PERFORMANCE ANALYSIS OF OFDM BASED COOPERATIVE COGNITIVE RADIO NETWORK IN THE PRESENCE OF NARROWBAND INTERFERENCE

A THESIS

Submitted by

RAJKUMAR S

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

FACULTY OF INFORMATION AND COMMUNICATION ENGINEERING

ANNA UNIVERSITY

CHENNAI 600 025

OCTOBER 2015

ANNA UNIVERSITY, CHENNAI - 600 025
CENTRE FOR RESEARCH

CERTIFICATE

1. This is to certify that no corrections/suggestions were pointed out by the Indian/Foreign Examiner(s) in the Thesis titled " PERFORMANCE ANALYSIS OF OFDM BASED COOPERATIVE COGNITIVE RADIO NETWORK IN THE PRESENCE OF NARROWBAND INTERFERENCE " submitted by Mr./Ms. Rajkumar.S

OR

2. This is to certify that all corrections and suggestions pointed out by the Indian /Foreign Examiner(s) are incorporated in the Thesis titled " PERFORMANCE ANALYSIS OF OFDM BASED COOPERATIVE COGNITIVE RADIO NETWORK IN THE PRESENCE OF NARROWBAND INTERFERENCE " submitted by Mr./Ms. Rajkumar.S

Signature of the Joint Supervisor
(if applicable)

S.J.Thiruvengadam
Signature of the Supervisor

Dr. S.J. THIRUVENGADAM
Professor, ECE Department
Thiagarajar College of Engineering
Madurai-625 015

Place : Madurai

Date : 29.10.2015

Proceedings of the Ph.D. Viva-Voce Examination of Mr./Ms. Rajkumar.S held at 11:00 AM on 29.10.2015 in Seminar Hall Department of ECE Thiagarajar College of Engineering Madurai

The Ph.D. Viva-Voce Examination of Mr./Ms. Rajkumar.S (Reg. No. 2009740118) on his/her Ph.D. Thesis Entitled " PERFORMANCE ANALYSIS OF OFDM BASED COOPERATIVE COGNITIVE RADIO NETWORK IN THE PRESENCE OF NARROWBAND INTERFERENCE " was conducted on **29.10.2015** at 11:00 AM in the Seminar Hall Department of ECE Thiagarajar College of Engineering Madurai.

The following Members of the Oral Examination Board were present:

1.	Dr. Rajib Kar,Assistant Professor,Department of Electronics & Communication Engineering,National Institute of Technology,Durgapur	Indian Examiner
2.	Dr. D.Sriram Kumar,Associate Professor & Head,Department of Electronics & Communication Engineering,National Institute of Technology,Tiruchirappalli	Subject Expert
3.	Dr. Thiruvengadam.S J,Professor,Thiagarajar College of Engineering,Madurai	Supervisor & Convenor

The research scholar, Mr./Ms. Rajkumar.S presented the salient features of his/her Ph.D. work. This was followed by questions from the board members. The questions raised by the Foreign and Indian Examiners were also put to the scholar. The scholar answered the questions to the full satisfaction of the board members.

The corrections suggested by the Indian/Foreign examiner have been carried out and incorporated in the Thesis before the Oral examination.

Based on the scholars research work, his/her presentation and also the clarifications and answers by the scholar to the questions, the board recommends that Mr./Ms.Rajkumar.S be awarded Ph.D. degree in the **Faculty of Information and Communication Engineering**.

1.	[Dr. RAJIB KAR]	Indian Examiner
2.	[Dr. D. SRIRAMKUMAR]	Subject Expert
3.	S.J. Thiruvengadam [Dr. S.J. THIRUVENGADAM]	Supervisor & Convenor
4.	Dr. S.J. THIRUVENGADAM Professor, ECE Department Thiagarajar College of Engineering Madurai-625 015	Joint Supervisor (If any)

ANNA UNIVERSITY
CHENNAI 600 025

CERTIFICATE

The research work embodied in the present Thesis entitled "**PERFORMANCE ANALYSIS OF OFDM BASED COOPERATIVE COGNITIVE RADIO NETWORK IN THE PRESENCE OF NARROWBAND INTERFERENCE**" has been carried out in the Department of Electronics and Communication Engineering, Thiagarajar College of Engineering, Madurai. The work reported herein is original and does not form part of any other thesis or dissertation on the basis of which a degree or award was conferred on an earlier occasion or to any other scholar.

I understand the University's policy on plagiarism and declare that the thesis and publications are my own work, except where specifically acknowledged and has not been copied from other sources or been previously submitted for award or assessment.

RAJKUMAR S
RESEARCH SCHOLAR

Dr. S. J. THIRUVENGADAM
SUPERVISOR
Professor
Department of ECE
Thiagarajar College of Engineering
Madurai - 625 015, Tamil Nadu.

ABSTRACT

In recent years, the demand for high data rate wireless systems is growing more and more for the transmission of voice, video and web contents. But, it requires large bandwidth and high transmit power. The use of the large bandwidth introduces Inter Symbol Interference (ISI) in high data rate wireless systems and the wireless channel becomes frequency selective in nature. Orthogonal Frequency Division Multiplexing (OFDM) system uses the concept of cyclic or zero prefix to remove ISI and converts the frequency selective fading channel into set of parallel orthogonal flat fading channels. Due to its simplicity, OFDM is adopted in high data rate wireless communication systems and standards such as IEEE 802.11a, g, ac, ad, Worldwide Interoperability for Microwave Access (WiMAX) and Long Term Evolution (LTE). However, in practice, one or more subcarriers of OFDM symbol are highly susceptible to Narrow Band Interference (NBI) from wireless devices such as Bluetooth, Zigbee and cordless phones. Further, the Carrier Frequency Offset (CFO) of NBI spreads the NBI power over all the subcarriers of the OFDM symbol. CFO reduces the system capacity and increases the bit error probability. Mathematically, NBI is modeled in time domain using the parameters such as interference power, original symbol period, random phase, sampling frequency and subcarrier frequency. It can also be modeled as a sparse vector in frequency domain, which contains only few non-zero elements and in this work, it is assumed that quasi-static over one OFDM symbol. NBI is a common problem in many of the wireless communication systems especially in OFDM based dynamic spectrum management systems.

Cognitive Radio (CR) is the intersection of personal wireless technology and computational intelligence. It provides solution to the spectrum congestion problem with the opportunistic usage of frequency bands that are not

occupied by the primary users. As OFDM scheme has the better sensing, spectrum shaping, scaling and interoperable capabilities, it has been best adopted in CR network. The major goal of this thesis is to minimize the effect of NBI using the concept of cooperative communication in an OFDM based CR network.

Cooperative communication implements communication process in a distributed fashion using relays. It virtually forms an antenna array to combat fading effect and hence to improve the received signal power. Fixed relaying, selection relaying and incremental relaying are the basic low complexity cooperative strategies. In fixed relaying, the relays can either amplify the received signal subject to the power constraint and forward it to next node or decode the received signal and transmit the re-encoded signal to next node. In selection relaying strategy, transmit nodes can choose either cooperative or non-cooperative transmission based on the channel condition between transmit and receive nodes. Incremental relaying strategy uses relaying only when it is necessary based on the limited feedback from receive node. In the half duplex mode of signal transmission through relays, signal is received at relay in the first time slot and transmitted to the destination in the second time slot. In full duplex mode, signal is transmitted and received simultaneously through relays.

The first contribution of this thesis is the development of a communication process in OFDM based CR network in a cooperative manner using relay to suppress the effect of NBI. From information theoretic concept, the performance of the proposed OFDM based cooperative CR network is analyzed with respect to achievable rate and outage probability in the presence of NBI.

The second major contribution of this thesis is the performance analysis of the proposed communication process incorporating relaying strategies such as fixed, selection and incremental strategies and transmission modes such as half duplex and full duplex in the presence of NBI. In half duplex Amplify and Forward (AF) relaying strategy, amplification factor at the relay is chosen such that the relay node transmit power is the same as the source node transmit power. In half duplex Decode and Forward (DF) relaying strategy, the receive signal is decoded at the relay and re-transmitted to the destination node. In half duplex mode, output Signal to Interference plus Noise Ratio (SINR) at the destination node is improved by exploiting the spatial diversity through Maximal Ratio Combining (MRC) over the two time slots. In full duplex mode transmission, the system performance is improved at high channel degrees of freedom despite the presence of loop interference in the relay. The Cumulative Distribution Functions (CDF) and Probability Density Functions (PDF) of the SINR between source to relay, relay to destination and source to destination are derived using the magnitudes of desired channel following exponential PDF and interference channel following Gamma PDF. Analytical expressions for the end-to-end outage and BER performance of the proposed communication process in an OFDM based CR network are derived in the presence of NBI. Numerical results show that the performance of AF relaying strategy is better than DF relaying. The reason is that the cost of decoding at relay node depends on the SINR between source and relay nodes in DF relaying strategy.

The third major contribution of the thesis is the introduction of multiple full duplex AF relay nodes in the proposed communication process of OFDM based CR network to suppress the effect of NBI. Further, the relay is selected based on maximum SNR criterion at each subcarrier. The outage performance of the OFDM based CR network is analyzed in the presence of NBI using multiple relays over Nakagami-m fading channel. The CDF and PDF of SINR between the source to relay and relay to destination are derived using the magnitudes of desired channel following the Nakagami PDF and interference channel following Moschopoulos multi-variate Gaussian PDF. Numerical results show that performance of the OFDM based CR multiple relays network is improved by increasing the number of relays and fading figure parameter of the Nakagami-m fading channel.

The fourth contribution of this thesis is the introduction of multiple antennas at the full duplex DF relaying node and at the destination node in the proposed communication process. The use of multiple antennas improves the received signal strength through array gain by coherent combining and diversity gain by combining the number of independent signal fading paths. As the loop interference and NBI dominate noise, the concept of optimum combining is applied to minimize the effect of interference, instead of maximum ratio combining. SINR expressions are well approximated to Signal to Interference Ratio (SIR). The CDF and PDF of SIRs between source to relay and relay to destination are obtained using Hotelling's T^2 distribution. Closed form analytical expression is derived for the end-to-end outage performance of the OFDM based cooperative DF multi-antennas relay network using the hypergeometric functions. Numerical results show that the outage performance of the OFDM based multiple antennas relay network using optimum combining achieves the better performance than the network with maximal ratio combining.

ACKNOWLEDGEMENT

I express my heartfelt gratefulness to my supervisor Dr. S. J. Thiruvengadam for his excellent guidance, insight, and scholarly disposition without which this thesis would not have seen the light of the day. I am grateful to Mr. Karumuttu T. Kannan, Chairman, Thiagarajar College of Engineering, for providing an excellent research environment. I extend my thankfulness to our beloved Principal Dr.V.Abhaikumar and Head of the Department Dr. (Mrs).S. Raju. I also express my sincere gratitude to the Doctoral Committee members Dr.V.Abhaikumar and Dr.S.Ramakrishnan, for their valuable suggestions. I acknowledge and appreciate the precious help offered to me by Dr.M.N.Suresh, Dr.S.Mohammed Mansoor Roomi, Dr.V.N.Senthilkumaran and Mr.P.G.S.Velmurugan. I thank my well wishers Dr.S.Rajaram, Dr.N.B.Balamurugan, Dr.G.Ananthi, Dr.M.S.K.Manikandan, Dr.K.Rajeswari and the faculty members of ECE Department of TCE for extending their whole hearted support. I owe very much to my parents whose great care and concern sustained me throughout and especially in this venture. I thank my brothers, sisters and other friends for their support. My special thanks to my wife Mrs.R.Sheema and my loving son R.Rokin Kumar who have been very helpful and understanding.

RAJKUMAR S

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

AWGN	-	Additive White Gaussian Noise
ρ	-	Amplification factor at relay node
AF	-	Amplify and Forward
I_{DF}	-	Average mutual information of DF relay
BPSK	-	Binary Phase Shift Keying
BER	-	Bit Error Rate
r	-	Cardinality of NBI signal vector
CFO	-	Carrier Frequency Offset
$\boldsymbol{A}_{bd}^{(n)}$	-	CFO matrix of NBI signal at destination
β	-	CFO parameter of NBI signal
CFR	-	Channel Frequency Response
CNR	-	Channel gain to Noise power Ratio
CIR	-	Channel Impulse Response
B	-	Circulant amplification matrix
$\bar{\boldsymbol{H}}_{sd}$, $\bar{\boldsymbol{H}}_{bd}$	-	Circulant channel matrices
$\boldsymbol{C}_d$	-	Circulant matrix of CFO
$\bar{\boldsymbol{h}}_{br}$	-	CIR vector between NBI and relay
$\bar{\boldsymbol{h}}_{sr}$	-	CIR vector between source and relay
CR	-	Cognitive Radio
${}_1F_1(a,b;x)$	-	Confluent Hypergeometric function
CMMOE	-	Constrained Minimum Mean-Output-Energy
$\boldsymbol{R}_1$	-	Covariance matrix of the loop interference and NBI
CDF	-	Cumulative Distribution Function
R	-	Data rate
DF	-	Decode and Forward
DAB	-	Digital Audio Broadcasting

DVB-T	-	Digital Video Broadcasting Terrestrial
DFT	-	Discrete Fourier Transform
DSB-AM	-	Double Side Band/Amplitude Modulation
EM	-	Expectation Maximization
EVD	-	Eigen Value Decomposition
FB1,FB2,FB3	-	Frequency Bands
$\boldsymbol{b}_{r_i}^{(n)}$	-	Frequency domain NBI signal vector
$\boldsymbol{z}_{r_i}^{(n)}$	-	Frequency domain noise vector
$\boldsymbol{y}_{rd}^{DF}$	-	Frequency domain received signal vector
$\Gamma(\cdot)$	-	Gamma function
${}_2F_1(\cdot,\cdot;\cdot;-x)$	-	Gauss hypergeometric function
GPS	-	Global Positioning System
I-AF	-	Incremental Amplify and Forward
i.i.d	-	independent identically distributed
σ_h^2	-	Instantaneous squared norm of the CIR vector
IPPS	-	Interference Power Per Subcarrier
$INR_1,\ INR_2$	-	Interference to Noise Ratio
ISI		Inter Symbol Interference
IDFT	-	Inverse Discrete Fourier Transform
L^{-1}	-	Inverse Laplace Transform
σ_{br}^2	-	IPPS of NBI term
LMMSE	-	Linear Minimum Mean Square Error
$\bar{\boldsymbol{h}}_{rr}$	-	Loop/echo interference at relay
LTE	-	Long Term Evolution
MRC	-	Maximal Ratio Combining
$\mu_{sr_i}(k),\quad \mu_{r_id}(k)$		Mean values of the signal channels
$d_{\min}$	-	Minimum distance

MBWA	-	Mobile Broadband Wireless Access
MGF	-	Moment Generating Function
MIMO	-	Multiple Input Multiple Output
MUD	-	Multiuser Detection
NBI	-	Narrow Band Interference
σ_{zd}^2	-	Noise variance at destination
L	-	Number of Channel taps
M_R, M_D	-	Number of receiving antennas at relay and destination nodes
M	-	Number of relay nodes
N	-	Number of Subcarriers
N_{br}, N_{bd}	-	Number of Subcarriers with NBI at relay and destination nodes
OC	-	Optimum Combining
$\boldsymbol{w}_{OC,k}^{D}$	-	Optimum combining weight vector at destination
OFDM	-	Orthogonal Frequency Division Multiplexing
PDF	-	Probability Density Function
$Q(.)$	-	Queue function
RF	-	Radio Frequency
K	-	Rician fading parameter
Ω_{sr_i}	-	Scale parameter of Nakagami-m fading
SU_d	-	Secondary Destination Node
SU_r	-	Secondary Relay Node
SU_s	-	Secondary Source Node
I	-	Set of cooperating relays
m_{sr_i}	-	Shape parameter of Nakagami-m fading
SINR	-	Signal to Interference plus Noise Ratio
SIR	-	Signal to Interference Ratio

SNR	-	Signal to Noise Ratio
P_s	-	Source Power
ς_d	-	Squared norm of CFO
k	-	Subcarrier element of OFDM symbol
SEP	-	Symbol Error Probability
γ_1	-	Threshold SINR
UWB	-	Ultra Wideband
α	-	Uniformly distributed random variable
USB	-	Universal Serial Bus
VHF	-	Very High Frequency
WiFi	-	Wireless Fidelity
WLAN	-	Wireless Local Area Network
WMAN	-	Wireless Metropolitan Area Network
WiMAX	-	Worldwide Interoperability for Microwave Access
v	-	Zero padded guard subcarriers

CHAPTER 1

INTRODUCTION

In recent years, the demand for high data rate wireless systems is growing more and more due to its support of voice, text, video and multimedia web contents. Future wireless communications focus on developing new algorithms for wireless broadband multimedia communication systems. However, the major challenges are the limited Radio Frequency (RF) bandwidth and the requirement of high transmission power. The limitation of RF bandwidth in high data rate wireless systems introduces Inter Symbol Interference (ISI) and the wireless channel becomes frequency selective in nature. Orthogonal Frequency Division Multiplexing (OFDM) is a promising technology that uses the concept of cyclic or zero prefix to remove ISI. It converts the frequency selective fading channel into a set of parallel flat fading channels (Prasad & Nee 2000). This makes the equalization process much simpler at the receiver side. Earlier, the principle of frequency division multiplexing is applied for data transmission in general switched telephone network and cellular radio (Bingham 1990). As OFDM has several distinct features, it becomes most powerful tool in high data rate communications. But, in practice, one or more subcarriers of OFDM symbol are highly susceptible to Narrow Band Interference (NBI) from wireless devices such as Bluetooth, Zigbee and cordless phones. Further, the Carrier Frequency Offset (CFO) of NBI spreads the NBI power over all the subcarriers of the OFDM symbol and it reduces the system capacity and increases the bit error probability (Batra & Zeidler 2009). NBI is a common problem in many of the wireless communication systems, especially in OFDM based dynamic spectrum management systems. In this thesis, three major technologies namely

Cognitive Radio (CR), Cooperative communications and Multiple Input Multiple Output (MIMO) are considered for NBI suppression.

1.1 NBI IN OFDM SYSTEMS

1.1.1 Baseband OFDM System

OFDM is an overlapping multicarrier modulation technique which reduces the ISI while keeping the orthogonality between subcarriers.

Figure 1.1 shows the block schematic of transmitter and receiver part of the baseband OFDM system. At transmitter side, input data stream is first converted from serial to parallel data and then synthesized using Inverse Discrete Fourier Transform (IDFT). Then cyclic prefix is added with IDFT coefficients and again resulting data are converted from parallel to serial data stream before transmission. Similarly, at the receiver side, first cyclic prefix is removed from the received signal and converted from serial to parallel data. Then it is demodulated using Discrete Fourier Transform (DFT) and then passed to detector where desired data is detected and again converted from parallel to serial form to recover the original signal.

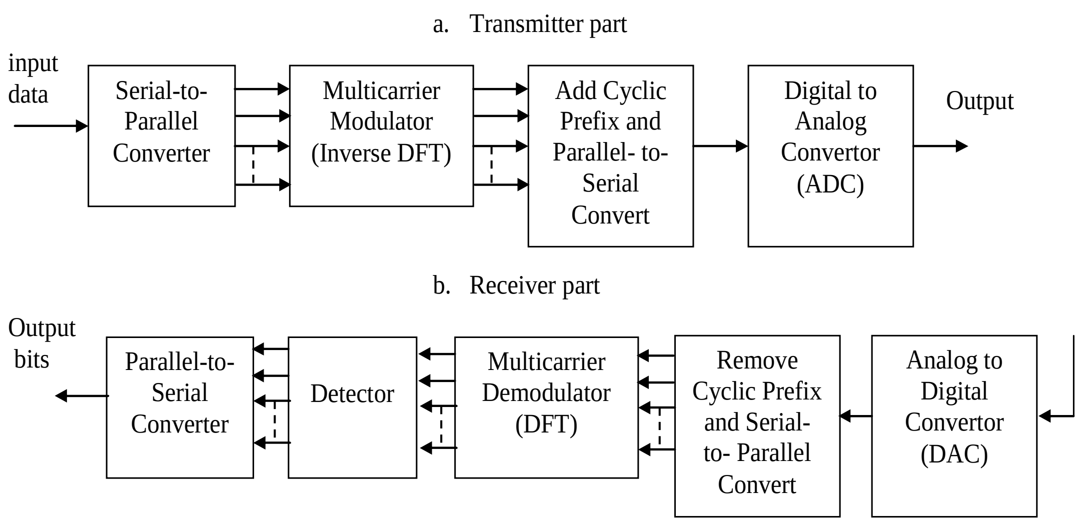

Figure 1.1 Block schematic of baseband OFDM system

The major advantage of OFDM is the mitigation of the multipath fading at reduced implementation complexity. Hence, OFDM is adopted in many wireless standards such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting terrestrial (DVB-T), Wireless Fidelity (WiFi), Worldwide Interoperability for Microwave Access (WiMAX), Bluetooth, Zigbee, Wireless Universal Serial Bus (USB), Mobile Broadband Wireless Access (MBWA), Wireless Regional Area Network and Long Term Evolution (LTE). Currently, the standard IEEE 802.11ay is in progress and aimed to support the data rate of up to 100 Gbps with coverage area of 1000 meters. Due to the underlying sensing, spectrum shaping, scaling and interoperable capabilities of OFDM, it has been adapted as a best transmission technology for CR systems [Mahmoud et al 2009]. However, OFDM is more sensitive to timing and frequency offset, phase noise and NBI. Further it has relatively large peak to average power ratio which reduces the power efficiency of the RF amplifier.

1.1.2 Narrowband Interference

NBI is a major RF impairment in OFDM system which affects only few of the subcarriers. Due to the limited bandwidth in the radio frequency spectrum, it is unavoidable for OFDM based wireless communication system to be free from narrowband interferers (Chi & Das 2009). For instance, in OFDM based Ultra Wideband (UWB) wireless systems, some of the subcarriers are affected by the WiMAX systems and in other cases, Bluetooth devices act as a narrowband interferers for Wireless Local Area Network (WLAN) systems (Batra & Zeidler 2008). Other narrowband systems such as voice Double Side Band/Amplitude Modulation (DSB-AM) signals and Very High Frequency (VHF) digital links affects the VHF spectrum of OFDM systems (Marey & Steendom 2007). As UWB systems transmit the OFDM signal at low power and wider bandwidth, they are inherently affected by other wireless devices such as Global Positioning System (GPS), WLAN and WiMAX (Shi et al 2007). Popular standards such as WLAN and Wireless Metropolitan Area Network (WMAN) are operating in unlicensed spectrum and therefore they must be coexisting with other unlicensed systems including cordless phones, garage door openers, baby monitors and microwave ovens (Coulson 2004). Such interferences are modeled as an unmodulated complex sinusoid, however, this model is not suitable when bandwidth of the NBI is larger than the subcarrier spacing in OFDM systems. Later, Batra & Zeidler (2008) proposed a new model for NBI using the parameters such as interference power, symbol period, random phase and sampling frequency which is widely used to describe the NBI in OFDM systems.

1.2 COGNITIVE RADIO

Cognitive Radio uses the available radio spectrum known as spectrum holes opportunistically to cope with the growing number of bandwidth limited wireless services (Haykin 2005). Spectrums are assigned to primary users with respect to time or frequency or geo-graphical locations, however, primary users are not always using these spectrums. CR systems utilize these unused spectrums by sensing the environments.

