09/01/2018 Wireless Communications 2

TS 436

chapters 5, 6, 7, 8, 10

coursework is based on final one. (journal)

coherant combining ?

12/01/2018

Study the channel capacity equation

$C = \frac{W}{2} \log \left(1 + \frac{P}{NW}\right)$?

slide 15

response of c th ~~Subcarr~~ channel

$\max \sum_{c=1}^{N_c} \log_2 \left(1 + \frac{|h_c|^2 P_c}{N_0}\right)$ power of the subcarr

$\sum_{c=1}^{N_c} P_c \leq P_T$

$P_c \geq 0$

slide 16

How each user will get max capacity. When user 1 is decoded, that info is used for user 2. So he will get single user capacity.

slide 19

Singular value decomposition. of this 4 x 4 matrix

No of R

No of T

slide 20

No of Streams - Degrees of freedom

16/01/2018

slide 4 $H(x,y) \leq H(x) + H(y)$

		0	1
x	0	½	0
	1	0	½

example explained.

		Y	
X	3/8	1/8	4/8
	1/8	3/8	4/8
	4/8	4/8	

~~¾~~ ~~¼~~ $H(x,y) = 2 \times \frac{1}{8} \log 8 + 2 \times \frac{3}{8} \log \frac{8}{3} = 1.81$

$P(x|y) =$ $H(x) = 1$

$H(y) = 1$

	Y		
X	3/8	1/8	4/8
	1/8	3/8	4/8
	4/8	4/8	

$H(X,Y) = 1.811$

$H(X) = 1$

$H(Y) = 1$

$P(Y|X)$

3/4	1/4
1/4	3/4

$P(X|Y)$

3/4	1/4
1/4	3/4

	Y		
X	3/8	1/8	4/8
	3/8	1/8	4/8
	6/8	2/8	

$H(X,Y) = 1.811$

$H(X) = 1$

$H(Y) = 0.811$

$P(X,Y)$

X \ Y	0	1	
0	3/8	1/8	4/8
1	3/8	1/8	4/8
	6/8	2/8	

$P(y|x)$

X \ Y	0	1
0	3/4	1/4
1	3/4	1/4

↑

This case X & Y are independent

if X = 0 the Y is probability is ~~agn~~ ? $\Rightarrow$

For this distribution

$P(y|x) = \left(\frac{3}{4}, \frac{1}{4}\right)$ if x is given 0 or 1

$P(y|x) = P(y)$. hence independent.

same for $P(x|y) = P(x)$

slide 10

$I(X,Y) = H(Y) - H(Y|X)$

$$= \underbrace{\sum_{i \in Y} P_Y(i) \log \frac{1}{P_Y(i)}}_{\text{concave}} + \underbrace{\sum_{i \in X} P_X(i)\, H(Y|X=i)}_{\text{linear } f^n}$$

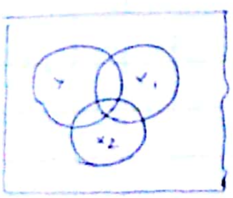
19/01/2018

slide 7

$I(Y; X_1, X_2)$ mutual info between Y and joint distribution of X_1 & X_2

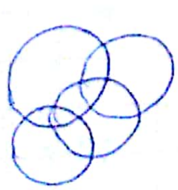
$I(Y; X_1, X_2) = H(Y) - H(Y|X_1, X_2)$

$= I(Y; X_2) + I(Y|X_2 ; X_1)$



$= I(X_2 ; Y) + I(X_1, Y|X_2)$

$= I(X_1 ; Y) + I(X_2 ; Y|X_1)$ ← once X_1 is decoded we can remove it ~~from~~ and decode X_2



$I(X_1, X_2, X_3 ; Y) = H(Y) - H(Y|X_1, X_2, X_3)$

$= I(X_3 ; Y) + I(X_2, Y|X_3) + I(X_1 ; Y|X_3, X_2)$

decode X_3
use it to decode X_2
use both ~~to~~ decode X_1

↖ can write in 6 ways

slide 14

prove 3

$\frac{P}{\sigma^2} = SNR$ — In complex we have I and Q channels. So it will be multiplied by 2

$C = \log\left(1 + \frac{P}{\sigma^2}\right)$

$h(w) = h(Re(w)) + h(Im(w))$

$= \frac{1}{2}\log 2\pi e \frac{N_0}{2} + \frac{1}{2}\log \pi e N_0$

variance is now split between I & Q

$= \log \pi e N_0$

slide 15

if $Y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad r_y = \sqrt{x_1^2 + x_2^2}$

$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ $\quad r_y = \sqrt{y_1^2 + y_2^2 \dots y_n^2}$

$= \sqrt{(x_1+w_1)^2 + (x_2+w_2)^2 \dots (x_n+w_n)^2}$

$= \sqrt{\sum_{i=1}^{n} x_i^2 + \underbrace{\sum_{i=1}^{n} x_i w_i}_{\text{when } n \text{ is large} \to 0} + \sum_{i=1}^{n} w_i^2}$

$\approx$

$= \sqrt{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} w_i^2}$

$= \sqrt{N.P + N\cdot\sigma^2}$ ← when n goes to inf.

hardening effect.

$\frac{1}{N}\log\left(\frac{\text{vol of } y}{\text{vol of } w}\right)$

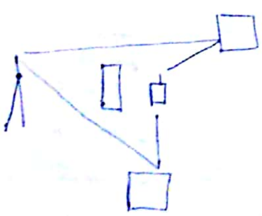
slide 17

Example

1 look back.

2

3



slide 16

$W \log\left(1 + \frac{P}{N_0 W}\right)$

$W \lll 1$

$W \log \frac{P}{N_0 W}$

W↑ almost linear w.r.t W

P↑ log increaste w.r.t P

$W \ggg 1$

$W \cdot \frac{P}{N_0 W} \log_2 e$

$= \frac{P}{N_0} \log_2 e$

linear w.r.t. P

Scanned by CamScanner

23/01/2018

Slide 22 one sub channel $\log\left(1 + \frac{P_n |h_n|^2}{N_0}\right)$

↓

add them all

slide 25

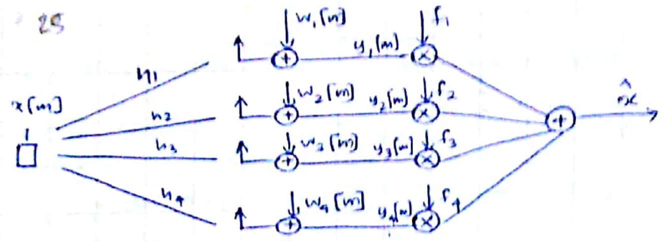
Example. compare both algorithms

↙ ordering

↘ bisection.

Lecture 3

Slide 20. 25



What is the best strategy to combine this 4 spatial samples?

How to design $f_1, \dots, f_4$ complex values as maximize capacity ???

$$\hat{x} = \sum_{l=1}^{L} f_l^* \left(h_l x[m] + w_l[m]\right)$$

$$\underline{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_L \end{bmatrix} \qquad \underline{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix}$$

- apply MRC multiply each antenna by complex conjugate of channel estimate

$f_1 = h_1^H$

$$\hat{x} = \underline{f}^H \underline{h}\, x[m] + \underline{f}^H \underline{w}[m]$$

peak power constraint	Sum Power Constraint
can't exceed certain power level	MRC $\|f\|_2^2 = 1$
co-phase each transmitted signal with the channel s.t. antenna specific beamformer weight is 1 phase can be matched to channel	
power per antenna can vary	

- Since Capacity is $C = \log_2(1 + SNR)$. SNR ???

$$SNR = \frac{E\left[|\underline{f}^H \underline{h}\, x[m]|^2\right]}{E\left[|\underline{f}^H w[m]|^2\right]}$$

$$= \frac{|\underline{f}^H \underline{h}|^2 E\{|x[m]|^2\}}{\cancel{\underline{f}^H E\{|w[m]|^2\}}\ E\left[\underline{f}^H \underbrace{\underline{w}\,\underline{w}^H}_{N_0 I} \underline{f}\right]}$$

$$= \frac{|\underline{f}^H \underline{h}|^2 P}{E\ N_0 \|\underline{f}\|^2}$$

To maximize SNR by cauchy Schwarz inequality.

$$|\underline{f}^H \underline{h}| \le \|\underline{f}^H\| \cdot \|\underline{h}\|$$

also, optimal when $\underline{f} = \underline{h}$

$$= \frac{\|\underline{f}^H\|^2 \|\underline{h}\|^2 P}{N_0 \|\underline{h}\|^2}$$

~~optimal~~ when $= \frac{\|\underline{h}\|^2 P}{N_0}$ $\therefore C = \log\left(1 + \frac{\|h\|^2 P}{N_0}\right)$

In practical

pilot sequence

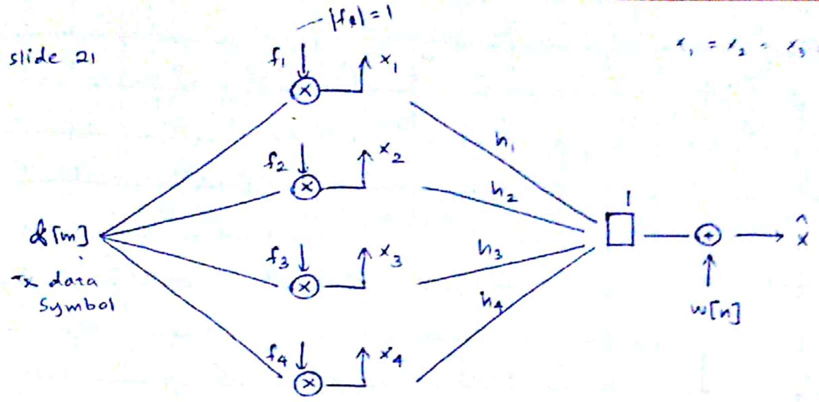
→ get channel estimate

→ take complex conjugate

→ use it as optimum multiplier

→ maximize SNR and capacity.

slide 21



$x_1 = x_2 = x_3 = x_4$ (same symbol)

Problem
How should we pre multiply / pre code the antenna to maximize the signal at the receiver
design f such that received signal is added coherently.

x is drawn from complex gaussian distribution $x \sim CN(0, P)$ ← average power of transmitted signal is P

$$SNR = \frac{E[|\underline{h}^H \underbrace{\underline{x}[m]}_{\underline{f}\, d[m]}|^2]}{E[|\underline{w}|^2]} = \frac{E[|\underline{h}^H \underline{f}\, d[m]|^2]}{N_0} = \frac{|\underline{h}^H \underline{f}|^2 P}{N_0}$$

Now to maintain power constraint, normalize $\underline{f}$

$$SNR = \frac{|\underline{h}^H \underline{f}|^2 P}{\|\underline{f}\|^2 N_0} \quad ??$$

$$= \frac{\|h\|^2 P}{N_0}$$

again cauchy schwarz $\underline{f}$ should be $\underline{h}^H$ (matched) — in TDD

② design some codebooks. transmitter sends precoded pilot. user goes through the codebook. calculate product and choose the one which maximizes the product, reports back index of the codebook

① In FDD we have antenna specific pilots. user estimates the antenna specific channel, quantize it, report it back

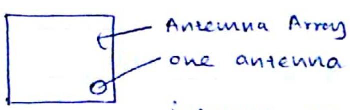
one practical difference
TDD in the receiver we get the channel knowledge for free
need to have the pilot of course

36:00 - Assignment question hint

38:01 USE ... channel ... with TDD

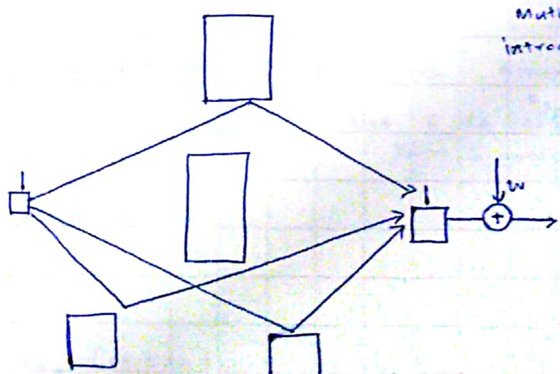
③ if you have TDD and reciprocity and the channel is slowly fading it can be assumed that the channel remains constant for UL & DL

Practical example (Power Constraint P)

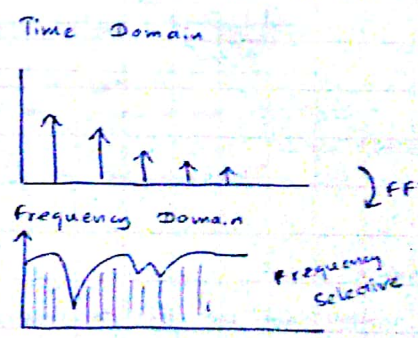


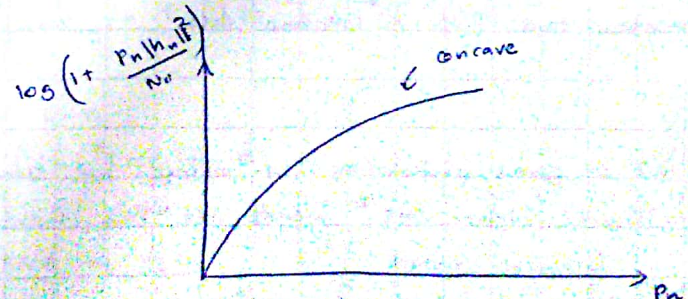
increase no. of antennas → Power per antenna ↓ . simplified bus for this

slide 22



Multipath fading in time domain introduces fading in f domain





23/. **Lecture 3** 31-10

Slide 30/01/18

slide 4

$$P_{out}(R) = 1 - e^{-\left(\frac{2^R - 1}{SNR}\right)}$$

sli

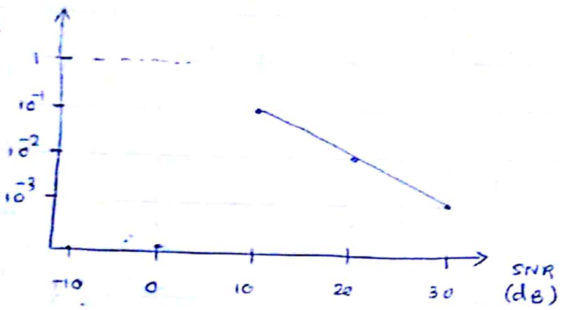
$$1 - e^{-x}$$

$$= 1 - \left[1 + (-x) + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \dots\right]$$

SNR $\uparrow$ $x \downarrow$

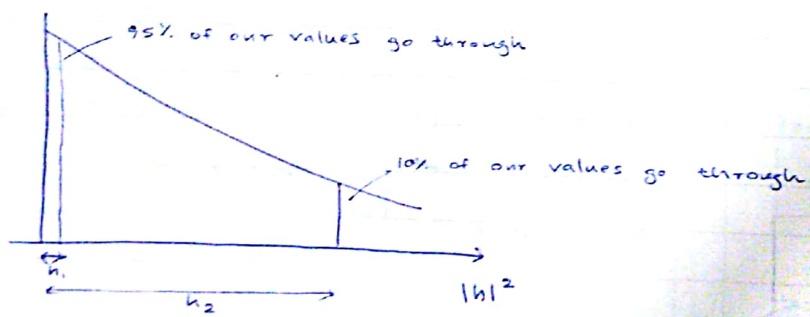
$$= x$$

lec / sli $R = 1$ $P_{out} = \frac{1}{SNR}$



P_{out} can't go above 1

slide 5

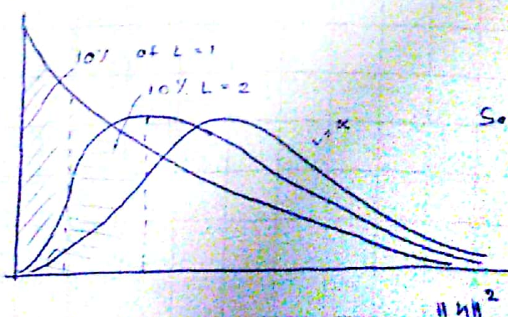


Throughput $= (1-\varepsilon)\, C_\varepsilon$

slide 6

what can we do to improve

compare 1 antenna case vs 2 antenna case vs 4 antenna case. ← 8 dgrees of freedom



So for a given probability of outage we can have higher $\|h\|^2$ means we can have higher rates

$$C = \log(1 + \|h\|^2 SNR)$$

$$P_{out}(R) \approx \frac{(2^R - 1)^L}{L!\,SNR^L}$$

for SNR = 10 dB

$P_{out} \approx \frac{1}{2! \cdot 10^2} = 0.005$

$\frac{1}{200}$

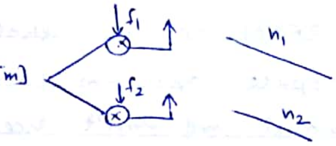
20 dB $= \frac{1}{2 \cdot 10^4} = 5 \times 10^{-5}$

$R = 1 \quad P_{out}(R) = \frac{1}{L!\,SNR^L}$

vary L & SNR to see how $P_{out}(R)$ behaves.

3.7 Transmiter without CSIT

if we know the channel.



Since we know the channel

$f_1 = h_1^* \rightarrow |h_1|^2$

$f_2 = h_2^* \rightarrow |h_2|^2$

$$\frac{P(|h_1|^2 + |h_2|^2)}{N_0}$$

but if we do not know

$\hat{u}_1 = h_1^* y_1 + h_2 \cancel{{}^*} y_2$

$= h_1^* [h_1 u_1 + h_2 u_2] + h_2 \cancel{{}^*} [h_1 (-u_2^*) + h_2 u_1^*] + h_1^* w_1 + h_2^* w_2$

← conjugate moved

$= h_1^* h_1 u_1 + \underbrace{h_1^* h_2 u_2 - h_2 h_1^* u_2}_{=0} + h_2 h_2^* u_1 + h_1^* w_1 + h_2^* w_2$

$|h_1|^2 u_1 + |h_2|^2 u_1 + w_1 + w_2$

in Alamouti scheme for each transmission we consider 2 symbols u_1 & u_2

$\hat{u}_2 = h_2^* y_1 + h_1 y_2$

compare L = 2 reciver case with Alamonti Scheme. 3 dB → loss $\frac{SNR}{2}$

3 dB. 3 dB loss.

from 2 antenna full CSIT to 2 antenna no CSIT

but still better than single antenna case

3.9

channel is slowly varying.

