DEPARTMENT OF MATHEMATICS, I.I.T. MADRAS

Max Marks: 30 Online Exam, July 2, 2020; MA – 6270, Exam TIME – 75 MINS

Max time allowed is 2 hrs including sending PDF from 10 AM to 12 Noon

Q1. Using Crank-Nicolson F-D scheme, find $\boldsymbol{U(x,1)}$ for $\boldsymbol{x = 0, 1, 2}$ and $\boldsymbol{3}$ when $\boldsymbol{U(x,t)}$ satisfies $\frac{\partial U}{\partial t} = 4\,\frac{\partial^2 U}{\partial x^2}$, $0 < x < 3$; **(10 M)**

$$U(3,t) = 54\,t, \quad t > 0\,; \quad \frac{\partial U}{\partial x}(0,t) = U(0,t), \quad t \geq 0;$$

$$U(x,0) = 18x, \qquad 0 \leq x \leq 3$$

(Use central difference formula for derivative boundary condition)

Q2. Let $\boldsymbol{U(x,y)}$ satisfies

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = U\,;\ 0 < x < 4\,; 0 < y < 4$$

With $\quad U(x,0) = U(x,4) = 100x\,; \qquad 0 \leq x \leq 2$

$\qquad\qquad = 100(4-x);\ 2 \leq x \leq 4$

$\quad U(0,y) = U(4,y) = 200y\,; \qquad 0 \leq y \leq 2$

$\qquad\qquad = 200(4-y); 2 \leq y \leq 4$

If $\boldsymbol{U(1,1) = 109.41}$ is fixed, Find the values $\boldsymbol{U(1,2), U(2,1)}$, $\boldsymbol{U(2,2), U(2,3)}$ and $\boldsymbol{U(3,2)}$ at intermediate grid points using central difference scheme with equal step lengths as $\Delta x = \Delta y = 1$. **(10 M)**

Q3. Use the method of Characteristics to find the first approximation to the solution of $\frac{\partial^2 U}{\partial x^2} - (2x+y)^2\,\frac{\partial^2 U}{\partial y^2} = 0$, at the first characteristic grid point between $x = 0.2$ and $x = 0.3$, $y > 0$,
where $U(x,0) = 2 + 5\,x^2$ and $\frac{\partial U}{\partial y}(x,0) = 6\,x$, $\ 0 \leq x \leq 1$. **(10 M)**

NOTE: Send your answer booklet in PDF through email and if email does NOT work, then photo in whatsup to 9444636946. Any one is enough, do not send both modes.