Two classes of secondary spectrum access methods are spectrum overlay and spectrum underlay. In spectrum overlay approach, the secondary users access the band only when no primary users are available for transmission and therefore all the nodes operating in the wireless network environments are secondary users. In underlay approach, primary accepts the possibility of interference up to a predefined threshold, where the secondary users share the spectrum of the primary users (Wyglinski et al 2010). After reliable detection of spectrum is achieved, CR systems adaptively employ the appropriate transmission schemes preferably OFDM scheme for high data rate communications. However, the challenge here is that there is no guarantee for data transmission without interference in all the times.

This thesis addresses the problem of NBI suppression in OFDM based overlay CR network.

1.3 COOPERATIVE COMMUNICATIONS

In recent years, researchers have developed new wireless network architectures that depart from the traditional individual point-to-point communications. Due to the broadcast nature of the wireless channel, any wireless transmission from an end-user can be received and processed by other nodes, rather than being considered as an interference, and send to the destination node (Liu et al 2009). Cooperative communication achieves the same advantages as those found in multi-antennas systems. Fixed relaying, selection relaying and incremental relaying are the basic low complexity cooperative strategies (Laneman et al 2004).

1.3.1 Fixed Relaying

In fixed relaying, the relays can either amplify the received signal subject to the power constraint and forward it to next node or decode the received signal and transmit the re-encoded signal to next node.

1.3.1.1 Amplify and Forward (AF) relaying

In this relaying strategy, source terminal transmits the OFDM signal to the relay in first time slot. During this interval, relay node process the amplification of the received signal. In second time slot, source node becomes silent while relay node transmits the amplified signal to the destination node. The amplification factor is chosen such that relay node transmit power is same as source node transmit power. However, the major drawback in AF relaying strategy is that during the amplification at the relay node the noise is enhanced.

1.3.1.2 Decode and Forward (DF) relaying

In this relaying strategy, source terminal transmits the signal to the relay in first time slot. During this time, relay node decode the received signal. In second time slot, source node keeps silent while relay node transmits the re-encoded signal to the destination node. In addition, relay node can either fully decode the received signal or perform the symbol-by-symbol decoding and allow the destination node for full decoding.

1.3.2 Selection Relaying

In selection relaying strategy, transmit nodes can choose either cooperative or non-cooperative transmission based on the channel condition between transmit and receive nodes. Relay nodes adapt their transmission according to the realized source to relay channel coefficient. If the measured channel magnitudes fall below the threshold, then source node continues its transmission to the destination node. If the measured channel magnitude is above the threshold, then the relay transmits the message that it received from the source node to the destination node using either AF relaying or DF relaying.

1.3.3 Incremental Relaying

Incremental relaying strategy uses relaying only when it is necessary based on the limited feedback from receive node. It operates at data rate of R b/s/Hz, if source to destination transmission is successful or it operates at the data rate of R/2 b/s/Hz, if relay node repeats the source transmission.

1.4 MULTIPLE INPUT MULTIPLE OUTPUT (MIMO) SYSTEMS

MIMO is another popular technology in wireless communication which improves the data rates, range and reliability. MIMO technology constitutes a breakthrough in wireless communication system that meets the challenges impaired by wireless multipath channel as well as resource constraints (Biglieri et al 2007). Multiple antennas improve the received signal quality and increase the data communication speed by using digital signal processing techniques to shape and combine the transmitted signals from multiple wireless paths created by the use of multiple receive and transmit antennas (Liu et al 2009). Using multi-antennas systems, the multipaths are effectively converted into benefit in wireless systems and take the advantage of multipaths delay spread to increase the data rates (Oestges & Clercks 2007). By employing the multiple antennas at both ends, the capacity of the wireless link is increased (Paulraj & Kailath 1994). The capacity of the wireless communication systems is dramatically improved without additional spectrum (Telatar 1999). The importance of multi-antennas is identified by exploiting the spatial dimension in addition with the exploitation of time and frequency dimension as in the single antenna system. The benefits of multi-antennas are array gain, spatial diversity gain, spatial multiplexing gain and interference reduction.

1.4.1 Array Gain

Array gain is the coherent combining effect of multiple received signals at the receiver. It improves the output Signal to Noise Ratio (SNR) and resistance to noise. In this, coherent combining represents the spatial processing at transmit antennas and/or at receiver antennas. Array gain improves the coverage and range of the wireless network.

1.4.2 Spatial Diversity Gain

The spatial diversity gain is achieved by receiving the multiple copies of the transmitted signal with respect to space, time or frequency. By increasing the number of independent copies, the probability of at least one of the copies not going to deep fade increases, thereby improve the quality and reliability of the wireless systems.

In addition, the diversity gain is achieved at receiver by employing multiple antennas. Receiver diversity can be achieved via two combining methods: 1. selection combining 2. gain combining.

1.4.2.1 Selection combining

In this method, the combiner selects the branch with high SNR among the multiple received antennas signals. With the assumption that all the received channels are independent and identically distributed with unit energy and the noise levels are equal on each antenna, selection combining comes to compare the instantaneous amplitude of each channel and choose the branch with largest SNR $\Gamma_{\max}$. The probability that $\Gamma_{\max}$ falls below the threshold γ is given by (Janaswamy 2000)

$$\Pr\left(\Gamma_{\max} < \gamma\right) = \left[1 - e^{-\gamma^2}\right]^{M_R} \tag{1.1}$$

where M_R is the number of received antennas. For Binary Phase Shift Keying (BPSK) modulation, the average error probability of the selection combining is expressed as (Simon & Alouini 2000).

$$\bar{P} = \int_0^\infty Q\left(\sqrt{2\bar{\Gamma}_{\max}\gamma}\right) f_{\Gamma_{\max}}(\gamma)\, d\gamma \tag{1.2}$$

where $f_{\Gamma_{\max}}(\gamma)$ is the probability density function (PDF) of $\Gamma_{\max}$, $\bar{\Gamma}_{\max}$ is the average SNR. The PDF of $\Gamma_{\max}$ is determined as

$$f_{\Gamma_{\max}}(\gamma) = M_R 2\gamma e^{-\gamma^2}\left[1 - e^{-\gamma^2}\right]^{M_R - 1} \tag{1.3}$$

1.4.2.2 Gain combining

Gain combining is further divided into two types: 1. Equal gain combining, 2. Maximal Ratio combining. In first method, weight factor at k^{th} receive antenna is fixed to $w_k = e^{-j\phi_k}$ where ϕ_k is the co-phase of the received signal. With knowledge of signal phases of each received antennas, the signals are combined. In second method, the weights are chosen as $w_k = h_k^*$, where * indicate the complex conjugate and h_k is the channel coefficient of the k^{th} path.

1.4.3 Spatial Multiplexing Gain

The spatial multiplexing gain increases the data rate linearly by transmitting the multiple, independent data streams within the bandwidth of operation. Under the appropriate channel conditions and rich scattering environments, the receiver can able to separate the data streams. The number of data stream supported by the multi-antennas channels is equal to the minimum number of transmit and receive antennas.

1.4.4 Interference Reduction

Interference in wireless networks occurs whenever the resources of one user like time or frequency is shared by other user. The interference in multi-antennas systems is mitigated by exploiting the spatial dimension. For instance, array gain increases the tolerance to noise and interference power, thereby improves the Signal to Interference plus Noise Ratio (SINR). In addition, the spatial dimension is used to direct the signal towards the desired user and minimize the interference to other users.

By combining the cooperative relays with multiple antennas, the capacity of the wireless systems is further improved. Further, multi-antennas system is virtually formed through cooperative relaying. This thesis addresses the issue of NBI suppression in OFDM based CR network through the cognitive AF relaying which improves the output SINR at the destination node by exploiting the spatial diversity. Another method to minimize the NBI in OFDM based multi-antennas CR network is the optimum combining. In this case, the received signals are optimally weighted and combined in order to improve the output SINR which gives better outage performance than maximal ratio combining.

1.5 LITERATURE SURVEY

1.5.1 NBI Suppression Techniques

The widely used interference cancellation techniques include the filter-based approach, transform- domain approach, cyclostationarity-based approach, higher order statistics-based approach, joint detection/Multiuser Detection (MUD), and spatial processing. NBI is estimated by measuring the output signal energy and receiver windowing is proposed to reduce the effect of NBI in OFDM system (Redfem 2002). Unmodulated subcarriers are used to measure the narrowband interference power using Linear Minimum Mean Square Error (LMMSE) based estimator (Nilsson et al 2003). As the energy of NBI is varied on each subcarrier, it is considered as a colored noise and compensated using an iterative decoder based on Expectation Maximization (EM) algorithm (Sanguinetti et al 2010). Constrained Minimum Mean-Output-Energy (CMMOE) based algorithm is developed for NBI estimation with the assumption that the second order statistics of the received signal is known at the receiver (Darsena et al 2007). Compressive sensing is used to estimate the NBI signal by exploiting the sparsity of the NBI signal in frequency domain (Gomaa et al

2011). The effect of NBI is analyzed in ultra wideband cooperative relay network (Maichalernnukul et al 2011). In coded OFDM system, the NBI effect is reduced by whitening the narrowband interference spectrum using Prediction Error Filter (PEF) (Batra & Zeidler 2008). Filter based approach is used to suppress NBI in OFDM based conventional CR network (Coulson 2006). In this method, interfering carrier frequency is estimated and cancelled using notch filter. However, the success of this method is based on the accuracy in estimating the interfering carrier frequency. Higher order statistics based Bayesian decision theoretic approach is used to suppress the partial band interference in frequency hop communication system (Baum 1992).

Recently, few of the articles are found in analyzing the effect of NBI in OFDM based CR systems. For instance, in multiband UWB systems, WLAN and WiMAX devices act as a NBI which affects some portion of the spectrum in UWB systems (Batra & Zeidler 2008). The primary user interference is minimized in CR system using Opportunistic Interference Cancellation method (Popovski 2007). Joint detection and multi user detection-based approach is used for detecting secondary user symbols in the presence of primary user signal, at the cost of high complexity (Hassan & Hossain 2013). Spectrum reshaping based-approach is used for suppressing the primary user interference in CR network (Yamaguchi 2004). However, some portion of the spectrum becomes unused while reshaping is applied. Interference in OFDM based CR network is reduced through phase adjustment approach (Alian 2013).

1.5.2 Outage Performance Analysis of Single Relay Network

Outage probability is one of the major parameters to analyze the performance of OFDM based CR systems. It is defined as the probability that the instantaneous error probability exceeds a specified value or equivalently the probability of the output SNR, Γ falls below a certain threshold SNR γ. Mathematically, it can be represented by

$$P_{out} = \int_0^{\gamma} f_{\Gamma}(\gamma) d\gamma \tag{1.4}$$

where $f_{\Gamma}(\gamma)$ is the Probability Density Function (PDF) of the SNR Γ.

Hasna & Alouini (2004) derived closed form expressions for the statistics of the harmonic mean of the two independent and identically distributed gamma variates to study performance of the wireless communication systems with non regenerative relays over Nakagami-m fading channels.

Karagiannidis et al (2005) persents a closed form bounds for outage and average error probability of multihop transmissions with non-generative relays over Nakagami-m fading channels. Novel closed form expressions are derived for moment generating functions, probability density functions and cumulative distribution functions.

The exact Symbol Error Probability (SEP) derived for the multi-hop transmission with regenerative relays where hard decision is performed before forwarding to destination [Müller & Speidel 2007].

Zhu et al (2008) derived the analytical expression for outage probability of the full duplex DF relay network over rician fading channel. However, the performance of the interference affected network is not considered.

Zhang et al (2009) proposed the cooperative relay based CR network to improve the throughput of the secondary users by increasing the spatial diversity and spectrum diversity.

Kwon et al (2010) analyzed the tradeoff between full-duplex and half-duplex modes for DF relay network. It is shown that full duplex mode works well at low Signal to Interference Ratio (SIR) and Signal to Noise Ratio (SNR) region for a given target outage probability. Although, interference is considered in this work, analysis is carried out only in flat fading environment.

Lee et al (2011) derived the outage probability of cognitive relay networks with cooperation between secondary users based on the underlay approach, while adhering the to the interference constraint on the primary users.

Yan et al (2011) proposed a cognitive relay networks with the maximum transmit power limit in a spectrum sharing scenario. Exact outage probability expression is derived in Rayleigh fading channels.

Li et al (2011) analyzed the tradeoff between the Diversity and Multiplexing for single user with multihop relays through space-time AF relaying scheme.

Xu et al (2012) derived outage probability of the decode and forward cognitive relay network in the presence of primary user interference. Asymptotic outage probability also derived at high SNR regime.

Yu et al (2012) analyzed the outage probability of multi branch dual hop decode and forward cooperative relay systems over non-identical Nakagami fading channels. It is concluded that DF relaying systems are more vulnerable at low and medium SINR region.

Baranwal et al (2013) analyzed the outage performance of multi-hop full duplex relay networks where echo interference is minimized by isolating transmission from the reception.

Aloqlah & Badarneh (2013) analyzed the average symbol error Probability of coherent and non-coherent modulation for the dual-hop DF relay network.

Fan et al (2013) proposed a two phase two way relay network consisting of two sources and single amplify and forward relay. Distributed switch and stay combining technique is employed to exploit the direct link to achieve the full diversity order.

1.5.3 Outage Performance Analysis of Multiple Relays Network

Suraveera et al (2006) derived the outage probability of the cooperative multiple relays network over Nakagami-m fading channels. At m=1, results are applicable to Rayleigh fading channels.

Zhao et al (2007) proposed a new optimal power allocation scheme to minimize the outage probability of amplify and forward cooperative diversity system based on the channel state information and channel statistics. Best relay selection scheme is employed to achieve the full diversity order.

Hu & Beaulieu (2007) studied the performance of the DF relay network using selection combining in independent identically distributed (i.i.d) channels and independent non-identically distributed channels.

Seddik et al (2007) analyzed the outage probability for multi-node amplify and forward relay network where harmonic mean of two exponential random variables are well approximated to single exponential random variable at high SNR.

Kim et al (2008) proposed an optimal resource allocation strategy for multi-hop OFDMA cooperative relay networks. Dual method employed to solve the problem efficiently and optimally.

Michalopoulos & Karagiannidis (2008) analyzed the outage probability and average error probability of threshold based opportunistic relaying and selection cooperation at arbitrary SNRs.

Datsikas et al (2008) derived a closed form expression for outage probability for an L-relays dual hop decode and forward relay network. Further, the network performance is analyzed over Nakagami-m fading channels.

Zhang et al (2009) develop an algorithm for optimal power and time adaptation policy to minimize the outage probability under long term total power constraint and derived minimum short term power required to meet the target transmission rate.

Adinoyi et al (2009) proposed two-hop multi-relays communication networks. Selection relaying is employed to achieve the full diversity order. Exact expressions for outage probability and capacity are derived.

Ikki & Ahmed (2010) developed a general mathematical probability model to study the outage performance of the adaptive decode and forward cooperative relay networks using the best relay selection scheme.

Su & Liu (2010) analyzed the outage probability of selection relaying protocols and compared with cooperative wireless networks. Depnding on the quality of the source to relay channel, relay node choose either decode and forward or amplify and forward or direct transmission.

Selvaraj & Mallik (2010) proposed a new model based on the scaled selection combining which perform the selection process using the deterministic scale factor that incorporates the effect of source to relay links.

Michalopoulos et al (2010) studied the rate at which a relay switching occurs in selective cooperative relaying applications over time varying channels. Closed form expressions are derived for relay switching rate for opportunistic relaying and distributed switch and stay combining.

Ropokis et al (2011) analyzed the BER performance of the DF and optimal DF relay network using the optimal weighting factor selection concept, in which decision at relay is properly weighted based on the reliability of the source to relay link.

Krikidis et al (2012) studied a relay selection problem in a finite buffer aided decode and forward cooperative wireless networks. A relay selection policy use the buffering ability of the relay nodes in order to maximize the achieved diversity gain.

Xu et al (2014) proposed a distribution link selection scheme for cognitive amplify and forward relay networks. Closed form lower bound outage probability expression is derived by approximating the instantaneous SNR of the dual hop relaying links.

Zou et al (2015) analyzed the Intercept probability and outage probability of the cognitive radio systems using relay selection scheme in the presence of eavesdropper.

1.5.4 Outage Performance Analysis of Multi-antennas relay Network

Shah & Hoimovich (1998) studied the optimum combining for space diversity reception in digital cellular mobile radio systems with Rayleigh fading in the presence of multiple interferences.

Shah & Hoimovich (2000) studied the maximal ratio combining for space diversity reception in digital mobile radio systems and compared with optimum combining. In this, the signal fading follows the Rayleigh or Rician distribution, cochannel interference follows the Rayleigh distribution.

Aalo & Zhang (2000) analyzed the average error probability of the digital mobile radio system with optimum combiner in the Rayleigh fading environment with cochannel interference.

Yue et al (2006) derived the exact closed form expression for output SINR and outage probability of optimum combiner based wireless system in Rayleigh fading environment.

Yue & Zhang (2006) analyzed the Outage performance of the Multi Input Multi Output (MIMO) system employing optimum combiner with and without cochannel interference in Rayleigh fading environment.

Yuksel & Erkip (2007) analyzed the performance of the multiple antennas wireless system through cooperative relays.

Adinoyi & Yanikomeroglu (2007) presented distributed decode and forward fixed relays which are used in cooperation in a two-hop wireless network. Multiple antennas are employed on distributed relays. Threshold based maximal ratio combining and threshold based selection combining are studied.

Riihonen et al (2011) analyzed the performance of the multi-antennas full duplex decode and forward relay network.

1.6 SCOPE AND OBJECTIVES OF THE THESIS

In OFDM based CR network, many of the research works are proposed to improve the system performance and resource utilization which includes the optimal power allocations with respect to channel state information, spectrum sensing and primary interference avoidance. However, when OFDM based CR network operating in overlay mode (i.e., all nodes are secondary users), it is necessary to take care of existing narrowband interferences which will affect the one or many subcarriers of the OFDM symbol. In this thesis, the issue of narrowband interference is addressed in OFDM based CR network.

The cognitive AF relay network operating with different bands is introduced in OFDM based CR networks to minimize the effect of NBI. Further, performance of the OFDM based CR networks are analyzed with various relaying strategies such as AF relaying strategy and DF relaying strategy and various duplex modes such as half duplex and full duplex modes.

Full diversity in OFDM based CR network is achieved through multiple relays. To reduce the complexity, relay selection scheme is applied in OFDM based CR network.

Throughput and reliability of the OFDM based CR network is improved through multiple antennas which improves the received signal gain. The concept of optimum combining is used in this model to suppress the NBI.

The objectives of the thesis are,

1. To suppress the NBI in OFDM based cognitive radio systems through cooperative AF relay network and different operating bands.
2. To analyze the outage performance and average error performance of the proposed OFDM based cognitive AF relay network in the presence of NBI.
3. To analyze the outage performance and average error performance of the proposed OFDM based cognitive full duplex AF relay network in the presence of NBI.

4. To analyze the outage performance of the proposed OFDM based cognitive AF multiple relays network in the presence of NBI.
5. To analyze the outage performance of the proposed OFDM based multi-antennas full duplex DF relay network in the presence of NBI.

1.7 ORGANIZATION OF THE THESIS

In chapter 2, outage performance and average error performance of the proposed OFDM based cognitive AF relay network are analyzed in the presence of NBI. Further, the performance of the proposed network is compared with OFDM based cognitive DF relay network and OFDM based non cooperative relay network. In chapter 3, outage performance and average error performance of the OFDM based cognitive full duplex AF relay network is analyzed in the presence of NBI and compared with OFDM based cognitive full duplex DF relay network and OFDM based cognitive half duplex AF relay network without direct link. In chapter 4, outage performance of the OFDM based cognitive full duplex AF multiple relays network is analyzed in the presence of NBI and compared with OFDM based cognitive half duplex AF multiple relays network and OFDM based cognitive half duplex DF multiple relays network. Further, the performance of the proposed AF multiple relays network is analyzed over Nakagami-m fading channels. In chapter 5, outage performance of the OFDM based multi-antennas full duplex DF relay network is analyzed in the presence of NBI. The concept of optimum combining is employed to suppress the NBI. The performance of the network with optimum combining is compared with maximal ratio combining. Chapter 6 concludes the thesis with consolidations of the results reported.

CHAPTER 2

OFDM BASED HALF DUPLEX COGNITIVE RELAY NETWORK FOR SUPPRESSING THE EFFECT OF NBI

2.1 PREAMBLE

The usages of modern wireless systems are growing rapidly due to its support of multimedia communications and mobile telephony. OFDM is widely used in current high data rate wireless communication systems and standards due to its ability of mitigating the multipath fading effect of the broadband received signal and improving the spectral efficiency of the system (Prasad & Nee 2000). Cognitive radio is another popular technology developed to utilize the unoccupied spectrums of the primary users by the secondary users at right location and time (Haykin 2005). Further, to improve the throughput of the CR system, OFDM scheme is introduced due to its inherent capabilities. However, in practice, one or many subcarriers of the OFDM signal are affected by the existing narrowband interferers. Narrowband interference is modeled as a sparse vector in frequency domain and it is assumed quasi-static over one OFDM symbol. Due to the presence of NBI, the performance of the OFDM based CR system is drastically reduced.

The concept of spectrum sensing in OFDM based CR network is addressed through random subcarrier allocation (Ekin et al 2012). Using optimal and suboptimal power allocation schemes, downlink transmission capacity is maximized in OFDM based CR system (Bansal et al 2008). OFDM based underlay CR system sharing the subcarriers of Orthogonal Frequency Division Multiplexing Access (OFDMA) based primary system is studied using optimal power allocation (Kang et al 2010). Variable subband Nulling method is used to reduce the primary user interference in OFDM based CR system (Park et al 2011). Although many of the works are proposed in OFDM based CR network, the issue of NBI effect in OFDM based CR network is not considered so far. In this chapter, the effect of NBI in OFDM based CR network is addressed. Further, to reduce the effect of NBI, a fixed Amplify and Forward (AF) relay which operates at a frequency band that is different from the frequency band of direct mode is introduced in OFDM based CR network. It is assumed that OFDM based cooperative CR network consists of three nodes denoted by source, relay and destination and operate in overlay mode which ensures that all nodes are secondary users. In half duplex mode, source node transmits the OFDM signal to relay and to destination node in first time slot. Relay node amplify the received signal and transmits to the destination node in second time slot. Using Maximal Ratio Combining (MRC), the received signals are combined at the destination node over the two time slots. Hence, output SINR at the destination node is improved by exploiting the spatial diversity and frequency diversity. The performance of the OFDM based cognitive half duplex AF relay network is compared with the performance of the non-cooperative OFDM based CR network and OFDM based cognitive half duplex Decode and Forward (DF) relay network.

2.2 SYSTEM MODEL

Consider an OFDM based cognitive half duplex AF relay network with three secondary nodes shown in Figure 2.1. The OFDM signal is assumed to have N subcarriers and v zero padded guard subcarriers to suppress ISI. Zero padded OFDM signal eliminate the inter block interference and guarantees symbol recovery regardless of the channel zero locations (Muquet et al 2002). The total number of subcarriers in an OFDM signal is $P = N + v$.

The network can operate in two modes namely, direct mode and relay mode. In direct mode of operation, the secondary source node SU_s

transmits OFDM signal to secondary destination node SU_d in a single time slot. In relay mode of operation, the relay node SU_r can choose one among the relaying strategies namely AF relaying, DF relaying and Incremental Amplify and Forward (I-AF). In AF relaying strategy, received OFDM signal at relay node SU_r during first time slot is amplified, subject to the power constraints, and forwarded to destination node SU_d in the second time slot. When relay node SU_r is operating with DF relaying strategy, received signal at SU_r is decoded and the re-encoded signal is transmitted to the destination node SU_d in second time slot. In incremental AF relaying strategy, relay node SU_r is chosen only when it is necessary, based on the feedback from the destination node SU_d.