3.11

$C_{AWGN} = \log_2(1 + SNR)$

$C_{fading} = E_h[\log(1 + |h|^2 SNR)]$

ray (0,1) → $E[|h|^2] = 1$

$E[\log_2 |h|^2] < 1$ because of log.

low SNR

$E_h[|h|^2 SNR \log_2 e]$

$E_h[|h|^2]\ SNR \log_2 e$

$1 \cdot SNR \log_2 e$

high SNR

$E_h[(\log_2 |h|^2 SNR)]$

$E_h[\log_2 |h|^2] + \cancel{SN} \log_2 SNR$

-0.83

constant

(The graph is 0.83 bits lower in slide 11)

If there is lot of power, water level is high. we will be transmitting all the time. Power difference is asymptotically equal.

If we have small power start focusing the power on peaks only

when SNR goes to infinitly small, most of the time not transmitting.

The difference to OFDM case is that we can averge over all fading states., we can compute optimum water level given the average power constraint beforehand we don't need to do any bisection. pre-compute the water level depending on the SNR, fix it and transmit.

02/02/2018

32.10

slide 1

UE1, UE2 communicate with BS at the Same time using same resource what is the best strategy for communication?

<u>Terms used in this course</u>

uplink - MAC (multiple access channel) multiple users

downlink → BC (Broadcast channel) (one base station)

in MAC → individual power constraints (per user)

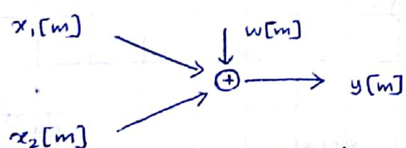
two antennas. two amplifies

in BC → Sum power constraint.

one amplifier

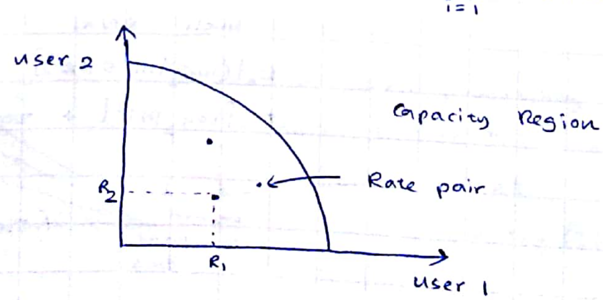
Optimal strategy is to serve the two users at the same time. successive interference cancellation.

slide 2



for K users direct extension

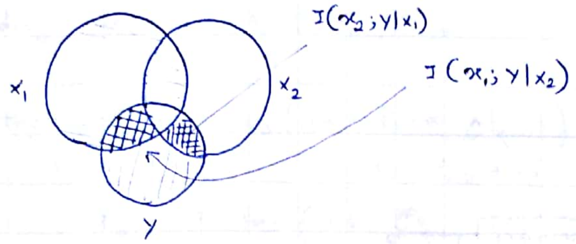
$$y[m] = \sum_{i=1}^{K} x_i[m] + w[m]$$



knowing $x_2[m]$ would result a single user channel for user 1

$$y[m] = x_1[m] + x_2[m] + w[m]$$

$$y[m] - x_2[m] = x_1[m] + w[m]$$



$$I(x_1,x_2;y) = I(x_1;y) + I(x_2;y|x_1) \qquad I(x_1,x_2;y) = I(x_2;y) + I(x_1;y|x_2)$$

$$I(x_1,x_2:y) = h(y) - h(y|x_1,x_2)$$

$$x_1 \sim CN(0, P_1)$$
$$x_2 \sim CN(0, P_2)$$
$$w \sim CN(0, N_0)$$

$$h(y) = \log_2 \pi e \{E[y^2[m]]\}$$
$$= \log_2 [\pi e \; E\{[x_1[m] + x_2[m] + w[m]]^2\}]$$
$$= \log_2 [\pi e \; E\{x_1^2[m] + x_2^2[m] + w^2[m]\}]$$ cross terms are zero.
$$= \log_2 [\pi e \; (P_1 + P_2 + N_0)]$$ due to independance.

$$h(y|x_1,x_2) = \log_2 \pi e \; E\{w^2[m]\}$$
$$= \log_2 \pi e N_0$$

$$I(x_1,x_2;y) = \log_2 \pi e (P_1 + P_2 + N_0) - \log_2 (\pi e N_0)$$
$$= \log_2 \left(1 + \frac{P_1 + P_2}{N_0}\right)$$

For gaussian distribution mutual information is equivalent to capacity.

$$R_1 \leq I(x_1;y|x_2)$$
$$= h(y|x_2) - h(y|x_1,x_2)$$
$$= \log_2 \pi e (P_1 + N_0) - \log_2 \pi e N_0$$
$$= \log_2 \left(1 + \frac{P_1}{N_0}\right)$$

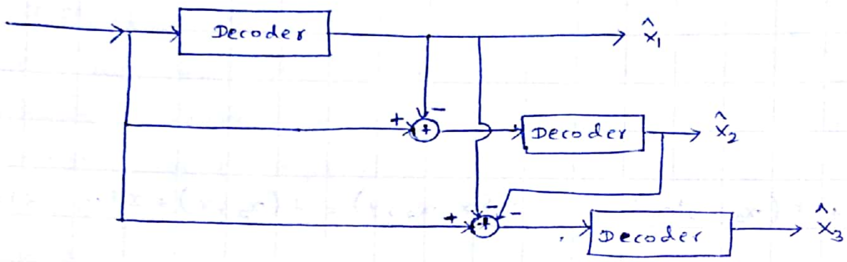
$$\therefore R_1 \leq \log_2 \left(1 + \frac{P_1}{N_0}\right)$$

similarly $R_2 \leq \log_2 \left(1 + \frac{P_2}{N_0}\right)$

slide 6

$$I(x_2;y) = h(y) - h(y|x_2)$$
$$= \log_2 \pi e(P_1 + P_2 + N_0) - \log_2 \pi e(P_1 + N_0)$$
$$= \log_2 \left(1 + \frac{P_2}{P_1 + N_0}\right)$$

$$I(x_1;y) = \log_2 \left(1 + \frac{P_1}{P_2 + N_0}\right)$$



* Interesting scenario

what if x_1 and x_2 are correlated

$$Y = x_1 + x_2 + w$$

in SNR

$E[(x_1 + x_2)^2]$ is the desired part

$$E[x_1^2 + x_2^2 + \underbrace{2x_1x_2}]$$

$= 0$ if independent.

$\neq 0$ if not independent.

$P_1 + P_2 + 2\sqrt{P_1P_2}$ ← fully correlated.

rate. $\log_2(1 + SNR)$

$$\log_2\left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1P_2}}{N_0}\right)$$

if $P_1 = P_2 = P$

$\log_2\left(1 + \frac{4P}{N_0}\right)$ while in uncorrelated we got $\log\left(1 + \frac{2P}{N_0}\right)$

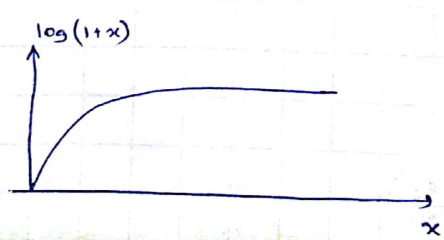
This is beamforming gain of 3 dB in SNR

a) in low SNR $\frac{P}{N_0} \lll 1$

$$\frac{C_{corr}}{C_{uncorr}} = 2 \quad (2 \text{ times higher capacity})$$

b) in high SNR

it is diminshing.



06/02/2018

slide 5 Rate Splitting

We divide Symbol x_1 into two.

user 1 $x_1 = \{x_{1,a}\ x_{1,b}\} \Longrightarrow x_{1,a} = P_1 - \alpha \leftarrow$ vary this parameter power α to achieve all the points between A and B

user 2 $x_2 = x_2$ $\qquad x_{1,b} = \alpha$

vary $\alpha = [0, P_1]$ to get different points.

decoding order $x_{1,a}, x_2, x_{1,b}$

$$\text{then } r_{1,a} = \log\left(1 + \frac{P_1 - \alpha}{P_2 + P_{1,b} + N_0}\right)$$

$$r_2 = \log\left(1 + \frac{P_2}{P_{1,b} + N_0}\right)$$

$$r_{1,b} = \log\left(1 + \frac{P_{1,b}}{N_0}\right)$$

$$r_{sum} = \log\left(1 + \frac{P_{1,a} + P_2 + P_{1,b}}{N_0}\right) = \log\left(1 + \frac{P_1 + P_2}{N_0}\right)$$

drawbaks - complex

advantages - achieve capacity, get different rates for users

slide 7

~~The~~ Target is to come up with some scheme where we don't have any interference.

TDMA - reciprocity, straight forward.

FDMA - assign sub channel, orthogonal resources in frequency.

Explained w.r.t time (TDMA)

with this, no interference. so sum rate is $R_1(\alpha) + R_2(\alpha)$

optimality

$$\text{sum rate} \Rightarrow R_1 = \left(\frac{P_1}{P_1+P_2}\right)\log\left(1 + \frac{P_1(P_1+P_2)}{P_1 N_0}\right)$$

$$R_2 = \left\{1 - \frac{P_1}{P_1+P_2}\right\}\log\left(1 + \frac{P_2}{N_0}\cdot\frac{1}{\left(1 - \frac{P_1}{P_1+P_2}\right)}\right)$$

$$= \left(\frac{P_2}{P_1+P_2}\right)\log\left(1 + \frac{P_2(P_1+P_2)}{N_0 P_2}\right)$$

$$\therefore R_1 + R_2 = \log\left(1 + \frac{P_1+P_2}{N_0}\right)$$

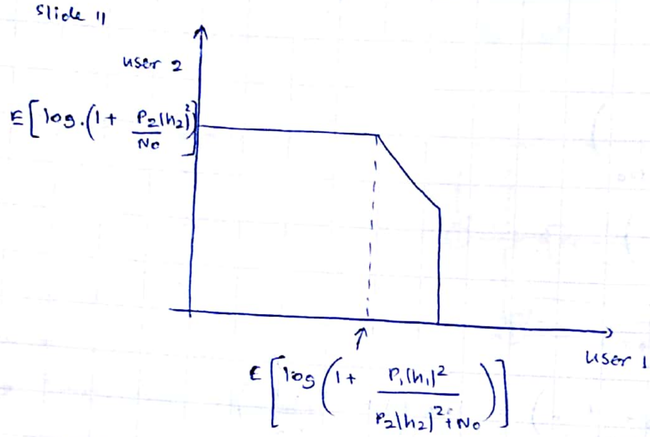
Scanned by CamScanner

slide 8. $2^K - 1$

example :- 3 users. 7 constraints

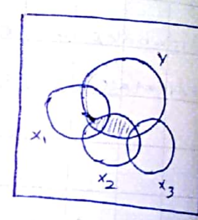
- 3 single user bounds
- 3 , two user bounds
- sum rate bound

slide 11



13/02/2018

slide 8



$$
\begin{aligned}
I(x_1,x_2,x_3;Y) &= I(x_1;Y) + I(x_2;Y|x_1) + I(x_3;Y|x_1,x_2) \\
&= I(x_3;Y) + I(x_2;Y|x_3) + I(x_1;Y|x_3,x_2) \\
&= H(Y) - H(Y|x_3) + H(Y|x_3) - H(Y|x_2,x_3) + H(Y|x_2,x_3) - H(Y|x_1,x_2,x_3) \\
&= \log \pi e(P_1+P_2+P_3+N_0) - \log \pi e(P_1+P_2+N_0) \\
&\quad + \log \pi e(P_1+P_2+N_0) - \log \pi e(P_1+N_0) \\
&\quad + \log \pi e(P_1+N_0) - \log \pi e N_0 \\
&= \log\left(1+\frac{P_3}{P_1+P_2+N_0}\right) + \log\left(1+\frac{P_2}{P_1+N_0}\right) + \log\left(1+\frac{P_1}{N_0}\right) \quad // \\
&= \log\left(1+\frac{P_1+P_2+P_3}{N_0}\right)
\end{aligned}
$$

if the decoding order is not 3, 2, 1 still the sum rate is same. individual rates for each user is different.

slide 12

$$E \log\left(1+\frac{KP|h_k|^2}{N_0}\right)$$

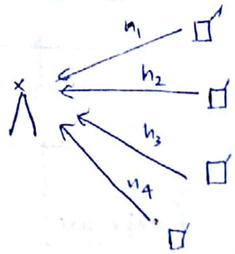
high SNR $\quad E\left[\log\left(\frac{KP|h_k|^2}{N_0}\right)\right] = \log \frac{KP}{N_0} + E \log |h_k|^2$

K user AWGN

-0.83 bits

penalty we pay in orthogonal (TDMA) case

slide 15



$$Y = h_1 x_1 + h_2 x_2 + h_3 x_3 + h_4 x_4 + w$$

$$I(x_1, x_2, x_3, x_4; Y) = h(Y) - h(Y | x_1, x_2, x_3, x_4)$$
$$= \log \pi e \left(|h_1|^2 P_1(m) + |h_2|^2 P_2(m) + |h_3|^2 P_3(m) + |h_4|^2 P_4(m) + N_0 \right) - \log \pi e N_0$$

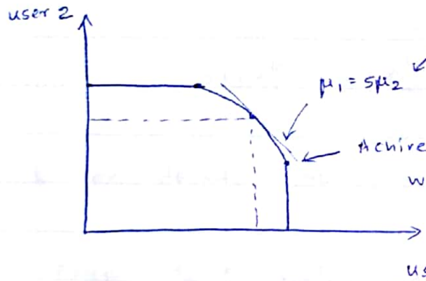
$$= \log \left(1 + \frac{\sum_{i=1}^{4} |h_i|^2 P_i(m)}{N_0} \right)$$ ← This is for one channel realization and for 4 users in the note this is generalized for k users

$$\frac{1}{L} \sum_{l=1}^{L} \log \left(1 + \frac{\sum_{i=1}^{k} |h_i|^2 P_i(m)}{N_0} \right)$$

average over all channel realizations and then $L \to \infty$ it becomes $E(\cdot)$

15/02/2018

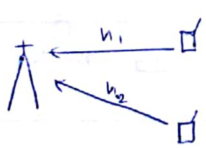
slide 18



Achive by first doing the waterfilling for 1 user, the user that we want to give the highest prority, we fix those powers and consider it as a fixed interference to the second user. and then we do waterfilling again to get this Rate pair, user 1 is decoded last hence interference free, user 2 is getting a rate such that user 1 is fully interfering.

$$\log \left(1 + \frac{P_{1,c} |h_{1,c}|^2}{N_0} \right)$$ — rate in c th subcarrier for user 1

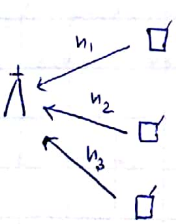
$$\log \left(1 + \frac{P_{2,c} |h_{2,c}|^2}{P_{1,c} |h_{1,c}|^2 + N_0} \right)$$ rate in c th subcarrier for user 2



$Y = h_{1,c} x_{1,c} + h_{2,c} x_{2,c} + w_c$ c could be subcarrier or time index

let's say user 2 is decoded first

slide 19



$w_1 > w_2 > w_3$

~~$\mu_1 > \mu_2 > \mu_3$~~

decoding order $3 \to 2 \to 1$

$R_1 = C_1$ ~~$R_2 = C$~~

$R_2 = C_{12} - R_1$ ← sum rate of user 1 & 2

$R_3 = C_{123} - R_1 - R_2$

↑ sum rate of user 1, 2, 3

$w_1 R_1 + w_2 R_2 + w_3 R_3$

$R_1 + R_2 = C_{12}$

$$w_1 C_1 + w_2 (C_{12} - C_1) + w_3 (C_{123} - C_{12})$$

s_1 $$w_3 C_{123} + (w_2 - w_3) C_{12} + (w_1 - w_2) C_1$$

$$w_3 \frac{1}{N_c} \sum_{k=1}^{N_c} \log\left(1 + \frac{P_{1,c}|h_{1,c}|^2 + P_{2,c}|h_{2,c}|^2 + P_{3,c}|h_{3,c}|^2}{N_0}\right)$$
$$+ (w_2 - w_3) \cdot \frac{1}{N_c} \sum_{k=1}^{N_c} \left\{ \log\left(1 + \frac{P_{1,c}|h_{1,c}|^2 + P_{2,c}|h_{2,c}|^2}{N_0}\right) \right.$$
$$+ (w_1 - w_2) \frac{1}{N_c} \sum_{k=1}^{N_c} \log\left(1 + \frac{P_{1,c}|h_{1,c}|^2}{N_0}\right)$$

objective

power constraints

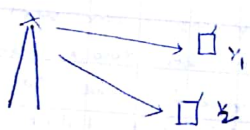
$$\frac{1}{N_c} \sum_{k=1}^{N_c} P_{1,c} \leq P_1$$

$$\frac{1}{N_c} \sum_{k=1}^{N_c} P_{2,c} \leq P_2$$

$$\frac{1}{N_c} \sum_{k=1}^{N_c} P_{3,c} \leq P_3$$

How the rates are defined? By the operator, premium customers ... etc.
weight due to a queue. low data in queue → low prioris

slide 21



Base station broadcasts.

$$y_1 = h_1 x_1 + h_1 x_2 + w$$
$$y_2 = h_2 x_1 + h_2 x_2 + w$$

In the DL channel we have one power constraint

$x \sim CN(0, P)$

$x_1 \sim CN(0, P_1)$
$x_2 \sim CN(0, P_2)$ } x_1 & x_2 are independent

can freely allocate power. All to user 1 or all to user 2 or any combination

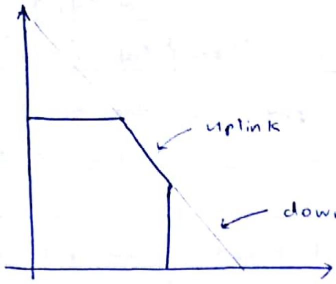
$P = P_1 + P_2$

$$SINR_1 = \cancel{\frac{h_1 x_1}{h_2 x_2}} \quad \frac{|h_1|^2 P_1}{|h_1|^2 P_2 + N_0}$$

$$SINR_2 = \frac{|h_2|^2 P_2}{|h_2|^2 P_1 + N_0}$$

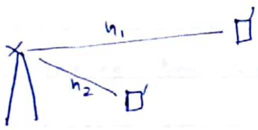
case 1 :- gains of h_1 and h_2 are same

$P_1 = P_2 = \frac{P}{2}$



slide 22

Now assume channel of user 2 is better



$$SINR_1^{(1)} = \frac{|h_1|^2 P_1}{|h_1|^2 P_2 + N_0}$$

$SINR_1^{(2)}$ (SINR of message 1 of user 2)

189
2.00

$$SINR_1^{(2)} = \frac{|h_2|^2 P_1}{|h_2|^2 P_2 + N_0}$$

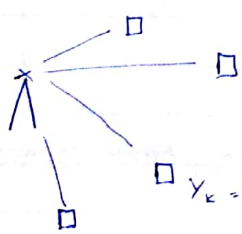
03.22 mute
16.04

$SINR_1^{(2)} > SINR_1^{(1)}$ because h_2 is larger than h_1

user 2 can decode the message intended to user 1 (x_1) and then subtract.

