain:

1. Rates of return on short- and long-dated bonds move together over time.
2. When short rates are low, the yield curve is likely to be upward sloping and vice versa.
3. Yield curves generally have a persistent upward slope.

We now examine a number of theories that try to explain these features.

The expectations hypothesis

The expectations hypothesis of the yield curve links the rate of return on short-term bonds to that on long-term bonds, essentially by assuming that short and long-dated bonds are perfect substitutes. Consider someone who wishes to save a fixed amount of money for n periods. She could either buy long-dated bonds now, at date t, that pay ${ }_{t} R_{t+n}$ per cent per year. The left-hand subscript denotes the time when the bond is bought and the right-hand subscript denotes when the bond will mature. Alternatively, she could buy a bond that matures at date $t+1$ (a one-period bond) paying a rate of return ${ }_{t} r_{t+1}$. When this matures she will buy another one-period bond (at date $t+1$ that matures at date $t+2$) paying a rate of return ${ }_{t+1} r_{t+2}$ and will continue doing this until date $t+n$. If the return from a portfolio made up of long-dated bonds is not identically equal to the expected return from continually rolling over one-period debt, then arbitrage opportunities will be exploited to bring the two returns together.

Suppose for example that a two-year bond paid a rate of return of 4% per year. If the rate of return on a one-year bond was 3% today and expected to remain at 3% next year, investors will rush in to b uy the longer-dated bond as it pays a higher rate of return. The increased demand for the two period bonds will push the price up and, as explained above, will push the rate of return down from 4%. Also, the reduction in demand for one-period bonds will push the price down, causing the one-period rate of return to increase from 3%. In equilibrium, under the expectations hypothesis, the total return from each portfolio must be equal to avoid arbitrage opportunities. Using the same notation as above, but noting that at date t when the saving decision is made, all future one-period rates are not known and have to be estimated, then the discount bond reads:

$$
\left(1+{ }_{t} R_{t+n}\right)^{n}=\left(1+{ }_{t} r_{t+1}\right)\left(1+{ }_{t+1} r_{t+2}^{e}\right)\left(1+{ }_{t+2} r_{t+3}^{e}\right) \cdots\left(1+{ }_{t+n-1} r_{t+n}^{e}\right)
$$

The left-hand side is the total return from holding n period bonds until they mature. The right-hand side is the total expected return from holding and continually rolling over oneperiod bonds, n times. The rates of return on all bonds bought at date $t+1$ onwards are not known at date t, hence the e (expectation) superscript. Taking logs of 13. The term structure of interest rates

(13.2), noting that $\ln (1+X) \approx X$ for small X, gives:

$$
\begin{aligned}
n \cdot{ }_{t} R_{t+n} & ={ }_{t} r_{t+1}+{ }_{t+1} r_{t+2}^{e}+{ }_{t+2} r_{t+3}^{e}+\cdots+{ }_{t+n-1} r_{t+n}^{e} \\
\Rightarrow \quad{ }_{t} R_{t+n} & =\frac{1}{n} \sum_{s=t}^{t+n-1}{ }_{s} r_{s+1}^{e}
\end{aligned}
$$

This is one of the main results of the expectations hypothesis; the long-term interest rate is an average of the current and all future expected short-term interest rates. The expectations hypothesis does have a number of criticisms, however:

- It cannot explain the empirical fact that the yield curve has a persistent upward slope. If the long rate is an average of current and expected future short rates, this can only be explained if the short rate increases through time. This clearly is not the case.
- If the long rate is an average of current and future short rates, the long rate must be a smoother series (when plotted through time) than the short rate. This is not true; the long rate is just as volatile.
- In order to avoid arbitrage opportunities and keep total returns equal, if the rate of return on long-term bonds is greater than the return on short bonds, then holding long-dated bonds must be accompanied by a capital loss (i.e. the price of long-term bonds must fall). If the price of long-term bonds falls, the rate of return must increase still further in the next period. Putting this another way: if the long rate is higher than the short rate, the long rate must increase. This does not happen in reality.

The expectations hypothesis and expected inflation

If the long rate is an average of the current and expected future short rates, then an upward-sloping yield curve suggests that the short rate is expected to increase in the future (see (13.3)). If the real interest rate is constant then the increase in expected future nominal interest rates must be associated with an increase in expected future inflation, as per the Fisher equation. The greater the slope of the yield curve, the more short-term rates are expected to rise and so the faster is inflation expected to increase.

The segmentation hypothesis

Whereas the expectations hypothesis assumes short- and long-term bonds are perfect substitutes so the decision as to whether to hold long- or short-dated debt depends entirely on expected returns, the segmentation hypothesis assumes short- and long-term bonds are not substitutes in any way. The rate of return on m period bonds will not depend on the market for, or the return on, $m-j$ period bonds at all. Instead, its rate of return will depend entirely on the demand for and supply of credit that matures in m periods' time. If the demand for m period bonds increased, caused by more people wanting to save for m periods, then the price of m period bonds will increase and the rate of return will fall. The markets for $m, m+1, m+2$, etc. period debt are said to be segmented.

Whereas the expectations hypothesis could not explain the persistent upward sloping nature of the yield curve, this feature can be easily explained if we use the segmentation hypothesis. If people generally prefer to hold short-dated debt (i.e. save by buying shortterm bonds), then this will cause the price of short-term bonds to be high, relative to longterm bonds, and so the rate of return on short-term debt will then be lower than the longterm rate, implying an upward-sloping yield curve.

Activity 13.2 Why do you think people would prefer to hold short-term, rather than long-term, debt?

(Hint: think of the desire to lock up money in long-term bonds and the risk of capital gains or losses when such holdings have to be liquidated.)

Despite being able to explain persistent upward-sloping yield curves, the segmentation hypothesis cannot explain the fact that interest rates move together since the markets are completely segmented. Also, the theory cannot explain why yield curves are upward (downward)-sloping when short rates are low (high).

Preferred habitat theory

The preferred habitat theory lies in between the expectations and segmentations hypotheses. It assumes bonds are neither fully substitutable nor non-substitutable. Instead, people have a preferred bond maturity they wish to hold (as in the segmentation hypothesis) but will be willing to move to another bond maturity if the gains from doing so are significant (so exploiting excess arbitrage opportunities as in the expectations hypothesis). The preferred habitat theory can be represented as (13.4):

$$
{ }_{t} R_{t+n}=\frac{1}{n} \sum_{s=t}^{t+n-1} s r_{s+1}^{e}+{ }_{t} k_{n} .
$$

This is exactly the same as the expectations hypothesis, (13.3), except for the inclusion of a term premium, ${ }_{t} k_{n}$. If n period bonds were not the bond of choice, then people would need an extra rate of return in order to encourage them to hold such debt. In this case ${ }_{t} k_{n}$ would be positive. If people generally tend to prefer short-dated debt then the term premium will be a monotonic function of maturity; in order to hold longer and longer dated debt, an increasing term premium must be offered. When combined with the 'expectations hypothesis' component, (13.4) can explain the persistent upward slope of the yield curve, along with the other empirical regularities: the co-movement of short and long rates, and the fact the yield curve tends to be upward (downward)-sloping when short rates are low (high). However, using the term premium to fix the problems of the expectations and segmentations hypotheses, is arguably a not a proper solution. A theory of the term premium should be provided, rather than simply assumed in order to make the data consistent with the 'fixed' theory. For a recent paper on a theory of term premium that builds upon maturity transformation risk faced by the banks, see Aksoy and Basso (2014). 13. The term structure of interest rates

A reminder of your learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

\item explain what a yield curve is and why it is important for policy makers

\item list and discuss the different empirical regularities of the yield curve

\item list and describe the three different theories of the term structure outlined in this chapter, noting any differences and similarities and stating which theories can explain which empirical facts

\item describe the relationship between rates of return and bond prices and why this is so important for the absence of arbitrage conditions on which the expectations hypothesis focuses.