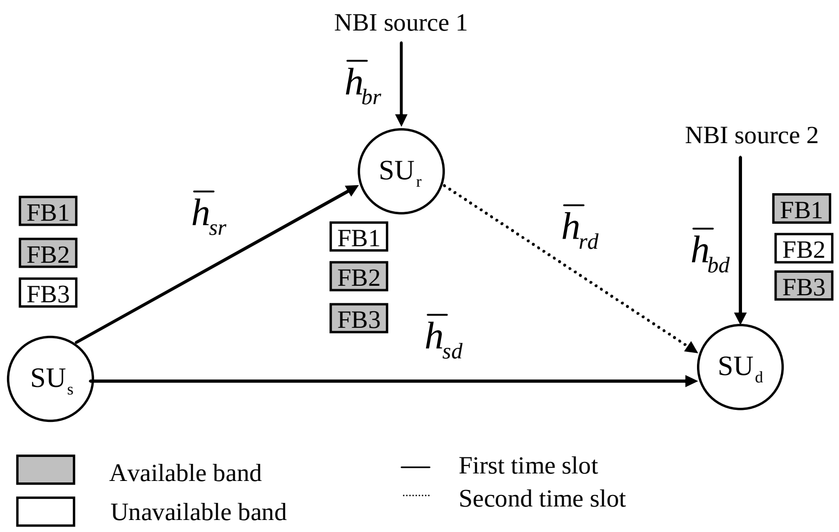

Figure 2.1 Cognitive half duplex Amplify and Forward relay network

The use of cooperative relaying strategies in OFDM based CR network provides increased SNR at the destination node and better performance in low spectral efficiency regime. However, the half duplex cooperative relaying strategies require twice the bandwidth of direct transmission for a given rate. This problem is jointly addressed with suppressing the effect of NBI by exploiting the available spectrum bands in the network in a cognitive manner. Since the subcarriers affected by NBI at the relay node would be different from subcarriers at the destination node, the effect of NBI can be minimized effectively by cooperative relaying strategies.

Let FB_1, FB_2 and FB_3 be available frequency bands for transmission/reception in the network. The available frequency bands at each node have also been marked in Figure 2.1.

2.2.1 Direct Mode

In this mode, source node SU_s transmits the OFDM signal to destination node SU_d using the frequency band FB_1. Let $\boldsymbol{x}_s$ be a $N \times 1$ data vector at N subcarriers of OFDM signal in source node SU_s and $\bar{\boldsymbol{b}}_d$ be a $P \times 1$ NBI signal vector. The $P \times P$ diagonal matrix of the CFO in NBI is $\boldsymbol{A}_{fo,d}^{(1)} = diag\left[1, \exp\left(j2\pi\alpha/P\right), ..., \exp\left(j2\pi\alpha(P-1)/P\right)\right]$ at the frequency band FB_1 and α is uniformly distributed random variable over the interval $[-0.5 \;\; 0.5]$ (Batra et.al 2009). The $L_1 \times 1$ Channel Impulse Response (CIR) vector between source node SU_s and destination node SU_d is defined as $\bar{\boldsymbol{h}}_{sd} = \left[\bar{h}_{sd}(0), \bar{h}_{sd}(1), .., \bar{h}_{sd}(L_1 - 1)\right]^T$. The $L_2 \times 1$ CIR vector between second NBI source and destination node SU_d is defined as $\bar{\boldsymbol{h}}_{bd} = \left[\bar{h}_{bd}(0), \bar{h}_{bd}(1), .., \bar{h}_{bd}(L_2 - 1)\right]^T$. Let $\bar{\boldsymbol{H}}_{sd}$ and $\bar{\boldsymbol{H}}_{bd}$ be $P \times P$ circulant matrices of CIR vectors $\bar{\boldsymbol{h}}_{sd}$ and $\bar{\boldsymbol{h}}_{bd}$ respectively. The $P \times 1$ time-domain receive signal vector at SU_d is given by

$$\bar{\boldsymbol{y}}_{sd} = \bar{\boldsymbol{H}}_{sd}\boldsymbol{F}_{zp}\boldsymbol{x}_s + \boldsymbol{A}_{fo,d}^{(1)}\bar{\boldsymbol{H}}_{bd}\bar{\boldsymbol{b}}_d + \bar{\boldsymbol{z}}_d \tag{2.1}$$

where $\boldsymbol{F}_{zp}$ is a $P \times N$ zero padded IDFT matrix at SU_s, it is defined as $\boldsymbol{F}_{zp} = \left[\boldsymbol{F}_N \;\; \boldsymbol{0}_{N \times v}\right]^H$. $\bar{\boldsymbol{z}}_d$ is $P \times 1$ zero-mean complex white Gaussian noise vector whose elements are independent and identically distributed with variance σ_{zd}^2. By taking DFT, the frequency domain representation of receive signal vector $\bar{\boldsymbol{y}}_{sd}$ in Equation (2.1) is given by

$$\boldsymbol{y}_{sd} = \boldsymbol{F}_p \bar{\boldsymbol{y}}_{sd} = \boldsymbol{F}_p \left(\bar{\boldsymbol{H}}_{sd} \boldsymbol{F}_{zp} \boldsymbol{x}_s + A_{fo,d}^{(1)} \bar{\boldsymbol{H}}_{bd} \bar{\boldsymbol{b}}_d + \bar{\boldsymbol{z}}_d \right) \tag{2.2}$$

Using Eigen Value Decomposition (EVD), the circulant matrices $\bar{\boldsymbol{H}}_{sd}$ and $\bar{\boldsymbol{H}}_{bd}$ are decomposed as $\bar{\boldsymbol{H}}_{sd} = \boldsymbol{F}_P^H \Lambda_{sd} \boldsymbol{F}_P$, $\bar{\boldsymbol{H}}_{bd} = \boldsymbol{F}_P^H \Lambda_{bd} \boldsymbol{F}_P$ (Gray 2006). The elements of the diagonal matrices Λ_{sd} and Λ_{bd} are eigen value λ. They are defined as $\Lambda_{sd} = diag\left[\lambda_{sd}(0), \lambda_{sd}(1), ..., \lambda_{sd}(P-1)\right]$ and $\Lambda_{bd} = diag\left[\lambda_{bd}(0), \lambda_{bd}(1), ..., \lambda_{bd}(P-1)\right]$. The diagonal elements of the matrices Λ_{sd} and Λ_{bd} are obtained by DFT of $P \times 1$ CIR vectors $\bar{\boldsymbol{h}}_{sd}$ and $\bar{\boldsymbol{h}}_{bd}$ respectively. Now, Equation (2.2) can be simply written as

$$\boldsymbol{y}_{sd} = \Lambda_{sd} \boldsymbol{V} \boldsymbol{x}_s + \boldsymbol{C}_d \Lambda_{bd} \boldsymbol{b}_d + \boldsymbol{z}_d \tag{2.3}$$

where $\boldsymbol{V} = \boldsymbol{F}_P \boldsymbol{F}_{zp}$ is the precoding matrix, $\boldsymbol{C}_d = \boldsymbol{F}_P \boldsymbol{A}_{fo,d}^{(1)} \boldsymbol{F}_P^H$ is a circulant matrix of CFO at $\mathrm{SU_d}$, $\boldsymbol{b}_d$ and $\boldsymbol{z}_d$ are $P \times 1$ NBI signal and noise vectors in frequency domain respectively. The NBI signal vector $\boldsymbol{b}_d$ is sparse in nature and it indicates that the vector contains only few non-zero elements. The k^{th} subcarrier element $b_d(k) \neq 0$ if $k \in I_{bd}$, where I_{bd} is a set which contains index of the non-zero elements.

2.2.2 Relay Mode

In this mode, relay node can operate with one of the relaying strategies namely AF relay, DF relay and I-AF relay. Source node $\mathrm{SU_s}$ transmits OFDM signal to relay node $\mathrm{SU_r}$ using the frequency band $\mathrm{FB_2}$ in the first time slot. Let $\bar{\boldsymbol{b}}_r$ be a $P \times 1$ NBI signal vector at secondary relay node $\mathrm{SU_r}$. $\boldsymbol{A}_{fo,r}^{(2)}$ is the CFO of NBI signal in band $\mathrm{FB_2}$. The $L_3 \times 1$ CIR vector between source node $\mathrm{SU_s}$ and relay node $\mathrm{SU_r}$ is defined as $\bar{\boldsymbol{h}}_{sr} = \left[\bar{h}_{sr}(0), \bar{h}_{sr}(1), .., \bar{h}_{sr}(L_3 - 1)\right]^{\mathrm{T}}$. The $L_4 \times 1$ CIR vector between first NBI source and relay is defined as $\bar{\boldsymbol{h}}_{br} = \left[\bar{h}_{br}(0), \bar{h}_{br}(1), .., \bar{h}_{br}(L_4 - 1)\right]^{\mathrm{T}}$. $\bar{\boldsymbol{H}}_{sr}$ and $\bar{\boldsymbol{H}}_{br}$ are the $P \times P$ circulant matrices of the CIR vectors $\bar{\boldsymbol{h}}_{sr}$ and $\bar{\boldsymbol{h}}_{br}$ respectively. The $P \times 1$ time-domain receive signal vector $\bar{\boldsymbol{y}}_{sr}$ at relay node $\mathrm{SU_r}$ is given by

$$\bar{\boldsymbol{y}}_{sr} = \bar{\boldsymbol{H}}_{sr} \boldsymbol{F}_{zp} \boldsymbol{x}_s + A_{fo,r}^{(2)} \bar{\boldsymbol{H}}_{br} \bar{\boldsymbol{b}}_r + \bar{\boldsymbol{z}}_r \tag{2.4}$$

where $\bar{\boldsymbol{z}}_r$ is $P \times 1$ noise vector at $\mathrm{SU_r}$. By taking DFT, the frequency domain representation of the receive signal vector $\bar{\boldsymbol{y}}_{sr}$ is given by

$$\boldsymbol{y}_{sr} = \boldsymbol{F}_p \bar{\boldsymbol{y}}_{sr} = \boldsymbol{F}_p \left(\bar{\boldsymbol{H}}_{sr} \boldsymbol{F}_{zp} \boldsymbol{x}_s + A_{fo,r}^{(2)} \bar{\boldsymbol{H}}_{br} \bar{\boldsymbol{b}}_r + \bar{\boldsymbol{z}}_r \right) \tag{2.5}$$

Since $\bar{\boldsymbol{H}}_{sr}$ and $\bar{\boldsymbol{H}}_{br}$ are circulant matrices, using decomposition Equation (2.5) is simplified as

$$\boldsymbol{y}_{sr} = \Lambda_{sr} \boldsymbol{V} \boldsymbol{x}_s + \boldsymbol{C}_r \Lambda_{br} \boldsymbol{b}_r + \boldsymbol{z}_r \tag{2.6}$$

where Λ_{sr}, Λ_{br} are $P \times P$ diagonal matrices of the vectors λ_{sr} and λ_{br} respectively, $\boldsymbol{C}_r = \boldsymbol{F}_P \boldsymbol{A}_{fo,r}^{(2)} \boldsymbol{F}_P^H$ is a circulant matrix.

2.2.2.1 AF relay mode

In AF relaying strategy, the relay node $\mathrm{SU_r}$ amplifies the receive signal vector $\bar{\boldsymbol{y}}_{sr}$ and forwards it to $\mathrm{SU_d}$ using the band $\mathrm{FB_3}$. Let $\bar{\boldsymbol{b}}_d$ be a $P \times 1$ NBI signal vector. $\boldsymbol{A}_{fo,d}^{(3)}$ is the CFO of NBI signal in band $\mathrm{FB_3}$. The $L_5 \times 1$ CIR vector between relay node $\mathrm{SU_r}$ and destination node $\mathrm{SU_d}$ is defined as $\bar{\boldsymbol{h}}_{rd} = \left[\bar{h}_{rd}(0), \bar{h}_{rd}(1), .., \bar{h}_{rd}(L_5 - 1)\right]^{\mathrm{T}}$. The $L_6 \times 1$ CIR vector of second NBI source node and destination node is defined as

$\bar{\boldsymbol{h}}_{bd} = \left[\bar{h}_{bd}(0), \bar{h}_{bd}(1), .., \bar{h}_{bd}(L_5 - 1)\right]^{\mathrm{T}}$. Let $\bar{\boldsymbol{H}}_{rd}$ and $\bar{\boldsymbol{H}}_{bd}$ be $P \times P$ circulant matrices of the CIR vectors $\bar{\boldsymbol{h}}_{rd}$ and $\bar{\boldsymbol{h}}_{bd}$ respectively. The $P \times 1$ time domain receive signal vector $\bar{\boldsymbol{y}}_{rd}$ at destination node SU_{d} is given by

$$\bar{\boldsymbol{y}}_{rd} = \rho \bar{\boldsymbol{H}}_{rd} \left(\bar{\boldsymbol{y}}_{sr}\right) + A_{fo,d}^{(3)} \bar{\boldsymbol{H}}_{bd} \bar{\boldsymbol{b}}_d + \bar{\boldsymbol{n}}_d \tag{2.7}$$

where $\rho = \sqrt{\mathrm{P}_s / \left(\mathrm{P}_s \sigma_h^2 + \sigma_{br}^2 + \sigma_{zr}^2\right)}$ is the amplification factor at relay node SU_{r}, $\sigma_h^2 = E\left[\left|\bar{\boldsymbol{h}}_{sr}\right|^2\right]$ is the instantaneous squared-norm of the CIR vector $\bar{\boldsymbol{h}}_{sr}$, $\mathrm{P}_s = E\left[\left|\boldsymbol{x}_s\right|^2\right]$ is the transmitted source power which is same as in relay node transmit power, σ_{br}^2 is the Interference Power Per Subcarrier (IPPS) of the NBI signal vector $\bar{\boldsymbol{b}}_r$ and σ_{zr}^2 is the variance of the noise vector $\bar{\boldsymbol{z}}_r$, and $\bar{\boldsymbol{n}}_d$ represents the noise vector at SU_{d}. Substituting Equation (2.4) in Equation (2.7), the receive signal vector $\bar{\boldsymbol{y}}_{rd}$ is written as

$$\bar{\boldsymbol{y}}_{rd} = \rho \bar{\boldsymbol{H}}_{rd} \bar{\boldsymbol{H}}_{sr} F_{zp} \boldsymbol{x}_s + \rho \bar{\boldsymbol{H}}_{rd} A_{fo,r}^{(2)} \bar{\boldsymbol{H}}_{br} \bar{\boldsymbol{b}}_r + A_{fo,d}^{(3)} \bar{\boldsymbol{H}}_{bd} \bar{\boldsymbol{b}}_d + \rho \bar{\boldsymbol{H}}_{rd} \bar{\boldsymbol{z}}_r + \bar{\boldsymbol{n}}_d \tag{2.8}$$

By taking DFT, the frequency domain representation of the receive signal vector $\bar{\boldsymbol{y}}_{rd}$ is given by

$$\begin{aligned} \boldsymbol{y}_{rd} = \boldsymbol{F}_p \bar{\boldsymbol{y}}_{rd} = \boldsymbol{F}_p \Big\{ & \rho \bar{\boldsymbol{H}}_{rd} \bar{\boldsymbol{H}}_{sr} F_{zp} \boldsymbol{x}_s + \rho \bar{\boldsymbol{H}}_{rd} A_{fo,r}^{(2)} \bar{\boldsymbol{H}}_{br} \bar{\boldsymbol{b}}_r \\ & + A_{fo,d}^{(3)} \bar{\boldsymbol{H}}_{bd} \bar{\boldsymbol{b}}_d + \rho \bar{\boldsymbol{H}}_{rd} \bar{\boldsymbol{z}}_r + \bar{\boldsymbol{n}}_d \Big\} \end{aligned} \tag{2.9}$$

By decomposing the circulant matrices $\bar{\boldsymbol{H}}_{rd}$, $\bar{\boldsymbol{H}}_{sr}$, $\bar{\boldsymbol{H}}_{br}$ and $\bar{\boldsymbol{H}}_{bd}$, Equation (2.9) is simplified as

$$\boldsymbol{y}_{rd} = \rho \Lambda_{rd} \Lambda_{sr} \boldsymbol{V} \boldsymbol{x}_s + \rho \Lambda_{rd} \boldsymbol{C}_r \Lambda_{br} \boldsymbol{b}_r + \boldsymbol{E}_d \Lambda_{bd} \boldsymbol{b}_d + \rho \Lambda_{rd} \boldsymbol{z}_r + \boldsymbol{n}_d \tag{2.10}$$

Let $\boldsymbol{\lambda}_{rd} = \left[\lambda_{rd}(0), ..., \lambda_{rd}(P-1)\right]^{\mathrm{T}}$ and $\boldsymbol{\lambda}_{bd} = \left[\lambda_{bd}(0), ..., \lambda_{bd}(P-1)\right]^{\mathrm{T}}$

be P-point DFT vectors of $\bar{\boldsymbol{h}}_{rd}$ and $\bar{\boldsymbol{h}}_{bd}$ respectively. The elements of the diagonal matrices Λ_{rd} and Λ_{bd} are the vectors λ_{rd} and λ_{bd} respectively, $\boldsymbol{E}_d = \boldsymbol{F}_p \boldsymbol{A}_{fo,d}^{(3)} \boldsymbol{F}_p^H$ is a circulant matrix of CFO.

2.2.2.2 DF relay mode

In DF relay mode, the relay node $\mathrm{SU_r}$ decodes the received signal vector and transmits the re-encoded signal $\boldsymbol{x}_r$ to destination node $\mathrm{SU_d}$ in second time slot. Following Equation (2.3), the frequency domain received signal vector $\boldsymbol{y}_{rd}^{DF}$ at destination node $\mathrm{SU_d}$ is given by

$$\boldsymbol{y}_{rd}^{DF} = \Lambda_{sr} \boldsymbol{V} \boldsymbol{x}_r + \boldsymbol{C}_d \Lambda_{bd} \boldsymbol{b}_d + \boldsymbol{z}_d \tag{2.11}$$

2.2.2.3 Incremental AF relay mode

In this mode, the received signal vector $\boldsymbol{y}_{rd}^{I-AF}$ from the relay at destination node is same as in AF relay mode received signal. However, this received signal is available based on the destination node feedback. Destination node sends the feedback to relay and destination about success or failure of the received signal during first time slot. If the signal is received at high SINR, then relay do not transmit to destination. If the received signal has low SINR, then relay can amplify and forward to destination.

2.3 SINR ANALYSIS IN HALF DUPLEX RELAY NETWORK

In this section, analytical expressions for SINR of OFDM based CR network in direct mode, AF relay mode and DF relay mode are derived at relay and destination nodes.

2.3.1 SINR in Direct Mode

From Equation (2.3), the direct mode receive signal $\boldsymbol{y}_{sd}$ at k^{th} subcarrier is given by

$$y_{sd}(k) = \lambda_{sd}(k)v(k,k)x_s(k) + \lambda_{sd}(k)\sum_{q=0,q\neq k}^{N-1} v(k,q)x_s(q) + \sum_{\substack{m=0\\ m\in I_{bd}}}^{P-1} \lambda_{bd}(m)c_d\left(k,(m-k)_P\right)b_d(m) + z_d(k), \tag{2.12}$$

where $k = 0,1,2,...,P-1$. In Equation (2.12), the first term is the desired signal, $x_s(k)$ is the k^{th} subcarrier source data. The second and third terms denotes the residual zero padded interference and NBI signal respectively. Since $\boldsymbol{C}_d$ is a circulant matrix, third term is written as a circular convolution. Fourth term represents the noise.

Assuming that the CIR vectors λ_{sd} and λ_{bd} are perfectly known are perfectly known at destination node SU_d, the SINR of the direct mode receive signal $\boldsymbol{y}_{sd}$ at k^{th} subcarrier is given by

$$\Gamma_{sd}(k) = \frac{|\lambda_{sd}(k)|^2 |v(k,k)|^2 E\left(|x_s(k)|^2\right)}{\left\{|\lambda_{sd}(k)|^2 E\left(\left|\sum_{q=0,q\neq k}^{N-1} v(k,q)x_s(q)\right|^2\right) + E\left(\left|\sum_{\substack{m=0\\ m\in I_{bd}}}^{P-1} \lambda_{bd}(m)c_d\left(k,(m-k)_P\right)b_d(m)\right|^2\right) + E\left(|z_d(k)|^2\right)\right\}} \tag{2.13}$$

Using Cauchy – Schwartz inequality, first term in the denominator of Equation (2.13) can be expressed as

$$E\left(\left|\sum_{q=0,q\neq k}^{N-1} v(k,q)x_s(q)\right|^2\right) \leq \sum_{q=0,q\neq k}^{N-1} \left|v(k,q)\right|^2 E\left(\sum_{q=0,q\neq k}^{N-1} \left|x_s(q)\right|^2\right) \tag{2.14}$$

Since $\sum_{q=0}^{N-1}\left|v(k,q)\right|^2 = 1$, the term $\sum_{q=0,q\neq k}^{N-1}\left|v(k,q)\right|^2$ is written as $1-\left|v(k,k)\right|^2$, then Equation (2.14) can be simplified as

$$E\left(\left|\sum_{q=0,q\neq k}^{N-1} v(k,q)x_s(q)\right|^2\right) \leq \left(1-\left|v(k,k)\right|^2\right)\mathrm{P}_s \tag{2.15}$$

Similarly, the second term in the denominator of Equation (2.13) can be expressed as

$$E\left(\left|\sum_{m=0,m\in I_{bd}}^{P-1} \lambda_{bd}(m)c_d\left(k,(m-k)_P\right)b_d(m)\right|^2\right) \leq \sum_{m=0,m\in I_{bd}}^{P-1} \left|\lambda_{bd}(m)\right|^2 \varsigma_d \sigma_{bd}^2 \tag{2.16}$$

where $\varsigma_d = \sum_{m=0,m\in I_{bd}}^{P-1}\left|c_d\left(k,(m-k)_P\right)\right|^2$ is the squared-norm of the CFO term and $\sigma_{bd}^2 = E\left(\sum_{m=0,m\in I_{bd}}^{P-1}\left|b_d(m)\right|^2\right)$ is the IPPS of NBI signal. Now, substituting Equation (2.15) and Equation (2.16) in Equation (2.13), the lower bound of $\Gamma_{sd}(k)$ is simply written as

$$\Gamma_{sd}(k) \geq \frac{\left|\lambda_{sd}(k)\right|^2\left|v(k,k)\right|^2\mathrm{P}_s}{\left|\lambda_{sd}(k)\right|^2\left(1-\left|v(k,k)\right|^2\right)\mathrm{P}_s + \sum_{\substack{m=0\\ m\in I_{bd}}}^{P-1}\left|\lambda_{bd}(m)\right|^2\varsigma_d\sigma_{bd}^2 + \sigma_{zd}^2} \tag{2.17}$$

where $\sigma_{zd}^2 = E\left(\left|z_d(k)\right|^2\right)$ is a noise variance. Dividing the numerator and denominator by the source power P_s, the lower bound SINR at destination node $\mathrm{SU_d}$ is given by

$$\Gamma_{sd}(k) \geq \frac{\left|\lambda_{sd}(k)\right|^2 \left|v(k,k)\right|^2}{\left|\lambda_{sd}(k)\right|^2 \left(1-\left|v(k,k)\right|^2\right) + \sum_{\substack{m=0 \\ m\in I_{bd}}}^{P-1} \left|\lambda_{bd}(m)\right|^2 \varsigma_d \frac{1}{SIR_1} + \frac{1}{SNR}} \tag{2.18}$$

where $SIR_1 = \mathrm{P}_s / \sigma_{bd}^2$ is Signal to Interference Ratio and $SNR = \mathrm{P}_s / \sigma_{zd}^2$.