$P = 10$ $P_2 = 9$ $P_1 = 1$ $|h_1|^2 = 1$ $|h_2|^2 = 10$

slide 24



$\underline{x} = \sum x_i$

$y_k = h_k \sum x_i + w_k$

user 3 decod 1 → subtrack
then " 2 → subtract
thn " 3 →
until it can decode own message without interference.

ordering is important

Symmetic case → TDMA is optimal because h_k is same

user 1 weakest user we have to decode user 1 first. Weak users can't decode stronger users but can do vice versa

every other user can decode the message intended to user 1

user 2 can do this but still has interference from 3 4 5 6.

user 6 → decode 1 - subtract
2 "
3 "
4 "
5 "

Comptationally compex
always optimal

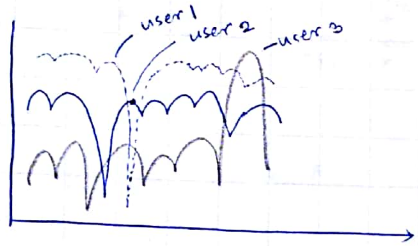
slide 25

We do not know the channel. we can't order. But if we need to order we ne[ed] channel cofficients ???

we can do time sharing - but sub optimal. Assign full power in that time slot

slide 26

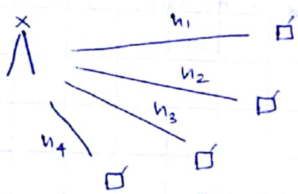
if we know CSIT



when there is sum power.

23/02/2018

slide 27



$C_{AWGN} = \log(1 + SNR)$

fading capacity

$$C_f = E\left[\log\left(1 + \frac{|h_k|^2 P(h)}{N_0}\right)\right]$$

$$= \frac{1}{N} \sum_{i=1}^{N} \log\left(1 + \frac{|h_k|^2 P(n)}{N_0}\right)$$

When we know the channel, optimum solution is waterfilling

$$P_n = \left(\frac{1}{V} - \frac{N_0}{|h_n|^2}\right)^+$$

if we don't know the channel P is fixed

$$C_{fading} = \frac{E\left[|h_k|^2\right] P \log_2 e}{N_0}$$

$$\left(\frac{P}{N_0} \log_2 e\right) \quad \leftarrow \text{Same as low SNR AWGN capacity.}$$

$$C_{sum} = E\left[\log\left(1 + \underbrace{\max_{k=1,\dots k} |h_k|^2}\, SNR\right)\right]$$

in single user case single realization
in multi user case max (always the best)

when we have large number of users to choose an SNR is large waterfilling is not so important in multiuser diversity.

slide 28

12.00

$$E[h^2] = 1$$

$$E[\log(1+|h|^2 SNR)] < \log(1+SNR)$$

20-30 users → just a fraction increase in gain

why ?

$$E[\log(1+\max_{k=1,\dots K}|h_k|^2 SNR)]$$

it can be shown that for Rayleigh Fading.

$$\frac{E[\max_{k=1,\dots K}|h_k|^2 SNR]}{\log_e K} \to 1 \quad \text{when } K \to \infty$$

Now taking the upper bound.. using jensen inequality.

$$E[\log(1+\max_{k=1,\dots K}|h_k|^2 SNR)] \le \log(1+E[\max_{k=1,\dots K}|h_k|^2 SNR])$$

$$= \log(1+\log_e K\, SNR)$$

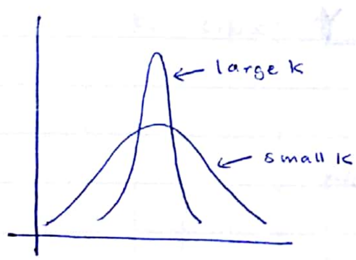
in high SNR

$$= \log_2(\log_e K).\ SNR$$

$$= \log_2 \log_e K + \log_2 SNR$$

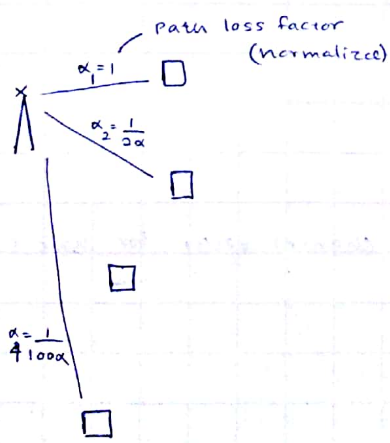
$$= \underbrace{\log_2 \log_e K}_{\text{double log} \to \text{saturates very quickly.}} + C_{AWGN}$$

For Rician fading.



$$\frac{E[\max_{k=1,\dots K}|h_k|^2]}{\log_e K} \to \frac{1}{K+1} \quad \text{as } K \to \infty$$

slide 29

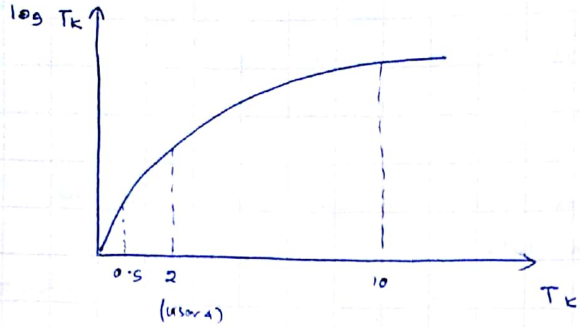


proportional fair sheduling is a very good strategy.

$$R_k[m] = \log(1+|h[k]|^2 SNR)$$

if served $\delta_k = 1$

not served $\delta_k = 0$



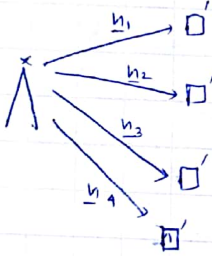
$$\max \; U[m] = \sum_{k=1}^{K} \log T_k[m]$$

equivalent utility $\max \; U[m+1] - U[m]$

$$= \max \sum_{k=1}^{K} \log T_k[m+1] - \log T_k[m]$$

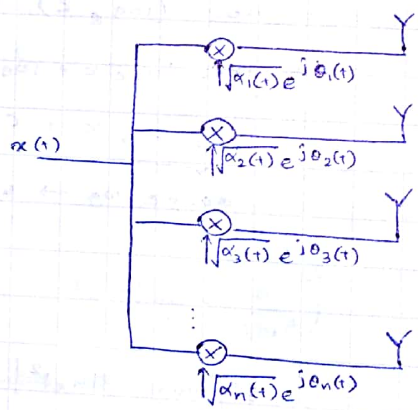
$$= \max \sum^{K} \log \left\{ \left(1 - \frac{1}{T_c}\right) T_k[m] + \frac{\delta}{T_c} R_k[m] \right\} - \log T_k[m]$$

slide 30 No CSIT



let's say FDD

Aim is to pick 1 user at a time



$$q = \begin{bmatrix} \sqrt{\alpha_1} e^{j\theta_1} \\ \sqrt{\alpha_2} e^{j\theta_2} \\ \vdots \\ \sqrt{\alpha_{N_r}} e^{j\theta_{N_r}} \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_{N_r} \end{bmatrix}$$

q is a complex weight

received signal for each user

$y_1 = h_1^T q x + w$ where $h_1 = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \end{bmatrix}$ channel vector for user 1

$$y_1 = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{1n} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} + w$$

what user does is user estimates the strength of the pilot $h_k q$ scalar.
depending on how well q match ~~k~~ with h we have a range from 0 to some value $q = \alpha h$

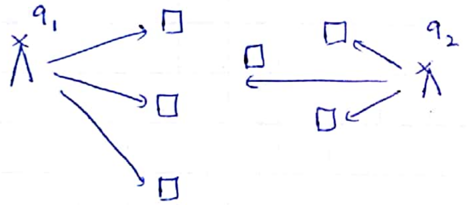
$$|h_k^T h|^2 \cdot \alpha$$

$$h_2 \|h\|_2^2 \cdot \alpha$$

If we know the channel, we compute this value $\|h\|_2^2$ and select the largest. But we don't know

Since we have large number of users, there is a high chance of one of the users has a good match. channel is well aligned with random beamformer.

Easy extension to multi cell opportuistic beamforming



now we have interference

now we have to evaluate not SNR but SINR.

$$SINR_k = \frac{|h_k^T q_1|^2 P_1}{|\tilde{h}_k^T q_2|^2 P_2 + N_0}$$

numerator maximize this when q_1 is aligned with h_k

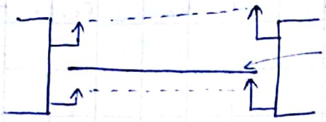
denominator minimize when q_2 is orthogonal.

So SINR $\uparrow$

2: 23/02/2018 - MIMO

slide 2

S1.

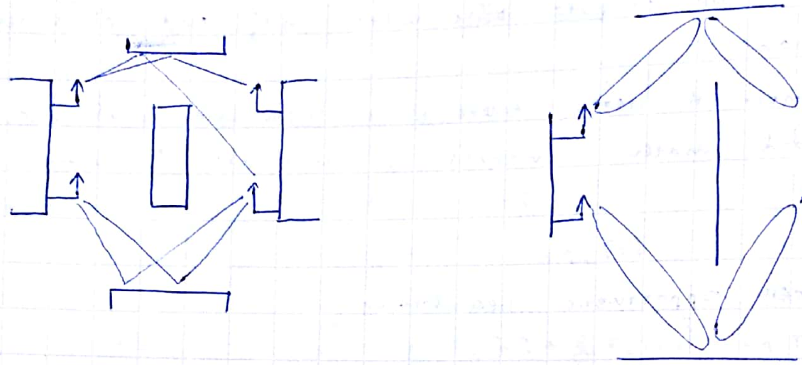


wall (~~sing~~ signal can't penetrate)

in this case we have two parallel channels.

No Interference. 2 parallel streams. Can double the

s

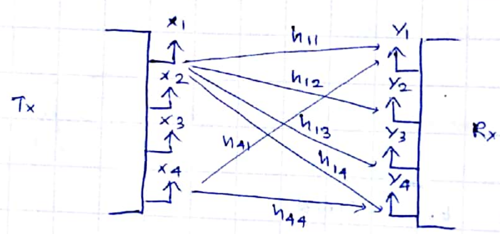


Spatial multiplexing gain

1/c

s

slide 3



$\underline{y}[m] = \underline{H}\,\underline{x}[m] + \underline{h}[m] \rightarrow \underline{Y} = \underline{H}\,\underline{X} + \underline{W}$

$$
\begin{array}{cc}
 & \xleftrightarrow{\quad Tx \quad} \\
\underline{H} = \; Rx & \begin{bmatrix} h_{11} & h_{21} & h_{31} & h_{41} \\ h_{12} & h_{22} & h_{32} & h_{42} \\ h_{13} & h_{23} & h_{33} & h_{43} \\ h_{14} & h_{24} & h_{34} & h_{44} \end{bmatrix}
\end{array}
$$

↑ vector corresponding to Tx antenna 1 and all receive antennas.

$y_1 = h_{11}x_1 + h_{21}x_2 + h_{31}x_3 + h_{41}x_4 + w_1$

$y_2 = h_{12}x_1 + h_{22}x_2 + h_{32}x_3 + h_{42}x_4 + w_2$

$y_3 =$

$\vdots$

$y_4 =$

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slide 3

if No. of Tx antennas > No. of Rx antennas $H = [\qquad\qquad]$ fat

No. of Tx antennas < No. of Rx antennas $H = \begin{bmatrix} \\ \\ \end{bmatrix}$ tall

slide 4

H = [1/sqrt(2)][rand (4,4) + 1j rand(4,4)]

(normalized) 4x4 matrix of complex coefficients

[U, L, V] = svd (H)

U Λ V

u & v are orthonormal

$U = n_r \times n_r$ matrix

$\Lambda = n_r \times n_t$ matrix

$V = n_t \times n_t$ matrix

if $n_t \neq n_r$ there ... null space ...

$$HH^H = (U\Lambda V^H)(U\Lambda V^H)^H$$
$$= U\Lambda \underbrace{V^H V}_{I} \Lambda U^H$$
$$= U\Lambda^2 U^H$$

$$(HH^H)^{-1} = (U^H)^{-1}\Lambda^{-2}U^{-1}$$ $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

$$= U\Lambda^{-2}U^H$$ $(U^H)^{-1} = U$ because orthonormal.

$$H^H H = (U\Lambda V^H)^H(U\Lambda V^H)$$
$$= V\Lambda^H U^H U\Lambda V^H$$
$$= V\Lambda^2 V^H$$

$(H^H H)^{-1} = V\Lambda^{-2}V^H$ (similar as above)

pseudoinverse

$H^+ = (HH^H)^{-1}H$ "fat"

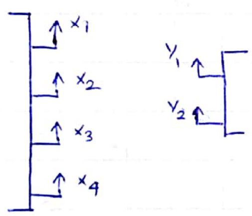
$H^+ = H(HH^H)^{-1}$ "skinny"

~~UA~~ $U\Lambda^{-2}\underbrace{U^H U}\Lambda V^H$

$U\underbrace{\Lambda^{-2}\Lambda}V^H$

$U\Lambda^{-1}V^H$

slide 5 consider $n_t = 4$ $n_r = 2$ so rank(H) = 2



$\tilde{x} = 2\times 1$

$\tilde{V} = 4\times 2$

So that $x = 4\times 1$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} \qquad \underline{V} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$H = U\Lambda V^H$

2×4 ; 2×2 ; 2×2 ; 2×4 (V is 4×2)

✓ null space is not ~~cos~~ considered here

in fact with null space

$$\left.\begin{array}{l} U = 2\times 2 \\ \Lambda = 2\times 4 \\ V^H = 4\times 4 \text{ making } V \text{ also } 4\times 4 \end{array}\right\} \text{general result}$$

$$\underline{y} = \underline{H}\,\underline{x} + \underline{w}$$
$$= \underline{H}\,\underline{V}.\tilde{x} + \underline{w}$$
$$= \underline{U}\,\underline{\Lambda}\,\underbrace{\underline{V}^H\underline{V}}\,\tilde{x} + \underline{w}$$
$$= \underline{U}\,\underline{\Lambda}\,\tilde{x} + \underline{w}$$

$$\hat{\tilde{x}} = U^H y = U^H U \Lambda \tilde{x} + U^H w$$

$$= \Lambda \tilde{x} + \underbrace{U^H w}$$

rotated version of the noise. let's say $\tilde{n}$

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

$$E\{nn^H\} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} N_0 = N_0 I$$

$$E\{\tilde{n}\tilde{n}^H\} = E[U^H n n^H U]$$

$$= U^H E[nn^H] U^H$$

$$= U^H N_0 I U$$

$$= N_0 U^H U$$

$$= N_0 I = N_0$$ (rotation does not changed the statistics of the noise)

$$\hat{\tilde{x}} = \Lambda \tilde{x} + U^H w$$

This is equivalent to two parallel channels which do not interfere a

To do this we need to know $H \rightarrow V$. Need full CSI.

In this case both Tx and Rx has to do SVD. because Tx needs $\underline{V}$ and R needs $\underline{U}$. Receiver, traditional way is to send pilots and estimate. ineffic

$$\underline{y} = Hx + w$$

$$y = \underbrace{HV}\tilde{x} + w$$

in this case the estimation can be done for the produ precoded pilot

$$y = U\Lambda V^H V \tilde{x} + w$$

$$= U\Lambda\tilde{x} + w$$

$$y = T\tilde{x} + w \qquad T = U\Lambda$$

after estimating T we can use

$$T^H y = T^H T \tilde{x} + T^H w$$

$$= \Lambda U^H \cdot U \Lambda \tilde{x} + \Lambda U^H w$$

$$= \Lambda^2 \tilde{x} + \underbrace{\Lambda U^H w}_{\tilde{n}}$$

$$E[\tilde{n}\tilde{n}^H] = \Lambda^2 N_0$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}$$

$$E[\tilde{n}\tilde{n}^H] = E[\Lambda U^H w w^H U \Lambda]$$

$$= \Lambda U^H E[ww^H] U\Lambda$$

$$= \Lambda U^H N_0 I U \Lambda$$

$$= N_0 \Lambda U^H U \Lambda$$

$$= N_0 \Lambda^2$$

$$y_1 = \lambda_1^2 \tilde{x}_1 + \tilde{n}_1$$

$$y_2 = \lambda_2^2 \tilde{x}_2 + \tilde{n}_2$$

$$SNR_1 = \frac{E[|\lambda_1^2 \tilde{x}_1|^2]}{E[|\tilde{n}_1|^2]} = \frac{\lambda_1^4 P_1}{\lambda_1^2 N_0} = \frac{\lambda_1^2 P_1}{N_0}$$

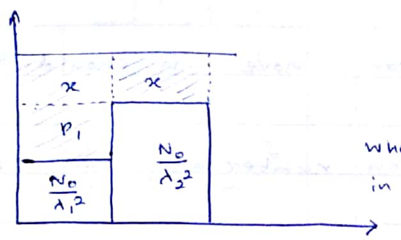
$$SNR_2 = \frac{\lambda_2^2 P_2}{N_0}$$

slide 6. Now we have parallel channels in Spatial domain.

waterfilling is the optimal solution.

$$C = \sum_{i=1}^{n_{min}} \log \left(1 + \frac{P_i \lambda_i^2}{N_0}\right)$$

optimization constraints $\sum_{i=1}^{n_{min}} P_i = P_T$; $P_i \geq 0 \ \forall i$



when you do SVD, the singular values are always in decending order. Her $\lambda_1 > \lambda_2$

$\therefore \frac{N_0}{\lambda_1^2} < \frac{N_0}{\lambda_2^2}$

Here we use ordering because only 2 channels.

$$(P_1 + x) + x = P$$

$$x = \frac{P - P_1}{2}$$

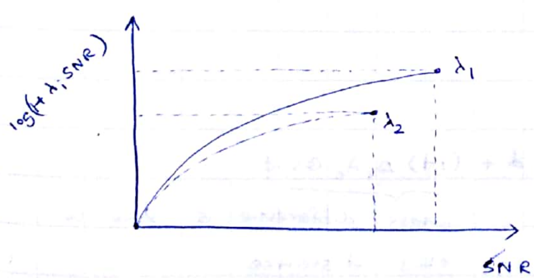
slide 7

high SNR

when SNR goes high power loading becomes asymptotically equal.

waterfilling with equal power loading.

