MM16B023 H. Vishal

(1)

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$$

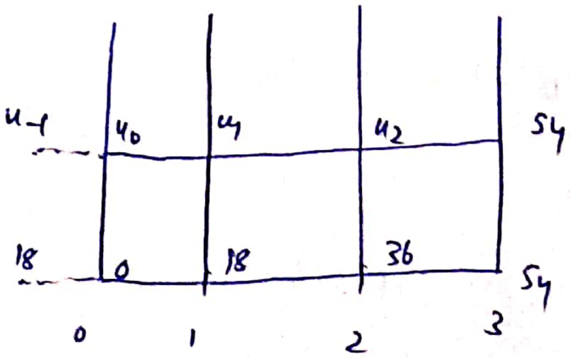
CN FD Scheme :

$$\Rightarrow \frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \frac{4}{2}\left[\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta x)^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta x)^2}\right]$$

$$\Delta x = \Delta t = 1 \quad \left(r = \frac{\Delta t}{(\Delta x)^2} = 1\right)$$

$$\Rightarrow u_{i,j+1} - u_{i,j} = 2\left[u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1} + u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right]$$

$$\Rightarrow +2u_{i-1,j+1} - 5u_{i,j+1} + 2u_{i+1,j+1} = -2u_{i-1,j} + 3u_{i,j} - 2u_{i+1,j}$$



$$\frac{u_1 - u_{-1}}{2} = u_0 \Rightarrow u_{-1} = u_1 - 2u_0$$

CN at $x = 0$, $t = 1$:

$$2u_{-1} - 5u_0 + 2u_1 = -36 - 0 - 36 = -72$$

$$\Rightarrow 2u_1 - 4u_0 - 5u_0 + 2u_1 = -72 \Rightarrow 4u_1 - 9u_0 = -72 \quad \text{—(1)}$$

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CN at $x = 1, t = 1$:

$$2u_0 - 5u_1 + 2u_2 = -0 + 54 - 72 = -18 \quad -(2)$$

CN at $x = 2, t = 1$:

$$2u_1 - 5u_2 + 108 = -36 + 108 - 108 = -36$$

Assembling the eqns,

$$4u_1 - 9u_0 = -72 \quad - (1)$$

$$2u_0 - 5u_1 + 2u_2 = -18 \quad - (2)$$

$$2u_1 - 5u_2 = -144 \quad - (3)$$

Solving the above system of linear eqns,

$$u_0 = 20.295$$

$$u_1 = 27.664$$

$$u_2 = 39.866$$

$\Rightarrow$ $u(x)$ at $t = 1$ is

$$\boxed{u(0,1) = 20.295, \; u(1,1) = 27.664, \; u(2,1) = 39.866 \\ u(3,1) = 54}$$

Also, the imaginary value $u_{-1} = -12.926$

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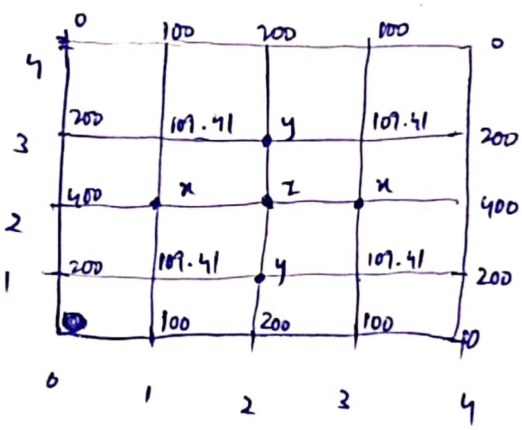
(2) $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u$$

$$\Rightarrow \frac{1}{(\Delta x)^2}\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i,j+1} - 2u_{i,j} + u_{i,j-1}\right] = u_{i,j}$$

$$\Delta x = 1$$

$$\Rightarrow u_{i,j} = \frac{1}{5}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right]$$



$$x = \frac{1}{5}\left(400 + 2(107.41) + z\right)$$

$$z = \frac{1}{5}\left(2x + 2y\right)$$

$$y = \frac{1}{5}\left(z + 2(107.41) + 200\right)$$

Rearranging the eqns,

$$5x - z = 618.82 \quad -(1)$$

$$2x + 2y - 5z = 0 \quad -(2)$$

$$5y - z = 418.82 \quad -(3)$$

Solving for x, y, z

$u(1,2) = y = 103.529$

$u(2,1) = x = 143.529$

$u(2,2) = z = 98.823$

$u(2,3) = x = 143.529$

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(3) $\frac{\partial^2 u}{\partial x^2} - (2x+y)^2 \frac{\partial^2 u}{\partial y^2} = 0$; $u(x,0) = 2+5x^2$,

$u_y(x,0) = 6x$

Given : $P(0.2, 0)$ $Q(0.3, 0)$

$p = \frac{\partial u}{\partial x} = 10x$, $q = \frac{\partial u}{\partial y} = 6x$

Comparing with $a u_{xx} + b u_{xy} + c u_{yy} + e = 0$,

$a = 1$, $b = 0$, $c = -(2x+y)^2$, $e = 0$

Let $\frac{dy}{dx} = m$

$\Rightarrow a\left(\frac{dy}{dx}\right)^2 - b\left(\frac{dy}{dx}\right) + c = 0$

$\Rightarrow m^2 - (2x+y)^2 = 0$

$\Rightarrow m = \pm(2x+y)$

$f = 2x+y$, $g = -(2x+y)$

$\Rightarrow f_P = 0.4$, $g_Q = -0.6$, $p_P = 2$, $p_Q = 3$,

$q_P = 1.2$, $q_Q = 1.8$, $a_P = 1$, $a_Q = 1$,

$c_P = -0.16$, $c_Q = -0.36$, $u_P = 2.2$, $u_Q = 2.45$

First approximation :

$y_R - y_P = f_P(x_R - x_P)$; $y_R - y_Q = g_Q(x_R - x_Q)$

$\Rightarrow y_R = 0.4(x_R - 0.2)$,

$y_R = -0.6(x_R - 0.3)$

$\Rightarrow 0.4x_R - 0.08 = -0.6x_R + 0.18$

$\Rightarrow x_R = 0.26$, $y_R = 0.024$

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Now,

$$a_p f_p (p_R - p_p) + c_p (q_R - q_p) + e_p (y_R - y_p) = 0$$

$$\Rightarrow \quad 0.4(p_R - 2) - 0.16(q_R - 1.2) = 0$$

$$\Rightarrow \quad 0.4\, p_R - 0.16\, q_R = 0.608$$

and

$$a_Q g_Q (p_R - p_Q) + c_Q (q_R - q_Q) + e_Q (y_R - y_Q) = 0$$

$$\Rightarrow \quad -0.6(p_R - 3) - 0.36(q_R - 1.8) = 0$$

$$\Rightarrow \quad 0.6\, p_R + 0.36\, q_R = 2.448$$

$$\Rightarrow \quad \boxed{p_R = 2.544, \quad q_R = 2.56}$$

Therefore,

$$u_R = u_p + \frac{1}{2}(p_R + p_p)(x_R - x_p) + \frac{1}{2}(q_R + q_p)(y_R - y_p)$$

$$= 2.2 + \frac{1}{2}(2.544 + 2)(0.26 - 0.2) + \frac{1}{2}(2.56 + 1.2)(0.024 - 0)$$

$$= 2.2 + \frac{1}{2}(4.544)(0.06) + \frac{1}{2}(3.76)(0.024)$$

$$= 2.2 + 0.13632 + 0.04512$$

$$= 2.3814$$