\end{itemize}

\subsection{Sample examination questions}
\section{Section A}
Specify whether the following statement is true, false or uncertain. Explain your answer in a short paragraph.

\begin{enumerate}
\item 'Under the segmentation hypothesis, the slope of the yield curve tells us nothing about expected inflation.'
\end{enumerate}

\section{Section B}
\begin{enumerate}
\setcounter{enumi}{1}
\item Empirical fact: yield curves tend to have an especially steep upward slope when short rates are low and a downward slope when short-term interest rates are high. Can you explain this fact with the theories of the term structure analysed in this chapter?

\item Assuming that the expectations hypothesis is the correct theory of the term structure, calculate the interest rates for one to five year bonds and plot the resulting yield curves when one year interest rates over the next five years look as follows:

\end{enumerate}

(a) $5 \%, 7 \%, 7 \%, 7 \%, 7 \%$.

(b) $5 \%, 4 \%, 4 \%, 4 \%, 4 \%$.

How would your yield curves change if people preferred shorter-term bonds over longer-term bonds?

\begin{enumerate}
\setcounter{enumi}{3}
\item If the yield curve was steeply upward sloping, what is the market predicting about the movements of future short-term interest rates? What might the market predict about the inflation rate in the future?

\item How can the preferred habitat theory explain the appearance of downward-sloping yield curves if ${ }_{t} k_{n}$, the term premium, is positive?

\end{enumerate}

\subsection{Feedback to Sample examination questions}
\section{Section B}
\begin{enumerate}
\setcounter{enumi}{2}

\item From the expectations hypothesis, and in particular (13.3), we can calculate the 2, 3, 4 and 5-year interest rates from the one year rates given in the question.
\end{enumerate}

(a) Using (13.3), we see that:

2 year rate; ${ }_{t} R_{t+2}=1=2(5+7)=6 \%$ per year.

3 year rate; ${ }_{t} R_{t+3}=1=3(5+7+7)=6.33 \%$ per year.

The 4 - and 5-year rates can be calculated in a similar way to obtain ${ }_{t} R_{t+4}=6.5 \%$ per year and ${ }_{t} R_{t+5}=6.6 \%$ per year. The yield curve is therefore upward sloping (notice that an upward-sloping yield curve suggests interest rates will rise in the future; 5% to 7%).

(b) Again using (13.3), we can calculate the 2, 3, 4 and 5-year interest rates to obtain ${ }_{t} R_{t+2}=4.5 \%$ per year, ${ }_{t} R_{t+3}=4.33 \%$ per year, ${ }_{t} R_{t+4}=4.25 \%$ per year and ${ }_{t} R_{t+5}=4.2 \%$ per year. The yield curve is therefore downward sloping (a downward-sloping yield curve suggests interest rates will fall; 5% to 4%).

If people preferred shorter-term bonds then they would have to receive an increased rate of return on longer-term debt (over and above that implied by the expectations hypothesis) in order to encourage them to hold such assets. In case (a), the yield curve would be even steeper and in part (b), the yield curve will also be rotated anticlockwise, so becoming more flat, or perhaps becoming positively sloped if the term premium was large enough. 13. The term structure of interest rates

\section{Appendix A Sample examination paper}
\section{Important note}
This commentary reflects the examination and assessment arrangements for this course in the academic year 2014-15. The format and structure of the examination may change in future years, and any such changes will be publicised on the virtual learning environment (VLE).

Time allowed : three hours

Candidates should answer ELEVEN of the following THIRTEEN questions: EIGHT from Section A (5 marks each) and THREE from Section B (20 marks each).

Candidates are strongly advised to divide their time accordingly.

If more questions are answered than requested, only the first answers attempted will be counted.

\section{Section A}
Answer ALL EIGHT questions from this section. (5 marks each)

Indicate whether the following statements are true or false, or uncertain and give a short explanation. Points are only given for a well reasoned answer.

\begin{enumerate}
\item The advantage of using fiat money vs. a barter economy is that fiat money no longer requires trust between transaction partners.

\item The demand of money is positively related to the price uncertainty of assets.

\item Under the preferred habitat theory the slope of the yield curve tells us nothing about the expected inflation.

\item In Tobin's portfolio selection model individuals hold either money or bonds.

\item Parameter uncertainty in the Brainard model implies more aggressive fiscal policy action.

\item The quantity theory of money states that the total value of all monetary transactions must be equal to the real value of goods and services it buys.

\item According to the standard real business cycle (RBC) theory, money is not neutral.

\item Recessions are best dealt with by conservative central bankers.

\end{enumerate}

\section{A. Sample examination paper}
\section{Section B}
Answer THREE out of FIVE questions from this section. (20 marks each)

\begin{enumerate}
\setcounter{enumi}{8}
\item Consider a classical-Patinkin economy where a representative household utility is a function of con
sumption \$(C)\$ and real money balances \$(M / P)\$ such that:
\end{enumerate}

$$
U=C^{\alpha}(M / P)^{1-\alpha}
$$

and the household is subject to a budget constraint given by:

$$
C+M / P \leq Y+M_{0} / P
$$

where \$Y\$ stands initial endowment of goods, \$M_{0}\$ stands for initial money balances.

(a) Calculate optimal goods and money demand. Provide intuition. Would your results change qualitatively
if elements (that are real money balances and consumption) in the utility function were additive, instead o
f multiplicative?

(b) Is there a role for monetary policy in this setting? What happens when the economy faces a money su
pply increase? Show graphically and provide intuition.