$P_i = \frac{P}{k}$ (k is the rank of the channel) $k \leq \min(n_t, n_r)$



Trace

$$Tr(HH^H) = Tr(\Lambda V^H V \Lambda U^H U)$$

$$= Tr(\Lambda^2)$$

$$= \sum_{i=1}^{n_{min}} \lambda_i^2$$

$$Tr(AB) = Tr(BA)$$

equality is achieved only when the eigen values are equal. So we should aim to make the eigen values equally as possible. When we increase the number of antennas, eigen values tend to be more equal.

09/03/2018

slide 7

High SNR

$$C = \sum_{i=1}^{K} \log\left(1 + \frac{P\lambda_i^2}{K N_0}\right) = \sum_{i=1}^{K} \log\left(\frac{P\lambda_i^2}{K N_0}\right)$$

$$= \sum_{i=1}^{N} \log \frac{P}{N_0} + \sum_{i=1}^{K} \cancel{\lambda_i} \log \frac{\lambda_i^2}{K}$$

$$= K \log \frac{P}{N_0} + \sum_{i=1}^{K} \log \frac{\lambda_i^2}{K}$$

$K \rightarrow$ how many parallel streams we can have. It should be the rank of H

condition number $= \frac{\max \lambda_i}{\min \lambda_i}$ if condition number $= 1$, max capacity

slide 8

low SNR

$$C = \cancel{\sum_{i=1}^{K} \log\ 1 + \frac{P\lambda_i^2}{K}}$$

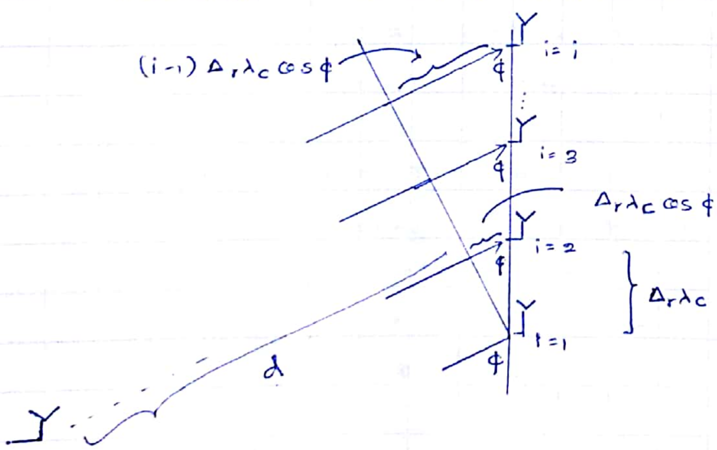
$$C = \log\left(1 + \frac{P}{N_0} \max \lambda_i^2\right) = \frac{P}{N_0} \max \lambda_i^2 \cdot \log_2 e$$

choose always the strongest channel

slide 10

Assuming $d \ggg \Delta_r \lambda_c$ (seperation between antennas)

all signals arrive at an angle of ϕ



$$d_i = d + \underbrace{(i-1)\,\Delta_r \lambda_c \cos\phi}$$

phase difference is due this distance.

$$u = \frac{h}{\|h\|}$$

$$u^H h = \frac{1}{\sqrt{n_r}}\left[1 \quad e^{j2\pi\Delta_r\Omega_r} \quad \cdots \quad e^{j2\pi(n_r-1)\Delta_r\Omega_r}\right] \frac{1}{\sqrt{n_r}}\, a\sqrt{n_r}\, e^{-\frac{j2\pi d}{\lambda_c}} \begin{bmatrix} 1 \\ e^{-j2\pi\Delta_r\Omega_r} \\ \vdots \\ e^{-j2\pi(n_r-1)\Delta_r\Omega_r} \end{bmatrix}$$

$$= \frac{a}{n_r}\sqrt{n_r}\, e^{-\frac{j2\pi d}{\lambda_c}} \left[\underbrace{1 + 1 + \cdots + 1}_{n_r \text{ times}}\right]$$

$$= a\sqrt{n_r}\, e^{-\frac{j2\pi d}{\lambda_c}}$$

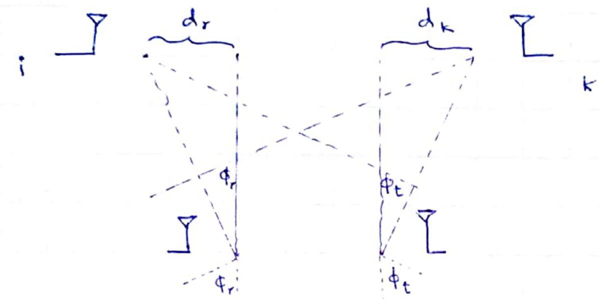
~~slide 12~~

$$\tilde{y} = u^H h \sqrt{P} + u^H n$$

$$SNR = \frac{|u^H h \sqrt{P}|^2}{N_0} = \frac{a^2 n_r P}{N_0}$$

slide 12

kth Tx antenna   ith Rx antenna

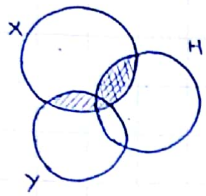


16/03/2018

slide 3

$$\underline{y} = \underline{H}\,\underline{x} + \underline{n}$$

$$I(x; y, H) = ?$$

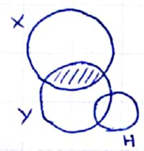


$$I(x; y, H) = I(x; H) + I(x; y|H)$$

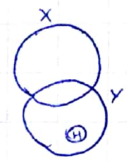
x & H are independent $\therefore I(x; H) = 0$

$$I(x; y, H) = I(x; y|H)$$

so it will look like



or



$$I(x; y|H) = h(y) - h(y|x) \quad \text{or} \quad h(y|H) - h(y|x, H)$$

$$h(y) = \log|\pi e K_y| \qquad y \sim CN(0, K_y)$$

need to compute covariance matrix of y

$$K_y = E[y y^H] = \frac{1}{N}\sum_{n=1}^{N}[y(n)\, y(n)^H] = E[(Hx+n)(Hx+n)^H]$$

$$= E[H x x^H H^H + \underbrace{H x n^H}_{0} + \underbrace{n x^H H^H}_{0} + n n^H]$$

$$= H E[x x^H] H^H + E[n n^H]$$

$$= H K_x H^H + E[n n^H]$$

$$= H K_x H^H + N_0 I$$

$$\cancel{h(y) = \log} |\pi e (N_0 I \cancel{+ H K_x H^H})|$$

~~K_x is diagonal.~~ $\cancel{h(y) = \log|\pi e\, K|}$

$$h(y) = \log \cancel{\pi e I + \log K}$$

$$h(y) = \log|\pi e\, K_y|$$

$$= \log|\pi e| + \log|K_y|$$

$$= \log\begin{vmatrix}\pi e & & \\ & \pi e & \\ & & \ddots \\ & & & \pi e\end{vmatrix} + \log|K_y|$$

$$= \log(\pi e)^{n_r} + \log|K_y|$$

$$h(y|x) = h(n)$$

$$= \log|\pi e N_0 I|$$

$$= \log\begin{vmatrix}\pi e N_0 & & \\ & \pi e N_0 & \\ & & \ddots \\ & & & \pi e N_0\end{vmatrix}$$

$$= \log(\pi e N_0)^{n_r}$$

$$= \log(\pi e)^{n_r} + \log|N_0 I|$$

$$\therefore h(y) - h(n) = \log|K_y| - \log|N_0 I|$$

$$= \log\frac{|K_y|}{|N_0 I|} = \log|K_y (N_0 I)^{-1}|$$

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$$= \log \left| \left(H K_x H^H + N_0 I\right) \cdot (N_0 I)^{-1} \right|$$

$$= \log \left| I_{n_r} + \frac{1}{N_0} H K_x H^H \right|$$

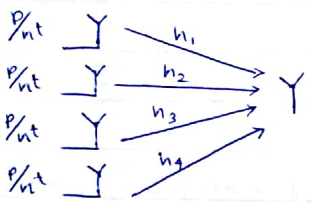
slide 4

$n_r = 1$ $\qquad H K_x H^H$

$(1 \times n_t)\ (n_t \times n_t)\ (n_t \times 1)$

becomes a scalar

$\sum_{k=1}^{n_t} |h_k|^2 P_k$ (if we don't know anything we allocate power evenly P_k)



$$\therefore\ C = E\left[\log\left(1 + \frac{1}{N_0} \sum_{k=1}^{n_t} |h_k|^2 P_k\right)\right]$$

optimum receiver in this case is SIC.

In the transmitter we can use transmit beamforming matched with channel

$\mathrm{svd}(\underline{h}) = U \Lambda V^H$ $\qquad (n_t = 4)$

$1 \times 1 \quad 1 \times n_t \quad n_t \times n_t$

null space.

$$1 \cdot [\lambda_1\ 0\ 0\ 0] \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \end{bmatrix}$$

same as $\frac{h}{\|h\|}$

slide 5

$$C = E\left[\log \left| I_{n_r} + \frac{1}{N_0} H K_x H^H \right|\right]$$

if we allocate equal power $K_x = \frac{P}{n_t} I$

$$C = E\left[\log \left| I_{n_r} + \frac{P}{n_t N_0} H H^H \right|\right] \qquad H H^H = U \Lambda V^H V \Lambda^H U^H = U \Lambda^2 U^H$$

$$C = E\left[\log \left| I_{n_r} + \underbrace{\frac{P}{n_t N_0} U}_{A} \underbrace{\Lambda^2 U^H}_{B} \right|\right]$$

using det rule $\quad |I + AB| = |I + BA|$

$$C = E\left[\log \left| I_{n_r} + \frac{P}{n_t N_0} \Lambda^2 \underbrace{U^H U}_{I} \right|\right] = E\left[\log \left| I_{n_r} + \frac{P}{n_t N_0} \Lambda^2 \right|\right]$$

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This matrix will look like

$$\begin{bmatrix} 1+\frac{P}{n_t N_0}\lambda_1^2 & & & \\ & 1+\frac{P}{n_t N_0}\lambda_2^2 & & \\ & & \ddots & \\ & & & 1+\frac{P}{n_t N_0}\lambda_n^2 \end{bmatrix}$$

$$C = E \log\left(1+\frac{P}{n_t N_0}\lambda_1^2\right)\left(1+\frac{P}{n_t N_0}\lambda_2^2\right)\cdots\left(1+\frac{P}{n_t N_0}\lambda_n^2\right)$$

$$= \sum_{i=1}^{n_{min}} E\left[\log\left(1+\frac{P}{n_t N_0}\lambda_i^2\right)\right]$$

if we know the channel.

$$\log \left| I_{n_r} + \frac{1}{N_0} H K_x H^H \right|$$

$$= \log \left| I_{n_r} + \frac{1}{N_0} \cdot \underbrace{U \Lambda V^H V}_{I} P \underbrace{V^H V}_{I} \Lambda U^H \right|$$

$$= \log \left| I_{n_r} + \frac{1}{N_0} U \Lambda P \Lambda U^H \right|$$

$$= \log \left| I_{n_r} + \frac{1}{N_0} \Lambda^2 P \right| \quad \text{using det rule}$$

$$= \sum \log\left(1+\frac{1}{N_0} P_i \lambda_i^2\right)$$

High SNR

CSIR

$$C = \sum E \log\left(\frac{P}{n_t N_0}\lambda_i^2\right)$$

$$= \sum_{i=1}^{n_{min}} \log\frac{P}{n_t N_0} + E\left[\sum_{i=1}^{n_{min}} \log(\lambda_i^2)\right]$$

$$\underbrace{n_{min} \log\frac{P}{n_t N_0}}_{\text{don't know CSIR}} + E\left[\sum_{i=1}^{n_{min}} \log(\lambda_i^2)\right]$$

Full CSI

$$C = \sum_{i=1}^{n_{min}} E \log\left(\frac{P(\lambda_i)\lambda_i^2}{N_0}\right)$$

$$= \sum_{i=1}^{n_{min}} \left\{ \log\frac{P_i(\lambda_i)}{N_0} + E(\log \lambda_i^2) \right.$$

$$P_c = \frac{P}{n_{min}}$$

$$= \sum_{i=1}^{n_{min}} \left\{ \log\frac{P}{n_{min} N_0} + E[\log(\lambda_i^2)] \right.$$

$$= \underbrace{n_{min} \log\frac{P}{n_{min} N_0}}_{} + \sum_{i=1}^{n_{min}} E[\log(\lambda_i^2)]$$

know the channel
Full CSI

Compare

Full CSI with $n_t > n_r$?

" " " $n_t < n_r$?

CSIR " $n_t > n_r$?

CSIR " $n_t < n_r$?

slide 8

1x1 case $\quad c = E\left[\log\left(1 + |h|^2 SNR\right)\right]$

1x4 case $\quad c = E\left[\log\left(1 + \|h\|^2 SNR\right)\right]$

4x4 case $\quad c = E\left[\log\left|\left(I + \frac{P}{n_t N_0} HH^H\right)\right|\right]$

slide 9

<u>low SNR</u>

$$C = \sum_{i=1}^{n_{min}} E\left[\log\left(1 + \frac{SNR}{n_t}\lambda_i^2\right)\right]$$

$$= \sum_{i=1}^{n_{min}} E\left[\frac{SNR}{n_t}\lambda_i^2 \log_2 e\right]$$

$$= \sum_{i=1}^{n_{min}} \frac{SNR}{n_t}\log_2 e \,.\, E\left[(\lambda_i^2)\right]$$

$$= \frac{SNR}{n_t}\log_2 e \quad E\left[\sum_{i=1}^{n_{min}} \lambda_i^2\right]$$

$$= \frac{SNR}{n_t}\log_2 e \quad E\left[Tr\left(HH^H\right)\right]$$

$$Tr\left(HH^H\right) = Tr\left(U\Lambda V^H V \Lambda U^H\right)$$

$$= Tr\left(\Lambda^2\right)$$

$$= \sum_{i=1}^{n_{min}} \lambda_i^2$$

$$\therefore C = \frac{SNR}{n_t}\log_2 e \quad E\left[\sum_{i,j}|h_{ij}|^2\right]$$

$$= \frac{SNR}{n_t}\log_2 e \,.\, (n_r \times n_t)$$

$$\begin{array}{cc} & n_t \\ n_r & \begin{bmatrix} h_{ij} & \\ & \end{bmatrix} \end{array}$$

$E|h_{ij}|^2 = 1$

$\therefore \sum E|h_{ij}|^2 = n_r \times n_t$

$$= n_r SNR \log_2 e$$

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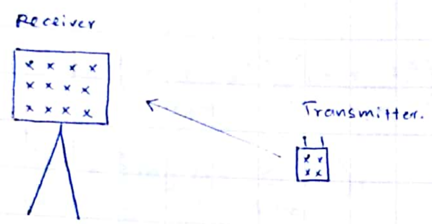
Slide 13

Slic Average capacity. $\frac{E\left[\log |I + SNR\, HH^H|\right]}{n}$

with $n = 8$ we ~~get~~ almost get $n \to \infty$

S1 Slide 15



1R/S1

$|I + AB| = |I + BA|$

$= \log \left| I_{n_r} + \frac{P}{n_t N_0} HH^H \right|$

$= \log \left| I_{n_t} + \frac{P}{n_t N_0} H^H H \right|$ $H^H H$ becomes $n_r I_{n_t}$ because of i.i.d.

$= \log \left| I_{n_t} + \frac{P}{n_t N_0} n_r I \right|$

This is diagonal matrix. Det is product of diagonal element. log of pro[duct]

sum of log.

$= \underbrace{\log\left(1 + \frac{P n_r}{n_t N_0}\right) + \log\left(1 + \frac{P n_r}{n_t N_0}\right) + \dots + \left(\log 1 + \frac{P n_r}{n_t N_0}\right)}_{n_t \text{ terms}}$

$= n_t \log\left(1 + \frac{P n_r}{n_t N_0}\right)$

consider $n_t = 1$ special case. we get SIMO channel

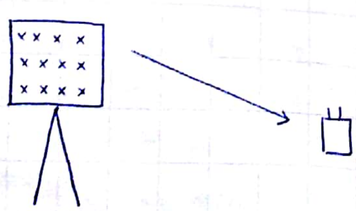
$c = \log\left(1 + SNR\, \|h\|^2\right)$

if n is large $\|h\|^2 \to n_r$

So $c = \log\left(1 + SNR \cdot n_r\right)$

$c = 1 \cdot \log\left(1 + \frac{SNR}{1} \cdot n_r\right)$ } similar.

Slide 16



slide 17

MISO case

$\log(1 + SNR\|h\|^2)$

when $n \to \infty$ can be approximated with AWGN capacity.

$\log(1+1)$

$= \log 2 = 1$

SIMO case

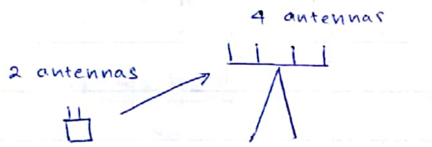
$= \log(1 + SNR\|h\|^2)$

$=$ as $n \to \infty$ $\log(1 + SNR \cdot n_r)$ logarithmic of n_r

MIMO case

$n \cdot c^*(SNR)$ linear

slide 19.



$H = [\;]_{4\times 2}$

$$H = \begin{array}{cc} \text{Tx antenna1} & \text{Tx antenna 2} \\ \left[h_1 \right. & \left. h_2 \right] \end{array}$$

let's say $H = [h_1 \vdots h_2 \vdots h_3 \vdots h_4]$

$Y = H\underline{x} + \underline{n}$

$$[\underline{h}_1 \vdots \underline{h}_2 \; \underline{h}_3 \; \underline{h}_4] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$Y = \sum_{i=1}^{4} \underline{h}_i x_i + \underline{n}$$

let's say we want to decode stream 2

$$Y = \underline{h}_2 x_2 + \underbrace{\sum_{i=1,3,4} \underline{h}_i x_i}_{\text{interference}} + n$$

if we can cancel interference.. the it will be SIMO model

23 slide 20

one way to create the projection matrix is SVD

Slic

consider one stream at a time; collect all the channels into a matrix, exc. the channel of interest, form Q_K and use it.

$q_K\, n[m]$ does not have effect. Q_K has orthonormal columns. statistical

sl properties of $\underline{n}$ remains constant.

$$y_K = \underline{h}_K x_K + \sum_{i \neq K} \underline{h}_i x_i + \underline{n}$$

$$\hat{y}_K = Q_K h_K x_K + \underbrace{Q_K \sum(\quad\quad)}_{0} + Q_K \underline{n}$$

/K

sl

$$\hat{x}_K = \underbrace{(h_K^H Q_K^H)(Q_K h_K)}_{c_K^H \text{ vector}} x_K + 0 + h_K^H Q_K^H Q_K \underline{n}$$

$$\hat{x}_K = c_K^H h_K x_K + c_K^H \underline{n}$$

$$SNR = \frac{P |c_K^H h_K|^2}{\|c_K\|_2^2 N_0}$$ simplify this

Geometric interpretation with 2 streams

if h_1 and h_2 are orthogonal, projecting y to null space of h_1 would be exactly h_2.