2.3.2 SINR in AF Relay Mode

In AF relaying strategy, signal passes through the two statistically independent and identically distributed channels $\bar{\boldsymbol{h}}_{sr}$ and $\bar{\boldsymbol{h}}_{rd}$ with amplification factor ρ at relay node $\mathrm{SU_r}$. At k^{th} subcarrier, the SINR between the source node $\mathrm{SU_s}$ and destination node $\mathrm{SU_d}$ through relay node $\mathrm{SU_r}$ in AF relay mode is given by (Rabiei et.al 2011).

$$\Gamma_{srd}(k) = \left\{\Gamma_{sr}(k)\Gamma_{rd}(k)\right\} / \left\{\Gamma_{sr}(k) + \Gamma_{rd}(k) + 1\right\} \tag{2.19}$$

where $\Gamma_{ij}(k)$ is the SINR between the link $\mathrm{SU_i}$ and $\mathrm{SU_j}$, $i \in (s,r)$ and $j \in (r,d)$.

The SINR between the source node $\mathrm{SU_s}$ and relay node $\mathrm{SU_r}$ at k^{th} subcarrier is determined as follows. From Equation (2.6), the receive signal vector $\boldsymbol{y}_{sr}$ at relay node $\mathrm{SU_r}$ in k^{th} subcarrier is given by

$$\begin{aligned} y_{sr}(k) = {} & \lambda_{sr}(k) v(k,k) x_s(k) + \lambda_{sr}(k) \sum_{q=0,q\neq k}^{N-1} v(k,q) x_s(q) \\ & + \sum_{\substack{m=0 \\ m\in I_{br}}}^{P-1} \lambda_{br}(m) c_r\left(k,(m-k)_P\right) b_r(m) + z_r(k), k = 0.1...,P-1 \end{aligned} \tag{2.20}$$

In Equation (2.20), the first term is the desired signal, second and third term denote the residual zero padded interference and NBI signal respectively, and fourth term is the noise. Assuming that λ_{sr} and λ_{br} are perfectly known at relay node $\mathrm{SU_r}$, SINR at k^{th} subcarrier is defined as

$$\Gamma_{sr}(k)=\frac{|\lambda_{sr}(k)|^2|v(k,k)|^2E\left(|x_s(k)|^2\right)}{\left\{|\lambda_{sr}(k)|^2E\left(\left|\sum_{q=0,q\neq k}^{N-1}v(k,q)x_s(q)\right|^2\right)+E\left(\left|\sum_{m=0,m\in I_{br}}^{P-1}\lambda_{br}(m)c_r\left(k,(m-k)_P\right)b_r(m)\right|^2\right)+E\left(|z_r(k)|^2\right)\right\}} \tag{2.21}$$

Using Cauchy-Schwartz inequality, the first and second terms in the denominator of Equation (2.21) can be simplified as

$$E\left(\left|\sum_{q=0,q\neq k}^{N-1}v(k,q)x_1(q)\right|^2\right)\leq\left(1-|v(k,k)|^2\right)\mathrm{P}_s \tag{2.22}$$

$$E\left(\left|\sum_{\substack{m=0\\ m\in I_{br}}}^{P-1}\lambda_{br}(m)c_r\left(k,(m-k)_P\right)b_r(m)\right|^2\right)\leq\sum_{\substack{m=0\\ m\in I_{br}}}^{P-1}|\lambda_{br}(m)|^2\varsigma_r\sigma_{br}^2 \tag{2.23}$$

where $\varsigma_r=\sum_{m=0,m\in I_{br}}^{P-1}\left|c_r\left(k,(m-k)_P\right)\right|^2$ is the squared-norm of the CFO term, $\sigma_{br}^2=\left(\sum_{m=0,m\in I_{br}}^{P-1}|b_r(m)|^2\right)$ is the IPPS of NBI term. Substituting Equation (2.22) and Equation (2.23), Equation (2.21) is simply written as

$$\Gamma_{sr}(k)\geq\frac{|\lambda_{sr}(k)|^2|v(k,k)|^2}{|\lambda_{sr}(k)|^2\left(1-|v(k,k)|^2\right)+\sum_{\substack{m=0\\ m\in I_{br}}}^{P-1}|\lambda_{br}(m)|^2\varsigma_r\frac{1}{SIR_2}+\frac{1}{SNR}} \tag{2.24}$$

where $SIR_2=\mathrm{P}_s/\sigma_{br}^2$ and $SNR=\mathrm{P}_s/\sigma_{zr}^2$, σ_{zr}^2 is the noise variance. Given λ_{rd} and λ_{bd} at destination node $\mathrm{SU_d}$, SINR of the receive signal vector $\boldsymbol{y}_{rd}$ at k^{th} subcarrier is defined as

$$\Gamma_{rd}(k)=\frac{|\lambda_{rd}(k)|^2|v(k,k)|^2E\left(|x_s(k)|^2\right)}{\left\{|\lambda_{rd}(k)|^2E\left(\left|\sum_{q=0,q\neq k}^{N-1}v(k,q)x_s(q)\right|^2\right)+E\left(\left|\sum_{m=0,m\in I_{bd}}^{P-1}\lambda_{bd}(m)e_d\left(k,(m-k)_P\right)b_d(m)\right|^2\right)+E\left(|n_d(k)|^2\right)\right\}} \tag{2.25}$$

The first and second terms in the denominator of Equation (2.25) are simplified using Cauchy- Schwartz inequality as

$$E\left(\left|\sum_{q=0,q\neq k}^{N-1}v(k,q)x_s(q)\right|^2\right)\leq\left(1-|v(k,k)|^2\right)\mathrm{P}_s \tag{2.26}$$

$$E\left(\left|\sum_{\substack{m=0\\ m\in I_{bd}}}^{P-1}\lambda_{bd}(m)e_d\left(k,(m-k)_P\right)b_d(m)\right|^2\right)\leq\sum_{\substack{m=0\\ m\in I_{bd}}}^{P-1}|\lambda_{bd}(m)|^2\xi_d\sigma_{bd}^2 \tag{2.27}$$

where $\xi_d=\sum_{m=0,m\in I_{bd}}^{P-1}\left|e_d\left(k,(m-k)_P\right)\right|^2$ is the squared norm of the CFO, $\sigma_{bd}^2=\left(\sum_{m=0,m\in I_{bd}}^{P-1}|b_d(m)|^2\right)$ is the IPPS of NBI term. Substituting Equation (2.26) and Equation (2.27) in Equation (2.25), the lower bound SINR at the destination node $\mathrm{SU_d}$ is given by

$$\Gamma_{rd}(k)\geq\frac{|\lambda_{rd}(k,k)|^2|v(k,k)|^2}{|\lambda_{rd}(k,k)|^2\left(1-|v(k,k)|^2\right)+\sum_{\substack{m=0\\ m\in I_{bd}}}^{P-1}|\lambda_{bd}(m)|^2\xi_d\frac{1}{SIR_3}+\frac{1}{SNR}} \tag{2.28}$$

where $SIR_3=\mathrm{P}_s/\sigma_{bd}^2$, $SNR=\mathrm{P}_s/\sigma_{nd}^2$ and the noise variance $\sigma_{nd}^2=E\left(|n_d(k)|^2\right)$.

In AF relay mode, the overall SINR at k^{th} subcarrier between source and destination node is written as

$$\Gamma_{srd}\left(k\right)=\frac{\Gamma_{sr}(k)\Gamma_{rd}(k)}{\Gamma_{sr}(k)+\Gamma_{rd}(k)+1}\leq\min\left(\Gamma_{sr}(k),\Gamma_{rd}(k)\right) \tag{2.29}$$

2.3.3 SINR in DF Relay Mode

Using Equation (2.11), given λ_{rd} and λ_{bd}, SINR of the receive signal vector $\boldsymbol{y}_{rd}$ at destination node $\mathrm{SU_d}$ in k^{th} subcarrier is given by

$$\Gamma_{rd}^{DF}\left(k\right)=\frac{\left|\lambda_{rd}\left(k\right)\right|^2\left|v\left(k,k\right)\right|^2 E\left(\left|x_r\left(k\right)\right|^2\right)}{\left\{E\left(\left|\sum_{m=0,m\in I_{bd}}^{P-1}\lambda_{bd}\left(m\right)c_d\left(k,\left(m-k\right)_P\right)b_d\left(m\right)\right|^2\right)+E\left(\left|z_d\left(k\right)\right|^2\right)\right\}}$$

(2.30)

After simplification, Equation (2.30) can be written as

$$\Gamma_{rd}^{DF}\left(k\right)\geq\frac{\left|\lambda_{rd}\left(k,k\right)\right|^2\left|v\left(k,k\right)\right|^2\mathrm{P}_r}{\left|\lambda_{rd}\left(k,k\right)\right|^2\left(1-\left|v\left(k,k\right)\right|^2\right)\mathrm{P}_r+\sum_{\substack{m=0\\ m\in I_{bd}}}^{P-1}\left|\lambda_{bd}\left(m\right)\right|^2\xi_d\sigma_{bd}^2+\sigma_{zd}^2} \tag{2.31}$$

where $\mathrm{P}_r=E\left(\left|\boldsymbol{x}_r\right|^2\right)$ is the relay node transmit power $\xi_d=\sum_{m=0,m\in I_{bd}}^{P-1}\left|c_d\left(k,\left(m-k\right)_P\right)\right|^2$ is the squared norm of the CFO, $\sigma_{bd}^2=\left(\sum_{m=0,m\in I_{bd}}^{P-1}\left|b_d\left(m\right)\right|^2\right)$ is the IPPS of NBI term.

2.4 OUTAGE ANALYSIS IN HALF DUPLEX RELAY NETWORK

In this section, analytical expressions for outage probability of OFDM based CR network in direct mode, AF relay mode, DF relay mode and I-AF relay mode are derived at subcarrier level.

2.4.1 Outage Probability in Direct Mode

The outage probability of the OFDM based CR network in direct mode at k^{th} subcarrier, for the given data rate of R b/s/Hz, is defined as

$$P_{out,k}^{dir}(R) = Pr\left(\log_2\left(1+\Gamma_{sd}(k)\right) < R\right) \tag{2.32}$$

For the data rate of R b/s/Hz, the threshold SINR is given by $\gamma_1 = 2^R - 1$. Then, $P_{out,k}^{dir}(R)$ can be written as

$$P_{out,k}^{dir}(R) = Pr\left(\Gamma_{sd}(k) < \gamma_1\right) \tag{2.33}$$

Let $\Gamma_{sd}(k)$ be defined as

$$\Gamma_{sd}(k) = \frac{aX}{bY+cZ+d} \tag{2.34}$$

where $X = Y = \left|\lambda_{sd}(k)\right|^2$, $Z = \sum_{m=0, m\in I_{bd}}^{P-1} \left|\lambda_{bd}(m)\right|^2$, $a = \left|v(k,k)\right|^2$, $b = \left(1-\left|v(k,k)\right|^2\right)$, $c = \varsigma_d\left(1/SIR_1\right)$ and $d = 1/SNR$. The random variables X and Y are exponentially distributed with variance $\sigma_x^2 = \sigma_y^2 = 0.5$ and Z is chi-square distributed with n degrees of freedom (Proakis 2008). Let $r = n/2$ is the cardinality of the NBI signal vector. The PDF of the variables X, Y and Z are given by

$$f_X(x) = \left(1/2\sigma_x^2\right)\exp\left(-x/2\sigma_x^2\right), x \ge 0 \tag{2.35}$$

$$f_Y(y) = \left(1/2\sigma_y^2\right)\exp\left(-y/2\sigma_y^2\right), y \ge 0 \tag{2.36}$$

$$f_Z(z) = \frac{1}{\sigma_z^n 2^{n/2}\Gamma\left(\frac{n}{2}\right)} z^{\frac{n}{2}-1} \exp\left(-z/2\sigma_z^2\right), z \ge 0 \tag{2.37}$$

Since the desired signal channel λ_{sd} and the interference channel λ_{rd} are negatively quadrant dependent (Lehmann 1966), the outage probability of X for the given random variables Y and Z is given by

$$P_{out,k}^{dir}(\gamma_1) \le \int_0^\infty \int_0^\infty Pr\left(X < \gamma_1 \frac{(by+cz+d)}{a} \Big|_{Y=y,Z=z} \right) f_Y(y)\, f_Z(z)\, dydz \qquad (2.38)$$

Since $a \approx 1$, Equation (2.38) is simplified as,

$$P_{out,k}^{dir}(\gamma_1) \le \int_0^\infty \int_0^\infty \int_0^{(by+cz+d)\gamma_1} f_X(x) f_Y(y)\, f_Z(z)\, dxdydz \qquad (2.39)$$

Evaluating the inner integral by substituting Equation (2.35), it becomes

$$P_{out,k}^{dir}(\gamma_1) \le \int_0^\infty \int_0^\infty \left[1-\exp\left(-\gamma_1(by+cz+d)\right)\right] f_Y(y)\, f_Z(z)\, dydz \qquad (2.40)$$

Now, Equation (2.40) can be rewritten as

$$P_{out,k}^{dir}(\gamma_1) \le 1 - e^{-\gamma_1 d} \int_0^\infty e^{-\gamma_1 by} f_Y(y)\, dy \int_0^\infty e^{-\gamma_1 cz} f_Z(z)\, dz \qquad (2.41)$$

Substituting Equation (2.36) and Equation (2.37) in Equation (2.41) then integrating, the outage probability is determined as

$$P_{out,k}^{dir}(\gamma_1) \le 1 - \exp(-\gamma_1 d) \frac{\left[1+(\gamma_1 c/r)\right]^{-r}}{\left[1+\gamma_1 b\right]} \qquad (2.42)$$

Substituting b, c, d and γ_1, the outage probability $P_{out,k}^{dir}(R)$ is given by

$$P_{out,k}^{dir}(R) \le 1 - \exp\left(-\left(2^R-1\right)/SNR\right) \frac{\left[1+\left(\left(2^R-1\right)\varsigma_d / rSIR_1\right)\right]^{-r}}{\left[1+\left(2^R-1\right)\left(1-\left|v(k,k)\right|^2\right)\right]}$$

(2.43)

By using the approximations $\left(1+(a/r)\right)^{-r} \approx \exp(-a)$ and $\left|v(k,k)\right|^2 \approx 1$, Equation (2.43) can be simplified as

$$P_{out,k}^{dir}(R) \le 1 - \exp\left(-(1/SNR) - \left(\varsigma_d / SIR_1\right)\right)\left(2^R-1\right) \qquad (2.44)$$

2.4.2 Outage Probability in AF Relay Mode

The end-to-end outage probability of the OFDM based cognitive half duplex AF relay network at k^{th} subcarrier, for the given data rate of R b/s/Hz , is defined as

$$P_{out,k}^{AF-HDD}(R) = Pr\left(\left(1/2\right)\log_2\left(1+\Gamma_{AF-HDD}(k)\right) < R\right) \tag{2.45}$$

Since MRC is applied at destination in the second time slot, AF relay mode SINR Γ_{AF-HDD} is defined as $\Gamma_{AF-HDD}(k) = \Gamma_{sd}(k) + \Gamma_{srd}(k)$. Now, Equation (2.45) can be written as

$$P_{out,k}^{AF-HDD}(R) = Pr\left(\left[\Gamma_{sd}(k) + \Gamma_{srd}(k)\right] < 2^{2R} - 1\right) \tag{2.46}$$

Given the threshold SINR γ_1 for $\Gamma_{sd}(k)$, the lower bound AF outage probability is given by

$$P_{out,k}^{AF-HDD}(R) \geq \int_0^{2^{2R}-1} Pr\left(\Gamma_{srd}(k) < 2^{2R} - 1 - \gamma_1\right) f_{\Gamma_{sd}}(\gamma_1)\, d\gamma_1 \tag{2.47}$$

Let $\gamma_2 = 2^{2R} - 1 - \gamma_1$. Then, Equation (2.47) can be written as

$$P_{out,k}^{AF-HDD}(R) \geq \int_0^{2^{2R}-1} \int_0^{2^{2R}-1-\gamma_1} f_{\Gamma_{srd}}(\gamma_2)\, f_{\Gamma_{sd}}(\gamma_1)\, d\gamma_1 d\gamma_2 \tag{2.48}$$

where $f_{\Gamma_{srd}}(\gamma_2)$ and $f_{\Gamma_{sd}}(\gamma_1)$ are PDFs of the $\Gamma_{srd}(k)$ and $\Gamma_{sd}(k)$ respectively. Since $\Gamma_{srd}(k)$ is the minimum of $\Gamma_{sr}(k)$ and $\Gamma_{rd}(k)$, the Cumulative Distribution Function (CDF) of $\Gamma_{srd}(k)$ can be written as (Rabiei et al 2011).

$$F_{\Gamma_{srd}}(\gamma_2) = 1 - \left[\left(1 - F_{\Gamma_{sr}}(\gamma_2)\right)\left(1 - F_{\Gamma_{rd}}(\gamma_2)\right)\right] \tag{2.49}$$

Using Equation (2.44), the CDF of $\Gamma_{sr}(k)$ can be determined as

$$F_{\Gamma_{sr}}(\gamma_1) = 1 - \exp\left(-\left(1/SNR\right) - \left(\varsigma_r / SIR_2\right)\right)\gamma_1 \tag{2.50}$$

Similarly, the CDF of $\Gamma_{rd}(k)$ is determined as

$$F_{\Gamma_{rd}}(\gamma_1) = 1 - \exp\left(-(1/SNR) - (\xi_d / SIR_3)\right)\gamma_1 \qquad (2.51)$$

Substituting Equation (2.50) and Equation (2.51) in Equation (2.49), the CDF of $\Gamma_{srd}(k)$ is obtained as

$$F_{\Gamma_{srd}}(\gamma_2) = 1 - \exp\left(-\left(\frac{2}{SNR} + \frac{\varsigma_r}{SIR_2} + \frac{\xi_d}{SIR_3}\right)\right)\gamma_2 \qquad (2.52)$$

Let $g_2 = \left(\frac{2}{SNR} + \frac{\varsigma_r}{SIR_2} + \frac{\xi_d}{SIR_3}\right)$. Then, it can be written as

$$F_{\Gamma_{srd}}(\gamma_2) = 1 - \exp(-g_2\gamma_2) \qquad (2.53)$$

Differentiating Equation (2.53) with respect to γ_2, the PDF of $\Gamma_{srd}(k)$ is determined as

$$f_{\Gamma_{srd}}(\gamma_2) = d\left(F_{\Gamma_{srd}}(\gamma_2)\right)/d\gamma_2 = g_2 \exp(-g_2\gamma_2) \qquad (2.54)$$

Using Equation (2.44), the CDF of $\Gamma_{sd}(k)$ is determined as

$$F_{\Gamma_{sd}}(\gamma_1) = 1 - \exp\left(-(1/SNR) - (\varsigma_d / SIR_1)\right)\gamma_1 \qquad (2.55)$$

It can be simplified as

$$F_{\Gamma_{sd}}(\gamma_1) = 1 - \exp(-g_1\gamma_1) \qquad (2.56)$$

where $g_1 = \left((1/SNR) + (\varsigma_d / SIR_1)\right)$. Differentiating Equation (2.56) with respect to γ, the PDF of $\Gamma_{sd}(k)$ is determined as

$$f_{\Gamma_{sd}}(\gamma_1) = d\left(F_{\Gamma_{sd}}(\gamma_1)\right)/d\gamma_1 = g_1 \exp(-g_1\gamma_1) \qquad (2.57)$$

Substituting the PDF of $\Gamma_{srd}(k)$ from Equation (2.54) in Equation (2.48) and evaluating the inner integral, the lower bound outage probability becomes

$$P_{out,k}^{AF-HDD}(R) \geq \int_0^{2^{2R}-1} \left[1 - \exp\left(-g_2\left(2^{2R} - 1 - \gamma_1\right)\right)\right] f_{\Gamma_{sd}}(\gamma_1)\, d\gamma_1 \qquad (2.58)$$

Substituting the PDF of $\Gamma_{sd}(k)$ from Equation (2.57) in Equation (2.58), the lower bound outage probability of AF mode is determined as

$$P_{out,k}^{AF-HDD}(R) \geq \left[1-\exp\left(-g_1\left(2^{2R}-1\right)\right)\right]-\left(g_1/\left(g_1-g_2\right)\right)\times \\ \exp\left(-g_2\left(2^{2R}-1\right)\right)\left[1-\exp\left(-\left(g_1-g_2\right)\left(2^{2R}-1\right)\right)\right] \tag{2.59}$$

2.4.3 Outage Probability in DF Relay Mode

The outage probability of OFDM based cognitive half duplex DF relay network, for the given data rate R b/s/Hz, is given by

$$P_{out,k}^{DF-HDD}(R) = Pr\left(\frac{1}{2}\left[\min\left(\log\left(1+\Gamma_{sr}(k)\right),\log\left(1+\Gamma_{sd}(k)+\Gamma_{rd}(k)\right)\right)\right]<R\right) \tag{2.60}$$

It can be written as

$$P_{out,k}^{DF-HDD}(R) = Pr\left(\Gamma_{sr}(k)<2^{2R}-1\right)+\left[1-Pr\left(\Gamma_{sr}(k)<2^{2R}-1\right)\right] \\ \times Pr\left(\Gamma_{sd}(k)+\Gamma_{rd}(k)<2^{2R}-1\right) \tag{2.61}$$

Let the last term of Equation (2.61) be defined as $P_{out,k}^{\Gamma_a(k)}(R) = Pr\left(\Gamma_{sd}(k)+\Gamma_{rd}(k)<2^{2R}-1\right)$. Conditioned on the source to destination SINR $\Gamma_{sd}(k)$, it is written as

$$P_{out,k}^{\Gamma_a(k)}(R) \leq \int_0^{2^{2R}-1}\int_0^{2^{2R}-1-\gamma_1} f_{\Gamma_{rd}(k)}(\gamma_2)\, f_{\Gamma_{sd}(k)}(\gamma_1)\, d\gamma_1 d\gamma_2 \tag{2.62}$$

where $f_{\Gamma_{rd}(k)}(\gamma_2)$, $f_{\Gamma_{sd}(k)}(\gamma_1)$ are PDFs of $\Gamma_{sd}(k)$ and $\Gamma_{rd}(k)$ respectively. Using Equation (2.51), the PDF of $\Gamma_{rd}(k)$ is written as

$$f_{\Gamma_{rd}}(\gamma_2) = \left(\left(1/SNR\right)+\left(\xi_d/SIR_3\right)\right)\exp\left(-\left(\left(1/SNR\right)+\left(\xi_d/SIR_3\right)\right)\gamma_2\right) \tag{2.63}$$

Substituting $f_{\Gamma_{rd}(k)}(\gamma_2)$ and $f_{\Gamma_{sd}(k)}(\gamma_1)$ from Equation (2.63) and Equation (2.57) in Equation (2.62), then evaluating the inner integral, it is determined as

$$P_{out,k}^{\Gamma_a(k)}(R) \le 1 + \frac{\mu_c(k)}{(\mu_a(k) - \mu_c(k))} \exp\left(-\mu_a(k)\left(2^{2R} - 1\right)\right) - \frac{\mu_a(k)}{(\mu_a(k) - \mu_c(k))} \exp\left(-\mu_c(k)\left(2^{2R} - 1\right)\right) \tag{2.64}$$

where $\mu_a(k) = \left(1/[SNR] + \varsigma_d / SIR_1(k)\right)$, $\mu_c(k) = \left(1/[SNR] + \varsigma_d / SIR_3(k)\right)$

Using Equation (2.57), the PDF of $\Gamma_{sr}(k)$ is obtained as

$$f_{\Gamma_{sr}}(\gamma_2) = \left((1/SNR) + (\varsigma_r / SIR_r)\right) \exp\left(-\left((1/SNR) + (\varsigma_r / SIR_r)\right)\gamma_2\right) \tag{2.65}$$

Substituting Equation (2.65) and Equation (2.64) in Equation (2.61), the outage probability of the OFDM based cognitive half duplex DF relay network is determined as

$$P_{out,k}^{DF-HDD}(R) \le 1 + \frac{\mu_c(k)}{(\mu_a(k) - \mu_c(k))} \exp\left(-(\mu_b(k) + \mu_a(k))\left(2^{2R} - 1\right)\right) - \frac{\mu_a(k)}{(\mu_a(k) - \mu_c(k))} \exp\left(-(\mu_b(k) + \mu_c(k))\left(2^{2R} - 1\right)\right) \tag{2.66}$$

where $\mu_b(k) = \left(1/[SNR] + \varsigma_r / SIR_2(k)\right)$

2.4.4 Outage Probability in Incremental AF Relaying

In AF relay mode, relay transmits the amplified receive signal at the data rate of R b/s/Hz through a specified channel in all the time of signal transmission. Indirectly, it looses the degrees of freedom of the channel. But, in incremental AF relaying protocol, data is transmitted at either $R/2$ b/s/Hz or R b/s/Hz. The AF relay transmits data at the rate of $R/2$ b/s/Hz only if the direct mode transmission fails to send the data at the data rate of R b/s/Hz, thereby incremental AF relay network utilize the relay channel effectively and improve the system performance (Laneman et al 2004). The outage probability of the OFDM based CR network in incremental AF relay mode is given by

$$\begin{aligned} P_{out,k}^{IAF-HDD}(R) &= Pr\left(\Gamma_{sd}(k) < R\right) \\ &\times Pr\left(\left(1/2\right)\log_2\left(1+\Gamma_{AF}(k)\right) < \left(R/2\right)\Big|_{\Gamma_{sd}(k)<R}\right) \end{aligned} \tag{2.67}$$

In incremental AF relaying protocol, the intersection of the direct mode and AF relay mode outage probabilities is exactly the AF relay mode outage probability at half the rate. Therefore, it can be written as

$$P_{out,k}^{IAF-HDD}(R) = Pr\left(\left(1/2\right)\log_2\left(1+\Gamma_{AF}(k)\right) < \left(R/2\right)\right) \tag{2.68}$$

Using Equation (2.59), the lower bound outage probability of the I- AF relay mode is determined as

$$\begin{aligned} P_{out,k}^{IAF-HDD}(R) \geq& \left[1-\exp\left(-g_1\left(2^R-1\right)\right)\right] - \left(g_1/\left(g_1-g_2\right)\right) \\ &\times \exp\left(-g_2\left(2^R-1\right)\right)\left[1-\exp\left(-\left(g_1-g_2\right)\left(2^R-1\right)\right)\right] \end{aligned} \tag{2.69}$$

2.5 AVERAGE BER ANALYSIS IN HALF DUPLEX RELAY NETWORK

In this section, analytical expressions for average Bit Error Rate (BER) of OFDM based CR network in direct mode, AF relay mode and DF relay mode are derived. In this analysis, Binary Phase Shift Keying (BPSK) modulation is considered.

