MAT 141 HW # 3

#1. Find bijective map between C_1 & C_2

map: $f: C_1 \to C_2$ $\quad \Gamma_1 = \langle \tau_{(1,0)} \rangle$ $\Gamma_2 = \langle \tau(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \rangle$

~~$f([x]) = [\sqrt{2}x]$~~

~~suppose $f([x]) = f([y])$ $\forall$ $[x],[y] \in C_1$. Then $[\sqrt{2}x] = [\sqrt{2}y]$~~

~~$\Rightarrow \sqrt{2}(x-y) \in \Gamma 1 = \langle \tau(1,0) \rangle \in \mathbb{Z}$~~

~~given $x,y \in [0,1)$, $|x-y| < 1$, meaning $|\sqrt{2}(x-y)| < \sqrt{2}$~~

~~meaning that $x=y$, & every x maps to a y, meaning $f([x]) = [\sqrt{2}x]$ is injective~~

suppose $g: \Gamma_1 \to \Gamma_2$ = ~~Γ_{2}~~

C_1 is a cylinder of radius 1, and C_2 is a cylinder of radius $(\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = 1$ as well, meaning that a translation of C_1 by $t_{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})}$ maps to C_2.

C_3 however is a ~~radi~~ cylinder with radius $1^2 + 1^2 = 2$, meaning that a scaling factor is necessary to map C_1 to C_3, which is not an isometry, as curvature is not constant.

#2 assume torus $T = \mathbb{R}^2/\Gamma$ where $\Gamma = \langle \tau_1, \tau_2 \rangle$

because translations in the plane comute, given τ is a translation of the plane, $\tau\tau_1 = \tau_1\tau$ & $\tau\tau_2 = \tau_2\tau$, if τ is applied on torus T, τ will send points in Γ to other points in Γ, essentially mapping to itself. we also know this map is bijective as its inverse is τ^{-1}.

#3

i) we see that a rotation of a square by 90° sends one side to an adjacent side

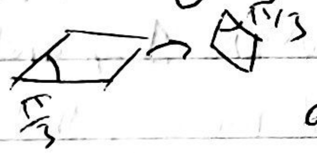
now assume this 90° rotation to be denoted by f

$f := r_{\frac{\pi}{4}}$ on a square plane

we can say $f^4 = f^{-4} = id$, meaning that f has an order of 4

ii)

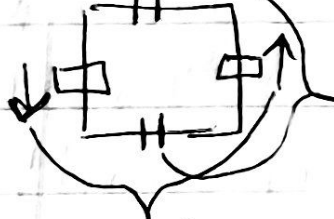
rotation by $\frac{\pi}{3}$ sends



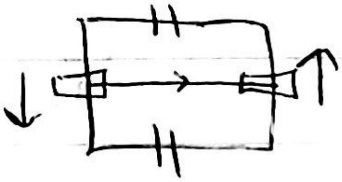
consider $f := r_{\frac{\pi}{3}}$ on a rhombus

$f^6 = r_{2\pi} = id$, there for the isometry f has an order of 6

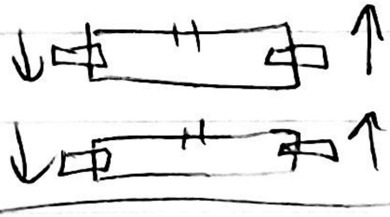
#4 the domain of a Klien bottle is a square, where the bottle is formed by connecting a pair of 2 opposite edges, and twisting to other two & connecting:



when cut in half, such a figure is formed



this results in 2 mobius strips, where opposite ends are twisted & connected =



#5 a) $(1,0,0)$ $(1,1,0)$ $(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

$S^2=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2=1\}\subset\mathbb{R}^3$

$1^2+0^2+0^2=1$ $\therefore$ $(1,0,0)$ exists on S^2

$1^2+1^2+0^2=2$ $\therefore$ $(1,1,0)$ doesn't exist on S^2

$0^2+(\frac{1}{\sqrt{2}})^2+(\frac{1}{\sqrt{2}})^2=1$ $\therefore$ $(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ exists on S^2

$\boxed{(1,0,0)\ \&\ (0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})\text{ exist on }S^2}$

b) $|a|=|b|=|c|$ $x^2+y^2+z^2=1$

(a,b,c) $|a|^2+|b|^2+|c|^2=1$

$=|a|^2+|a|^2+(|a|)^2=1$

$=3a^2=1$ $a=b=c=\sqrt{\frac{1}{3}}$

$\boxed{(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}})\text{ exists on }S^2}$