27/02/2018

slide 20

How to find Q_K

$$h_K = [h_1 \dots h_K \dots h_{nt}]$$

$\uparrow$ desired

$$\downarrow$$

$$\bar{H}_K = [h_1 \dots h_{K-1}, h_{K+1} \dots h_{nt}]$$

Ex:- $n_t = 3$ $n_r = 4$

$$h = [h_1\ h_2\ h_3]$$

we are interested in h_2

So $\bar{H}_K = [h_1\ h_3]$

$\bar{H}_K$ = is a 4×2 matrix

$$\downarrow \text{SVD}$$

$$\bar{H}_K = U \Lambda V^H$$

$$\downarrow$$

(4 x 4) matrix

two last columns forms the null space

slide 21

This is equivalent to SIMO channel

$$\tilde{y} = \tilde{h}_K x_K + \tilde{n}_K \qquad \tilde{h}_K = Q_K h_K$$

$$\hat{x} = \tilde{h}^H \tilde{y}$$

$$= \tilde{h}_K^H (\tilde{h}_K x_K + \tilde{n}_K)$$

$$= \underbrace{h_K^H Q_K^H Q_K h_K x_K}_{\text{desired}} + \underbrace{h_K^H Q_K^H n}_{\text{noise}}$$

$$SNR_K = \frac{P_K \| Q_K h_K \|^2}{N_0}$$

different method.

let's assume we have two streams $H = [h_1\ h_2]$

let's say we are interested in decoding 1st stream

So $\bar{H} = h_2$

$$c_K = (I - \bar{H}_K (\bar{H}_K^H \bar{H}_K)^{-1} H_K^H) h_K$$

$$c_1 = \left(I - \frac{h_2 h_2^H}{\|h_2\|^2}\right) h_1$$

$$h_2^H c_1 = h_2^H \left(I - \frac{h_2 h_2^H}{\|h_2\|^2}\right) h_1$$

$$= h_2^H h_1 - \frac{h_2^H h_2 h_2^H h_1}{\|h_2\|^2}$$

$$= h_2^H h_1 - \frac{\|h_2\|^2 h_2^H h_1}{\|h_2\|^2}$$

$$= h_2^H h_1 - h_2^H h_1$$

$$= 0$$

h_2^H and c_1 are orthogonal.

slide 22

high SNR

$$R_{decorr} = E\left[\sum_{k=1}^{n_{min}} \log\left(1 + \frac{SNR}{n_t} \|Q_k \underline{h}_k\|^2\right)\right]$$

$$= n_{min} \log\left(\frac{SNR}{n_t}\right) + E\left[\sum_{k=1}^{n_{min}} \|Q_k \underline{h}_k\|^2\right]$$

slide 23

$$\bar{y} = \underline{H}\underline{x} + \underline{n}$$

$\hat{\underline{x}}$ is the estimate of the transmitted signal. $\underline{y}$ is the received signal

find best estimate $\hat{x}$ such that the diff between $\underline{H}\hat{\underline{x}}$ and $\bar{y}$ is

$$\|r\|^2 = \|H\hat{x} - y\|^2$$

$$= (H\hat{x} - y)^H (H\hat{x} - y)$$

$$= \hat{x}^H H^H H \hat{x} - \underbrace{\hat{x}^H H^H y}_{\text{Scalar}} - \underbrace{y^H H \hat{x}}_{\text{Scalar}} + y^H y$$

$$= \hat{x}^H H^H H \hat{x} - 2\hat{x}^H H^H y + y^H y$$

w.r.t $\hat{x}$ (optimization variable)

$$\nabla_{\hat{x}} \|r\|^2 = 2H^H H \hat{x} - 2H^H y$$

setting $\nabla_{\hat{x}} \|r\|^2 = 0$

$$2H^H H \hat{x} = 2H^H y$$

$$\hat{x} = (H^H H)^{-1} H^H y$$

$$\hat{x} = (H^H H)^{-1} H^H (Hx + n)$$

$$\hat{x} = (H^H H)^{-1} (H^H H) x + (H^H H)^{-1} H^H n$$

$$\hat{x} = I x + (H^H H)^{-1} H^H n$$

$$\hat{x} = x + (H^H H)^{-1} H^H n$$

(noisy estimate of original

if $n = 0$ we get exact x

if $n \neq 0$ it is distroted

***

if H is not full rank

$$(H^H H)^{-1}$$

$$(V \Lambda \underbrace{U^H U}_{I} \Lambda V^H)^{-1}$$

$$V \Lambda^{-1} V^H$$

$$\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

dividing by 0. noise goes to infinity. nothing goes thr channels are linearly dep

$$\hat{y} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_4 \end{bmatrix}$$

slide 25

$$\underline{y} = \underline{H}\,\underline{x} + \underline{n}$$

$$\underline{y} = \cancel{\underline{h}_1 x_1} + \underline{h}_2 x_2 + \cdots \underline{h}_n x_n + \underline{n}$$

Start from 1st stream. apply decorrlator, regenerate $h_1 x_1$, remove it and pass it to decorrelator 2. For 2nd stream decorrlator has to null only stream 3 to n.

For the last stream

$$\underline{y}_{nt} = \underline{h}_{nt} x_{nt} + \underline{n}$$

has gain over linear decorrelator. (see graph)

$$C_{ZF-SIC} = E\left[\sum_{i=1}^{n_{min}} \log\left(1 + \frac{\|Q_k h_k\|^2 P}{n_t N_0}\right) \right]$$ ← same formula as ZF but size of Q_k is different

for 1st stream $k=1$ $\|Q_k h_k\|^2$ is a scalar $\sim \chi_2^2$

2 $k=2$ $\sim \chi_4^2$

$k=3$ $\sim \chi_6^2$

$k=8$ $\sim \chi_{16}^2$

beamforming gain.

*** matched filter for stream 3 is h_3^H

slide 26

$$\underline{y} = \underline{H}\,\underline{x} + \underline{n}$$

$$= \underline{h}_k x_k + \underbrace{\sum_{i \neq k} \underline{h}_i x_i + \underline{n}}_{\text{interference + noise}}$$

$$= \underline{h}_k x_k + \underline{z}_k$$

covariance matrix of $\underline{z}$

$$R_k = E[\underline{z}_k \underline{z}_k^H]$$

$$= \sum_{i \neq k} h_i E[x_i x_i^H] h_i^H + E[n n^H]$$

$$= \sum_{i \neq k} \frac{P_t}{n_t} h_i h_i^H + E[n n^H]$$

$$= \sum_{i \neq k} \frac{P_t}{n_t} h_i h_i^H + N_0 I$$

if SNR is low $N_0 I$ dominates. matched filter is optimal. If SNR is ~~low~~ high ~~n~~ interference dominates. ZF is optimal.

slide 27

Interference whitening.

sli $R_K^{-1/2} y$ (multiply y by $R_K^{-1/2}$)

$$R_K^{-1/2} y = R_K^{-1/2} h_K x_K + \underbrace{R_K^{-1/2} z_K}$$

si ~~$E\,[R_K^{-1/2} z_K]$~~ $E\left[R_K^{-1/2} \underbrace{z_K z_K^H}_{R_K} R_K^{-1/2}\right] = I$ (achieving whitening)

$$\underbrace{h_K^H R_K^{-1/2}}_{C_K^H} \cdot R_K^{-1/2} h_K x_K + \underbrace{h_K^H R_K^{-1/2}}_{C_K^H} \cdot R_K^{-1/2} z_K$$

1/2

sl $C_K^H = h_K^H R_K^{-1}$

03/04/2018

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$$\underline{y} = \underline{H}\,\underline{x} + \underline{n} = \sum_{i=1}^{n_t} \underline{h}_i x_i + \underline{n}$$

$$= \underline{h}_K x_K + \underbrace{\sum_{i \neq K} \underline{h}_i x_i + \underline{n}}_{\underline{z}_K}$$

$z_K \sim CN(0, R_K)$

z_K is colored noise. $\underline{n}$ is white. x_i is gaussian but it is multiplied w h_i. So z_K is white. covariance w is no longer diagonal.

$$\therefore R_K = E\left[z_K z_K^H\right]$$
$$= \sum_{i \neq K} h_i h_i \frac{P}{n_t} + N_0 I$$ prove

in order to whiten the colored interference.

$$\bar{\underline{y}}^K = R_K^{-1/2} \underline{y} = \underbrace{R_K^{-1/2} h_K}_{\bar{\underline{h}}_K} x_K + \underbrace{R_K^{-1/2} z_K}_{\bar{z}_K}$$ ← Covariance matrix of $\bar{z}_K$ will be diagonal (identity)

$$\bar{\underline{y}}_K = \bar{\underline{h}}_K x_K + \bar{z}_K$$

$$\hat{x}_K = \underbrace{\underline{h}_K^H R_K^{-1/2}}_{W_K^H} \cdot R_K^{-1/2} \underline{h}_K x_K + \underbrace{\underline{h}_K^H R_K^{-1/2}}_{W_K^H} R_K^{-1/2} z_K$$

W_K → (received beamformer)

$$W_K^H = h_K^H R_K^{-1}$$

receiver that maximizes the SNR of signal which was interfered by clolored interference (homework) linear MMSE receiver

$$w = \arg\min_{w} E\left[\| \underbrace{x}_{\text{transmitted signal}} - \underbrace{w^H y}_{\text{received estimate}} \|^2\right]$$

w - received beamformer matrix

In general - the solution is not linear. But when x and n are independent and they are gaussian, linear mmse is optimal.

How to write this:

$$Tr\left[E\left[(x - w^H y)(x - w^H y)^H\right]\right]$$

$$Tr\left[E[x x^H] - E[x y^H w] - E[w^H y x^H] + E[w^H y y^H w]\right]$$

$$Tr\left[(K_x) - K_x H^H w - w^H H K_x + w^H (H K_x H^H + N_0 I) w\right]$$

$$Tr(K_x) - Tr(K_x H^H w) - Tr(w^H H K_x) + Tr\left(w^H (H K_x H^H + N_0 I) w\right)$$

Differentiate w.r.t w

$$0 - \cancel{2} H K_x + \cancel{2} (H K_x H^H + N_0 I) w = 0$$

$$\therefore w = (H K_x H^H + N_0 I)^{-1} H K_x$$

first Tx antenna to all receive antennas

$$w_k = \left(\sum_{i=1}^{n_t} \frac{\rho}{n_t} (h_i h_i^H + N_0 I)\right)^{-1} h_k \frac{\rho}{n_t} \qquad w = [\underline{w}_1 \ \underline{w}_2 \ \dots \ \underline{w}_{n_t}]$$

max SNR scenario is different. It is a scaled version of MMSC

$$w_k = \left(\sum_{i \neq k} h_i h_i^H \frac{\rho}{n_t} + N_0 I\right)^{-1} h_k$$

MMSE is always better.

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What is the SINR ?

$$\underline{y} = \underline{H}\,\underline{x} + \underline{n}$$

$$\hat{x} = w_k^H h_k x_k + w_k^H \left(\sum_{i \neq k} \underline{h}_i x_i + \underline{n}\right)$$

$$\gamma_k^{mmse} = \frac{E\left[|w_k^H h_k x_k|^2\right]}{E\left[|w_k^H (\sum_{i \neq k} \underline{h}_i x_i + \underline{n})|^2\right]}$$

$$= \frac{E\left[(w_k^H h_k x_k)(w_k^H h_k x_k)^H\right]}{E\left[\{w_k^H (\sum_{i \neq k} \underline{h}_i x_i + \underline{n})\}\{w_k^H (\sum_{i \neq k} \underline{h}_i x_i + \underline{n})\}^H\right]}$$

$$= \frac{w_k^H h_k E[x_k x_k^H] h_k^H w_k}{E\left[\{w_k^H (\sum_{i \neq k} h_i x_i + n)\}\{(\sum_{i \neq k} (h_i x_i + n))^H w_k\}\right]} = \frac{P_k w_k^H h_k h_k^H w_k}{w_k^H R_k w_k}$$

23. Plug in $w_k^H = h_k^H R_k^{-1}$

Slic

$$\gamma_k^{mmse} = \frac{P_k h_k^H R_k^{-1} h_k h_k^H R_k^{-1} h_k}{h_k^H \underbrace{R_k^{-1} R_k}_{I} R_k^{-1} h_k}$$

R is symmetric

$R^H = R$

$$= P_k \underbrace{\underbrace{h_k^H}_{\substack{\text{row}\\\text{vector}}} \underbrace{R_k^{-1}}_{\text{matrix}} \underbrace{h_k}_{\substack{\text{column}\\\text{vector}}}}_{\text{Scalar}}$$

multiplying by $(h_k^H R_k^{-1} h_k)^{-1}$

sl.

34.00 (homework)

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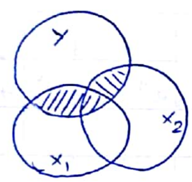
lec

Sl:



$$I(Y; x_1, x_2) = I(x_1; y) + I(x_2; y|x_1)$$

By extending this

$$I(\underline{x}; \underline{y}) = I(x_1, x_2, \dots x_{nt}; \underline{y})$$
$$= I(x_1; \underline{y}) + I(x_2; \underline{y}|x_1) + I(x_3; \underline{y}|x_1, x_2) + \dots \quad I(x_{nt}; \underline{y}|x_1 \dots)$$

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$$\underline{y} = \underline{H}\,\underline{x} + \underline{n}$$

$$I(\underline{x}; \underline{y}) = h(\underline{y}) - h(\underline{y}|\underline{x})$$
$$= h(\underline{y}) - h(\underline{n})$$
$$= \log \pi e \left| E[yy^H] \right| - \log \pi e \left| N_0 I \right|$$
$$= \log \pi e \left| E[(Hx+n)(Hx+n)^H] \right| - \log \pi e \left| N_0 I \right|$$
$$= \log \pi e \left| E[Hxx^H H^H + nn^H] \right| - \log \pi e \left| N_0 I \right|$$
$$= \log \pi e \left| H\,E[xx^H] H^H + N_0 I \right| - \log \pi e \left| N_0 I \right|$$
$$= \log \pi e \left| \frac{P}{n_t} HH^H + N_0 I \right| - \log \pi e \left| N_0 I \right|$$

if P is equally distributed

$$= \log \left| I + \frac{P}{n_t N_0} HH^H \right|$$

Generally $= \log \left| I + \sum_{i=1}^{n_t} \frac{P_i}{N_0} h_i h_i^H \right|$

Seperate the first term which corresponds to the first stream

$$= \log \left| \underbrace{I_{n_r} + \sum_{i=2}^{n_t} \frac{P_i}{N_0} h_i h_i^H}_{R_1} + \frac{P_1}{N_0} h_1 h_1^H \right|$$

$$= \log \left| R_1 + \frac{P_1}{N_0} h_1 h_1^H \right|$$

$$= \log \left| \underbrace{R_1^{1/2}}_{A} \underbrace{\left(I + \frac{P_1}{N_0} R_1^{-1/2} h_1 h_1^H R_1^{-1/2}\right) R_1^{1/2}}_{B} \right|$$

Scanned by CamScanner

$$= \log \left| \left(I + \frac{P_1}{N_0} R_1^{-1/2} h_1 h_1^H R_1^{-1/2}\right) R_1 \right| \qquad |AB| = |BA|$$

$$= \log \left| I + \frac{P_1}{N_0} \underbrace{R_1^{-1/2} h_1}_{\substack{\text{column}\\ \text{vector}\\ (c)}} \underbrace{h_1^H R_1^{-1/2}}_{\substack{\text{row}\\ \text{vector}\\ (r)}} \right| + \log |R_1| \qquad \log|AB| = \log|A| + \log|B|$$

** $|I + \text{col. vector} \times \text{row vector}| = 1 + \text{row vector} \times \text{col. vector}$ (for rank 1 matrix)

This is a special case of $|I + AB| = |I + BA|$ (for general rank matrix)

$$= \log \left(1 + \frac{P_1}{N_0} h_1^H R_1^{-1/2} R_1^{-1/2} h_1\right) + \log |R_1|$$

$$= \log \left(1 + \underbrace{\frac{P_1}{N_0} h_1^H R_1^{-1} h_1}_{SINR_1^{\,mmsc\text{-}sic}}\right) + \cancel{\log |R_1|} \log \left| I + \sum_{i=2}^{n_t} \frac{P_i}{N_0} (h_i h_i^H) \right|$$

↖ SINR of output of the mmsc receiver

following the same process

$$= \log(1 + SINR_1) + \log(1 + SINR_2) + \dots$$

↑ SINR of output of the mmsc receiver after removing stream 1

34.40
lec 7

Question :- in this case $x_1, x_2 ..$ are independent. what if they are dependent ?