(c) Is the classical theory convincing? Are you aware of alternative macro theories where monetary policy can be an effective output stabilisation tool? Discuss analytically one such model.

\begin{enumerate}
\setcounter{enumi}{9}
\item Consider a McCallum economy with sticky prices where the aggregate demand expression given a
s:
\end{enumerate}

$$
y_{t}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}\right)+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}\right]+v_{t}
$$

where y_{t}, m_{t} and p_{t} are the logs of real output, nominal money balances and the price level respectively at date t, v_{t} is an i.i.d. normal aggregate demand shock with $v_{t} \sim\left(0, \sigma_{v}^{2}\right)$ are positive parameters and E is the expectations operator. To construct the aggregate supply assume the following:

(i) the market clearing price is denoted by p_{t}^{*}

(ii) the prices are set by firms at $t-1$ and are only effective in t. These are expected to be the market clearing price in t, i.e. $p_{t}=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$

(iii) real output consistent with natural rate of unemployment, $\left(y^{*}\right)$, is determined by the following law of motion that captures hysteresis

$$
y_{t}^{*}=\delta_{0}+\delta_{1} t+\delta_{2} y_{t-1}^{*}+u_{t}
$$

where t being time trend, $\delta_{0}, \delta_{1}, \delta_{2}$ positive parameters and u_{t} is an i.i.d. normal aggregate supply shock with $u_{t} \sim\left(0, \sigma_{u}^{2}\right)$

(iv) the monetary policy is characterised by the expression

$$
m_{t}=\mu_{0}+\mu_{1} m_{t-1}+e_{t}
$$

where e_{t} is an i.i.d. normal money supply shock with $e_{t} \sim\left(0, \sigma_{e}^{2}\right)$. (a) Solve for the output gap, i.e. deviations of real output from the market clearing level.

(b) Are unanticipated monetary policy changes effective? Show analytically and provide intuition.

(c) Are anticipated monetary policy changes effective? Show analytically and provide intuition.

(d) What is the Lucas critique? Evaluate the critique based on the output gap equation you have derived above.

11. Suppose that the economy of Pool-land is characterised by the following IS schedule

$$
Y=a-b R+\varepsilon
$$

and the following LM schedule

$$
M=c-d R+e Y+\eta
$$

where Y represents the real output, R the short term interest rate, M the real money balances, coefficients a, b, c, d are positive constants. The economy is subject to additive shocks. ε and η are IS and LM shocks with the following properties $\varepsilon \sim$ i.i.d. $\left(0, \sigma_{\varepsilon}^{2}\right)$ and $\eta \sim$ i.i.d. $\left(0, \sigma_{\eta}^{2}\right)$. Note that the uncertainty arises onl y due to additive shocks.

Suppose that the monetary authority goal is to minimise the variance of output.

(a) Is it better to use short term interest rates instead of real money balances to stabilise the economy when the economy is subject to IS shocks only? Show analytically.

(b) Is it better to use short term interest rates instead of real money balances to stabilise the economy when the economy is subject to LM shocks only? Show analytically.

(c) Now consider the case of multiplicative uncertainty, where parameters of the model, a, b, c, d are subject to uncertainty. Would this model modification affect your policy results? Discuss verbally.

\begin{enumerate}
  \setcounter{enumi}{11}
  \item A treasurer has to manage the cash position of a small London college. At the beginning of the year the college receives $\pounds 5$ million in tuition fees. The college is expected to spend all of that money in the year, at an approximately constant daily rate. The treasurer can either hold the financial wealth of the college in bonds, yielding an interest rate of 4% or in cash, which is assumed to not receive any interest. The accrued interest on the bond holdings is received at the end of the year and is not compounded inbetween. The brokerage fees of exchanging bonds for money are $\pounds 1,000$ per transaction. Note that the first transaction takes place at the beginning of the year, when the treasurer buys bonds, the transactions during the rest of the year will compose of selling bond holdings. The treasurer wants to minimize the costs of cash management, C, which are composed of the interest foregone on the cash holdings and the brokerage fees.
\end{enumerate}

\section{A. Sample examination paper}
(a) Draw the time profile of the college's holdings of cash and bonds.
(b) Set up the cost minimization problem for the treasurer and calculate the how much money, Z, should be transferred each time the treasurer sells off bonds.
(c) Calculate the interest elasticity of money.
(d) The problem outlined above is based on the Baumol-Tobin inventory theoretic model. State and discuss two criticisms of the inventory theoretic model.

\begin{enumerate}
  \setcounter{enumi}{12}
  \item Assume that the banking sector is described as follows:
\end{enumerate}

$$
\begin{aligned}
D & =d_{0}-d_{1}\left(i-i_{D}\right) \\
L & =l_{0}-l_{1}\left(i-i_{L}\right)
\end{aligned}
$$

where L stands for bank loans, D stands for bank deposits, i_{L} the loan rate, i_{D} deposit rate and i is the market interest rate. Assume that banks do not have operating costs or are not required to hold reserves.

(a) Calculate the competitive equilibrium. Show graphically. How do your results change when the government sets deposit rates equal to $i_{D}=a$, with $a<i_{L}$? Provide intuition.

(b) Now suppose that government introduces a mandatory reserve ratio, r^{*}, such that $R=r^{*} D$. H ow do your results change? What are the implications of such a reserve ratio policy on prices and quantiti es? Provide intuition.

(c) Sometimes, it is suggested that the reserve ratio policy can be an alternative to targeting interest rates or monetary aggregates. Can this be an effective policy to stabilise output and inflation fluctuations?

\section{Appendix B
Examiners' commentary to Sample examination paper}
\section{Section A}
Answer ALL EIGHT questions from this section. (5 marks each)

Indicate whether the following statements are true or false, or uncertain and give a short explanation. Poin ts are only given for a well reasoned answer.

\begin{enumerate}
  \item The advantage of using fiat money vs. a barter economy is that fiat money no longer requires trust between transaction partners.
\end{enumerate}

False.

Reading for this question: the subject guide, Chapter 2.

Direct barter is the only method of transaction that does not require trust. Fiat money is intrinsically worthl ess: its value derives primarily from the trust that people put in it (Trust Monetary authority/government). T he advantages of fiat money rather lie in the fact that it is durable, portable, homogeneous, and acceptabl e.

\begin{enumerate}
  \setcounter{enumi}{1}
  \item The demand of money is positively related to the price uncertainty of assets.
\end{enumerate}

True.

Reading for this question: the subject guide, Chapter 3.

Higher uncertainty implies a higher opportunity cost of holding assets when agents are risk averse, hence reducing the demand for assets and thus increasing the demand for money.

\begin{enumerate}
  \setcounter{enumi}{2}
  \item Under the preferred habitat theory the slope of the yield curve tells us nothing about the expected i nflation.
\end{enumerate}

False.

Reading for this question: the subject guide, Chapter 13.

The PH Theory states that the yield of longer maturities is partially determined by risk premia and partially by expectations about future short term interest rates. Given a theory on the nature of risk premia (eg. hal f a percent per year of maturity) and the relation between short term interest rates and inflation (e.g. real i nterest rates are constant) one can use PH theory to make inference about expected inflation .

B. Examiners' commentary to Sample examination paper

4. In Tobin's portfolio selection model individuals hold either money or bonds. False.

Reading for this question:

1. The subject guide, Chapter 3 .
2. Tobin, J. (1956), 'The interest elasticity of transactions demand for cash', Review of Economics and Statistics, 38(3), pp. 241-47.
3. Tobin, J. (1958), 'Liquidity preference as behaviour towards risk', Review of Economic Studies, 25(1), pp. 65-86.

Unlike the Keynesian model, where agents are assumed to hold either only money or only bonds Tobin's model explains why people hold both.

