

Normal equation derivation:

The best fit line is given by:  $\theta_0 x_0 + \theta_1 x_1 + \dots + \theta_n x_n$   
 $\Rightarrow \theta^T x$

The motive in linear regression is to minimize the cost function:

$$J(\theta) = \frac{1}{n} \sum_{i=1}^n (\theta^T x_i - y_i)^2$$

Since we cannot ~~not~~ minimize  $n$ , we try to minimize,  
 $(\theta^T x_i - y_i)^2$

which can be written as

$$(X\theta - y)^T (X\theta - y) = J(\theta)$$

Since,  $\theta$  is the independent variable we find minima of  $J(\theta)$  w.r.t  $\theta$  by equating derivative of cost function equal to 0.

$$\rightarrow \frac{\partial J}{\partial \theta} = \frac{\partial [(X\theta - y)^T (X\theta - y)]}{\partial \theta} = 0$$

$$\Rightarrow 2X^T X\theta - 2X^T y = 0$$

$$\Rightarrow 2X^T X\theta = 2X^T y$$

$$\Rightarrow (X^T X)^{-1} (X^T X)\theta = (X^T X)^{-1} \cdot (X^T y)$$

$$\Rightarrow \theta = (X^T X)^{-1} \cdot (X^T y)$$