DEPARTMENT OF MATHEMATICS, I.I.T. MADRAS

Date:21–02–2020 ; Time : 08:00–08:50 am ;Quiz – I (20 Marks) ; MA6270

Q1. Using Crank-Nicolson F-D scheme, find $U(x, 0.1)$ for $x = 0, \frac{1}{3}$ and $\frac{2}{3}$ when $U(x,t)$ satisfies

$$\frac{\partial U}{\partial t} = \frac{1}{3}\frac{\partial^2 U}{\partial x^2}, 0 < x < 1; \frac{\partial U}{\partial x}(0,t) = U(0,t) ; U(1,t) = 0 , t > 0 ;$$

$$U(x,0) = 18x(1-x), \quad 0 \le x \le 1 \qquad \text{(8 M)}$$

(Use forward difference formula for derivative boundary condition)

Q2. Derive the stability condition of the explicit F-D scheme when applied to $\frac{\partial U}{\partial t} = \frac{1}{3}\frac{\partial^2 U}{\partial x^2}$, $0 < x < 1$ with $\Delta x = h$, $\Delta t = k$ and $r = \frac{k}{h^2}$, given that $U(x,t)$ is known at $t = 0$ and on $x = 0$ and $x = 1$ for $t > 0$. **(Use matrix method)** **(6M)**

Q3. Derive the local truncation error $T_{i,j}$ when fully implicit backward time scheme is applied to $\frac{\partial U}{\partial t} = \frac{1}{2}\frac{\partial^2 U}{\partial x^2}$, $0 < x < 1$ with known initial and boundary values for $U(x,t)$. **(6M)**

$(2+2r)$ $-r$ $\frac{k^2}{6}\left(\frac{\partial^3 U}{\partial t^3} - \frac{1}{2}\frac{\partial^4 U}{\partial t^2 \partial x^2}\right)$ $\frac{k^2}{6}\frac{\partial^3 U}{\partial t^3} - \frac{1}{2}\frac{k^2}{6}\frac{\partial^4 U}{\partial}$

r

$-r$ $+(2-2r)$ $-r.$

$= \frac{k^2}{6}\frac{\partial U}{\partial t}\left(\frac{\partial^2 U}{\partial t^2} - \partial^4\right)$ $\frac{\partial}{\partial t}\left(\frac{\partial^4 U}{\partial t^4} - \frac{1}{2}\frac{\partial^4 U}{\partial t^2 \partial x^2}\right)$ 0. 24 48

$3 \times 2 = 6$

$\frac{\partial^4 U}{\partial t^4}\left(\frac{\partial^2 U}{\partial t^2} - \frac{1}{2}\frac{\partial^3 U}{\partial t \partial x^2}\right)$ $\frac{\partial U}{\partial t}\left(\frac{\partial U}{\partial t}\left(\frac{\partial}{\partial t} - \frac{1}{2}\frac{\partial^2 U}{\partial x^2}\right)\right)$