2.5.1 Average BER of Direct Mode

The average BER of the OFDM based CR network in direct mode at k^{th} subcarrier can be expressed as

$$\tilde{P}_e^{dir} = E_{\gamma_{sd}}\left(Q\left(\sqrt{2\Gamma_{sd}(k)}\right)\right) = \int_0^\infty Q\left(\sqrt{2\gamma_{sd}}\right) f_{\Gamma_{sd}(k)}\left(\gamma_{sd}\right) d\gamma_{sd} \tag{2.70}$$

where $Q(.)$ - Q function and $f_{\Gamma_{sd}(k)}\left(\gamma_{sd}\right)$ is the PDF of the source to destination SINR at k^{th} subcarrier. Substituting the PDF $f_{\Gamma_{sd}(k)}\left(\gamma_{sd}\right)$ from Equation (2.57) in Equation (2.70), it is written as

$$\tilde{P}_e^{dir} = \int_0^{\infty} Q\left(\sqrt{2\gamma}\right) g_1 e^{-g_1\gamma}\, d\gamma \tag{2.71}$$

where γ is the dummy variable. Using the integral Q-function formula, $\int_0^{\infty} x e^{-x^2/2} Q\left(\frac{x}{\sigma}\right) dx = \frac{1}{2}\left(1 - \frac{1}{\sqrt{\sigma^2+1}}\right)$,

Equation (2.71) is written as

$$\tilde{P}_e^{dir} \le \frac{1}{2}\left(1 - \frac{1}{\sqrt{g_1+1}}\right) \tag{2.72}$$

After substituting for g_1 and simplifying, the average BER of the OFDM based CR network in direct mode is determined as

$$\tilde{P}_e^{dir} \le \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{SNR + INR_2 + 1}}\right) \tag{2.73}$$

where $INR_2 = \sigma_{bd}^2 / \sigma_{zd}^2$ is the Interference to Noise Ratio.

2.5.2 Average BER of AF Relay Mode

The average BER of the OFDM based cognitive half duplex AF relay network can be expressed as

$$\tilde{P}_e^{AF-HDD} = E_{\gamma_{af}}\left(Q\left(\sqrt{2\Gamma_{AF-HDD}(k)}\right)\right) = \int_0^{\infty} Q\left(\sqrt{2\gamma_{af}}\right) f_{\Gamma_{AF-HDD}(k)}\left(\gamma_{af}\right) d\gamma_{af} \tag{2.74}$$

where $f_{\Gamma_{AF-HDD}(k)}\left(\gamma_{af}\right)$ is the PDF of the overall SINR of the cognitive half duplex AF relay network. The PDF of this mode can be computed as follows. The CDF of overall SINR $\Gamma_{AF-HDD}(k)$ can be expressed as

$$F_{\Gamma_{AF-HDD}(k)}\left(\gamma\right) = Pr\left(\Gamma_{sd}(k) + \Gamma_{srd}(k) < \gamma\right) \tag{2.75}$$

Now, the CDF $F_{\Gamma_{AF-HDD}(k)}(\gamma)$ conditioned on $\Gamma_{sd}(k)$ can be written as

$$F_{\Gamma_{AF-HDD}(k)}(\gamma) = \int_0^{\gamma}\int_0^{\gamma-\gamma_1} f_{\Gamma_{srd}(k)}(\gamma)\, f_{\Gamma_{sd}(k)}(\gamma_1) d\gamma_1 d\gamma \tag{2.76}$$

Using Equation (2.59), the CDF of $\Gamma_{AF-HDD}(k)$ can be determined as

$$F_{\Gamma_{AF-HDD}(k)}(\gamma) = 1 + \psi_1(k)\exp(-\mu_a(k)\gamma) - \psi_2(k)\exp(-\mu_d(k)\gamma) \tag{2.77}$$

where $\psi_1(k) = (\mu_d(k)/[\mu_a(k) - \mu_d(k)])$, $\psi_2(k) = (\mu_a(k)/[\mu_a(k) - \mu_d(k)])$

$\mu_d(k) = (1/[SNR] + \varsigma_r/SIR_2(k)) + (1/[SNR] + \varsigma_d/SIR_3(k))$

By differentiating Equation (2.77) with respect to γ, the PDF of $\Gamma_{AF-HDD}(k)$ can be determined as

$$f_{\Gamma_{AF-HDD}(k)}(\gamma) = -\psi_1(k)\mu_a(k)\exp(-\mu_a(k)\gamma) + \psi_2(k)\mu_d(k)\exp(-\mu_d(k)\gamma) \tag{2.78}$$

Substituting Equation (2.78) in Equation (2.74), then the average BER of the OFDM based cognitive half duplex AF relay network can be written as

$$\begin{aligned}\tilde{P}_e^{AF-HDD} \geq -\psi_1(k)\mu_a(k)\int_0^{\infty} Q\left(\sqrt{2\gamma}\right)\exp(-\mu_a(k)\gamma)d\gamma + \\ \psi_2(k)\mu_d(k)\int_0^{\infty} Q\left(\sqrt{2\gamma}\right)\exp(-\mu_d(k)\gamma)d\gamma\end{aligned} \tag{2.79}$$

After integration and simplification, the average BER of cognitive half duplex AF relay network is determined as

$$\tilde{P}_e^{AF-HDD} \geq -\psi_1(k)\left[\frac{1}{2}(1-\phi_1(k))\right] + \psi_2(k)\left[\frac{1}{2}(1-\phi_2(k))\right] \tag{2.80}$$

where $\phi_1(k) = \sqrt{\dfrac{SNR}{SNR + INR_2 + 1}}$, $\phi_2(k) = \sqrt{\dfrac{SNR}{SNR + INR_1 + INR_2 + 2}}$

$INR_1 = \sigma_{br}^2/\sigma_{zr}^2$.

2.5.3 Average BER of DF Relay Mode

Although the received signal power is improved by the diversity gain at destination node, overall error probability of the half duplex DF relay network depend on the source to relay error probability. The average BER of the OFDM based cognitive half duplex DF relay network can be expressed as

$$\tilde{P}_e^{DF-HDD} = \tilde{P}_e^{\Gamma_{sr}} + \left(1 - \tilde{P}_e^{\Gamma_{sr}}\right)\tilde{P}_e^{\Gamma_a} \tag{2.81}$$

For BPSK modulation, Equation (2.81) can be written as

$$\tilde{P}_e^{DF-HDD} = E_{\gamma_{sr}}\left(Q\left(\sqrt{2\Gamma_{sr}(k)}\right)\right) + \left(1 - E_{\gamma_{sr}}\left(Q\left(\sqrt{2\Gamma_{sr}(k)}\right)\right)\right)E_{\gamma_a}\left(Q\left(\sqrt{2\Gamma_a(k)}\right)\right) \tag{2.82}$$

Using Equation (2.73), the average BER of the source to relay SINR is determined as

$$\tilde{P}_e^{\Gamma_{sr}} \le \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{SNR + INR_1 + 1}}\right) \tag{2.83}$$

Using (2.64), the CDF of Γ_a can be written as

$$F_{\Gamma_a(k)}(\gamma) = 1 + \psi_3(k)\exp(-\rho_1\gamma) - \psi_4(k)\exp(-\rho_2\gamma) \tag{2.84}$$

where $\psi_3(k) = \mu_c(k)/\left[\mu_a(k) - \mu_c(k)\right]; \psi_4(k) = \mu_a(k)/\left[\mu_a(k) - \mu_c(k)\right]$ $\rho_1(k) = (\mu_a(k))$ and $\rho_2(k) = (\mu_c(k))$.

Differentiating Equation (2.84) with respect to γ, the PDF of Γ_a is obtained as

$$\begin{aligned} f_{\Gamma_a(k)}(\gamma) = & -\rho_1(k)\psi_3(k)\exp\left(-\rho_1(k)\gamma\right) \\ & + \rho_2(k)\psi_4(k)\exp\left(-\rho_2(k)\gamma\right) \end{aligned} \tag{2.85}$$

Using Equation (2.85), the average BER $\tilde{P}_e^{\Gamma_a}$ can be written as

$$\tilde{P}_e^{\Gamma_a} \le -\rho_1(k)\psi_3(k)\int_0^\infty Q\left(\sqrt{2\gamma}\right)\exp\left(-\rho_1(k)\gamma\right)d\gamma$$
$$+\rho_2(k)\psi_4(k)\int_0^\infty Q\left(\sqrt{2\gamma}\right)\exp\left(-\rho_2(k)\gamma\right)d\gamma \tag{2.86}$$

After evaluating the integral, and simplifying, the average BER $\tilde{P}_e^{\Gamma_a}$ is determined as

$$\tilde{P}_e^{\Gamma_a}(k) \le -\rho_1(k)\psi_3(k)\left[0.5\left(1-\phi_3(k)\right)\right]$$
$$+\rho_2(k)\psi_4(k)\left[0.5\left(1-\phi_4(k)\right)\right] \tag{2.87}$$

where $\phi_3(k) = \sqrt{SNR/\left[INR_2 + SNR + 2\right]}$, $\phi_4(k) = \sqrt{SNR/\left[INR_2 + SNR + 2\right]}$.

Substituting Equation (2.83) and Equation (2.87) in Equation (2.82), the end-to-end average BER of the OFDM based cognitive half duplex DF relay network is obtained.

2.6 RESULTS AND DISCUSSION

In this section, outage and average BER performances of the OFDM based cognitive half duplex AF and DF relay networks are analyzed in the presence of NBI. The Numerical parameters of proposed cognitive half duplex AF relay network are listed in Table 2.1.

Table 2.1 Numerical parameters of the proposed cognitive half duplex AF relay network

Symbol	Parameter	Value
N	Number of subcarriers in OFDM signal	64
v	Guard sequence length	16
L	Number of channel taps	10
r	Number of subcarriers with NBI	1,3,5
β	Carrier frequency offset parameter of NBI signal	0.5
R	Data rates (in bits/s/Hz)	0.5,1,2,3
	Signal to NBI Interference ratio (in dB)	10,15,20,25
	Modulation	BPSK

2.6.1 Outage Performance of OFDM based CR Relay Network

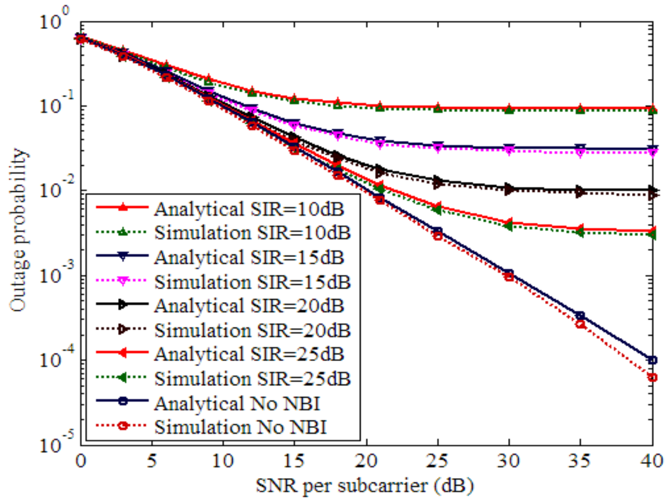

Figure 2.2 Outage performance of OFDM based CR network in direct mode at 1 b/s/Hz

The outage performance of the proposed cognitive AF relay network in direct mode is shown in Figure 2.2 for the data rate of 1 b/s/Hz. The SIR is varied from 10dB to 25dB. It is observed that the analytical results are very closer to the simulation results. As the noise dominates NBI at low SNR, the effect of NBI is more significant at high SNR. In direct mode, as the SIR increases from 20dB to 25dB, the outage probability decreases from 0.01 to 0.003 at the data rate of 1 b/s/Hz and SNR per-subcarrier of 30dB.

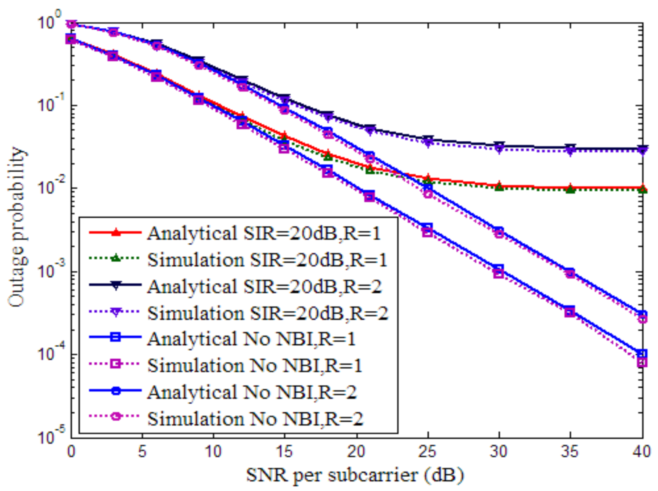

Figure 2.3 Outage performance of OFDM based CR network in direct mode at 1 b/s/Hz and 2 b/s/Hz

Figure 2.3 shows the outage performance of OFDM based CR network in direct mode for the data rates at 1 b/s/Hz and 2 b/s/Hz. It is observed that when the data rate is increased from 1 b/s/Hz to 2 b/s/Hz, the minimum SNR requirement of the OFDM based CR network in direct mode is increases from 20dB to 25dB in the absence of NBI at the maximum outage probability of 0.01. When network operate with SIR of 20dB, the minimum SNR requirement of OFDM based CR network in direct mode is rises to 30dB SNR with data rate of 1 b/s/Hz.

The outage performance of OFDM based CR network in direct mode for the data rates between 0.5 b/s/Hz and 3 b/s/Hz is shown in Figure 2.4. At low data rate, the outage is exactly reduced by 0.1 for every 5dB increase in SIR level. For the maximum outage probability of 0.01, the CR network supports the data rate up to 2 b/s/Hz at 25 dB SIR.

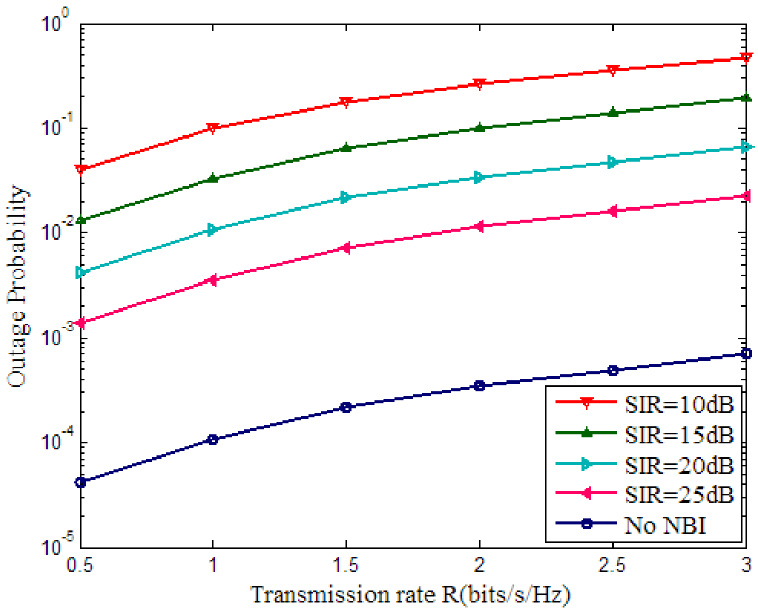

Figure 2.4 Outage performance of Direct mode vs transmission rates for the fixed SNR of 40dB

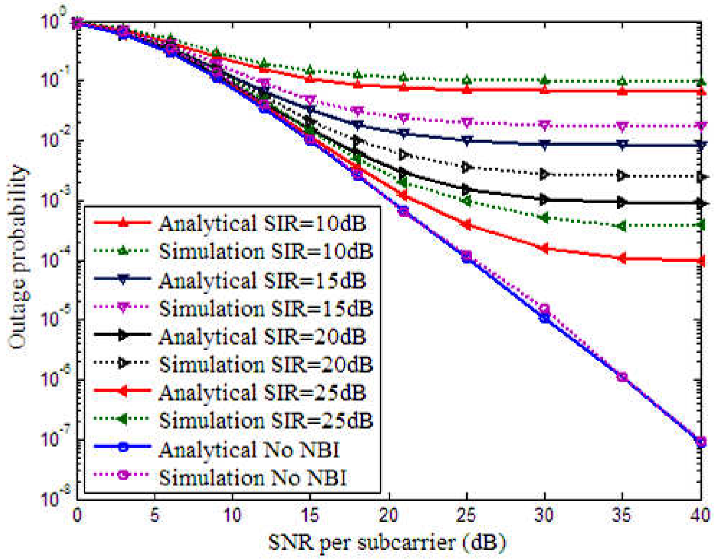

Figure 2.5 Outage performance of AF mode transmission at the data rate of 1b/s/Hz

The outage performance for the proposed OFDM based cognitive AF relay network in AF relay mode is shown in Figure 2.5 at the data rate of 1 b/s/Hz for the different SIR values. Since the MRC provides diversity gain at the destination node, the minimum SNR requirement decreases to 15dB from 20dB in the direct mode transmission at the outage probability of 0.01. In AF relay mode at 30dB SNR, as the SIR increases from 20dB to 25dB, the outage probability decreases from 0.001 to 0.0001 at the data rate of 1b/s/Hz.

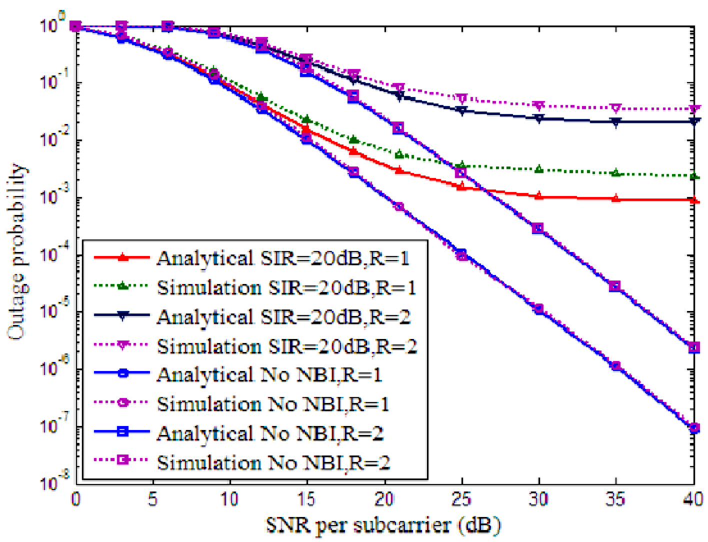

Figure 2.6 Outage performance of OFDM based CR network in AF relay mode at the data rates of 1 b/s/Hz and 2 b/s/Hz

The outage performance of the proposed cognitive half duplex AF relay network in AF relay mode is shown in Figure 2.6 for the data rates of 1 b/s/Hz and 2 b/s/Hz. It is observed that when the data rate moves from 1 b/s/Hz to 2 b/s/Hz, the minimum SNR requirement of the cognitive AF relay network increases from 15dB to 22dB in the absence of NBI at the maximum outage probability of 0.01 and for the network operate with SIR of 20dB it further rises to 17dB SNR with data rate of 1 b/s/Hz and exceed the limit at data rate of 2 b/s/Hz.

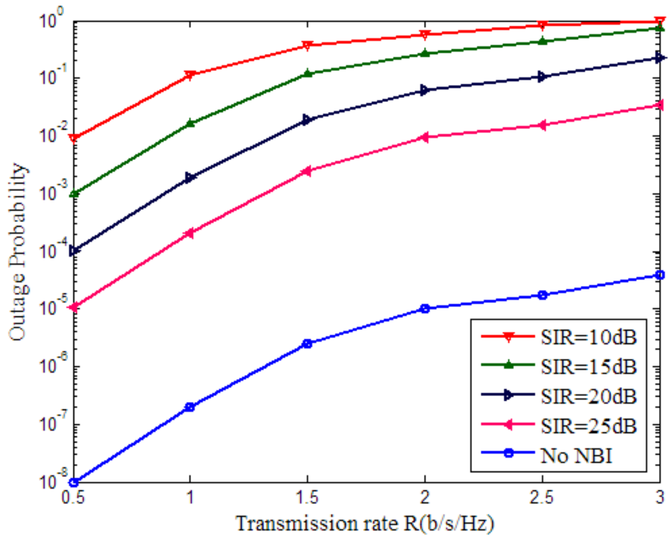

Figure 2.7 Outage performance of cognitive half duplex AF relay network vs transmission rates for the fixed SNR of 40dB

The outage performance of AF relay mode for the data rates between 0.5 b/s/Hz and 3 b/s/Hz is shown in Figure 2.7. At low data rate, the outage is exactly reduced by 0.1 for every 5dB increase in SIR level. For the maximum outage probability of 0.01, the CR network supports the data rate up to 2 b/s/Hz at 25 dB SIR.