Tobin modelled the demand for money and bonds as a utility maximisation of a portfolio of assets. Cash is assumed to have no risk and zero return, bonds positive expected return and strictly positive risk. The portfolio is chosen such that it maximizes a mean-variance utility function, which will normally yields a portfolio containing both cash and risky assets. Note that for non-trivial diversification of risk to take place, the number of risky assets in the portfolio should be strictly larger than 1.

5. Parameter uncertainty in the Brainard model implies more aggressive fiscal policy action.

False.

Reading for this question:

1. The subject guide, Chapter 12 .
2. Brainard, W. (1967), 'Uncertainty and effectiveness of policy', American Economic Review (papers and proceedings), 57 (2), pp. 411-425.

In Brainard's model of multiplicative uncertainty it is shown that if there is parameter uncertainty in the models the central bank uses, it will pursue a less aggressive monetary policy objective. This happens because the government has to trade off lower output volatility against higher output (as close to y^{*} as possible). Due to the parameter uncertainty, choosing higher levels of y will result in a higher level of output volatility. See Figure 12.2.

$$
\min _{X} X^{2} \sigma_{g}^{2}+\left(y^{*}-g X\right)^{2}, \quad g X=\frac{g^{2} y^{*}}{\sigma_{g}^{2}+g^{2}}<y^{*}
$$

$$

\begin{enumerate}
  \setcounter{enumi}{5}
  \item The quantity theory of money states that the total value of all monetary transactions must be equal to the real value of goods and services it buys.
\end{enumerate}

False.

Reading for this question: the subject guide, Chapter 5.

It refers to nominal rather than real value of goods and services it buys. 7. According to the standard real business cycle (RBC) theory, money is not neutral. False.

Reading for this question: the subject guide, Chapter 8 .

Standard RBC suggests the opposite as an extension of the classical model. It is based on the premise of a frictionless economy where all agents make optimal decisions given their preferences and constraints. Money is just a veil.

\begin{enumerate}
  \setcounter{enumi}{7}
  \item Recessions are best dealt with by conservative central bankers.
\end{enumerate}

False.

Reading for this question: the subject guide, Chapter 11.

Literature on inflation bias and time consistency suggests that a conservative central bank who is less concerned with output variations than society can reduce the inflation bias. According to this literature, given that inflation/output trade-offs are short term phenomena and agents rationally infer from the actions of the central banks temptation to generate inflation to raise output in the short run and act accordingly.

\section{Section B}
Answer THREE out of FIVE questions from this section. (20 marks each)

\begin{enumerate}
  \setcounter{enumi}{8}
  \item Consider a classical-Patinkin economy where a representative household utility is a function of consumption (C) and real money balances (M / P) such that:
\end{enumerate}

$$
U=C^{\alpha}(M / P)^{1-\alpha}
$$

and the household is subject to a budget constraint given by:

$$
C+M / P \leq Y+M_{0} / P
$$

where Y stands initial endowment of goods, M_{0} stands for initial money balances.

(a) Calculate optimal goods and money demand. Provide intuition. Would your results change qualitatively if elements (that are real money balances and consumption) in the utility function were additive, instead of multiplicative?

(b) Is there a role for monetary policy in this setting? What happens when the economy faces a money supply increase? Show graphically and provide intuition.

(c) Is the classical theory convincing? Are you aware of alternative macro theories where monetary policy can be an effective output stabilisation tool? Discuss analytically one such model.

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Reading for this question: the subject guide, Chapter 5 .

(a) Here we assume that the utility depends not only goods consumption but also on the level of real money balances. The justification, due to Patinkin, is that even if households plan to balance their budgets so that planned purchases are equal in value to planned sales, it may be convenient to buy and sell goods at different times. The more money they hold, the greater the extent to which they can purchase goods ahead of making sales. Money holdings stand as a proxy for the more convenient sequence of transactions they make possible. Thus, money is in the utility function.

Assume a household's utility depends on the quantity of goods consumed, X, and on real money balances, M / P. Let the household have initial endowments Y of goods and M_{0} of nominal money balances. The budget constraint faced by the household is then, in nominal terms:

$$
P X+M \leq P Y+M_{0}
$$

So that the nominal expenditure on goods, $P X$, plus the holdings of nominal money balances, M, must not be greater than the nominal value of the endowments of goods and money. Writing the budget constraint in real terms (dividing by the price level) gives:

$$
X+\frac{M}{P} \leq Y+\frac{M_{0}}{P}
$$

For the sake of simplicity we assume that $\alpha \leq 1 / 2$. Then, the household's utility function takes the specific form $U=\bar{X}^{1 / 2}(M / P)^{1 / 2}$. In order to determine the demands for goods and real money balances, we maximise the utility function subject to the budget constraint. To do this we form the Lagrangian:[1]

$$
L=X^{1 / 2}\left(\frac{M}{P}\right)^{1 / 2}+\lambda\left(Y+\frac{M_{0}}{P}-X-\frac{M}{P}\right)
$$

Differentiating with respect to the two choice variables, X and M / P, gives the first order conditions of:

$$
\begin{aligned}
\frac{\partial L}{\partial X} & =\frac{1}{2} X^{-1 / 2}\left(\frac{M}{P}\right)^{1 / 2}-\lambda=0 \\
\frac{\partial L}{\partial(M / P)} & =\frac{1}{2} X^{1 / 2}\left(\frac{M}{P}\right)^{-1 / 2}-\lambda=0
\end{aligned}
$$

from which we obtain

$$
X=\frac{M}{P}
$$

Substituting into the budget constraint will give solutions for the demands for goods and nominal money balances of:

$$
X=\frac{Y+M_{0} / P}{2} \quad \text { and } \quad M=\frac{P Y+M_{0}}{2}
$$

[1] We also impose an equality in the budget constraint as this implies no wastage of goods or money. Assume now that the economy consists of n households each identical to the one described above. The market clearing condition in the goods market then becomes:

$$
n\left(\frac{Y+M_{0} / P}{2}\right)=n Y
$$

In other words, total demand equals total supply. Solving for the price level gives:

$$
P=\frac{M_{0}}{Y}
$$

Alternatively, we can write down the market clearing condition for the money market:

$$
n\left(\frac{P Y+M_{0}}{2}\right)=n M_{0}
$$

If we solve for the price level here, we obtain:

$$
P=\frac{M_{0}}{Y}
$$

In this economy, money is neutral. Real output per household is fixed at Y as it depends on endowments.

Modifying the utility function such that elements are additive (i.e. such that $\left.U=X^{1 / 2}+(M / P)^{1 / 2}\right)$ does not change result.

(b) From the solution of the price level, a change in the money supply will only lead to a proportional increase in prices. Real money balances and 'production' of goods do not change. An increase in money, M_{0}, will shift the demand function for good X outwards in Figure 4.2 but this simply causes the price level to increase.

(c) In standard classical models money is just a veil, while we know that money may have real effects at least in the short run. However, it is possible even with the classical paradigm to obtain short term real effects of money. There are a wide range of alternatives here. Students could solve Lucas' misperceptions model or a simple cash in advance model for cases of classical model or a Keynesian (such as McCallum) model to make their point.

