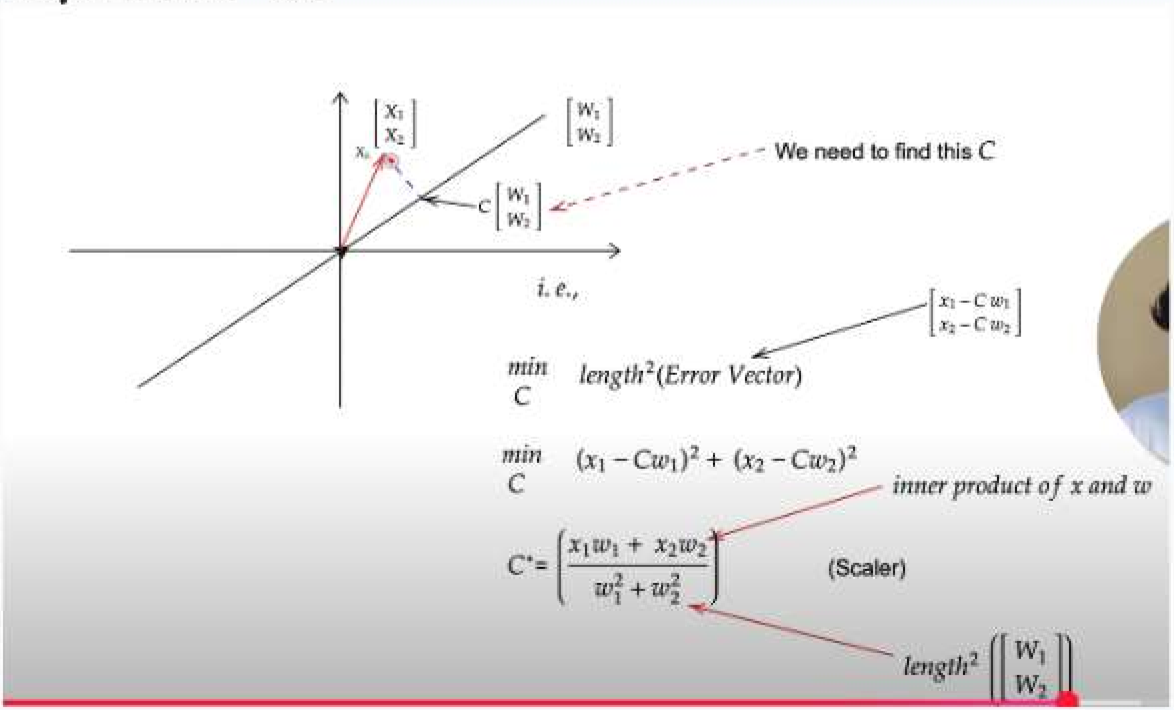
:::ML Notes:::

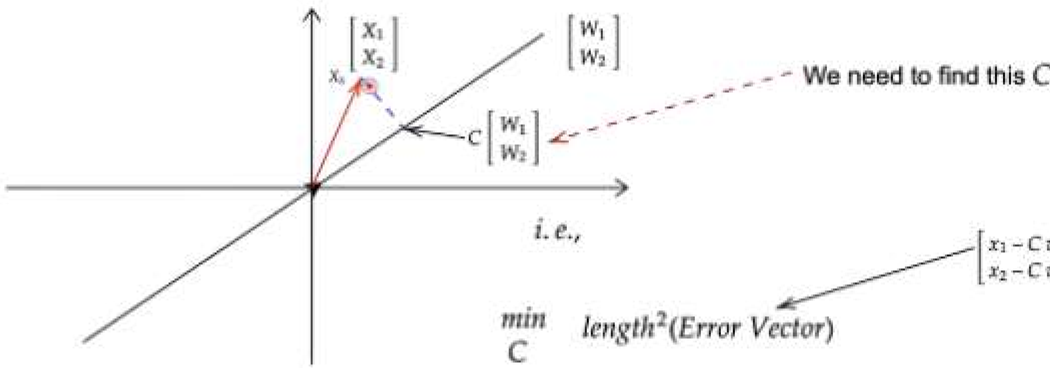
TOPIC1:Representation Learning

For Using matrics in ML suppose we have 2n matrix [a,b] now to make the data less we use a relation between all the n uppr data like [1,x] matix multiply to produce a new n lenght upr element set. not all mat are mutiple of x so for to avoid that other points proxy or the foot of perpendicular is used to represent it..