The outage performance for the OFDM based cognitive DF relay network is shown in Figure 2.8 at the data rate of 1 b/s/Hz. In this figure, SIR is varied from 20dB to 30dB. At 30dB SNR, as the SIR increases from 20dB to 25dB, the outage probability of cognitive DF relay network decreases from

0.073 to 0.025. Further increasing the SIR to 30dB, outage probability of cognitive DF relay network decreases to 0.01. However, the outage performance of the half duplex DF relay is less when comparing the with cognitive half duplex AF relay network.

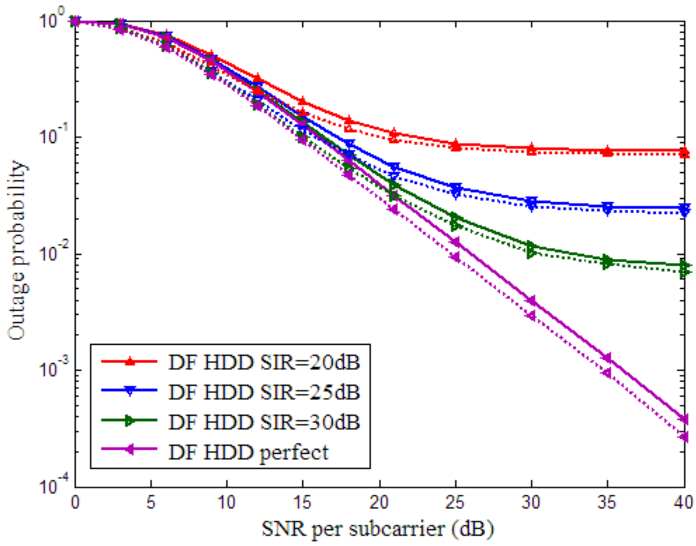

Figure 2.8 Outage performance of OFDM based cognitive DF relay network at the data rate of 1b/s/Hz

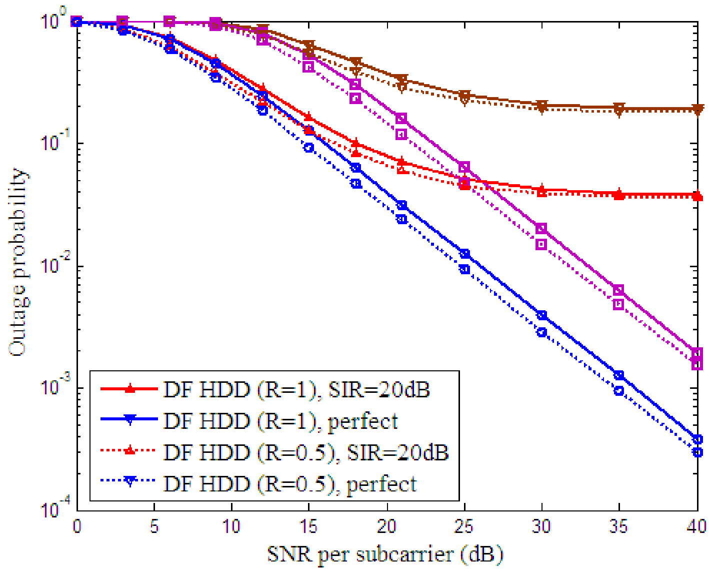

Figure 2.9 Outage performance of OFDM based cognitive DF relay network at the data rates of 1 b/s/Hz and 2 b/s/Hz

The outage performance of the cognitive half duplex DF relay network is shown in Figure 2.9 for the data rates of 1 b/s/Hz and 2 b/s/Hz. It is observed that when the data rate moves from 1 b/s/Hz to 2 b/s/Hz, the minimum SNR requirement of the cognitive AF relay network increases from 15dB to 22dB in the absence of NBI at the maximum outage probability of 0.01 and for the network operate with SIR of 20dB it further rises to 17dB SNR with data rate of 1 b/s/Hz and exceed the limit at data rate of 2 b/s/Hz.

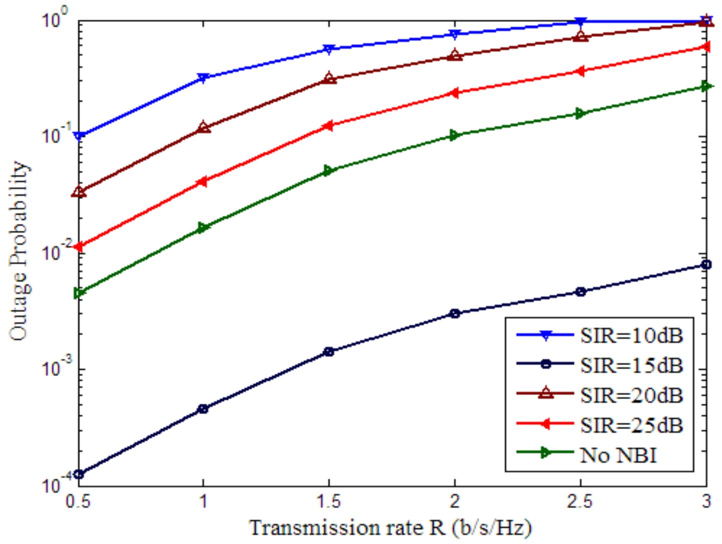

Figure 2.10 Outage performance of OFDM based cognitive DF relay network vs transmission rates for the fixed SNR of 40dB

The outage performance of cognitive half duplex DF relay network for the data rates between 0.5 b/s/Hz and 3 b/s/Hz is shown in Figure 2.10. At low data rate, the outage is exactly reduced by 0.1 for every 5dB increase in SIR level. For the maximum outage probability of 0.01, the CR network supports the data rate up to 2 b/s/Hz at 25 dB SIR.

Figure 2.11 shows the outage performance of OFDM based CR network in direct mode, AF relay mode, DF relay mode and Incremental AF relay mode at the data rate at 1 b/s/Hz. It is observed that incremental AF relay mode of OFDM based CR network achieves the better performance than all other modes. However, in incremental AF relaying, performance of the network is based on the reliable feedback from destination node. Further, it is noted that the performance of the OFDM based CR network in DF relay mode is less than all other relaying modes. At 30dB SNR and SIR of 20 dB, the outage probability of the incremental AF relay is 0.0001, in AF relay mode is 0.0008, in direct mode is 0.008 and in DF relay mode is 0.03.

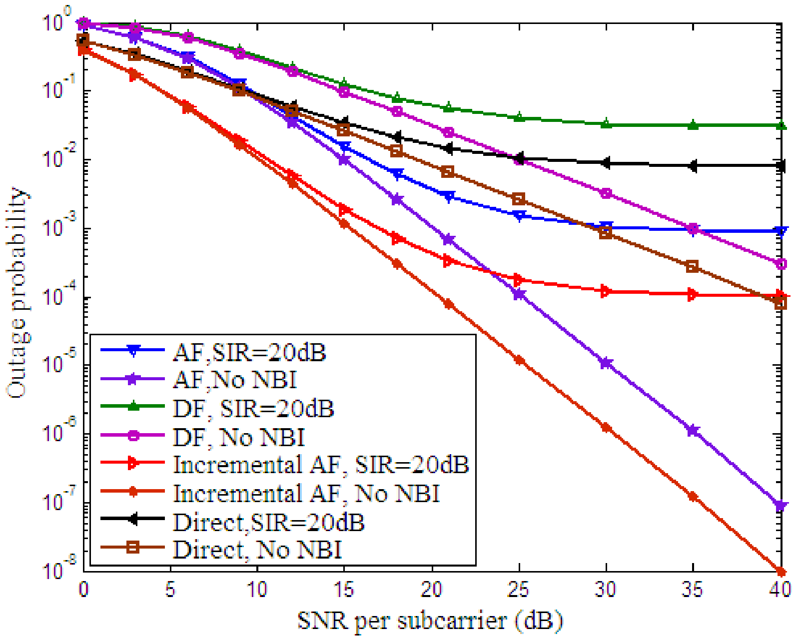

Figure 2.11 Outage performance of OFDM based CR network in direct mode, AF relay mode, DF relay mode and Incremental AF relay mode

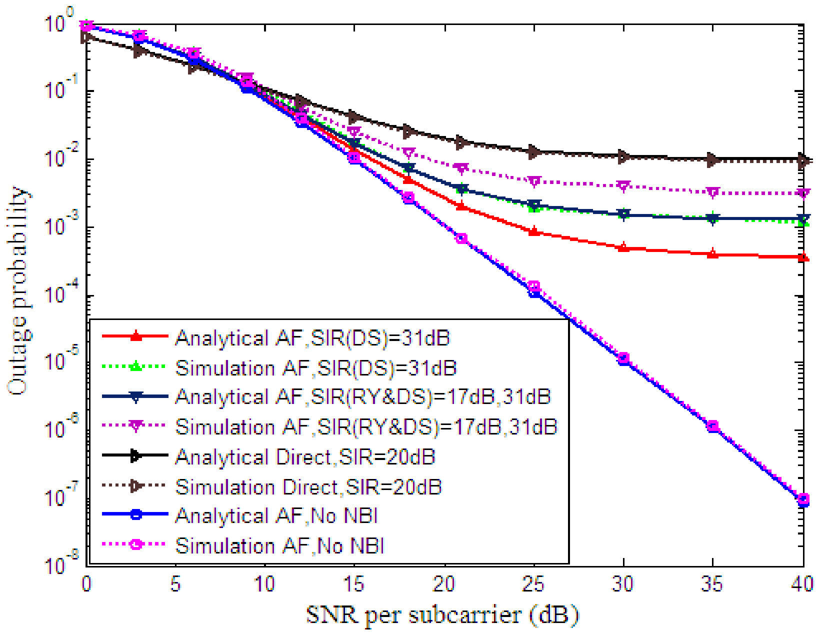

Figure 2.12 Comparison of outage performances in AF relay mode and direct mode at the data rate of 1b/s/Hz with and without NBI at relay

In Figure 2.12, the outage performance of the proposed OFDM based CR network in AF relay mode is compared with direct mode for various relay interference values. The data rate is fixed to 1 b/s/Hz. In this Figure, RY and DS denote the relay and destination nodes. It is noted that the outage probability decreases from 0.001 to 0.0004 when relay node operates with no NBI, at 30dB SNR per-subcarrier. In direct mode, the outage probability of the network is 0.01 for SIR of 20 dB and SNR of 30dB. It indicates that proposed network with limited interference at relay perform better than the direct mode.

2.6.2 Average BER Performance of OFDM based CR Network

The average error probability of OFDM based CR network in direct mode is shown in Figure 2.13. In this figure, SIR varied from 20dB to 30dB. BPSK modulation is used. The mean of desired signal channel $\overline{h}_{sd}$ is fixed to 0dB. For the mean values of the interference channel $\overline{h}_{bd,1}$ at $-20\,\text{dB}$ and SNR at $30\,\text{dB}$, the average BER of the OFDM based CR network in direct mode is 0.002. For mean of the interference channel at $-25\,\text{dB}$ and SNR of 30dB, the average BER of the network in direct mode is 0.001.

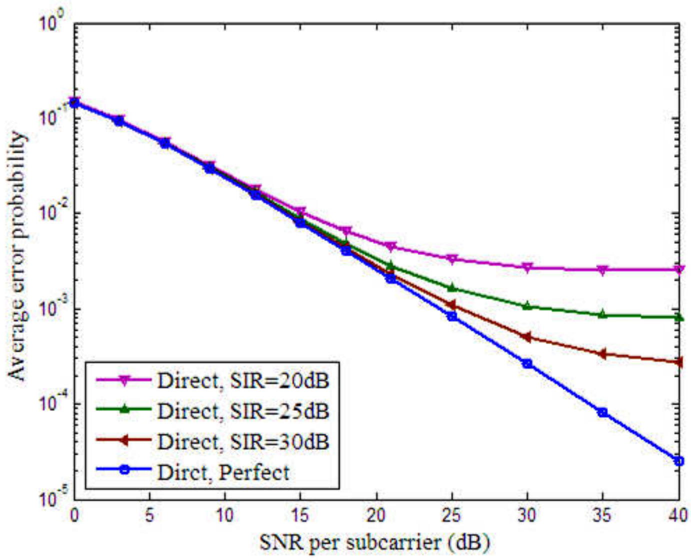

Figure 2.13 Average BER of the OFDM based CR network in direct mode

The average error probability of OFDM based cognitive half duplex AF relay network is shown in Figure 2.14. In this figure, HDD denote the half duplex with diversity. The mean of desired signal channels $\bar{h}_{sd}$ is fixed to 1dB and $\bar{h}_{sr}$, $\bar{h}_{rd}$ are fixed to 0dB. For mean values of the interference channels $\bar{h}_{br}$, $\bar{h}_{bd}$ and $\bar{h}_{bd,1}$ at -20 dB and SNR at 30 dB, the average BER of the OFDM based cognitive AF relay network is 0.0008. For mean of the interference channels at -25 dB and SNR of 30dB, the average BER of the network decreases to 0.0002.

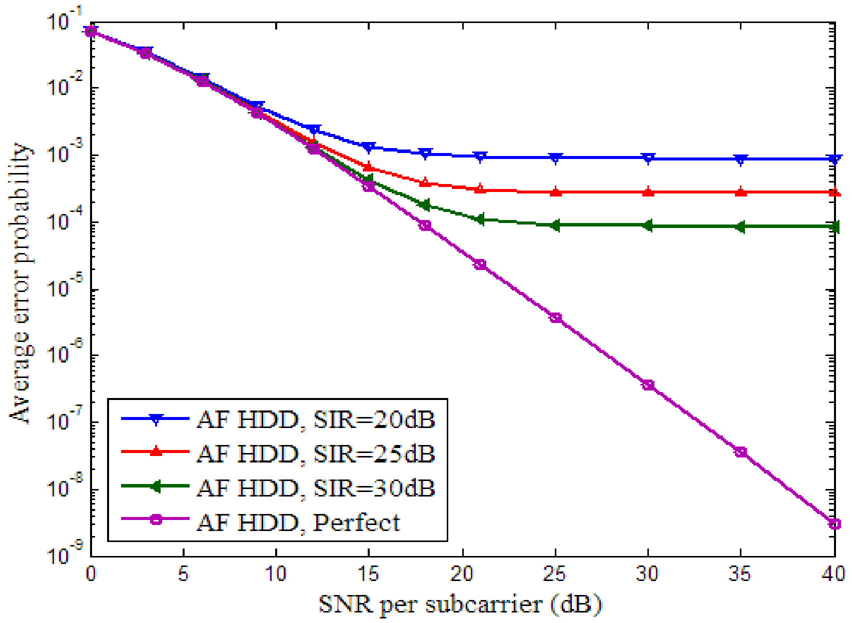

Figure 2.14 Average error probability of cognitive half duplex AF relay network in the presence of NBI

The average error probability of OFDM based cognitive half duplex DF relay network is shown in Figure 2.15. The mean of desired signal channels are same as in previous Figure 2.14. For mean values of the interference channel $\bar{h}_{br}$, $\bar{h}_{bd}$ and $\bar{h}_{bd,1}$ at -20 dB and SNR at 30 dB, the average BER of the OFDM based cognitive DF relay network is 0.0001. For mean of the interference channels at -25 dB and SNR of 30dB, the average BER of the network further decreases to 1.32×10^{-5}.

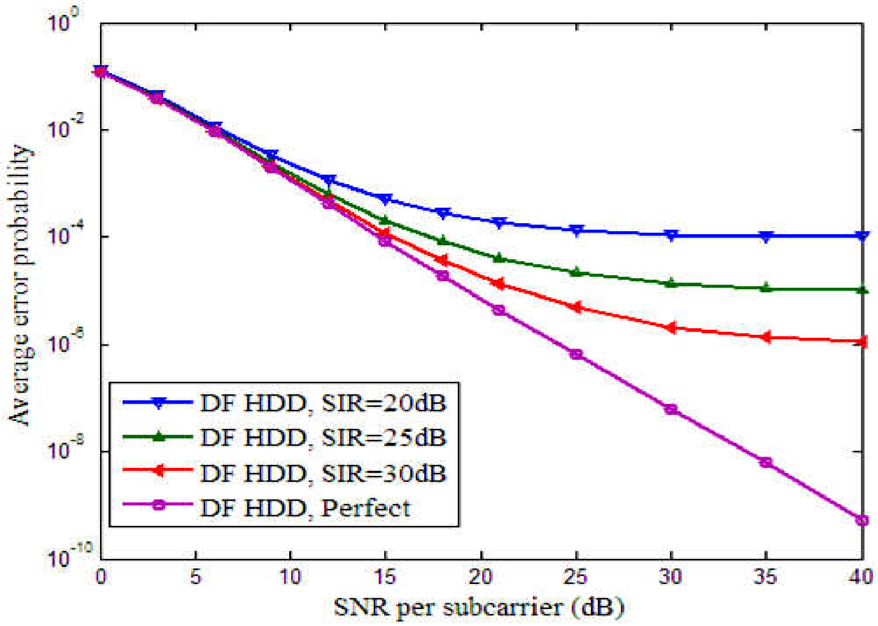

Figure 2.15 Average error probability of cognitive half duplex DF relay network in the presence of NBI

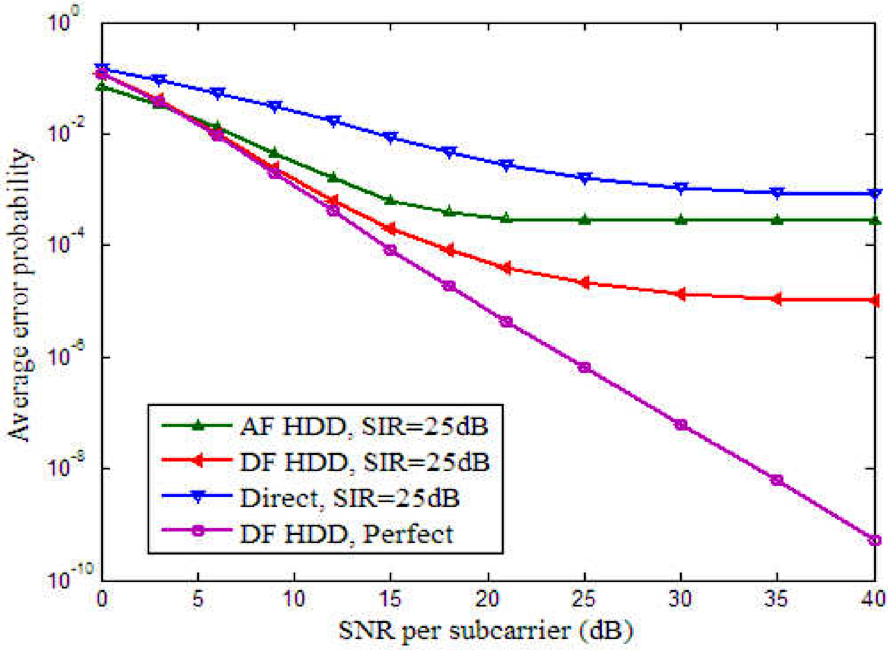

Figure 2.16 Average error probability of OFDM based CR network in AF relay, DF relay and direct mode

In Figure 2.16, the average error probability of OFDM based cognitive half duplex AF relay network is compared with DF relay and Direct mode. The mean of desired signal channels are same as in previous Figure 2.14. For mean values of the interference channel $\bar{h}_{br}, \bar{h}_{bd}$ and $\bar{h}_{bd,1}$ at -20 dB and SNR at 30 dB, the average BER of the OFDM based cognitive AF relay network in direct mode is 0.001, in AF relay mode is 0.0002 and in DF relay mode is 1.33×10^{-5}. It concludes that the OFDM based DF relay network achieves better performance than the OFDM based cognitive half duplex AF relay network.

Figure 2.17 shows the average BER performance of cognitive DF relay network compared with AF relay network for different values of transmitted source power. The mean values of the interference channels $\bar{h}_{br}$, $\bar{h}_{bd}$ and $\bar{h}_{bd,1}$ are fixed to 20 dB. At 25 dB Channel gain to Noise power Ratio γ' (CNR) and source power of 30 dBm, the average BER of AF relay meets the DF relay with source power at 20 dBm. It is evident that the performance of the AF relay improved over the DF relay by increasing the source power. However, the impact of source power is less at high CNR region due to dominant of interference power.

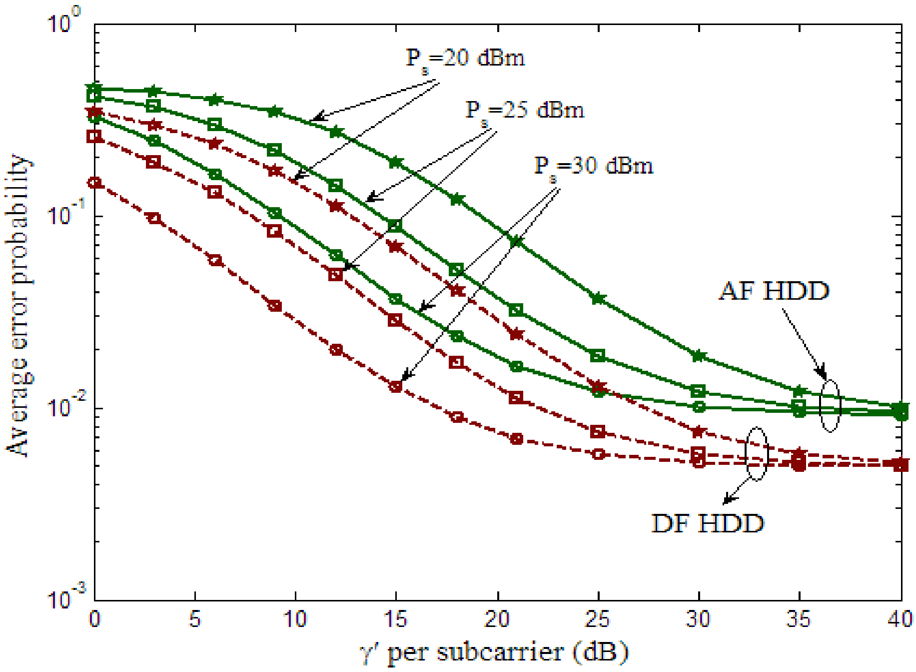

Figure 2.17 Average error probability comparisons in cognitive DF and AF relay network with diversity for different values of source power P_s

2.7 SUMMARY

In this chapter, the performance of OFDM based cognitive half duplex AF relay network is analyzed in the presence of NBI. It is assumed that NBI affect at relay and destination nodes. Closed form analytical expressions for outage probability and average error probability are derived for the proposed OFDM based cognitive half duplex AF relay network. Performance of the proposed network is compared with OFDM based half duplex DF relay network and OFDM based non-cooperative CR network. Simulation results conclude that performance of the proposed network outperforms the OFDM based non-cooperative CR network. Further, outage performance of the OFDM based cognitive DF relay network is less than the proposed CR network.