10. Consider a McCallum economy with sticky prices where the aggregate demand expression given as:

$$
y_{t}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}\right)+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}\right]+v_{t}
$$

where y_{t}, m_{t} and p_{t} are the logs of real output, nominal money balances and the price level respectively at date t, v_{t} is an i.i.d. normal aggregate demand shock with $v_{t} \sim\left(0, \sigma_{v}^{2}\right)$ are positive parameters and E is the expectations operator. To construct the aggregate supply assume the following:

(i) the market clearing price is denoted by p_{t}^{*}

(ii) the prices are set by firms at $t-1$ and are only effective in t. These are expected to be the market clearing price in t, i.e. $p_{t}=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$

B. Examiners' commentary to Sample examination paper

(iii) real output consistent with natural rate of unemployment, $\left(y^{*}\right)$, is determined by the following law of motion that captures hysteresis

$$
y_{t}^{*}=\delta_{0}+\delta_{1} t+\delta_{2} y_{t-1}^{*}+u_{t}
$$

where t being time trend, $\delta_{0}, \delta_{1}, \delta_{2}$ positive parameters and u_{t} is an i.i.d. normal aggregate supply shock with $u_{t} \sim\left(0, \sigma_{u}^{2}\right)$

(iv) the monetary policy is characterised by the expression

$$
m_{t}=\mu_{0}+\mu_{1} m_{t-1}+e_{t}
$$

where e_{t} is an i.i.d. normal money supply shock with $e_{t} \sim\left(0, \sigma_{e}^{2}\right)$.

(a) Solve for the output gap, i.e. deviations of real output from the market clearing level.

(b) Are unanticipated monetary policy changes effective? Show analytically and provide intuition.

(c) Are anticipated monetary policy changes effective? Show analytically and provide intuition.

(d) What is the Lucas critique? Evaluate the critique based on the output gap equation you have derived above.

Reading for this quesiton: the subject guide, Chapter 9 and McCallum (1989), Chapters 9 and 10.

(a) First consider aggregate demand. By combining the IS and LM equations, we can derive an AD expression of the form:

$$
y_{t}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}\right)+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}\right]+v_{t}
$$

y_{t}, m_{t} and p_{t} are the logs of real output, nominal money balances and the price level, respectively, at date t. v_{t} is a random demand shock with zero mean (i.e. an element of aggregate demand that is not picked up by real money balances or expected inflation). β_{0}, β_{1} and β_{2} are positive parameters and E_{t-1} is the rational expectations operator with the $t-1$ subscript denoting the date the expectation was formed. In rational expectations, people not only take all available information into account when they form their expectations, but their expectations are also consistent with the way in which the variables actually evolve. Rational expectations are sometimes also known as 'model consistent' expectations. The equation states that aggregate demand depends positively on real money balances and positively on expected inflation; for any given nominal interest rate, higher inflation implies a lower real interest rate, making investment cheaper.

Aggregate supply is a little more difficult to specify because of the assumption that prices are set at the beginning of the period, with supply being demand determined. Denote the price that clears the market at date t as p_{t}^{*}. We assume that the prices the firms set at date $t-1$, to be operational in the market at date t, p_{t}, are the prices they expect to clear the market at date t, i.e.:

$$
p_{t}=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]
$$

Also, denote the output level that clears the market at date t as y_{t}^{*}. Then, if the price equals that which allows markets to clear, by definition markets must clear and so y_{t}^{*} must equal the demand when $p_{t}=p_{t}^{*}$.

$$
y_{t}^{*}=\beta_{0}+\beta_{1}\left(m_{t}-p_{t}^{*}\right)+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}^{*}\right]+v_{t} .
$$

Rearranging (7.8) to obtain p_{t}^{*} on the left-hand side gives:

$$
p_{t}^{*}=\frac{\beta_{0}-y_{t}^{*}+\beta_{1} m_{t}+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}\right]+v_{t}}{\beta_{1}+\beta_{2}}
$$

noting that $\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$ equals p_{t} from (B.13) and that in the situation of market clearing, $p_{t}=p_{t}^{*}$. We now need an expression telling us how the market clearing/full employment level of output, y_{t}^{*}, evolves over time. Here we will generalise the process used in McCallum (1989) so that y_{t}^{*} increases gradually through time but also depends positively on last period's full employment output level. This is commonly known as a hysteresis term, which allows persistence of full employment output. If full employment output is high today, it is likely to be high tomorrow.

$$
y_{t}^{*}=\delta_{0}+\delta_{1} t+\delta_{2} y_{t-1}^{*}+u_{t}
$$

u_{t} is a zero mean, random supply, shock, which could include the discovery of oil reserves or other raw materials or a sudden technological breakthrough that can increase the productive capacity of the economy.

(B.13), (B.15) and (B.16) together can be regarded as constituting aggregate supply. Along with aggregate demand, (B.12), we can solve for the output level, y_{t}, in terms of deviations from y_{t}^{*}, and also for the price level, p_{t}. The solution method is given in McCallum, Chapter 10, and you should read the relevant section in order to see how the solution is derived. However, a brief outline is given below.

Take expectations of (B.16) conditional on information available at date $t-1$:

$$
\begin{aligned}
\mathrm{E}_{t-1}\left[y_{t}^{*}\right] & =\delta_{0}+\delta_{1} t+\delta_{2} y_{t-1}^{*} \\
\Rightarrow \quad \mathrm{E}_{t-1}\left[y_{t}^{*}\right] & =y_{t}^{*}-u_{t} .
\end{aligned}
$$

Do the same for (B.14), noting that $\mathrm{E}_{t-1}\left[v_{t}\right]=0$:

$$
\mathrm{E}_{t-1}\left[y_{t}^{*}\right]=\beta_{0}+\beta_{1} \mathrm{E}_{t-1}\left[m_{t}-p_{t}^{*}\right]+\beta_{2} \mathrm{E}_{t-1}\left[p_{t+1}-p_{t}^{*}\right]
$$

Equating (B.17) and (B.18), noting that $\mathrm{E}_{t-1}\left[p_{t+1}-p_{t}\right]$ in (B.18) is equal to p_{t} (from (B.13)) gives:

$$
y_{t}^{*}=\beta_{0}+\beta_{1}\left(\mathrm{E}_{t-1}\left[m_{t}\right]-p_{t}\right)+\beta_{2}\left(\mathrm{E}_{t-1}\left[p_{t+1}\right]-p_{t}\right)+u_{t} .
$$

If we subtract (B.19) from (B.12), we will have an expression for $y_{t}-y_{t}^{*}$ (i.e. deviations of output from the market clearing level).

$$
y_{t}-y_{t}^{*}=\beta_{1}\left(m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]\right)+v_{t}-u_{t}
$$

(b) Imagine that the money supply set by the authorities changes according to:

$$
m_{t}=\mu_{0}+\mu_{1} m_{t-1}+e_{t}
$$

where e_{t} is a zero mean, random error representing monetary policy shocks. Taking expectations of (B.21) conditional on information at date $t-1$ will give us an expression for the unexpected money supply at date $t, m_{t}-\mathrm{E}_{t-1}\left[m_{t}\right]$, which simply equals e_{t}. Substituting this into (B.20) will give us a solution for the output deviation, $y_{t}-y_{t}^{*}$.

$$
y_{t}-y_{t}^{*}=\beta_{1} e_{t}+v_{t}-u_{t}
$$

Notice that the systematic component of monetary policy $\left(\mu_{0}+\mu_{1} m_{t-1}\right)$ does not have any real effects here. This is because at date $t-1$, when prices for date t are set, firms take into consideration what they expect the monetary authorities will do. If they expect the money supply to increa se, knowing that money should have no real effects, they will increase their prices for date t accordingly. Only the random component of monetary policy, the monetary policy shock e_{t}, will have real effects since this is realised after the prices have been set. This result, that the systematic component of monetary policy has no real effect, will be discussed in more detail in the next chapter, in the section 'policy ineffectiveness proposition'.