::How to find a line with least distance or errors in proxies::

To find the minimum distance line from a point:



$i.e.,$

$$\min_{C} \; length^2(\text{Error Vector}) \qquad \text{Error Vector} = \begin{bmatrix} x_1 - C w_1 \\ x_2 - C w_2 \end{bmatrix}$$

$$\min_{C} \; (x_1 - Cw_1)^2 + (x_2 - Cw_2)^2$$

inner product of x and w

$$C^* = \left(\frac{x_1 w_1 + x_2 w_2}{w_1^2 + w_2^2} \right) \qquad \text{(Scaler)}$$

$length^2\left(\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}\right)$

To find the sumation of many min diatance to find a proxy line:

$$\max_{\substack{w \\ ||w||^2 = 1}} \; g(w) = \frac{1}{n}\sum_{i=1}^{n} \left(x_i^T w\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} \left(w^T x_i\right)\left(x_i^T w\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} w^T \left(x_i x_i^T\right) w$$

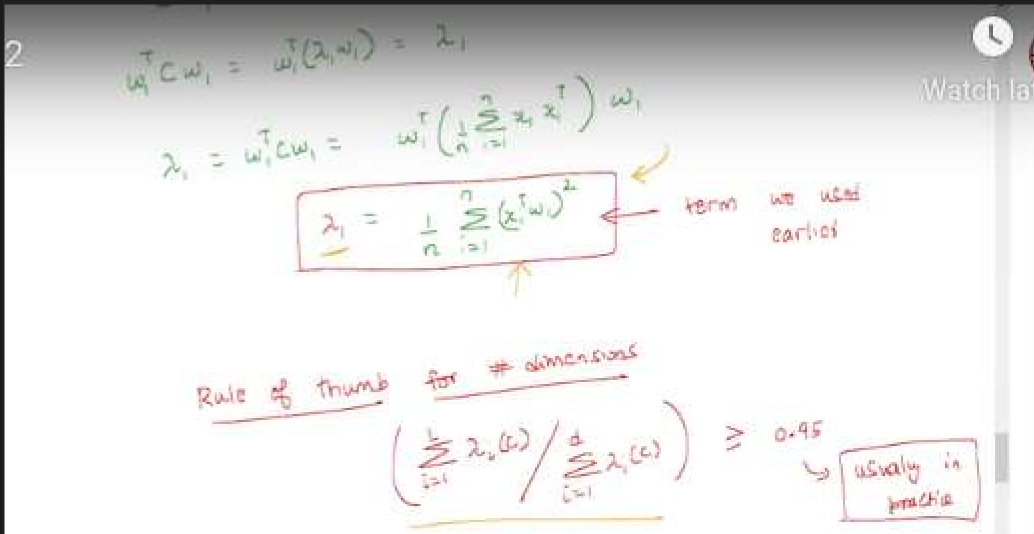
$$= w^T \left(\frac{1}{n}\sum_{i=1}^{n} \left(x_i x_i^T\right)\right) w$$

$$\boxed{\text{equivalently,} \quad \max_{\substack{w \\ ||w||^2 = 1}} \; w^T C\, w \qquad \text{Where, } C = \left(\frac{1}{n}\sum_{i=1}^{n} \left(x_i x_i^T\right)\right)}$$

Covarience Matrix

W is the Eigen vector corresponding to maximum eigenvalue of the covariance matrix C

See in real world requirements data is not at origin so basically we have to center the data and use that as a origin...



How to find a residue:

Step 1: Projection vector

Formula:

$$\mathrm{proj}_w(x) = \frac{x^\top w}{w^\top w}\, w.$$

- $x^\top w = 2(1) + 5(1) = 7.$
- $w^\top w = 1^2 + 1^2 = 2.$

So,

$$\mathrm{proj}_w(x) = \frac{7}{2}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3.5\\3.5\end{bmatrix}.$$

Step 2: Residue

$$r = x - \mathrm{proj}_w(x) = \begin{bmatrix}2\\5\end{bmatrix} - \begin{bmatrix}3.5\\3.5\end{bmatrix} = \begin{bmatrix}-1.5\\1.5\end{bmatrix}.$$

Reconstruction Error:

Step 1: Residue (already found earlier)

$$r = x - \mathrm{proj}_w(x) = \begin{bmatrix} -1.5 \\ 1.5 \end{bmatrix}.$$

Step 2: Reconstruction error

The reconstruction error is the squared norm of the residue:

$$E = \|r\|^2 = (-1.5)^2 + (1.5)^2 = 2.25 + 2.25 = 4.5.$$

If the problem instead asks for the **Euclidean distance** (not squared), then:

$$\|r\| = \sqrt{4.5} \approx 2.12.$$