CHAPTER 3

OFDM BASED FULL DUPLEX COGNITIVE RELAY NETWORK IN THE PRESENCE OF NBI

3.1 PREAMBLE

The main drawback in half duplex transmission is that it loses the temporal efficiency while transmitting over the two time slots thereby reducing the data rate by half. One way to achieve the performance improvement without loss of temporal efficiency is the introduction of full duplex relay in OFDM based CR network. In full duplex mode, the OFDM signal is transmitted and received at relay in a single time slot. The advantage of full duplex mode is that it has full channel degrees of freedom thereby throughput of the network is highly improved. However, the overall system performance is affected due to the presence of loop interference at the relay node and the NBI. By keeping loop interference power at certain level in such a way that the performance of the OFDM based cognitive full duplex relay network in the presence of NBI is highly improved.

In chapter 2, it is observed that outage performance of the OFDM based cognitive AF relay network outperforms the non cooperative OFDM based CR network and the OFDM based cognitive half duplex DF relay network in the presence of NBI. At first, full duplex AF relay is introduced in OFDM based CR network in the presence of NBI. Then, outage performance of the OFDM based cognitive full duplex AF relay network is compared with OFDM based cognitive full duplex DF relay network. Further, performance of the OFDM based cognitive full duplex AF relay network is compared with cognitive half duplex AF relay network without diversity. In addition, analytical expression for the average BER of the OFDM based cognitive full duplex AF relay network is derived and error performance is compared with cognitive full duplex DF relay network.

3.2 SYSTEM MODEL

Consider an OFDM based cognitive full duplex AF relay network with three secondary nodes shown in Figure 3.1. The full duplex relay network can operate in either AF relay mode or DF relay mode. In AF relay mode, relay node $\mathrm{SU_r}$ receives the OFDM signal and at the same time slot it forwards the amplified received OFDM signal in the previous time slot to destination node $\mathrm{SU_d}$. In DF relay mode, relay node $\mathrm{SU_r}$ receives the OFDM signal and at the same time slot, it forward the re-encoded OFDM signal to destination node $\mathrm{SU_d}$ which is decoded in previous time slot. The system is operational only when a common band is available at all the three nodes.

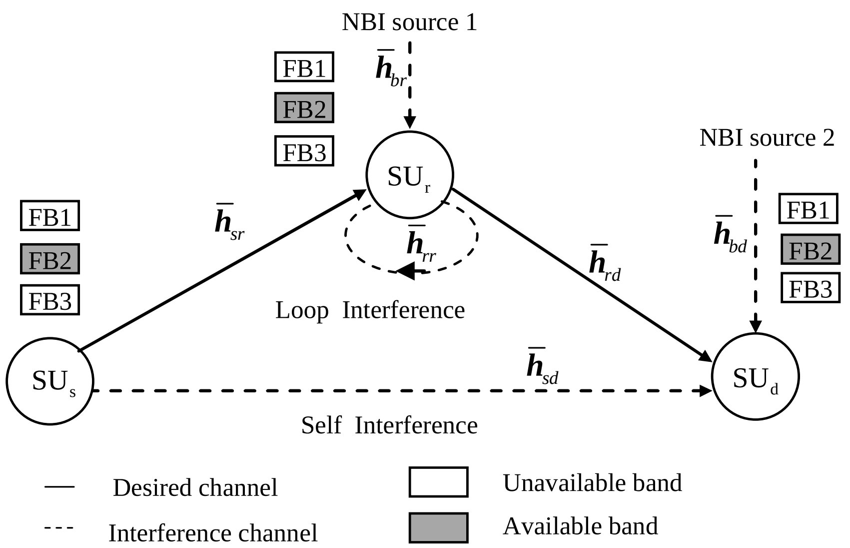

Figure 3.1 Cognitive full duplex Amplify and Forward relay network

3.2.1 AF Relay Mode

The $L_{sr} \times 1$ CIR vector between the source node $\mathrm{SU_s}$ and the relay node $\mathrm{SU_r}$ is denoted by $\bar{\boldsymbol{h}}_{sr}$. The $L_{rr} \times 1$ CIR vector of loop/echo interference at

SU_r is denoted by $\bar{\boldsymbol{h}}_{rr}$. The $L_{br} \times 1$ CIR vector between the NBI and SU_r is denoted by $\bar{\boldsymbol{h}}_{br}$. It is assumed that channel coefficients in CIR vectors $\bar{\boldsymbol{h}}_{sr}$, $\bar{\boldsymbol{h}}_{rr}$ and $\bar{\boldsymbol{h}}_{br}$ are Rayleigh distributed with zero mean and variance σ^2. The OFDM signal contains N data subcarriers appended with v guard subcarriers and the total length is $P = N + v$. The guard length is chosen greater than the length of the channel taps to remove the ISI, that is $v \geq \max[L_{sr}, L_{rr}, L_{br}]$. After removing cyclic prefix, the $N \times 1$ time-domain receive signal vector $\bar{\boldsymbol{y}}_{sr}$ at relay node SU_r in n^{th} signal duration is given by

$$\bar{\boldsymbol{y}}_{sr}^{(n)} = \bar{\boldsymbol{H}}_{sr} \boldsymbol{F}^H \boldsymbol{x}_s^{(n)} + \bar{\boldsymbol{H}}_{rr} \bar{\boldsymbol{x}}_r^{(n)} + \bar{\boldsymbol{A}}_{br} \bar{\boldsymbol{H}}_{br} \bar{\boldsymbol{b}}_r^{(n)} + \bar{\boldsymbol{z}}_r^{(n)} \tag{3.1}$$

where $\bar{\boldsymbol{H}}_{sr}$, $\bar{\boldsymbol{H}}_{rr}$ and $\bar{\boldsymbol{H}}_{br}$ are $N \times N$ circulant matrices of CIR vectors $\bar{\boldsymbol{h}}_{sr}$, $\bar{\boldsymbol{h}}_{rr}$ and $\bar{\boldsymbol{h}}_{br}$ respectively. $\boldsymbol{F}^H$ is $N \times N$ IDFT matrix, $\boldsymbol{x}_s^{(n)}$ is $N \times 1$ data vector from source node in n^{th} duration, $\bar{\boldsymbol{x}}_r^{(n)} = \mathbf{B}\bar{y}_{sr}^{(n-1)}$ is a loop interference signal where $\mathbf{B}$ is a circulant amplification matrix at relay node, $\mathbf{B} = \boldsymbol{F}^H \mathbf{E} \boldsymbol{F}$ and $\mathbf{E}$ is a $N \times N$ diagonal matrix, $\mathbf{E} = diag\left[\beta(1), \beta(2), \ldots \beta(N-1)\right]$, $\beta(k)$ is an amplification factor at k^{th} subcarrier, $\bar{\boldsymbol{b}}_r^{(n)}$ represents the $N \times 1$ NBI signal vector at relay. The CFO of the NBI signal is denoted as $N \times N$ diagonal matrix, $\bar{\boldsymbol{A}}_{br} = diag\left(1, \exp\left(j2\pi\alpha/N\right), \ldots, \exp\left(j2\pi\alpha(N-1)/N\right)\right)$ at relay node, where α is uniformly distributed random variable over the interval $[-0.5, 0.5]$, $\bar{\boldsymbol{z}}_r^{(n)}$ is $N \times 1$ complex white Gaussian noise vector whose elements are independent and identically distributed with zero mean and variance σ_{zr}^2.

After taking DFT, the $N \times 1$ frequency domain receive signal vector $\boldsymbol{y}_{sr}^{(n)}$ is given by

$$\boldsymbol{y}_{sr}^{(n)} = \Lambda_{sr} \boldsymbol{x}_s^{(n)} + \Lambda_{rr} \mathrm{E} \boldsymbol{y}_{sr}^{(n-1)} + \boldsymbol{C}_{br} \Lambda_{br} \boldsymbol{b}_r^{(n)} + \boldsymbol{z}_r^{(n)} \tag{3.2}$$

where $\Lambda_{sr} = \boldsymbol{F}\overline{\boldsymbol{H}}_{sr}\boldsymbol{F}^{H}$, $\Lambda_{rr} = \boldsymbol{F}\overline{\boldsymbol{H}}_{rr}\boldsymbol{F}^{H}$ and $\Lambda_{br} = \boldsymbol{F}\overline{\boldsymbol{H}}_{br}\boldsymbol{F}^{H}$ are the frequency domain diagonal matrices whose elements are N-point DFT of the channel vectors $\overline{\boldsymbol{h}}_{sr}$, $\overline{\boldsymbol{h}}_{rr}$ and $\overline{\boldsymbol{h}}_{br}$ respectively. Mathematically, they are defined as $\Lambda_{xy} = diag\left(\lambda_{xy,0}, \lambda_{xy,1}, \ldots, \lambda_{xy,N-1}\right)$, $xy \in \left(sr, rr, rd\right)$, $\boldsymbol{C}_{br} = \boldsymbol{F}\overline{\boldsymbol{A}}_{br}\boldsymbol{F}^{H}$ is frequency domain circulant CFO matrix, $\boldsymbol{z}_{r}^{(n)} = \boldsymbol{F}\,\overline{\boldsymbol{z}}_{r}^{(n)}$ is frequency domain noise vector, $\boldsymbol{b}_{r}^{(n)} = \boldsymbol{F}\overline{\boldsymbol{b}}_{r}^{(n)}$ is $N \times 1$ NBI signal vector which is sparse in nature whose k^{th} subcarrier element $b_{r}^{(n)}\left(k\right) \neq 0$ if $k \in i_{x}$. The set 'i_x' has the indices of the subcarrier which are affected by NBI.

In the same time slot, the relay node $\mathrm{SU_r}$ transmits the amplified signal to the destination node $\mathrm{SU_d}$. In time domain, transmit signal at the relay node $\mathrm{SU_r}$ is given by

$$\overline{\boldsymbol{x}}_{r}^{(n)} = \mathbf{B}\overline{y}_{sr}^{(n-1)} \tag{3.3}$$

where $\overline{\boldsymbol{y}}_{sr}^{(n-1)}$ is $N \times 1$ receive signal vector at $\left(n-1\right)^{th}$ signal duration. By taking DFT of Equation (3.3) and using $\mathbf{B} = \boldsymbol{F}^{H}\mathrm{E}\boldsymbol{F}$, the transmit signal vector in frequency domain is given by

$$\boldsymbol{x}_{r}^{(n)} = \boldsymbol{F}\,\overline{\boldsymbol{x}}_{r}^{(n)} = \mathrm{E}\,\boldsymbol{y}_{sr}^{(n-1)} \tag{3.4}$$

Now, substituting Equation (3.2) in Equation (3.4), the transmit signal at k^{th} subcarrier in n^{th} duration is expressed as

$$\begin{aligned} x_{r}^{(n)}\left(k\right) = \beta\left(k\right)\Big[& \lambda_{sr,k}x_{s}^{(n-1)}\left(k\right) + \beta\left(k\right)\lambda_{rr,k}y_{sr}^{(n-2)}\left(k\right) \\ & + \sum_{\substack{l=0 \\ l \in i_x}}^{N-1} \lambda_{br,l}c_{br}\left(k, \left(l-k\right)_{N}\right)b_{r}^{(n-1)}\left(l\right) + z_{r}^{(n-1)}\left(k\right) \Big] \end{aligned} \tag{3.5}$$

By recursive substitution for the term $y_{sr}^{(n)}(k)$, Equation (3.5) can be expressed as

$$x_r^{(n)}(k)=\beta(k)\left[\sum_{j=1}^{n}\left(\left(\lambda_{rr,k}\beta(k)\right)^{j-1}\left\{\lambda_{sr,k}x_s^{(n-j)}(k)\right.\right.\right.$$
$$\left.\left.\left.+\sum_{\substack{l=0\\ l\in i_x}}^{N-1}\lambda_{br,l}c_{br}\left(k,(l-k)_N\right)b_r^{(n-j)}(l)+z_r^{(n-j)}(k)\right\}\right)\right] \tag{3.6}$$

For large values of n, the instantaneous transmit power at k^{th} subcarrier is expressed as

$$E\left(\left|x_r^{(n)}(k)\right|^2\right)\leq\beta^2(k)\left(\frac{1}{1-\left|\lambda_{rr,k}\right|^2\beta^2(k)}\right)$$
$$\left\{\left|\lambda_{sr,k}\right|^2P_s+\sum_{l=0,l\in i_x}^{N-1}\left|\lambda_{br,l}\right|^2\varsigma_{br}\sigma_{br}^2+\sigma_{zr}^2\right\} \tag{3.7}$$

where $P_s=E\left(\left|x_s^{(n-1)}(k)\right|^2\right)$ is the transmit power from source node $\mathrm{SU_s}$, $\sigma_{zr}^2=E\left(\left|z_r^{(n-1)}(k)\right|^2\right)$ is the noise variance. Using Cauchy – Schwartz inequality, the expectation of the second term in Equation (3.6) can be expressed as

$$E\left(\left|\sum_{l=0l\in i_x}^{N-1}\lambda_{br,l}c_{br}\left(k,(l-k)_N\right)b_r^{(n\text{-}1)}(l)\right|^2\right)\leq\sum_{l=0,l\in i_x}^{N-1}\left|\lambda_{br,l}\right|^2\varsigma_{br}\sigma_{br}^2 \tag{3.8}$$

where $\sigma_{br}^2=E\left(\sum_{l=0,l\in i_x}^{N-1}\left|b_r^{(n-1)}(l)\right|^2\right)$ is the Interference Power Per-Subcarrier (IPPS) of NBI signal and the squared-norm of CFO is $\varsigma_{br}=\sum_{l=0,l\in i_x}^{N-1}\left|c_{br}\left(k,(l-k)_N\right)\right|^2$.

The condition to guarantee finite relay transmit power P_r is $\beta^2(k)<\left(1/\left|\lambda_{rr}(k,k)\right|^2\right)$. The amplification factor $\beta(k)$ is given by

$$\beta(k) \geq \sqrt{\frac{P_r}{\left|\lambda_{sr,k}\right|^2 P_s + \left|\lambda_{rr,k}\right|^2 P_r + \sum_{l=0, l\in i_x}^{N-1} \left|\lambda_{br,l}\right|^2 \varsigma_{br}\sigma_{br}^2 + \sigma_{zr}^2}} \tag{3.9}$$

By substituting Equation (3.9) in Equation (3.7), the relay transmit power can be determined as $E\left(\left|x_r^{(n)}(k)\right|^2\right) = P_r$ which is same as source node transmit power P_s .

The relay node transmitted signal is received at destination node. It is noted that in full duplex mode, signal received from the source node act as a self interference at the destination node. The $L_{rd} \times 1$ CIR vector between relay node $\mathrm{SU_r}$ and destination node $\mathrm{SU_d}$ is denoted as $\bar{\boldsymbol{h}}_{rd}$. The $L_{sd} \times 1$ CIR vector between source node $\mathrm{SU_s}$ and destination node $\mathrm{SU_d}$ is denoted as $\bar{\boldsymbol{h}}_{sd}$. The $L_{bd} \times 1$ CIR vector between NBI source and destination node $\mathrm{SU_d}$ is defined as $\bar{\boldsymbol{h}}_{bd}$. Assume, $v \geq \max\left(L_{rd}, L_{bd}, L_{sd}\right)$. After removing the cyclic prefix, the time domain receive signal vector $\bar{\boldsymbol{y}}_{rd}^{(n)}$ is given by

$$\bar{\boldsymbol{y}}_{rd}^{(n)} = \bar{\boldsymbol{H}}_{rd}\bar{\boldsymbol{x}}_r^{(n)} + \bar{\boldsymbol{H}}_{sd}\bar{\boldsymbol{x}}_s^{(n)} + \bar{\boldsymbol{A}}_{bd}\bar{\boldsymbol{H}}_{bd}\bar{\boldsymbol{b}}_d^{(n)} + \bar{\boldsymbol{z}}_d^{(n)} \tag{3.10}$$

After taking DFT, the frequency domain representation of the received signal $\bar{\boldsymbol{y}}_{rd}^{(n)}$ is given by

$$\boldsymbol{y}_{rd}^{(n)} = \Lambda_{rd}\boldsymbol{x}_r^{(n)} + \Lambda_{sd}\boldsymbol{x}_s^{(n)} + \boldsymbol{C}_{bd}\,\Lambda_{bd}\boldsymbol{b}_d^{(n)} + \boldsymbol{z}_d^{(n)} \tag{3.11}$$

where $\Lambda_{rd} = \boldsymbol{F}\bar{\boldsymbol{H}}_{rd}\boldsymbol{F}^H$, $\Lambda_{sd} = \boldsymbol{F}\bar{\boldsymbol{H}}_{sd}\boldsymbol{F}^H$ and $\Lambda_{bd} = \boldsymbol{F}\bar{\boldsymbol{H}}_{bd}\boldsymbol{F}^H$ are the frequency domain diagonal matrices whose elements are N-point DFT of the channel vectors $\bar{\boldsymbol{h}}_{rd}$, $\bar{\boldsymbol{h}}_{sd}$ and $\bar{\boldsymbol{h}}_{bd}$ respectively. They are defined as $\Lambda_{xy} = diag\left(\lambda_{xy,0}, \lambda_{xy,2}, \ldots, \lambda_{xy,N-1}\right)$, $xy \in \left(rd, sd, bd\right)$, $\boldsymbol{C}_{bd} = \boldsymbol{F}\bar{\boldsymbol{A}}_{bd}\boldsymbol{F}^H$ is frequency domain circulant CFO matrix, $\boldsymbol{z}_d^{(n)} = \boldsymbol{F}\,\bar{\boldsymbol{z}}_d^{(n)}$ is frequency domain noise vector, $\boldsymbol{b}_d^{(n)} = \boldsymbol{F}\bar{\boldsymbol{b}}_d^{(n)}$ is $N \times 1$ NBI signal vector.

3.2.2 DF Relay Mode

In this mode, relay node decode the received OFDM signal from source node and transmit the re-encoded signal $\boldsymbol{x}_r^{(n)}$ to destination node while relay node receives the next OFDM signal. It is noted that in full duplex mode, signal received from the source node act as a self interference at the destination node.

Assume that and relay node transmit signal power is same as source node power like AF relaying. Therefore, the $N \times 1$ frequency domain received signal vector $\overline{\boldsymbol{y}}_{rd}^{(n),DF}$ in n^{th} signal duration is given by

$$\boldsymbol{y}_{rd}^{(n),DF} = \Lambda_{rd}\boldsymbol{x}_r^{(n)} + \Lambda_{sd}\boldsymbol{x}_s^{(n)} + \boldsymbol{C}_{bd}\,\Lambda_{bd}\boldsymbol{b}_d^{(n)} + \boldsymbol{z}_d^{(n)} \tag{3.12}$$

where Λ_{rd} , Λ_{sd} and Λ_{bd} are the frequency domain diagonal matrices, $\boldsymbol{C}_{bd}$ is frequency domain circulant CFO matrix, $\boldsymbol{z}_d^{(n)}$ is frequency domain noise vector and $\boldsymbol{b}_d^{(n)}$ is $N \times 1$ NBI signal vector.

3.3 SINR ANALYSIS OF OFDM BASED FULL DUPLEX COGNITIVE RELAY NETWORK

In this section, SINR expressions for OFDM based CR network in full duplex AF relay mode and full duplex DF relay mode are derived at subcarrier level.

3.3.1 SINR Analysis in Full Duplex AF Relay Network

Using Equation (3.2), the receive signal $\boldsymbol{y}_{sr}^{(n)}$ at k^{th} subcarrier is expressed as

$$\begin{aligned} y_{sr}^{(n)}(k) &= \lambda_{sr,k}x_s^{(n)}(k) + \lambda_{rr,k}x_r^{(n)}(k) \\ &\quad + \sum_{l=0,l\in i_x}^{N-1} \lambda_{br,l}c_{br}\left(k,(l-k)_N\right)b_r^{(n)}(l) + z_r^{(n)}(k) \end{aligned} \tag{3.13}$$

In this expression, the first term is the desired signal, second and third terms denote the loop interference and NBI respectively and fourth term is the noise at k^{th} subcarrier. Assuming that, $\lambda_{sr}, \lambda_{rr}$ and λ_{br} are known, the average SINR of the receive signal $y_{sr}^{(n)}(k)$ is given by

$$\Gamma_{sr}^{FD}(k) = \frac{\left|\lambda_{sr,k}\right|^2 E\left(\left|x_s^{(n)}(k)\right|^2\right)}{\left\{\left|\lambda_{rr,k}\right|^2 E\left(\left|x_r^{(n)}(q)\right|^2\right) + E\left(\left|\sum_{l=0,l\in i_x}^{N-1} \lambda_{br,l} c_{br}\left(k,(l-k)_P\right) b_r^{(n)}(l)\right|^2\right) + E\left(\left|z_r^{(n)}(k)\right|^2\right)\right\}} \tag{3.14}$$

Using Cauchy-Schwartz inequality, the second term in the denominator of Equation (3.14) can be simplified as

$$E\left(\left|\sum_{l=0,l\in i_x}^{N-1} \lambda_{br,l} c_{br}\left(k,(l-k)_P\right) b_r^{(n)}(l)\right|^2\right) \le \sum_{l=0,l\in i_x}^{N-1} \left|\lambda_{br,l}\right|^2 \varsigma_{br} \sigma_{br}^2 \tag{3.15}$$

where $\varsigma_{br} = \sum_{l=0,l\in i_x}^{N-1} \left|c_{br}\left(k,(l-k)_N\right)\right|^2$ is the squared-norm of the CFO term, $\sigma_{br}^2 = \left(\sum_{l=0,l\in i_x}^{N-1} \left|b_r^{(n)}(l)\right|^2\right)$ is the IPPS of NBI term. Substituting Equation (3.15) in Equation (3.14), the lower bound average SINR $\Gamma_{sr}^{FD}(k)$ is obtained as

$$\Gamma_{sr}^{FD}(k) \ge \frac{\left|\lambda_{sr,k}\right|^2 \mathrm{P}_s}{\left|\lambda_{rr,k}\right|^2 \mathrm{P}_r + \sum_{l=0,l\in i_x}^{N-1} \left|\lambda_{br,l}\right|^2 \varsigma_{br}\sigma_{br}^2 + \sigma_{zr}^2} \tag{3.16}$$

The receive signal $y_{rd}^{(n)}$ at k^{th} subcarrier in n^{th} signal duration is expressed as

$$\begin{aligned} y_{rd}^{(n)}(k) &= \lambda_{rd,k} x_r^{(n)}(k) + \lambda_{sd,k} x_s^{(n)}(k) \\ &+ \sum_{l=0,l\in i_y}^{N-1} \lambda_{bd,l} c_{bd}\left(k,(l-k)_N\right) b_d^{(n)}(l) + z_d^{(n)}(k) \end{aligned} \tag{3.17}$$

In this expression, the first term is the desired signal, second and third terms denote the loop interference and NBI respectively and fourth term is the noise at k^{th} subcarrier. Assuming that, λ_{rd}, λ_{sd} and λ_{bd} are known, the average SINR of the receive signal $y_{rd}^{(n)}(k)$ is given by

$$\Gamma_{rd}^{FD}(k) = \frac{\left|\lambda_{rd,k}\right|^2 E\left(\left|x_r^{(n)}(k)\right|^2\right)}{\left\{\left|\lambda_{sd,k}\right|^2 E\left(\left|x_s^{(n)}(q)\right|^2\right) + E\left(\left|\sum_{l=0,l\in i_y}^{N-1} \lambda_{bd,l} c_{bd}\left(k,(l-k)_P\right) b_d^{(n)}(l)\right|^2\right) + E\left(\left|z_d^{(n)}(k)\right|^2\right)\right\}} \tag{3.18}$$

Using Cauchy-Schwartz inequality, the second term in the denominator of Equation (3.17) can be simplified as

$$E\left(\left|\sum_{l=0,l\in i_y}^{N-1} \lambda_{bd,l} c_{bd}\left(k,(l-k)_P\right) b_d^{(n)}(l)\right|^2\right) \leq \sum_{l=0,l\in i_y}^{N-1} \left|\lambda_{bd}(l)\right|^2 \varsigma_{bd} \sigma_{bd}^2 \tag{3.19}$$

where $\varsigma_{bd} = \sum_{l=0,l\in i_y}^{N-1} \left|c_{bd}\left(k,(l-k)_N\right)\right|^2$ is the squared-norm of the CFO at destination, $\sigma_{bd}^2 = \left(\sum_{l=0,l\in i_y}^{N-1} \left|b_d^{(n)}(l)\right|^2\right)$ is the IPPS of NBI at destination. Substituting Equation (3.19) in Equation (3.18), the lower bound average SINR $\Gamma_{rd}^{FD}(k)$ is obtained as

$$\Gamma_{rd}^{FD}(k) \geq \frac{\left|\lambda_{rd,k}\right|^2 \mathrm{P}_r}{\left|\lambda_{sd,k}\right|^2 \mathrm{P}_s + \sum_{l=0,l\in i_y}^{N-1} \left|\lambda_{bd,l}\right|^2 \varsigma_{bd} \sigma_{bd}^2 + \sigma_{zd}^2} \tag{3.20}$$

3.3.2 SINR Analysis in Full Duplex DF Relay Network

The receive signal $y_{rd}^{(n),DF}$ at k^{th} subcarrier in n^{th} signal duration is expressed as

$$y_{rd}^{(n),DF}(k) = \lambda_{rd,k} x_r^{(n)}(k) + \lambda_{sd,k} x_s^{(n)}(k) + \sum_{l=0, l \in i_y}^{N-1} \lambda_{bd,l} c_{bd}\left(k,(l-k)_N\right) b_d^{(n)}(l) + z_d^{(n)}(k) \tag{3.21}$$

In this expression, the first term is the desired signal, second and third terms denote the loop interference and NBI respectively and fourth term is the noise at k^{th} subcarrier. Assuming that, λ_{rd}, λ_{sd} and λ_{bd} are known, the average SINR of the receive signal $y_{rd}^{(n),DF}(k)$ is given by

$$\Gamma_{rd}^{FD-DF}(k) \geq \frac{\left|\lambda_{rd}(k,k)\right|^2 \mathrm{P}_r}{\left|\lambda_{sd}(k,k)\right|^2 \mathrm{P}_s + \sum_{l=0, l \in i_y}^{N-1} \left|\lambda_{bd,l}\right|^2 \varsigma_{bd} \sigma_{bd}^2 + \sigma_{zd}^2} \tag{3.22}$$

3.4 OUTAGE ANALYSIS OF OFDM BASED FULL DUPLEX COGNITIVE RELAY NETWORK

In this section, analytical expression for end-to-end outage probability of OFDM based full duplex cognitive AF and DF relay networks are derived at subcarrier level.