(c) The Lucas critique refers to the instability of reduced-form expressions used for policy making or policy appraisal. In the sticky price model of Chapter 9 , the structural equations were given by the aggregate demand equation, the price equation, $p_{t}=\mathrm{E}_{t-1}\left[p_{t}^{*}\right]$ the equation showing how y_{t}^{*} evolves and the monetary policy reaction function showing how m_{t} depends on the state of the economy. When we solve for $y_{t}-y_{t}^{*}$, the equation we derive is a reduced-form equation; a mixture of aggregate demand, aggregate supply and the authorities' reaction function. For example, consider the sticky price model and the reduced-form expression, (B.22), $y_{t}-y_{t}^{*}=\beta_{1} e_{t}+v_{t}-u_{t}$. Instead of e_{t}, write $m_{t}=\mathrm{E}_{t-1}\left[m_{t}\right]=m_{t}-\mu_{0}-\mu_{1} m_{t-1}$.

$$
y_{t}-y_{t}^{*}=-\beta_{1} \mu_{0}+\beta_{1} m_{t}-\beta_{1} \mu_{1} m_{t-1}+\left(v_{t}-u_{t}\right)
$$

If this equation were given to an econometrician, he or she would run a regression of the form:

$$
y_{t}-y_{t}^{*}=\gamma_{0}+\gamma_{1} m_{t}+\gamma_{2} m_{t-1}+\eta_{1}
$$

and would find a positive coefficient for γ_{1} since we know $\gamma_{1}=\beta_{1}$ (only if the economy was accurately represented by the structural equations given above!). Since $\gamma_{1}>0$, we may then think that an increase in the money supply should cause an increase in output above y_{t}^{*}. For example if γ_{1} was found to equal 0.5, then increasing the money supply by 2% should cause a 1% increase in output. However, as we saw in Chapter 7, the systematic component of monetary policy had no real effects. Even though we may think an increase in the money supply will increase output (from the reduced form equation, (B.24)), in reality the change in people's expectations associated with this policy change will cause the reduced form to break down. If the authorities increased the money supply by increasing μ_{0}, indeed this will have a positive effect on output, via γ_{1}, but it will also have a negative effect on output since $\gamma_{0}=-\beta_{1} \mu_{0}$. The effect of an expansionary monetary policy will be purely inflationary. Quoting Romer (2001) p.251,

'If policymakers attempt to take advantage of statistical relationships, effects operating through expectations may cause the relationships to break down. This is the famous Lucas critique.'

11. Suppose that the economy of Pool-land is characterised by the following IS schedule

$$
Y=a-b R+\varepsilon
$$

and the following LM schedule

$$
M=c-d R+e Y+\eta
$$

where Y represents the real output, R the short term interest rate, M the real money balances, coefficients a, b, c, d are positive constants. The economy is subject to additive shocks. ε and η are IS and LM shocks with the following properties $\varepsilon \sim$ i.i.d. $\left(0, \sigma_{\varepsilon}^{2}\right)$ and $\eta \sim$ i.i.d. $\left(0, \sigma_{\eta}^{2}\right)$. Note that the uncertainty arises only due to additive shocks.

Suppose that the monetary authority goal is to minimise the variance of output.

(a) Is it better to use short term interest rates instead of real money balances to stabilise the economy when the economy is subject to IS shocks only? Show analytically.

(b) Is it better to use short term interest rates instead of real money balances to stabilise the economy when the economy is subject to LM shocks only? Show analytically.

(c) Now consider the case of multiplicative uncertainty, where parameters of the model, a, b, c, d are subject to uncertainty. Would this model modification affect your policy results? Discuss verbally.

Reading for this question: the subject guide, Chapter 12.

(a,b) (Note: Here, parts a and b are analysed together.) Uncertainty over the structure of the economic model, or reduced-form model instability, which manifests itself through the Lucas critique, is one form of model uncertainty that the monetary authorities must consider when deciding on policy.

Poole (1970) analysed the implication of adding shocks to both the IS and LM schedules. Such shocks could come about from trade union strikes or wars on the IS side and stock market crashes and financial crises such as the collapse of LTCM in 1998 on the LM side. Poole's model assumes that the parameters and structure of the model are known with certainty, which is an unrealistic assumption, but allows the IS and LM schedules to be subject to zero-mean random errors. The schedules can be described by (B.25).

$$
\begin{aligned}
Y & =a-b R+\varepsilon \\
M & =c-d R+e Y+\eta .
\end{aligned}
$$

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ε and η are additive IS and LM shocks whose variances are σ_{ε}^{2} and σ_{η}^{2} respectively, and for simplicity we ignore the price level in the LM curve. Alternatively, assume the monetary authorities can control real money balances, denoted by M. The authorities can either set the money supply, M, or the interest rate, R, but not both. With a downward-sloping money demand schedule, either R or M can be set and the other variable will have to change to allow the markets to clear. If the authorities set the interest rate, then from the IS schedule, the expected value of output, Y, given R, denoted $\left.\mathrm{E}[Y]\right|_{R}$ will be:

$$
\left.\mathrm{E}[Y]\right|_{R}=a-b R
$$

Assume that the goal of the policymakers is to minimise the variance of output. From (13.29) and (13.30), $Y-\left.\mathrm{E}[Y]\right|_{R}$ will simply be ε and so the variance of output given that the authorities set the interest rate will be $\mathrm{E}\left[Y-\left.\mathrm{E}[Y]\right|_{R}\right]^{2}=\sigma_{\varepsilon}^{2}$. Alternatively, the monetary authorities could set the money supply, M. In order to calculate the variance of output in this scenario, we need to calculate Y as a function of M only. From the

LM schedule, we can calculate R as a function of M, Y and η and then substitute this into the IS curve. Solving for Y will give:

$$
Y=\frac{a d-b c}{d+b e}+\frac{b M}{d+b e}+\frac{d \varepsilon-b \eta}{d+b e}
$$

We can then find $\left.\mathrm{E}[Y]\right|_{M}$ and the variance of output given that the authorities directly control the money stock. This is given in (13.32).