In this case we need to know the channel. If so,

$K_x = V P V^H$

$U \Lambda U^H = H$

$x = V P^{1/2} d$ ← Stream specific data

$Y = H V P^{1/2} d + n$

$= \tilde{H} d + n \qquad d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{n_{min}} \end{bmatrix}$

$\tilde{H}_d$ is the d receiver

$$\tilde{H}_d^H Y = \tilde{H}^H \tilde{H} d + \tilde{H}^H n$$

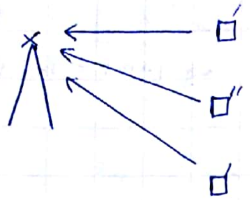
$$= P^{1/2} V^H H^H H V P^{1/2} + \tilde{H}^H n$$

$$= P^{1/2} \underbrace{V^H V}_{I} \Lambda \underbrace{U^H U}_{I} \Lambda \underbrace{V^H V}_{I} P^{1/2} + \tilde{H}^H n$$

$$= P^{1/2} \Lambda^2 P^{1/2} + \underbrace{\tilde{H}^H n}_{\tilde{n}}$$

06/04/2017 - Lecture 7

slide 3



$$\underline{y} = \sum_{i=1}^{K} \underline{h}_k x_k + \underline{n}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \sum_{i=1}^{K} \underbrace{\begin{bmatrix} h_{1k} \\ h_{2k} \\ h_{3k} \end{bmatrix}}_{\underline{h}_k} x_k + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

$$H = \begin{bmatrix} h_1 & h_2 & \cdots & h_K \end{bmatrix} \in \mathcal{CN}^{n_r \times K}$$

For gaussian uplink single antenna case

$$y = h_1 x_1 + h_2 x_2 + n$$

$$R_1 \leq \log_2 (1 + |h_1|^2 SNR_1)$$

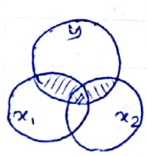
$$R_2 \leq \log_2 (1 + |h_2|^2 SNR_2)$$

$$R_1 + R_2 \leq \log_2 (1 + |h_1|^2 SNR_1 + |h_2|^2 SNR_2)$$

This can be calculated from mutual information

$$I(x_1, x_2 ; y) = h(y) - h(y | x_1, x_2)$$
$$= I(x_1 ; y) + I(x_2 ; y | x_1)$$

slide 5



in MIMO case variance → covariance

scalar → vector.

upper bound for R_1

$$R_1 \leq I(x_1 ; y | x_2)$$
$$= h(y|x_2) - h(y|x_1, x_2)$$
$$= \log_2 \pi e \left| P_1 h_1 h_1^H + N_0 I \right| - \log_2 \pi e |N_0 I|$$
$$= \log_2 \left| I + \frac{P_1 h_1 h_1^H}{N_0} \right|$$

Apply log and det rules $= \log_2 \left(1 + \frac{P_1}{N_0} h_1^H h_1\right)$ $\quad \det(I + CR) = 1 + RC$

$$= \log_2 \left(1 + \frac{P_1}{N_0} \|h_1\|^2\right)$$

The sum rate

$$R_1 + R_2 \leq I(x_1, x_2 : y)$$
$$= h(y) - h(y|x_1, x_2)$$
$$= \log \pi e \left| \{ P_1 h_1 h_1^H + P_2 h_2 h_2^H + N_0 I \} \right| - \log \pi e |N_0 I|$$
$$= \log \left| I + \underbrace{\frac{P_1}{N_0} h_1 h_1^H + \frac{P_2}{N_0} h_2 h_2^H} \right|$$

- $$K_x = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

$$\begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} h_1^H \\ h_2^H \end{bmatrix} = \begin{bmatrix} P_1 h_1 & P_2 h_2 \end{bmatrix} \begin{bmatrix} h_1^H \\ h_2^H \end{bmatrix}$$
$$= P_1 h_1 h_1^H + P_2 h_2 h_2^H$$

In the point to point case the user specific rate does not matter. Because we want to maximize the sum rate. But when users are geographically seperated, the decoding order matters.

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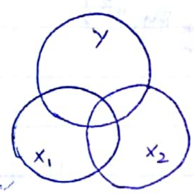
Rate of user 1 at point A = $R_{sum} - R_2$ ← Interference free rate



$$R_1 \leq I(x_1; \underline{y}) = h(\underline{y}) - h(\underline{y}|x_1)$$
$$= \log \pi e \left| P_1 h_1 h_1^H + P_2 h_2 h_2^H + N_0 I \right| - \log \pi e \left| P_2 h_2 h_2^H + N_0 I \right|$$
$$= \log \frac{\left| P_1 h_1 h_1^H + P_2 h_2 h_2^H + N_0 I \right|}{\left| P_2 h_2 h_2^H + N_0 I \right|}$$
$$= \log \left| (P_1 h_1 h_1^H + P_2 h_2 h_2^H + N_0 I)(P_2 h_2 h_2^H + N_0 I)^{-1} \right|$$
$$= \log \left| I + \underbrace{P_1 h_1 h_1^H}_{C} \underbrace{(P_2 h_2 h_2^H + N_0 I)^{-1}}_{R} \right|$$
$$= \log \left(1 + \underbrace{P_1 h_1^H (P_2 h_2 h_2^H + N_0 I)^{-1} h_1}_{\text{SINR at the o/p of MMSE receiver.}} \right) \qquad \log|I + CR| = \log(1 + RC)$$

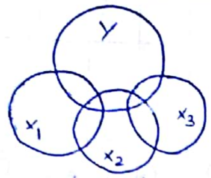
MMSE Receiver

$$\hat{x}_1 = w_1^H y$$
$$\cancel{= w_1^H h_1 x}$$

$$\gamma_1 = \frac{P_1 w_1^H h_1 h_1^H w}{w_1^H R_1 w_1} = P_1 h_1^H (P_2 h_2 h_2^H + N_0 I)^{-1} h_1$$

$$R_1 = P_2 h_2 h_2^H + N_0 I$$

Three user case



user 3 is decoded first

$$\underline{y} = \underline{h}_1 x_1 + \underline{h}_2 x_2 + \underline{h}_3 x_3 + \underline{n}$$

$$I(x_3 ; y) = h(y) - h(y|x_3)$$

$$= \log \pi e \left| E[yy^H] \right| - \log \pi e \left| E[(y|x_3)(y|x_3)^H] \right|$$

$$= \cancel{\log}$$

$$= \cancel{\log} = \log \pi e \left| P_1 h_1 h_1^H + P_2 h_2 h_2^H + P_3 h_3 h_3^H + N_0 I \right| - \log \pi e \left| P_1 h_1 h_1^H + P_2 h_2 h_2^H \ldots \right|$$

$$= \log \left| I + P_3 h_3 h_3^H \left(P_1 h_1 h_1^H + P_2 h_2 h_2^H + N_0 I \right)^{-1} \right|$$

$$= \log \left(1 + P_3 h_3^H \underbrace{\left(P_1 h_1 h_1^H + P_2 h_2 h_2^H + N_0 I \right)}_{R_3}^{-1} h_3 \right) \longrightarrow (*)$$

$$= \log (1 + \gamma_3)$$

then for user 2 $\quad R_2 = \log \left(1 + P_2 h_2^H \left(P_1 h_1 h_1^H + N_0 I \right)^{-1} h_2 \right)$

and then for user 1 $\quad R_1 = \log \left(1 + \dfrac{P_1 h_1^H h_1}{N_0} \right) = \log \left(1 + \dfrac{P_1}{N_0} \|h_1\|^2 \right)$

MMSE Receiver for user k

$$w_k = R_k^{-1} h_k$$

$$w_3 = R_3^{-1} h_3$$

$$= \left(P_1 h_1 h_1^H + P_2 h_2 h_2^H + N_0 I \right)^{-1} h_3$$

$$\gamma_3 = \frac{P_3 w_3^H h_3 h_3^H w_3}{w_3^H R_3 w_3} = \frac{P_3 h_3^H R_3^{-1} h_3 \cancel{h_3^H R_3^{-1} h_3}}{\cancel{h_3^H R_3^{-1} \underbrace{R_3 R_3^{-1}}_{I} h_3}}$$

$$= P_3 h_3^H R_3^{-1} h_3 \quad (\text{same as } *)$$

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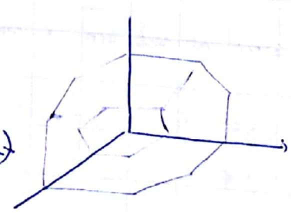
3 user constraints

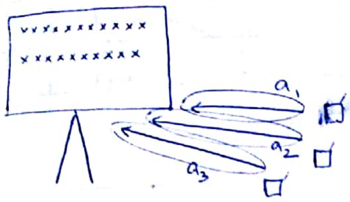
$2^3 - 1$

$$\begin{cases} R_1 \le \log \left(1 + \frac{P_1}{N_0} \|h_1\|^2 \right) \\ R_2 \le \\ R_3 \le \\ R_1 + R_2 \le \cancel{\log \left(1 + \frac{P_1}{N_0} \|h_1\|^2 + \frac{P_2}{N_0} \|h_2\|^2 \right)} \\ R_2 + R_3 \le \\ R_1 + R_3 \le \log \left| I + \frac{P_1}{N_0} h_1 h_1^H + \frac{P_3}{N_0} h_3 h_3^H \right| \\ R_1 + R_2 + R_3 \le \end{cases}$$



$n_r >>>> K$

slide 11



i.i.d is not a necessity to achieve massive MIMO. When number of antennas are high we can do beamforming.

$$K_x = \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix}$$

$$H = [h_1 \quad h_2 \quad h_3]$$

$$= \underbrace{\begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix}}_{3} \; 100 \qquad \text{rank} = 3$$

$^H + N_0 I|$

$$|I + \underbrace{\overset{100\times3}{A}\,\overset{3\times100}{B}}_{100\times100}| = I + \underbrace{\overset{3\times100}{B}\,\overset{100\times3}{A}}_{3\times3}$$

$$B \cdot A = [\quad]\,[\quad]$$

$$\frac{h_i^H h_k}{n_r} \longrightarrow 0 \quad \text{as } n_r \to \infty$$

$$\frac{a_1 h_1^H h_1}{n_r} = a_1 \frac{\|h_1\|^2}{n_r} = 1$$

$$\underline{h}_i = \sqrt{a_i}\,\bar{h}_i$$

$$\log_2 \left| I + \frac{n_r}{N_0} \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix} \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \right|$$

$$\log_2 \left| I + n_r \begin{bmatrix} \frac{P_1 a_1}{N_0} & 0 & 0 \\ 0 & \frac{P_2 a_2}{N_0} & 0 \\ 0 & 0 & \frac{P_3 a_3}{N_0} \end{bmatrix} \right| = \sum_{K=1}^{3} \log\left(1 + n_r \frac{P_K a_K}{N_0}\right) \qquad (7.16)$$

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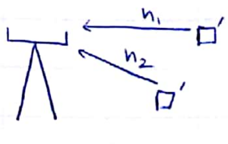

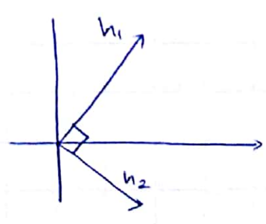

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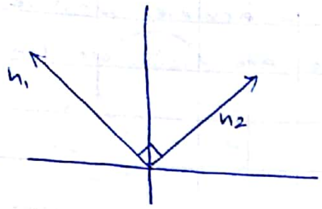
consider 2-dimensional case



we can use MF for user 1 and user 2 at the same time. This gives DoF gain.

In case they are not orthogonal use ZF structure. Here, we project user 1 to the null space of user 2, compensating a certain loss. ~~But~~ user 2 null space of user 1. again a loss.

** if the channels are overlapping we can't use, use SIC & or TDMA / FDMA

↑ highly unlikely with higher dimensions.

$\underline{y} = \underline{h}_1 \underline{x}_1 + \underline{h}_2 \underline{x}_2 + \underline{n}$

to receive $\underline{x}_1 \Rightarrow w_1 = (\underline{h}_2 \underline{h}_2^H + N_0 I)^{-1} \underline{h}_1$

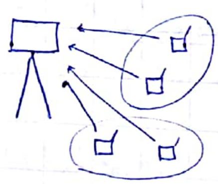
$\hat{x}_1 = w_1^H \; y = h_1^H h_1 x_1 + h_1^H h_2 x_2 + \underline{n}$

$\hat{x}_2 = w_2^H y$

what happens when there are 4 users? Still MMSE-SIC is optimal. Decide the decoding order. last user gets the interference free rate.

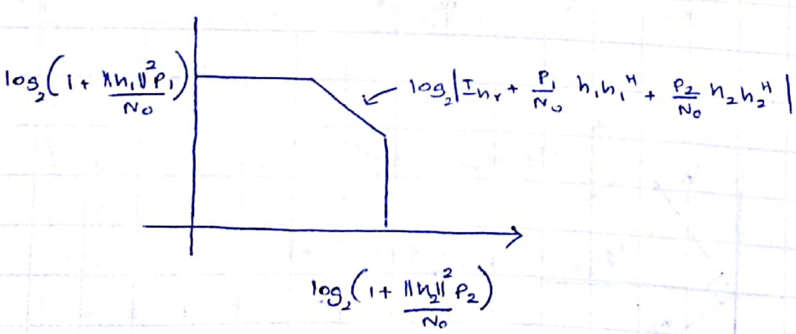
Practically -

Devide 4 users into 2 orthogonal groups. use timeslots for first 2 users and second 2 users



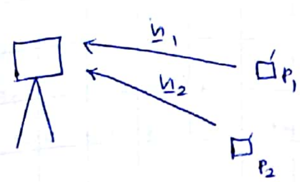
if we have DoF available in time or frequency ... ???

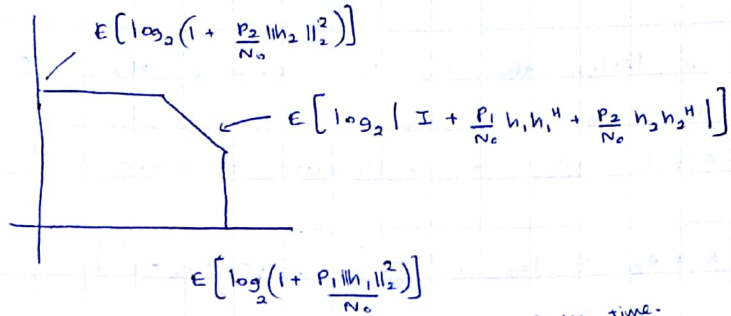
slide 18



2 user scenario

slide 19





infinite time.

To reach the corner points are practically impossible. Actual rates are always less. But can achieve closer points. Use MMSE-SIC for that

$$\underline{y}[m] = \underbrace{\underline{h}_1[m]\,\underline{x}_1[m]}_{\text{Decode first}} + \underline{h}_2[m]\,\underline{x}_2[m] + w[m]$$

Decode first $\rightarrow$ regenerate $\rightarrow$ remove

In practise LDPC 1000 - 10000 symbols will achieve very close rates.

slide 24

$$\max_k \frac{|h_k|}{\ln k} \rightarrow 1 \qquad k \rightarrow \infty$$

$$\log_2\left(1 + \max_k |h_k|\, SNR\right)$$

$$= \log_2\left(1 + \ln k \cdot SNR\right)$$

$$= \log_2 \log_e K + \log_2 SNR \qquad \text{(for high SNR)}$$

multiuser diversity is important when we have very little diversity in other domains.

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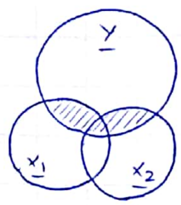
$$\underline{y}[m] = \sum_{k=1}^{K} \underline{H}_k \underline{x}_k[m] + \underline{n}[m]$$



Sum Rate $I(\underline{x}_1, \underline{x}_2 ; \underline{y}) = h(\underline{y}) - h(\underline{y}|\underline{x}_1, \underline{x}_2)$ $\qquad \underline{y} = \underline{H}_1\underline{x}_1 + \underline{H}_2\underline{x}_2 + \underline{n}$

$$= \log|\pi e(H_1 K_{x_1} H_1^H + H_2 K_{x_2} H_2^H + N_0 I)| - \log \pi e|N_0 I|$$

$E[\underline{y}\underline{y}^H] = \underline{H}_1 K_{x_1} \underline{H}_1^H + H_2 K_{x_2} H_2^H + N_0 I$ cross products disapper. due to independence.

$$I(\underline{x}_1 ; \underline{y}) + I(\underline{x}_2 ; \underline{y}|\underline{x}_1)$$

$$h(\underline{y}) - \cancel{h(\underline{y}|\underline{x}_1)} + \cancel{h(\underline{y}|\underline{x}_1)} - h(\underline{y}|\underline{x}_1, \underline{x}_2)$$

How to design the K_x to maximize the rate ?

A K_x gives a pentagon. So we get infinite number of pentagons for each channel realization. inefficient. to take the union of all pentagons.

$$R_1 + R_2 \le \log_2 \left| I + \sum_{k=1}^{2} \frac{1}{N_0} H_k K_{x_k} H_k^H \right|$$

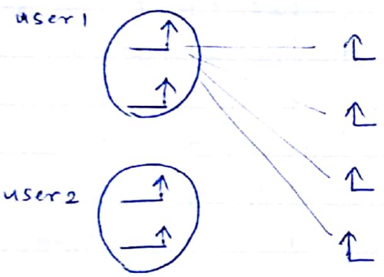
$$= \log_2 \left| I + H K_x H^H \right|$$

How to define H and K_x to get equality

$$H = [H_1 \;\; H_2]$$

$$K_x = \text{blockdiag}(K_{x_1}, K_{x_2}) \qquad K_x = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{K_{x_1}} & 0 & 0 \\ & & 0 & 0 \\ \hline 0 & 0 & \multicolumn{2}{c}{K_{x_2}} \\ 0 & 0 & & \end{array}\right]$$

2 users are seperated



$$\underline{y} = \underline{H}_1 \underline{x}_1 + \underline{H}_2 \underline{x}_2 + \underline{n}$$

$$= \underline{H}_1 \underbrace{\underline{U}_1 P_1^{1/2} \underline{d}_1}_{\underline{x}_1}$$

$$\tilde{x}_{11} = \sqrt{P_{11}}\, d_{11} \qquad \tilde{\underline{x}} = \begin{bmatrix} \tilde{x}_{11} \\ \tilde{x}_{12} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{x}_{11} \\ \tilde{x}_{12} \end{bmatrix} = \begin{bmatrix} \sqrt{P_{11}} & 0 \\ 0 & \sqrt{P_{12}} \end{bmatrix} \begin{bmatrix} d_{11} \\ d_{12} \end{bmatrix}$$

no. of streams $\le \min(n_t, n_r)$

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Now the discussion is for a given channel realization

$\underline{K}_x = U \underline{P} U^H$ ← eigen value decomposition

↑ eigen values corresponding to powers.

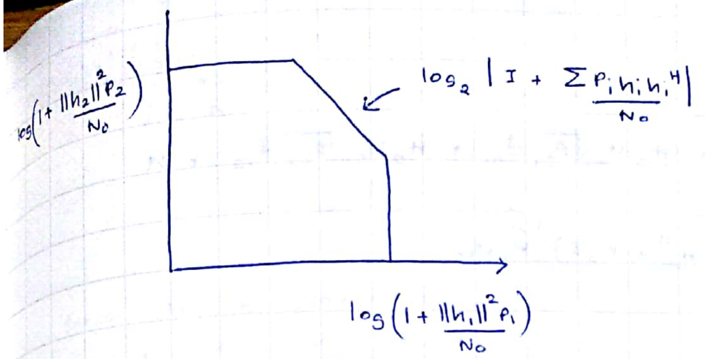
$$\underline{K}_x = [U_1 \,\vdots\, U_2] \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} U_1^H \\ \cdots \\ U_2^H \end{bmatrix}$$

$$\text{Tr}(\underline{K}_x) = \sum_{i=1}^{2} P_i \qquad \text{Tr}(U \underline{P} U^H) = \text{Tr}(P U^H U) = \text{Tr}(P) = \sum_{i=1}^{2} P_i$$

Now the rate region is not anymore a pentagon.

Earlier → { In the single antenna case, we can have only one stream per user

Rate region is given.



← This was the single antenna case.

