End Semester Examination
MA5470 Numerical Analysis
Department of Mathematics, IIT Madras

July 17, 2020 | Duration: 150 min from 9.30am | Max 50 Marks

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1. Write an algorithm for solving the linear system $Ax = b$ if $a_{ij} = 0$ whenever $j \leq n - i$ (assume A is non-singular). **(4)**

2. Assuming that the LU-factorization of A is available, write an algorithm to solve the equation $x^T A = b^T$. **(6)**

3. Prove that an iteration matrix for solving an algebraic system (of equations) $Ax = b$ can be written as $I - Q^{-1}A$. Also prove that, if $\delta = \|I - Q^{-1}A\| < 1$, **(5)**
$$\|x^{(k)} - x\| \leq \frac{\delta}{1-\delta}\|x^{(k)} - x^{(k-1)}\|$$

4. Starting with an initial approximation $(0, 0, 1)^T$, perform one Newton's iteration for the non-linear system $xy - z^2 = 1, \quad xyz - x^2 + y^2 = 2, \quad e^x - e^y + z = 3$ **(5)**

5. Find the truncation error of the approximation
$$f'(x) = \frac{1}{12\Delta x}\left(-f(x + 2\Delta x) + 8f(x + \Delta x) - 8f(x - \Delta x) + f(x - 2\Delta x)\right)$$
where Δx is the uniform step length. **(5)**

6. The quadrature formula $\int\limits_0^2 f(x)\ dx = c_0 f(0) + c_1 f(1) + c_2 f(2)$ is exact for all polynomials of degree less than or equal to 2. determine c_0, c_1 and c_2. **(5)**

7. Prove that, if f is a polynomial of degree k such that $k < n$, then the divided difference **(5)**
$$f[x_0, x_1, \cdots, x_n] = 0$$

8. Find the order of the numerical scheme (Δx is the uniform step length)

$$y_n - y_{n-2} = \frac{\Delta x}{3}\left(f_n + 4f_{n+1} + f_{n+2}\right)$$

used to solve the first order IVP $y' = f(x, y)$, $y(x_0) = y_0$. **(5)**

9. Derive the 2nd order Adam - Bashforth formula (Δx is the uniform step length)

$$y_{n+1} - y_n = \Delta x\left(\frac{3}{2}f_n - \frac{1}{2}f_{n-1}\right)$$

from a general linear miltistep method to solve the first order IVP $y' = f(x, y)$, $y(x_0) = y_0$. **(5)**

10. Describe the Newton's method for improving the choice of the value of the first derivative at the initial point while solving the boundary value problem $y'' = f(x, y, y')$, $y(a) = y_a$ and $y(b) = y_b$ using the non-linear shooting method. **(5)**

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