$$
\mathrm{E}\left[Y-\left.\mathrm{E}[Y]\right|_{M}\right]^{2}=\left(\frac{1}{d+b e}\right)^{2}\left(d^{2} \sigma_{\varepsilon}^{2}+b^{2} \sigma_{\eta}^{2}\right)
$$

We can now examine which policy instrument, when set by the authorities, will result in a lower output variance. Consider the case where there are no IS shocks, σ_{ε}^{2}. The variance of output under both interest rate and money targeting regimes is given in the top line of Table B.1. It is clear that setting the interest rate and allowing money to change to clear the market is the optimal strategy. However, if there are no LM shocks, σ_{η}^{2}, then output variance is smaller under a policy of fixed money supply, see the bottom line of Table B.1. Therefore, if an economy is prone to IS shocks, the authorities should keep the money supply constant. If the economy is prone to money market, LM, shocks, the interest rate should be the instrument of choice.

$$
\begin{array}{ll}
\text { Only LM shocks, } \sigma_{\varepsilon}^{2}=0 & \operatorname{Var}[Y \mid R]=0<\operatorname{Var}[Y \mid M]=\left(\frac{b}{d+b e}\right)^{2} \sigma_{\eta}^{2} \\
\text { Only IS shocks, } \sigma_{\eta}^{2}=0 & \operatorname{Var}[Y \mid R]=\sigma_{\varepsilon}^{2}>\operatorname{Var}[Y \mid M]=\left(\frac{d}{d+b e}\right)^{2} \sigma_{\varepsilon}^{2}
\end{array}
$$

Table B.1:

This is also shown in the subject guide Figures 12.1a and 12.1b. Figure 12.1a shows the case with IS shocks only. The output variation when the interest rate is fixed at R^{*} is shown by $\left.D Y\right|_{R}$, in which case the money supply has to change to clear the money markets causing the LM curve to shift so that equilibrium is at points A or B. If the money supply was kept fixed then output deviations are shown by $\left.D Y\right|_{M}$. Therefore, with IS shocks, in order to keep output variance at a minimum, it is best to keep the money supply fixed.

In Figure 12.1b, by keeping the interest rate fixed after LM shocks, equilibrium will be unchanged at point E and output variance will therefore be zero. By keeping the money supply fixed, however, LM shocks will cause equilibrium to move between points A and B and output variance will be positive, equal to $\left.\Delta Y\right|_{M}$. So, depending on the main source of economic shocks, whether they originate from the goods or money markets, will determine which monetary instrument the authorities should target.

(c) Whereas Poole considered the case where shocks were additive in nature, Brainard (1967) examined the case where the values of the parameters in the model were not known with certainty. This is, arguably, more realistic since any model must be estimated from data. An estimated model will not only give point estimates of the parameters but will also give standard errors since there will always be measurement error, model mis-specification and other problems that cause us not to know the exact structure of the economic model. Suppose output, y, depends on a policy mix, X, a vector containing fiscal and monetary in struments that the government can control. The relationship between y and X is given by:

$$
y=g X
$$

For simplicity, assume X is a single policy instrument so that g is a scalar parameter estimate with mean $\widehat{g}$ and variance σ_{g}^{2}. The aim of the authorities is to minimise the variance of y around some target level, y^{*}, full employment level of output for example, subject to the constraint (B.30), i.e.

$$
\min \mathrm{E}\left[y-y^{*}\right]^{2} \quad \text { s.t. } \quad y=g X \text {. }
$$

If $y=g X$, taking averages will give $\widehat{y}=\widehat{g} X$. The problem can then be written as:

$$
\begin{aligned}
& \min \mathrm{E}\left[(y-\widehat{y}) X-\left(y^{*}-\widehat{y}\right)\right]^{2} \quad \text { s.t. } \quad y=g X \\
\Rightarrow & \min \mathrm{E}\left[(g-\widehat{g}) X-\left(y^{*}-\widehat{g} X\right)\right]^{2} \\
\Rightarrow & \min \mathrm{E}\left[(g-\widehat{g})^{2} X^{2}+\left(y^{*}-\widehat{g} X\right)^{2}-2(g-\widehat{g}) X\left(y^{*}-\widehat{g} X\right)\right]
\end{aligned}
$$

Noting that $\mathrm{E}[g-\widehat{g}]^{2}=\sigma_{g}^{2}$ and that $\mathrm{E}[g-\widehat{g}]=\widehat{g}-\widehat{g}=0$, the problem then becomes:

$$
\min _{X} X^{2} \sigma_{g}^{2}+\left(y^{*}-\widehat{g} X\right)^{2}
$$

Differentiating this with respect to X, the choice variable, setting equal to zero and solving will give:

$$
X=\frac{\widehat{g} y^{*}}{\sigma_{g}^{2}+\widehat{g}^{2}} \quad \Rightarrow \quad \widehat{y}=\widehat{g} X=\frac{\widehat{g}^{2} y^{*}}{\sigma_{g}^{2}+\widehat{g}^{2}}<y^{*}
$$

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The implication of the model is that because of the presence of uncertainty, $\sigma_{g}^{2}>0$, the authorities will never push aggressively enough to make average output equal to the target level, y^{*}. To do so will simply cause output variation to increase to an intolerable level. The policy maker would rather have a stable level of output below the full employment level than very volatile output whose average was y^{*}. This is shown in the subject guide Figure 12.2.

From $y=g X$ and $\widehat{y}=\widehat{g} X$, it follows that $\sigma_{y}^{2}=\sigma_{g}^{2} X^{2}$ which implies $X=\sigma_{y} / \sigma_{g}$. Substituting into $y=g X$ will give the linear policy constraint in Figure 12.2. The authorities try to reach the indifference curve closest to y^{*}, the target level, subject to the policy constraint and as can be seen in Figure 12.2 , the presence of uncertainty, σ_{g}^{2}, causes the authorities to opt for a less aggressive policy stance, causing equilibrium output to be below y^{*}.

\begin{enumerate}
\setcounter{enumi}{11}
\item A treasurer has to manage the cash position of a small London college. At the beginning of the year the college receives $\pounds 5$ million in tuition fees. The college is expected to spend all of that money in the year, at an approximately constant daily rate. The treasurer can either hold the financial wealth of the college in bonds, yielding an interest rate of 4% or in cash, which is assumed to not receive any interest. The accrued interest on the bond holdings is received at the end of the year and is not compounded inbetween. The brokerage fees of exchanging bonds for money are $\pounds 1,000$ per transaction. Note that the first transaction takes place at the beginning of the year, when the treasurer buys bonds, the transactions during the rest of the year will compose of selling bond holdings. The treasurer wants to minimize the costs of cash management, C, which are composed of the interest foregone on the cash holdings and the brokerage fees.
\end{enumerate}

(a) Draw the time profile of the college's holdings of cash and bonds.

(b) Set up the cost minimization problem for the treasurer and calculate the how much money, Z, should be transferred each time the treasurer sells off bonds.

(c) Calculate the interest elasticity of money.

(d) The problem outlined above is based on the Baumol-Tobin inventory theoretic model. State and discuss two criticisms of the inventory theoretic model.

Reading for this question: the subject guide, Chapter 3 .