Now the rate region is a union of pentagon for all possible K_x

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$$\underline{y} = \sum_{k=1}^{K} \underline{H}_k \underline{x}_k + \underline{n}$$

↑ Transmitted vector

Ex:- two user case

$$\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \qquad \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}$$

user 1 user 2

Normalized beamformer matrix

$$\underline{y} = \sum_{k=1}^{K} H_k U P_k^{1/2} d_k$$

Power loading matrix - diagonal. $\begin{bmatrix} \sqrt{P_{1k}} & & \\ & \ddots & \\ & & \sqrt{P_{n_k}} \end{bmatrix}$

total streams. $n_k = \min\left(n_r, \sum_k n_{tk}\right)$

Beamformer.

$$x_k = U_k P_k^{1/2} d_k = \begin{bmatrix} u_{k_1} & u_{k_2} & u_{k n_k} \end{bmatrix} \begin{bmatrix} \sqrt{P_{k_1}} & & \\ & \ddots & \\ & & \sqrt{P_{k n_k}} \end{bmatrix} \begin{bmatrix} d_{k_1} \\ \vdots \\ d_{k n_k} \end{bmatrix}$$

← Transmitter Structure.

$$= \sum_{i=1}^{n_k} u_{ki} \sqrt{P_{ki}}\, d_{ki}$$

↑ power

↑ Symbol drawn from complex gaussian $\sim \mathcal{CN}(0,1)$

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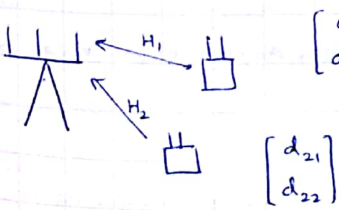
optimal receiver is MMSC-SIC



$$\begin{bmatrix} d_{11} \\ d_{12} \end{bmatrix} \longrightarrow \Rightarrow \underbrace{\begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sqrt{P_{11}} & 0 \\ 0 & \sqrt{P_{12}} \end{bmatrix} \begin{bmatrix} d_{11} \\ d_{12} \end{bmatrix}}_{}$$

$\underline{x}_1$

$$\begin{bmatrix} d_{21} \\ d_{22} \end{bmatrix}$$

Similarly of $\underline{x}_2$

Decoding order

stream 1 of user 1, stream 2 of user 1, stream 1 of user 2, Stream 2 of user 2

$$W_{11} = \underbrace{\left(P_{12} H_1 U_{12} U_{12}^H H_1^H + P_{21} H_2 U_{21} U_{21}^H H_2^H + P_{22} H_2 U_{22} U_{22}^H H_2^H + N_0 I\right)}_{R_{11}}^{-1} H_1 U_{11}$$

↑ Received beamformer

$$\tilde{d}_{11} = W_{11}^H \underline{y}$$

if the rate that we assign at the transmitter can be supported by the $\log(1+\text{SINR})$ ~~SINR~~ at the output of the receiver we can decode this. Then remove it to form $W_{..}$

$$\underline{Y} = \sum_{k=1}^{K} H_K U_K P_K^{1/2} d_K + n_K$$

$$= H_1 U_{11} P_{11}^{1/2} d_{11} + H_1 U_{12} P_{12}^{1/2} d_{12} + H_2 U_{21} \sqrt{P_{21}} d_{21} + H_2 U_{22} \sqrt{P_{22}} d_{22} + n$$

$$W_{12} = (P_{21} H_2 U_{21} U_{21}^H H_2^H + P_{22} H_2 U_{22} U_{22}^H H_2^H + N_0 I)^{-1} H_1 U_{12}$$

$$W_{21} = (P_{22} H_2 U_{22} U_{22}^H + N_0 I)^{-1} H_2 U_{21}$$

$W_{22} = \frac{H_2 U_{22}}{N_0}$ matched filter

Rate perspective → we can change order at the way we want. But sum rate will be same. if we want to ensure fairness to user we can do following order U1S1 → U2S1 → U1S2 → U2S2

if user 2 is more important U1S1 → U1S2 → U2S1 → U2S2

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$$R_1 \leq I(\underline{x}_1 ; \underline{y} | \underline{x}_2)$$
$$= h(\underline{y}|\underline{x}_2) - h(\underline{y}|\underline{x}_1, \underline{x}_2)$$
$$= \log \pi e |H_1 K_{x_1} H_1^H + N_0 I| - \log \pi e |N_0 I|$$
$$= \log |I + \frac{1}{N_0} H_1 K_{x_1} H_1^H|$$

similar for R_2

$$R_1 + R_2 \leq I(\underline{x}_1, \underline{x}_2 ; \underline{y})$$
$$= h(\underline{y}) - h(\underline{y}|\underline{x}_1, \underline{x}_2)$$
$$= \log \pi e |H_1 K_{x_1} H_1^H + H_2 K_{x_2} H_2^H + N_0 I| - \log \pi e |N_0 I|$$
$$= \log |I + \frac{1}{N_0} H_1 K_{x_1} H_1^H + \frac{1}{N_0} H_2 K_{x_2} H_2^H|$$

slide 30

For a single user case.

$H = U \Lambda V^H$ SVD

$K_x = V P V^H$, Right singular vector as precoders. Find P as per waterf[illing]

↑ Precoder.

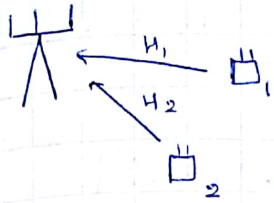
<u>To get full rate for user 1</u>

By MMSE-SIC precode user 2 first, remove it to achieve max rate for user 1

$$\underline{Y} = \underline{H}_1 \underline{x}_1 + \underline{H}_2 \underline{x}_2 + \underline{n}$$

$K_{x_1} = V_1 P_1 V_1^H$ (earlier we used U as the beamformer. So we switch to that notation

$K_{x_1} = U_1 P_1 U^H$



from user 2 perspective

$\underline{y} = H_1 x_1 + H_2 x_2 + \underline{n}$

$y_2 = \cancel{H_2 x_1}\ H_2 x_2 + z_2 \qquad z_2 = H_1 x_1 + n \quad . \quad K_{z_2} = H_1 K_{x_1} H_1^H + N_0 I$

$$R_2 = h(y_2) - h(y_2 | x_2)$$
$$= \log \pi e |H_2 K_{x_2} H_2^H + K_{z_2}| - \log \pi e |K_{z_2}|$$
$$= \log |I + H_2 K_{x_2} H_2^H K_{z_2}^{-1}|$$

$K_{z_2}^{-1/2} \cdot K_{z_2}^{-1/2}$

$|I + AB| = |I + BA|$

$$= \log |I + \underbrace{K_{z_2}^{-1/2} H_2}_{\tilde{H}_2} K_{x_2} \underbrace{H_2^H K_{z_2}^{-1/2}}_{\tilde{H}_2^H}|$$

equivalent point to point MIMO SVD and waterfilling. $\tilde{H}$ includes the impact of interference.

=

$K_{z_k} = Q \Lambda Q^H$

$K_{z_k}^{-1} = Q \Lambda^{-1} Q^H$

$$K_{z_k}^{-1} = K_{z_k}^{-1/2} \cdot K_{z_k}^{-1/2} = Q \Lambda^{-1} Q^H Q \Lambda^{-1} Q^H = Q \Lambda^{-1/2} \Lambda^{-1/2} Q^H$$

taking SVD of $\tilde{H}_k = A\ V \Lambda U^H$

$$P_{k_n} = \left(\frac{1}{\mu} - \frac{1}{\lambda_{kn}}\right)^+$$

$$K_{x_2} = U_2 P_2 U_2^H$$

This transmit covariance matrix is optimized for user 2 assuming full interference from user 1.

This is an extreme strategy. For user 1. Similar to user 2 we do the reverse. But this is not sum rate optimal. what is the sum rate optimal strategy ?

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slide 31

Say we have a given set of weights μ_1 and μ_2. for a 2 user case.

$\mu_1 R_1 + \mu_2 R_2$ $\qquad \mu_1 > \mu_2 \rightarrow$ user 1 decoded last.

$$= \mu_1 \log\left|I + \frac{1}{N_0} H_1 K_{x_1} H_1^H\right| + \mu_2 \left| I + H_2 K_{x_2} H_2^H (H_1 K_{x_1} H_1^H + N_0 I)^{-1}\right|$$

$$R_2 = I(x_2 ; y)$$
$$= h(y) - h(y|x_2)$$
$$= \log \pi e \left| H_1 K_{x_1} H_1^H + H_2 K_{x_2} H_2^H + N_0 I\right| - \log \pi e \left| H_1 K_{x_1} H_1^H + N_0 I\right|$$
$$= \log \underbrace{\left|I + H_2 K_{x_2} H_2^H (H_1 K_{x_1} H_1^H + N_0 I)^{-1}\right|}_{\text{neither convex or concave}}$$

$$R_2 = R_{sum} - R_1$$

$$\mu_1 R_1 + \mu_2 R_2$$
$$\mu_1 R_1 + \mu_2 (R_{sum} - R_1)$$
$$\underbrace{\mu_2 R_{sum}}_{\text{concave}} + \underbrace{(\mu_1 - \mu_2)}_{+} \underbrace{R_1}_{\text{concave}}$$

if $\mu_2 > \mu_1$ $\qquad \mu_1 R_{sum} + (\mu_2 - \mu_1) R_2$

slide 33 $\quad$ 3 user case. $\quad \mu_1 > \mu_2 > \mu_3$

decoding order $3 \rightarrow 2 \rightarrow 1$

$$\mu_1 R_1 + \mu_2 R_2 + \mu_3 R_3$$
$$\mu_1 R_1 + \mu_2 (R_{12} - R_1) + \mu_3 (R_{123} - R_{12})$$
$$\mu_3 R_{123} + (\mu_2 - \mu_3) R_{12} + (\mu_1 - \mu_2) R_1$$

when $\mu_1 = \mu_2 = \cdots = \mu_n$ then all $\mu_i - \mu_{i-1}$ goes to zero. only μR_{123} remains

special case.

24/04/2017

$$\log \left| Z_k + H_k K_{x_k} H_k^H \right|$$
$$= \log \left| Z_k^{1/2} (I_{n_r} + Z_k^{-1/2} H_k K_{x_k} H_k^H Z_k^{-1/2}) Z_k^{1/2} \right|$$
$$= \log \left| (I_{n_r} + Z_k^{-1/2} H_k K_{x_k} H_k^H Z_k^{-1/2}) Z_k \right| \qquad \text{using } |AB| = |BA|$$

$$= \log \left| I_{n_r} + \underbrace{Z_k^{-1/2} H_k}_{\tilde{H}_k} K_{x_k} \underbrace{H_k^H Z_k^{-1/2}}_{\tilde{H}_k^H} \right| + \log |Z_k|$$

$$= \log \left| I_{n_r} + \tilde{H}_k K_{x_k} \tilde{H}_k^H \right| + \log |Z_k| \rightarrow \text{equivalent optimization problem with white Gaussian noise.}$$

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what is the solution → How we find K_x

$K_x = V P V^H$ and V can be found from

$\tilde{H}_K$ ~~W~~ ↦ $\tilde{U}_K \Lambda^2 \tilde{V}_K^H$

$\Lambda^2 = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_{n_{min}} \end{bmatrix}$

$$p_i = \left[\frac{1}{\mu} - \frac{1}{\lambda_i}\right]^+$$

collect corresponding P values $P = \begin{bmatrix} p_1 & & \\ & \ddots & \\ & & p_{n\,min} \end{bmatrix}$ and then you have K_{x_K}.

and then iterate. (iterate between users)

So when we fixed the interference the KKT conditions are exactly the same as single user MIMO

(proj work (31·30)

$$\max_{K_{x_{K,c}}} \sum_{i=1}^{K} \log \left| N_0 I + \sum_{K=1}^{K} H_{K,c} K_{x_{K,c}} H_{K,c}^H \right|$$

↑ subcarrier index

$$\sum_{K=1}^{N_c} Tr\left(K_{x_{K,c}}\right) \leq P_T \quad \forall K$$

$K_{x_{K,c}} \succeq 0 \quad \forall K, c$ postitiveness constraint

initialize everything for zero. foll all sub channels

for loop over users

for loop over sub channels

Z_{Kc} for each sub channel (store in a 3D matrix)

whitening for sub channel. the terms inside the log det

get $\tilde{H}_{K,c}$ after whitening.

select the right singular vectors.

collect the λ values of each sub channel for waterfilling.

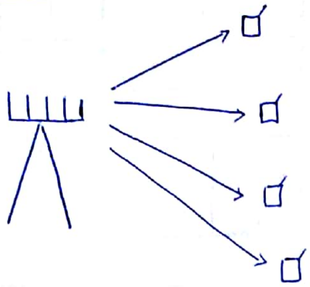
ex:- 2 antenna user and 64 sub carriers then 128 eigenvalues for that user.

run waterfilling.

When SNR is low $K_{x_{K,c}}$ could be zero for weak sub channels.

S1 Lecture 8

slide 1



SINR at each user $\geq$ SINR target.

slide 3

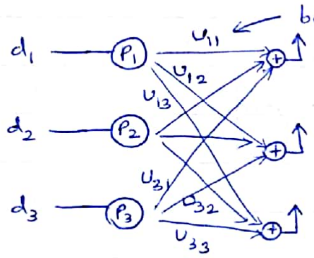
$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ with a sum power constraint
$$E\left[Tr(\underline{x}\underline{x}^H)\right] = \sum_{i=1}^{K} P_K \leq P$$

$$\underline{y}_K = \underline{h}_K^H \underline{x}[m] + w_K(m)$$

$$\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \cancel{n_1} \\ \cancel{n_2} \\ \cancel{n_3} \end{bmatrix} w_K(m)$$

$$\underline{x} = \sum \underline{u}_K \sqrt{P_K}\, d_K[m]$$

S1



beamformers

$$x_1 = \underbrace{u_{11} P_1 d_1}_{\text{desired for user 1}} + \underbrace{u_{21} P_2 d_2 + u_{31} P_3 d_3}_{\text{undesired for user 1}}$$

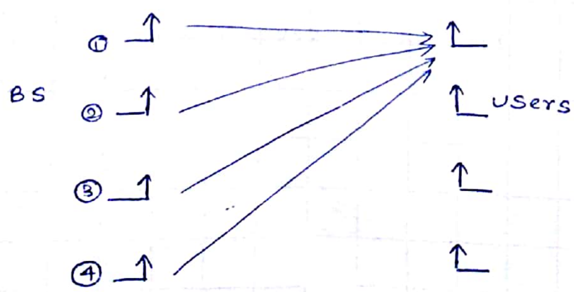
for this u_{11} should be aligned (Maximal ratio Tx)
u_{21}, u_{31} should be orthogonal.

How to achieve this in practise

1. pilots → assume reciprocity (TDD)
2. Feedback. Users estimate antenna specific pilots. and sends back to antenna. take a lot of signalling BW from uplink.

2. slide 4

Four antenna base station and 4 users.



1. $[h_1 \;\; h_2 \;\; h_3 \;\; h_4] = H$

point to
MIMO cha
SVD
DoF
= min

2. $h_i^H h_K = 0 \quad \forall\, i, k \quad i \neq k$

no interf
matched
same as
previ
orthogonal
channel

$$y_k = h_k^H \underbrace{\sum_{i=1}^{k} u_i \sqrt{P_i}\, d_i}_{x_i} + n$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_k \end{bmatrix}^H \begin{bmatrix} u_1 & u_2 & u_k \end{bmatrix}$$

data symbols

$$y = \underline{H}^H \underline{U}\, \underline{P}\, \underline{d} + \underline{n}$$ ← we have all users (all 4)

diagonal matrix of power

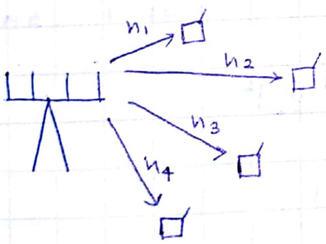
all beamformers

if U is selected to be $\cancel{HH^H}$

$$\underbrace{H^H H (H^H H)^{-1}}_{\text{becomes } I} \begin{bmatrix} \frac{P_1}{\|z_1\|^2} & & \\ & \frac{P_2}{\|z_2\|^2} & \\ & & \frac{P_K}{\|z_K\|^2} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_K \end{bmatrix}$$

27/04/2018

Sec 4



for orthogonal scheme $h_k^H h_i = 0$ for $k \neq i$

received signal at user k

$$y_k = \underline{h}_k^H \underline{u}_k \sqrt{P_k}\, d_k + \sum_{i \neq k} \underline{h}_i^H \underline{u}_i \sqrt{P_i}\, d_i + n_k$$

$$y_k = \underline{h}_k^H \frac{\underline{h}_k}{\|h_k\|} \sqrt{P_k}\, d_k + \underbrace{\sum_{i \neq k} \underline{h}_i^H \frac{\underline{h}_i}{\|h_i\|} d_i}_{0} + n_k$$

This disapperar if orthogonal.

then it appears as 4 parallel channels.

