MAT 141 HW#4

#1 assume great circles to be G_1 & G_2, which are 2 intersecting circles on a sphere S^2
let P_1 & P_2 be planes that encompas G_1 & G_2
Such that $S^2 \cap P_1 = G_1$ & $S^2 \cap P_2 = G_2$

Note that center of S^2 is $(0,0,0)$.
$G_2 \cap G_1 = S^2 \cap (P_2 \cap P_1)$
assume l to contain the points of $S^2 \cap (P_2 \cap P_1)$
l passes through the origin $\therefore$
$de(l \cap G_2) = de(l \cap G_1)$
meaning that $G_2 \cap G_1$ produces 2 antipodal points

#2 suppose $P = (x, y, z)$ & $Q = (-x, -y, z)$
P and Q are antipodal
we know $d_{S^2}(P, Q) = \pi$ (either side of the sphere)
we know that isometries do not change the distance on a surface, meaning $d_{S^2}(f(P), f(Q)) = d_{S^2}(P, Q)$
and $d_{S^2}(f(P), f(Q)) = \pi$, meaning $f(P)$ & $f(Q)$ are antipodal

#3 a) the map $f(x,y,z) = (-x,-y,-z)$ can be expressed as three reflections across the $x=0$ plane $y=0$ plane & $z=0$
c) $L = (x,y,z)$ such that $m \circ L = L$

b)

$(-x,-y,-z) = (x,y,z)$
$\therefore x = y = z = 0$, but
$S^2 \cap O = \emptyset$, meaning that m has no fixed point on S^2

#4 $\bar{m} = \bar{r}_x \circ \bar{r}_y \circ \bar{r}_z$

reflections are commutative

so $\bar{r}_x \circ \bar{r}_y \circ \bar{r}_z = \bar{r}_z \circ \bar{r}_y \circ \bar{r}_x$

$\bar{r}_x \circ f = f \circ \bar{r}_x$

therefore $\bar{m} \cdot f = f \circ \bar{m}$

$\bar{r}_x \circ \bar{r}_y \circ \bar{r}_z \circ f = f \circ \bar{r}_x \circ \bar{r}_y \circ \bar{r}_z$