(a) See Figure 3.2 in the subject guide.

(b) We have

$$
\begin{aligned}
\min _{z} C & =1,000 \times 5,000,000 / Z+0.04 Z / 2 \\
& =5,000,000,000 / Z+0.02 Z
\end{aligned}
$$

The first order condition gives

$$
\begin{aligned}
5,000,000,000 Z^{-2}+0.02 & =0 \\
Z & =\sqrt{5,000,000,000 / 0.02} \\
Z & =\pounds 500,000 .
\end{aligned}
$$

(c) Interest elasticity of money is defined as $\partial \ln M / \partial \ln i$.

$$
\begin{aligned}
M & =Z / 2=\sqrt{b T / 2 i} \\
\ln M & =0.5 \ln b+0.5 \ln T-0.5 \ln i-0.5 \ln 2 \\
\partial \ln M / \partial \ln i & =-0.5 .
\end{aligned}

$$

(d) A good answer points out that the inventory theoretic model does not hold well in practice. Two criticism are that the model assumes (1) perfect information about the pattern of expenditures and (2) does not take into account institutional factors. This leads the model to predict that firms hold very little cash, much less than empirically observed.

\begin{enumerate}
  \setcounter{enumi}{12}
  \item Assume that the banking sector is described as follows:
\end{enumerate}

$$
\begin{aligned}
D & =d_{0}-d_{1}\left(i-i_{D}\right) \\
L & =l_{0}-l_{1}\left(i-i_{L}\right)
\end{aligned}
$$

where L stands for bank loans, D stands for bank deposits, i_{L} the loan rate, i_{D} deposit rate and i is the market interest rate. Assume that banks do not have operating costs or are not required to hold reserves.

(a) Calculate the competitive equilibrium. Show graphically. How do your results change when the government sets deposit rates equal to $i_{D}=a$, with $a<i_{L}$? Provide intuition.

(b) Now suppose that government introduces a mandatory reserve ratio, r^{*}, such that $R=r^{*} D$. How do your results change? What are the implications of such a reserve ratio policy on prices and quantities? Provide intuition.

(c) Sometimes, it is suggested that the reserve ratio policy can be an alternative to targeting interest rates or monetary aggregates. Can this be an effective policy to stabilise output and inflation fluctuations?

Reading for this question: the subect guide, Chapter 4 .

(a) Competitive equilibrium: The profits made by the bank will equal the quantity of loans, L, multiplied by the interest earned on those loans, i_{L} (which is the bank's revenues) minus the costs faced by the bank. Costs will equal the interest paid to depositors in order to encourage them to hand over their funds to the bank. Costs then equal the quantity of deposits, D, times the interest rate paid on deposits, i_{D}. Noting that in a competitive equilibrium, profits equal zero:

$$
\Pi=L i_{L}-D i_{D} \quad \text { and } \quad \Pi=0 .
$$

When making the additional assumption that the bank holds no reserves, in order for the bank's balance sheet to balance and convenience assumption that constants $d_{0}=l_{0}$, loans (assets) must equal deposits (liabilities), $L=D$.

Substituting implies that in equilibrium:

$$
i_{D}=i_{L}
$$

B. Examiners' commentary to Sample examination paper

Substituting into bank condition and equating L with D gives:

$$
\left.d_{0}-d_{1}\left(i-i_{D}\right)=d_{0}+l_{(} i-i_{D}\right)
$$

Solving for the interest rate on deposits, which equals the interest rate charged on loans gives:

$$
i_{D}=i_{L}=i \text {. }
$$

This implies total deposits, D, which equals total loans, L, equals d_{0}. In equilibrium, the interest rate paid on deposits equals the interest rate charged on loans, which equals the market interest rate, i. The level of deposits, which in this model is equal to the level of the money stock,equals the amount of loans. (See e.g. Figure 4.1 in the subject guide.)

Government regulation of the deposit rate: One way that the government could reduce or control the money supply is by setting a limit on the interest rate paid on deposits. If the maximum interest rate banks can pay on deposits is less than i in the above example, then the amount of funds savers are willing to deposit with the bank will fall, resulting in a lower money supply. Note that this will also be associated with a lower quantity of loans that, despite the lower money supply, may be harmful to the economy if it means some investment opportunities are not undertaken. Consider the case where the government sets the interest rate on deposits equal to zero. From deposit condition, the quantity of deposits is fixed at $D=d_{0}-d_{1} i$. Diagrammatically, this is shown in Figure 4.2 in the subject guide.

The level of deposits, and hence of the money supply, has fallen from d_{0} to $d_{0}-d_{1} i$ because of the government regulation. Since deposits must equal loans in order for the bank's balance sheet to balance, the lower level of deposits means a lower level of loans, which is associated with a higher interest rate charged, $i_{L}>i$. Also, since there is a difference between the interest rate charged on loans and that paid on deposits (which has been set at zero), the government regulation has allowed the banks to make positive profits.

(b) Assume that the government introduces a mandatory reserve ratio, r^{*} (i.e. total reserves of the bank, R, equals r^{*} times $\left.D\right)$. The bank's assets now comprise loans, as before, but now include these reserves. Its liabilities are just the deposits of its customers. Since Assets = Liabilities, we can write:

$$
r^{*} D+L=D \quad \text { and } \quad L=\left(1-r^{*}\right) D
$$

Substituting this into the zero profit condition and rearranging will give the interest rate charged on loans to be:

$$
i_{L}=i_{D} /\left(1-r^{*}\right)>i_{D}
$$

The interest rate charged on loans is now higher than the interest rate paid on deposits. This is in order to compensate the banks for holding reserves, on which it earns no interest. Total loans are now less than total deposits so each dollar lent out must earn a higher return than that paid on each dollar deposited with the bank. i_{L} then needs to be greater than i_{D}.

By changing the mandatory reserve ratio, the government can change the total money supply through the activities of the banking sector. However, the way in which D varies with r^{*} is uncertain and depends on the parameters of the model. With no reserve requirements we equate L to D in order for the bank's balance sheet to balance. In this situation we equate L to $\left(1-r^{*}\right) D$. Therefore the new intersection results in a lower value of L but a higher interest rate charged, i_{L}. Total revenues for the bank, L times i_{L}, could then increase or decrease depending on the elasticity of the demand for loans function. If the demand for loans is elastic then, if L falls, revenue will fall. Banks will be forced to cut the interest paid on deposits, so i_{D} falls, resulting in fewer deposits and hence causing the money supply to fall. If the demand for loans is inelastic, a fall in L, caused by the reserve ratio, will increase revenues. The banks will increase i_{D} and so total deposits will increase.

(c) Unlikely for several reasons. First, such it is extremely cumbersome to implement reserve policy in relatively high frequency (say, monthly). Second, its effectiveness depends on the loan demand elasticity. Even under the assumption of time invariant elasticity, given the variety of loans offered in the banking system, it is very hard too anticipate the effects of the monetary policy on the transmission channels. Finally, it is a rather difficult one to communicate the policy changes to wider society, thus harming the transparency aspects of monetary implementation.

\end{document}