General Model.

$$y = \underline{H}^H \underline{U}\, \underline{P}^{1/2} \underline{d} + \underline{n}$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_k \end{bmatrix} \quad H = [h_1 \dots h_k] \quad U = [u_1 \; u_2 \; u_k] \quad P = \begin{bmatrix} P_1 & & \\ & P_2 & \\ & & \ddots \; P_k \end{bmatrix} \quad n = \begin{bmatrix} n_1 \\ \vdots \\ n_k \end{bmatrix}$$

If we select the $H(H^H H)^{-1}$ (this is normalized)

first column of $H(H^H H)^{-1}$ is z_1.

$$\underline{y} = \underline{H}^H \underline{U}\, \underline{P}^{1/2} \underline{d} + \underline{n}$$

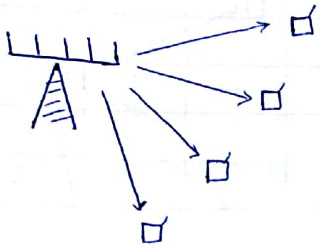
$$= \underline{H}^H \underline{H} (\underline{H}^H \underline{H})^{-1} \underline{P}^{1/2} \underline{d} + \underline{H}(\underline{H}^H \underline{H})^{-1} \underline{n}$$

$$= \underline{d} + \underline{H}(\underline{H}^H \underline{H})^{-1} \underline{n}$$

SNR

$$\gamma_K = \frac{P_K}{\|z_K\|^2 N_0}$$

slide 5



$d_k \sim CN(0,1)$

$$y_k = \underbrace{\underline{h}_k^H \underline{u}_k \sqrt{P_k}\, d_k}_{s_k} + \underbrace{\sum_{i\neq k} \underline{h}_k^H \underline{u}_i \sqrt{P_i}\, d_i + n_k}_{i_k}$$

how to find the beamformers $\underline{u}_i$ for each users and how do we allocate powers.

$$\gamma_k = \frac{E[|s_k s_k^*|^2]}{E[|i_k i_k^*|^2]}$$

$$= \frac{\underline{h}_k^H \underline{u}_k \sqrt{P_k}\, E[d_k d_k^*] \sqrt{P_k}\, \underline{u}_k^H \underline{h}_k}{\sum_{i\neq k} |\underline{h}_k^H \underline{u}_i|^2 P_i + N_0}$$

$$\gamma_k = \frac{|\underline{h}_k^H \underline{u}_k|^2 P_k}{\sum_{i\neq k} |\underline{h}_k^H \underline{u}_i|^2 P_i + N_0}$$

reorganizing.

$$\gamma_k N_0 + \gamma_k \left\{ \sum_{i\neq k} |\underline{h}_k^H \underline{u}_i|^2 P_i \right\} = P_k |\underline{h}_k^H \underline{u}_k|^2$$

$$\gamma_k N_0 + \gamma_k \sum_{i=1}^{K} |\underline{h}_k^H \underline{u}_i|^2 P_i = (1+\gamma_k) P_k |\underline{h}_k^H \underline{u}_k|^2$$

$$\frac{\gamma_k N_0}{(1+\gamma_k) P_k |\underline{h}_k^H \underline{u}_k|^2} + \frac{\gamma_k \sum_{i=1}^{K} |\underline{h}_k^H \underline{u}_i|^2 P_i}{(1+\gamma_k) P_k |\underline{h}_k^H \underline{u}_k|^2} = 1$$

$$a_k N_0 + a_k \sum_{i=1}^{K} |\underline{h}_k^H \underline{u}_i|^2 P_i = 1$$

$$1 - a_k \sum_{i=1}^{K} |\underline{h}_k^H \underline{u}_i|^2 P_i = a_k N_0 \quad \leftarrow \text{for kth user.}$$

let's define a scalar $G_{ki} = |h_k^H u_i|^2$

$$1 - a_k (G_{k1} P_1 + G_{k2} P_2 + \cdots G_{kK} P_K) = a_k N_0$$

$$1 - a_k \begin{bmatrix} G_{k1} & G_{k2} & \cdots & G_{kK} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_K \end{bmatrix} = a_k N_0 \quad \leftarrow \text{for the kth user}$$

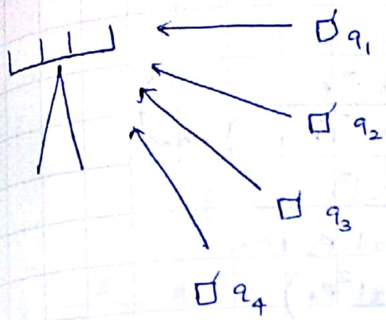
$$\begin{bmatrix} G_{1K1} & & G_{1K} \\ & & \\ G_{K1} & & G_{KK} \end{bmatrix}$$

for all users.

$\rightarrow (I_K - D_a G) P = N_0 a$

optimum Ps can be found by inverting.

let's look at the duel uplink



receive beamformer

$$\hat{d}_k = u_k^H y$$

received signal.

estimate of tx data at the output of beamformer

$$\hat{d}_k = \underbrace{\underline{u}_k^H \underline{h}_k \sqrt{q_k}\, d_k}_{S_k} + \sum_{i \neq k} \underline{u}_k^H \underline{h}_i \sqrt{q_i}\, d_i + \underline{u}_k^H \underline{n}_k$$

Here the channels are different

(in downlink) earlier the beamformers are different.

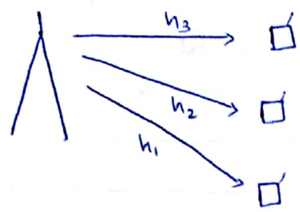
$$|u_k^H h_i|^2 = |h_k^H u_i|^2$$

$$a_{ki} = a_{ik}$$

we can end up with same form of matrix formulation. each row corresponds to 1 SINR. 1 user.

04/05/2018

$$y_k = h_k \underline{x} + n_k$$



$$R_1 \leq \log\left(1 + \frac{|h_1|^2 P_1}{|h_1|^2(P_2+P_3) + N_0}\right).$$

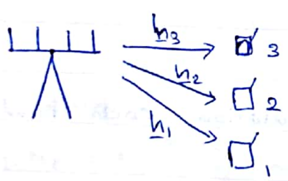
$$R_2 \leq \log\left(1 + \frac{|h_2|^2 P_2}{|h_2|^2 P_3 + N_0}\right)$$

$$R_3 \leq \log\left(1 + \frac{|h_3|^2 P_3}{N_0}\right)$$

superposition coding at Tx and. SIC at receiver. Needs ordering.

if we have multiple antennas at BS, now channels are vectors

$$y_k = \underline{h}_k^H \underline{U}_k \sqrt{P_k}\, d_k + h_k^H \sum_{i \neq k} \underline{U}_i \sqrt{P_i}\, d_i + n_k$$

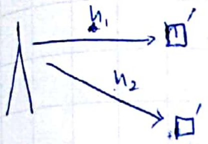


$$Y = X + S + W$$

if we can find $U = X - S$ at transmitter

$Y = X + W$. very inefficient.

capacity is $C = \frac{1}{2}\log\left(1 + \frac{P}{\sigma^2}\right)$



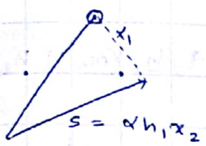
$$x = x_1 + x_2$$

$$s = \alpha h_1 x_2$$

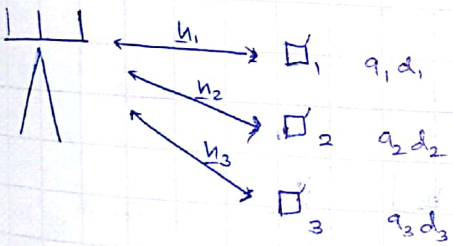
$$\alpha y_1 = \alpha h_1 x_1 + \alpha h_1 x_2 + n$$

$\vdots$

N



Dual uplink



$$\underline{y} = \sum_{i=1}^{K} \underline{h}_i x_i + \underline{w}$$

$$\underline{y} = \underline{h}_k x_k + \sum_{i \neq k} \underline{h}_i x_i + \underline{w}$$

$$\hat{d}_k = \underline{u}_k^H \underline{y}$$

Consider user 2. let's assume decoding order $1 \to 2 \to 3$

$$\hat{d}_2 = \underbrace{\underline{u}_2^H \underline{h}_2 x_2}_{\text{desired}} + \cancel{\underline{u}_2^H \underline{h}_1 x_1} + \underline{u}_2^H \underline{h}_3 x_3 + \underline{u}_2^H \underline{w}$$

$$\gamma_2 = \frac{|u_2^H h_2|^2 q_2}{|u_2^H h_3|^2 q_3 + N_0}$$

for general k

$$\gamma_k = \frac{|u_k^H h_k|^2 q_k}{\sum_{i>k} |u_k^H h_i|^2 q_i + N_0}$$

$$G_{ki} = |h_k^H u_i|$$

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}$$

for uplink we have to take a^T

$$a^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$\gamma_k^{mmse} = \frac{|u_k^H h_k|^2 q_k}{\sum_{i \neq k} |u_k^H h_i|^2 + N_o} \qquad \gamma_k^{mmse-SIC} = \frac{|u_k^H h_k|^2 q_k}{\sum_{i > k} |u_k h_i|^2 + N_o}$$

Sum Rate Formula

$$\sum_{k=1}^{2} \log \left(1 + \gamma_k^{UL-SIC}\right)$$

$$= \log\left(1 + \cancel{} q_1 \underline{h}_1^H (\underline{h}_2 \underline{h}_2^H q_2 + N_o I)^{-1} h_1\right) + \log\left(1 + q_2 \underline{h}_2^H (N_o I)^{-1} h_2\right)$$

$$= \log \left| I + \cancel{q_1} (h_2 h_2^H q_2 + N_o I)^{-1} \underline{h}_1 \underline{h}_1^H q_1 \right| + \log \left| I + \frac{q_2}{N_o} h_2 h_2^H \right|$$

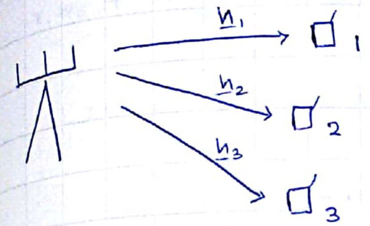
$$= \log \left| \left\{ I + (h_2 h_2^H q_2 + N_o I)^{-1} h_1 h_1^H q_1 \right\} \left\{ I + \frac{q_2}{N_o} h_2 h_2^H \right\} \right|$$

$$= \log \left| I + \frac{q_1 h_1 h_1^H}{N_o} + \frac{q_2 h_2 h_2^H}{N_o} + \right|$$

$$= \log \left| I + \frac{1}{N_o} H \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} H^H \right| \qquad H = [\underline{h}_1 \quad \underline{h}_2]$$

UL - DL Duality (Extra lec)

encoding order = 3, 2, 1



$$y_2 = \underline{h}_2^H \underline{u}_2 x_2 + \underbrace{\overbrace{\underline{h}_2^H \underline{u}_3 x_3}^{s}}_{\text{Side info known at Tx}} + \underbrace{\overbrace{\underline{h}_2^H \underline{u}_1 x_1 + n_2}^{z}}_{\text{Interference.}}$$

$$\gamma_2^{DL-DPC} = \frac{E\,|\underline{h}_2^H \underline{u}_2 x_2|^2}{E\,|zz^*|^2} = \frac{|\underline{h}_2^H \underline{u}_2|^2 P_2}{|\underline{h}_2^H \underline{u}_1|^2 P_1 + N_0}$$

$$y_3 = \underline{h}_3^H \underline{u}_3 x_3 + \underbrace{\underline{h}_3^H \underline{u}_1 x_1 + \underline{h}_3^H \underline{u}_2 x_2}_{z} + n_3$$

$$y_1 = \underline{h}_1^H \underline{u}_1 x_1 + \underbrace{\underline{h}_1^H \underline{u}_2 x_2 + \underline{h}_1^H \underline{u}_3 x_3}_{s} + \underbrace{n_1}_{z}$$

$$\gamma_1^{DL-DPC} = \frac{E\,|\underline{h}_1^H \underline{u}_1 x_1|^2}{E[nn^H]} = \frac{|h_1^H \underline{u}_1|^2 P_1}{N_0}$$

$$\gamma_3^{DL-DPC} = \frac{|\underline{h}_3^H \underline{u}_3|^2 P_3}{|\underline{h}_3^H u_2|^2 P_2 + |h_3^H u_1|^2 P_1 + N_0}$$

our problem is to maximize

$$\max_{u_k\ P_k} \sum_{k=1,2,3} \log(1+\gamma_k)$$

Subject to $\sum P_k \leq P$

When we try to solve, it cannot be solved. neither convex nor concave

what is the approach ?

Before going to dual uplink compute G matrix

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \quad 3\times3 \text{ matrix}$$

SINR for user 2 contains 2 G's G_{22} and G_{21}

user 3 G_{33}, G_{32}, G_{31}

user 1 G_{11}

Sum of DL powers = Sum of UL powers

Let's go to dual uplink

reverse the roles of Tx and Rx. Channels are same

Think of user 2

$$\hat{d}_2 = \underline{u}_2^H \underline{h}_2 x_2 + \cancel{\underline{u}_2^H \underline{h}_1 x_1} + u_2^H \underline{h}_3 x_3$$

$\underline{u}_2^H \underline{h}_2 = \underline{h}_2^H \underline{u}_2$ because they are scalars. ~~Th~~ This is also $\sqrt{G_{22}}$

In order to get the same G matrix, in the duel uplink we have to consider reverse decoding order. In this case $1 \to 2 \to 3$

* In duel uplink for user 2 we are interested in column 2
In that colunn G_{12} does not appear. Matrix remains same
for user 3 only G_{33} applies.

In the dual uplink when you write the SINR, you have to take transpose.

$$\gamma_2^{UL-SIC} = \frac{|\underline{u}_2^H h_2|^2 q_2}{|\underline{u}_2^H h_3|^2 q_3 + N_0} = \frac{G_{22} q_2}{G_{32} q_3 + N_0} \quad \text{for fixed beamformer}$$

$$\gamma_1^{UL-SIC} = \frac{G_{11} q_1}{G_{21} q_2 + G_{31} q_3 + N_0}$$

$$\gamma_3^{UL-SIC} = \frac{G_{33} q_3}{N_0}$$

Convert the SINR formulas $\to$ matrix

$$\gamma_1 N_o + \gamma_1(a_{21} q_2 + a_{31} q_3) = a_{11} q_1$$

$$\gamma_1 N_o + \gamma_1 \sum_{k=1,2,3} a_{k1} q_k = (1+\gamma) a_{11} q_1$$

Define $\quad a_k = \dfrac{\gamma_k}{(1+\gamma_k) a_{kk}}$

$$a_1 N_o + a_1 \begin{bmatrix} a_{11} & a_{21} & a_{31} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = q_1$$

similarly

$$\gamma_2 N_o + \gamma_2 a_{32} q_3 = a_{22} q_2$$

$$\gamma_2 N_o + \gamma_2 \sum_{k=2,3} a_{k2} q_k = (1+\gamma_2) a_{22} q_2$$

$$a_2 N_o + a_2 \begin{bmatrix} 0 & a_{22} & a_{32} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = q_2$$

$$a_3 N_o + \cancel{q}\, a_3 \begin{bmatrix} 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = q_3$$

$$\cancel{a_3 N_o}\; a_1 N_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} - a_1 \begin{bmatrix} a_{11} & a_{21} & a_{31} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$a_2 N_o = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} - a_2 \begin{bmatrix} 0 & a_{22} & a_{32} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$a_3 N_o = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} - a_3 \begin{bmatrix} 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Combining

$$N_o \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} - \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$N_o \underline{a} = \underline{I}_3 \underline{q} - \underline{D}_a \underline{A}^T \underline{q}$$

$$N_o \underline{a} = (\underline{I}_3 - \underline{D}_a \underline{A}^T) \underline{q}$$

In the dual uplink a's are b's. So $N_o \underline{b} = (\underline{I}_3 - \underline{D}_b \underline{A}^T) \underline{q}$

we can find q for a fixed u. u is the MMSC-SIC receiver here

$$\gamma_1^{UL-SIC} = \frac{|\underline{h}_1^H \underline{u}_1|^2 q_1}{|\underline{h}_2^H \underline{u}_1|^2 q_2 + |\underline{h}_3^H \underline{u}_1|^2 q_3 + N_0}$$

optimum MMSE receiver.

$$u_1 = \left(q_2 h_2 h_2^H + q_3 h_3 h_3^H + N_0 I\right)^{-1} \underline{h}_1$$

if we use this u_1

$$\gamma_1 = \underline{h}_1^H \left(q_2 h_2 h_2^H + q_3 h_3 h_3^H + N_0 I\right)^{-1} \underline{h}_1 \quad R_1 \, q_1$$

The SINR can be simplified it does not have u.

similarly

$$\gamma_2 = \underline{h}_2^H \left(q_3 h_3 h_3^H + N_0 I\right)^{-1} \underline{h}_2 q_2$$

$$\gamma_3 = \frac{\underline{h}_3^H \underline{h}_3 q_3}{N_0} = \frac{\|\underline{h}_3\|^2 q_3}{N_0} \leftarrow \text{matched filter.}$$

plug theses SINRs into sum rate. (Now the SINR depends only on power. not on beamformers)

we want to maximize the sum rate

$$\max \sum \log_2(1+\gamma_k)$$

$$\sum_{k=1}^{3} \log_2\left(1+\gamma_k^{UL-SIC}\right)$$

$$= \log_2\left(1+q_1 \underline{h}_1^H R_1^{-1} \underline{h}_1\right) + \log_2\left(1+q_2 \underline{h}_2^H R_2^{-1} \underline{h}_2\right) + \log_2\left(1+q_3 \underline{h}_3^H R_3^{-1} \underline{h}_3\right)$$

Apply $(1+RC) = |I+CR|$ first

$$= \log_2\left(1+q_1 \underline{h}_1^H R_1^{-1} \underline{h}_1\right) + \log_2\left(1+\frac{q_2 \underline{h}_2^H \underline{h}_2}{q_3 \underline{h}_3 \underline{h}_3^H + N_0 I}\right) \cdot \left(1+\frac{q_3 \underline{h}_3^H \underline{h}_3}{N_0 I}\right)$$

$$= \log_2\left(1+q_1 \underline{h}_1^H R_1^{-1} \underline{h}_1\right) + \log_2\left(\frac{q_2 \underline{h}_2^H \underline{h}_2 + q_3 \underline{h}_3^H \underline{h}_3 + N_0 \underline{I}}{N_0 \underline{I}}\right)$$

$$= \log_2\left(1+\frac{q_1 \underline{h}_1^H \underline{h}_1}{q_2 \underline{h}_2 \underline{h}_2^H + q_3 \underline{h}_3 \underline{h}_3^H + N_0 \underline{I}}\right) \cdot \left(\frac{q_2 \underline{h}_2 \underline{h}_2^H + q_3 \underline{h}_3 \underline{h}_3^H + N_0 \underline{I}}{N_0 \underline{I}}\right)$$

$$= \log_2\left(\frac{q_1 \underline{h}_1 \underline{h}_1^H + q_2 h_2 h_2^H + q_3 h_3 h_3^H + N_0 I}{N_0 I}\right)$$

$$= \log \left| I + \frac{1}{N_0} H Q H^H \right|$$

This is a concave objective. This is an optimization problem we can solve. Assuming sum power constraint. Because of MMSC-SIC the sum of log reduces to formula which does not depend on the beamformers. because for the beamformers we have a closed form solution. So closed form SINR formula → closed form sum of log formula → use $I + RC = I + CR$ → end up with concave function, optimize it subject to power constraint. → get optimal powers → then optimal beamformers.

And then we have to go back to downlink. we have everything

G matrix, SINR

1. compute uplink SINRs for the given decoding order.

3.2. " for those given SINRs, ~~dow~~ compute downlink powers.

3. use $(\underline{I}_K - \underline{D}_a \underline{G})\underline{p} = N_0 \underline{a}$ (use G instead of G^T now)

2. In order to compute power we assume the same SINR for downlink DPC

4. Final step is to calculate the optimal beamformer $m_k = \sqrt{p_k}\,\underline{u}_k$ for the given original encoding order

All steps.

1. Compute G matrix
2. Reverse ~~de~~ the encoding order to get UL decoding order
3. Solve the duel uplink power allocation
4. Compute MMSC receivers for given q
5. plus those powers to get optimal U
6. ~~for~~ compute SINR for given q and U
7. Fixed $SINR^{UL-SIC} = SINR^{DL-DPC}$
8. Compute DL power vector
9. Compute beamformers.
10. Compute SINRs for DL