3.4.1 Outage Probability of OFDM Based Cognitive Full Duplex AF Relay Network

The end-to-end outage probability of the cognitive full duplex AF relay network at k^{th} subcarrier for the given data rate of R b/s/Hz is defined as

$$P_{out,k}^{AF-FD}(R) = \Pr\left(\log\left(1 + \Gamma_{s\,r\,d}^{FD}(k)\right) < R\right) \tag{3.23}$$

where $\Gamma_{s\,r\,d}^{FD}(k)$ is the overall SINR of the cognitive full duplex AF relay network. Since, in AF relay network, maximum data rate of the network depends on minimum of $\Gamma_{sr}^{FD}(k)$ and $\Gamma_{rd}^{FD}(k)$, the overall SINR can be upper bounded as $\Gamma_{s\,r\,d}^{FD}(k) \le \min\left(\Gamma_{sr}^{FD}(k), \Gamma_{rd}^{FD}(k)\right)$.

Let the threshold SINR be $\gamma = 2^{R} - 1$, then the outage probability of cognitive full duplex AF relay network is written as

$$P_{out,k}^{AF-FD}(\gamma) \ge \int_{0}^{\gamma} f_{\Gamma_{srd}^{FD}(k)}(\gamma)\, d\gamma \tag{3.24}$$

The CDF of $\Gamma_{s\,r\,d}^{FD}(k)$ can be written as (Rabiei et al 2011)

$$F_{\Gamma_{s\,r\,d}^{FD}(k)}(\gamma) \ge 1 - \left[\left(1 - F_{\Gamma_{sr}^{FD}(k)}(\gamma)\right)\left(1 - F_{\Gamma_{rd}^{FD}(k)}(\gamma)\right)\right] \tag{3.25}$$

The CDF of the source to relay SINR $\Gamma_{sr}^{FD}(k)$ can be expressed as

$$F_{\Gamma_{sr}^{FD}(k)}(\gamma) = P\left(\Gamma_{sr}^{FD}(k) < \gamma\right) \tag{3.26}$$

Let $\Gamma_{sr}^{FD}(k) = \dfrac{X(k)}{Y(k) + \varsigma_{br} Z(k) + b}$, where $X(k) = \left|\lambda_{sr,k}\right|^{2}$ and $Y(k) = \left|\lambda_{rr,k}\right|^{2}$ are exponentially distributed with mean values $\mu_{sr}(k)$ and $\mu_{rr}(k)$, $Z(k) = \sum_{l=0, l \in i_x}^{N-1} \left|\lambda_{br}(k,l)\right|^{2}$ is chi-square distributed with $2r$ degrees of freedom with scaling factor of $\mu_{br}(k)/r$ and $b = (1/SNR) = \left(\sigma_{z_r}^{2} / P_s\right)$ (Proakis & Salehi 2008).

The CDF of $\Gamma_{sr}^{FD}(k)$ for the given random variables $Y(k)$ and $Z(k)$ is given by

$$F_{\Gamma_{sr}^{FD}}(\gamma) \le \int_{0}^{\infty}\int_{0}^{\infty} P\left(X(k) < \gamma\left(y(k) + \varsigma_{br} z(k) + b\right)\Big|_{Y(k)=y(k), Z(k)=z(k)}\right) f_{Y(k)}\left(y(k)\right) f_{Z(k)}\left(z(k)\right) dy(k)\, dz(k) \tag{3.27}$$

Then, Equation (3.27) can be simplified as

$$F_{\Gamma_{sr}^{FD}(k)}(\gamma) = \int_0^\infty \int_0^\infty \int_0^{\gamma(y(k)+\varsigma_{br}z(k)+b)} f_{X(k)}(x(k)) f_{Y(k)}(y(k)) \times f_{Z(k)}(z(k)) dx(k) dy(k) dz(k) \tag{3.28}$$

where $f_{X(k)}(x(k))$, $f_{Y(k)}(y(k))$ and $f_{Z(k)}(z(k))$ are the PDF of $X(k)$, $Y(k)$ and $Z(k)$ respectively. Substituting PDF of $X(k)$ in Equation (3.28) and evaluating the inner integral, it becomes

$$F_{\Gamma_{sr}^{FD}}(\gamma) \le \int_0^\infty \int_0^\infty \left[1 - \exp\left(-\gamma\left(y(k) + \varsigma_{br} z(k) + b\right)\right)\right] \times f_{Y(k)}(y(k)) f_{Z(k)}(z(k)) dy(k) dz(k) \tag{3.29}$$

Substituting the PDF of $Y(k)$ and $Z(k)$ in Equation (3.29) and then integrating, the CDF of $\Gamma_{sr}^{FD}(k)$ is determined as

$$F_{\Gamma_{sr}^{FD}(k)}(\gamma) = 1 - \frac{\exp\left(-\gamma / \mu_{sr}(k) SNR\right)\left(1 + \left(\gamma\varsigma_{br} / rSIR_1(k)\right)\right)^{-r}}{\left(\gamma / SIR_2(k)\right) + 1} \tag{3.30}$$

where $SNR = \mu_{sr}(k) P_s / \sigma_{zr}^2$ is the Signal to Noise Ratio, $SIR_1(k) = \mu_{sr}(k) / \mu_{br}(k)$, $SIR_2(k) = \mu_{sr}(k) / \mu_{rr}(k)$ is Signal to Interference Ratio, Using $\left(1 + (a/r)\right)^r \approx \exp(-a)$, it becomes

$$F_{\Gamma_{sr}^{FD}(k)}(\gamma) = 1 - \frac{\exp\left(-\gamma\left[\left(1/\mu_{sr}(k) SNR\right) + \left(\varsigma_{br} / SIR_1(k)\right)\right]\right)}{\left(\gamma / SIR_2(k)\right) + 1} \tag{3.31}$$

Let $\mu_a(k) = \left(1/\left(\mu_{sr}(k) SNR\right) + \varsigma_{br} / SIR_1(k)\right)$ and $\mu_b(k) = 1/SIR_2(k)$. Then, Equation (3.31) can be written as

$$F_{\Gamma_{sr}^{FD}(k)}(\gamma) = 1 - \left[\exp\left(-\mu_a(k)\gamma\right) / \left(\mu_b(k)\gamma + 1\right)\right] \tag{3.32}$$

Similarly, the CDF of the SINR $\Gamma_{rd}^{FD}(k)$ can be determined as

$$F_{\Gamma_{rd}^{FD}(k)}(\gamma) = 1 - \left[\exp\left(-\mu_c(k)\gamma\right) / \left(\mu_d(k)\gamma + 1\right)\right] \tag{3.33}$$

where $\mu_c(k) = \left(1/\left[\mu_{rd}(k)SNR\right] + \varsigma_{bd}/SIR_3(k)\right)$ and $\mu_d(k) = 1/SIR_4(k)$, $SIR_3(k) = \mu_{rd}(k)/\mu_{bd}(k)$, $SIR_4(k) = \mu_{rd}(k)/\mu_{sd}(k)$ and $SNR = \left(P_r/\sigma_{zd}^2\right)$.

By substituting Equation (3.32) and Equation (3.33) in Equation (3.25), the lower bound end-to-end outage probability of cognitive full duplex AF relay network is determined as

$$\begin{aligned} P_{out,k}^{AF-FD}(R) \geq 1 - \Big[& \left\{1 - \left(1 - \left[\exp\left(-\mu_a(k)\gamma\right)\left(1 - \mu_b(k)\gamma\right)\right]\right)\right\} \\ & \times \left\{1 - \left(1 - \left[\exp\left(-\mu_c(k)\gamma\right)\left(1 - \mu_d(k)\gamma\right)\right]\right)\right\} \Big] \end{aligned} \tag{3.34}$$

Finally, it can be simplified as

$$\begin{aligned} P_{out,k}^{AF-FD}(R) \geq 1 - & \exp\left(-\left(\mu_a(k) + \mu_c(k)\right)\gamma\right) \\ & \times \left(1 - \left(\mu_b(k) + \mu_d(k)\right)\gamma + \mu_b(k)\mu_d(k)\gamma^2\right) \end{aligned} \tag{3.35}$$

3.4.2 Outage Probability of OFDM Based Cognitive Half Duplex AF Relay Network without Direct Link

The end-to-end outage probability of the cognitive half duplex AF relay network at k^{th} subcarrier for the given data rate of R b/s/Hz is defined as

$$P_{out,k}^{AF-HD}(R) = \frac{1}{2}\Pr\left(\log\left(1 + \Gamma_{s\,r\,d}^{HD}(k)\right) < R\right) \tag{3.36}$$

where $\Gamma_{s\,r\,d}^{HD}(k)$ is the overall SINR of the cognitive half duplex AF relay network without diversity. Then, it can be rewritten as

$$P_{out,k}^{AF-HD}(R) \geq \int_0^{\gamma} f_{\Gamma_{srd}^{HD}(k)}(\gamma)\, d\gamma \tag{3.37}$$

Using Equation (3.35), the CDF of $\Gamma_{s\,r\,d}^{HD}(k)$ is determined as

$$F_{\Gamma_{srd}^{HD}}(\gamma) \geq 1 - \exp\left(-\left(\mu_a(k) + \mu_c(k)\right)\gamma\right) \tag{3.38}$$

Therefore, the lower bound outage probability of OFDM based cognitive half duplex AF relay network is determined as

$$P_{out,k}^{AF-HD}(R) \geq 1-\exp\left(-\left(\mu_a(k)+\mu_c(k)\right)\gamma\right) \tag{3.39}$$

3.4.3 Outage Probability of OFDM Based Cognitive Full Duplex DF Relay Network

The end-to-end outage probability of cognitive full duplex DF relay network at k^{th} subcarrier for the given data rate of R b/s/Hz is defined as

$$P_{out,k}^{DF-FD}(R) = \Pr\left(I_{DF}(k) < R\right) \tag{3.40}$$

where I_{DF} is the maximum average mutual information of full duplex DF relay network and it can be expressed as

$$I_{DF}(k) = \min\left[\log\left(1+\Gamma_{sr}(k)\right), \log\left(1+\Gamma_{rd}(k)\right)\right] \tag{3.41}$$

In DF relay network, outage occurs when SINR $\Gamma_{sr}(k)$ or $\Gamma_{rd}(k)$ does not support the required SINR. Therefore, the end-to-end outage probability of OFDM based cognitive DF relay network is given by

$$\begin{aligned} P_{out,k}^{DF-FD}(R) \leq & \Pr\left(\Gamma_{sr}(k) < 2^R-1\right) + \left[1-\Pr\left(\Gamma_{sr}(k) < 2^R-1\right)\right] \\ & \times \Pr\left(\Gamma_{rd}(k) < 2^R-1\right) \end{aligned} \tag{3.42}$$

Using Equation (3.32) and Equation (3.33), the end-to-end outage probability of the OFDM based cognitive full duplex DF relay network is determined as

$$\begin{aligned} P_{out,k}^{DF-FD}(R) \leq 1 - & \left[\exp\left(-\mu_a(k)\gamma\right)\left(1-\mu_b(k)\gamma\right)\right] \\ & + \left[\exp\left(-\mu_a(k)\gamma\right)\left(1-\mu_b(k)\gamma\right)\right] \\ & \times \left\{1-\left[\exp\left(-\mu_c(k)\gamma\right)\left(1-\mu_d(k)\gamma\right)\right]\right\} \end{aligned} \tag{3.43}$$

After multiplication, Equation (3.43) can be simplified as

$$P_{out,k}^{DF-FD}(R) \le 1 - \left[\exp\left(-\mu_a(k)\gamma\right)\left(1-\mu_b(k)\gamma\right)\right] \times \left[\exp\left(-\mu_c(k)\gamma\right)\left(1-\mu_d(k)\gamma\right)\right] \tag{3.44}$$

After simplification, the upper bound end-to-end outage probability of OFDM based cognitive full duplex DF relay network is obtained as

$$P_{out,k}^{DF-FD}(R) \le 1 - \exp\left(-\left(\mu_a(k)+\mu_c(k)\right)\gamma\right) \times \left(1-\left(\mu_b(k)+\mu_d(k)\right)\gamma + \mu_b(k)\mu_d(k)\gamma^2\right) \tag{3.45}$$

It is observed that in full duplex transmission, the end-to-end outage probability of the OFDM based cognitive DF relay network and cognitive AF relay network are same.

3.5 AVERAGE BER ANALYSIS OF OFDM BASED COGNITIVE FULL DUPLEX RELAY NETWORK

In this section, end-to-end average BER of OFDM based full duplex cognitive AF relay network and full duplex DF relay network is derived at subcarrier level.

3.5.1 Average BER of the OFDM Based Cognitive Full Duplex AF Relay Network

The average BER of the OFDM based cognitive full duplex AF relay network can be expressed as

$$\tilde{P}_e^{AF-FD} \ge \int_0^\infty \bar{N}_e Q\left(\sqrt{\left(d_{\min}^2/2\right)\gamma}\right) f_{\Gamma_{srd}^{FD}(k)}(\gamma)\, d\gamma \tag{3.46}$$

where $Q(.)$ is the Q-function, $d_{\min}$ is the minimum distance, , $\bar{N}_e$ is the number of nearest neighbors (Proakis & Salehi 2008) and $f_{\Gamma_{srd}^{FD}(k)}(\gamma)$ is the PDF of

$\Gamma_{srd}^{FD}(k)$. In this model, BPSK modulation is used that is $d_{\min}=2$. Differentiating Equation (3.44) with respect to γ , the PDF of $\Gamma_{srd}^{FD}(k)$ is obtained as

$$\begin{aligned} f_{\Gamma_{srd}^{FD}(k)}(\gamma) &= \eta_1(k)\exp(-\rho_1(k)\gamma) - \eta_2(k)\gamma\exp(-\rho_1(k)\gamma) \\ &\quad - \eta_3(k)\gamma^2\exp(-\rho_1(k)\gamma) \end{aligned} \tag{3.47}$$

where $\rho_1(k)=\mu_a(k)+\mu_c(k)$, $\eta_1(k)=\left[\mu_a(k)+\mu_c(k)+\mu_b(k)+\mu_d(k)\right]$,

$\eta_2(k)=\left[(\mu_a(k)+\mu_c(k))(\mu_b(k)+\mu_d(k))\right]$ and

$\eta_3(k)=\left[(\mu_a(k)+\mu_c(k))(\mu_b(k)\mu_d(k))\right]$

Substituting Equation (3.47) in Equation (3.46), then the average BER of the cognitive full duplex AF relay network at k^{th} subcarrier is written as

$$\begin{aligned} \tilde{P}_e^{AF-FD} \geq\ & \eta_1(k)\int_0^\infty Q\left(\sqrt{2\gamma}\right)\exp(-\rho_1(k)\gamma)\,d\gamma \\ & -\eta_2(k)\gamma\int_0^\infty Q\left(\sqrt{2\gamma}\right)\exp(-\rho_1(k)\gamma)\,d\gamma \\ & -\eta_3(k)\gamma^2\int_0^\infty Q\left(\sqrt{2\gamma}\right)\exp(-\rho_1(k)\gamma)\,d\gamma \end{aligned} \tag{3.48}$$

After evaluating the integral Q-function, the average BER of cognitive full duplex AF relay network is determined as

$$\tilde{P}_e^{AF-FD} \geq \frac{\eta_1(k)}{2\rho_1(k)}\Phi_1(k) - \frac{\eta_2(k)}{2\rho_1^2(k)}\Phi_2(k) - \frac{\eta_3(k)}{2\rho_1^2(k)}\Phi_3(k) \tag{3.49}$$

where $\Phi_1(k)=\left[1-\Phi(k)\right]$, $\Phi_2(k)=\left[\left(0.5(1-\Phi(k))^2(2+\Phi(k))\right)\right]$

$\Phi_3(k)=\left[(1-\Phi(k))^3(1+1.5(1+\Phi(k)))+1.5(1+\Phi(k))\right]$ in which

$\Phi(k)=\sqrt{SNR/[INR_1+INR_2+SNR+2]}$, $INR_1=\left(\sigma_{b_r}^2/\sigma_{z_r}^2\right)$ and

$INR_2 = \left(\sigma_{b_d}^2 / \sigma_{z_d}^2\right)$ is the Interference to Noise Ratio.

3.5.2 Average BER of the Cognitive Half Duplex AF Relay Network without Direct Link

The average error probability of the half duplex cognitive AF relay network without additional diversity can be expressed as

$$\tilde{P}_e^{AF-HD} \geq \int_0^\infty \bar{N}_e Q\left(\sqrt{\left(d_{\min}^2/2\right)\gamma}\right) f_{\Gamma_{srd}^{HD}(k)}(\gamma)\, d\gamma \qquad (3.50)$$

where $f_{\Gamma_{srd}^{HD}(k)}(\gamma)$ is the PDF of overall SINR $\Gamma_{srd}^{HD}(k)$.

By substituting the PDF of $\Gamma_{srd}^{HD}(k)$ from Equation (3.38) in Equation (3.50) and evaluating integral Q-function, the average BER of the OFDM based cognitive half duplex AF relay network without additional diversity is determined as

$$\tilde{P}_e^{AF-HD} \geq 0.5\left(1 - \sqrt{SNR/\left[INR_1 + INR_2 + SNR + 2\right]}\right) \qquad (3.51)$$

3.6 RESULTS AND DISCUSSION

In this section, outage and average BER performances of the OFDM based cognitive full duplex AF and DF relay networks are analyzed in the presence of NBI. The numerical parameters of cognitive full duplex AF relay network are listed in Table 3.1.

Table 3.1 Numerical parameters of cognitive full duplex AF relay network

Symbol	Parameters	Value
N	Number of subcarriers in OFDM signal	64
v	Guard sequence length	16
L	Number of channel taps	10
r	Number of subcarriers with NBI	1,3,5
β	Carrier frequency offset parameter of NBI signal	0.5
R	Data rates (in bits/s/Hz)	0.5,1,2,3
	Modulation	BPSK

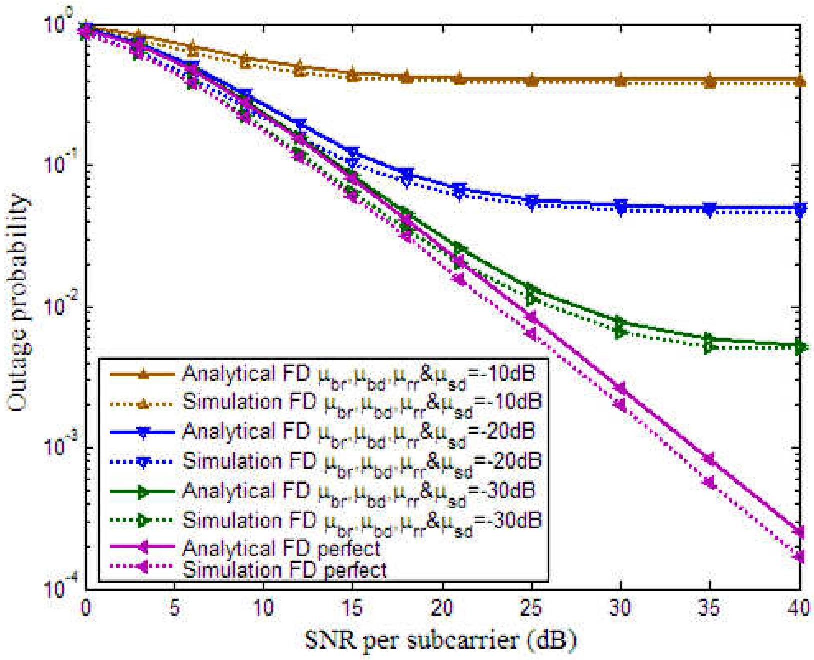

Figure 3.2 Outage performance of the proposed full duplex cognitive AF relay network at 1 b/s/Hz

The overall outage performance of the proposed cognitive DF FDR network at the data rate of $1\,\text{b/s/Hz}$ is shown in Figure 3.2. It is observed that at low SNR region, the effect of NBI and loop interferences is less and at high SNR region, it becomes more dominant. At 25dB SNR, when the mean values μ_{br}, μ_{bd}, μ_{rr} and μ_{sd} of the loop and NBI interference channels $\bar{\boldsymbol{h}}_{br}$, $\bar{\boldsymbol{h}}_{bd}$, $\bar{\boldsymbol{h}}_{rr}$ and $\bar{\boldsymbol{h}}_{sd}$ are increased from -30dB to -20dB, the outage probability increases from 0.013 to 0.057 correspondingly. Further, if the mean values of NBI and the loop interference channels are increased -10dB, the outage probability increases to 0.382 compared to 0.008 in the absence of NBI in the proposed network

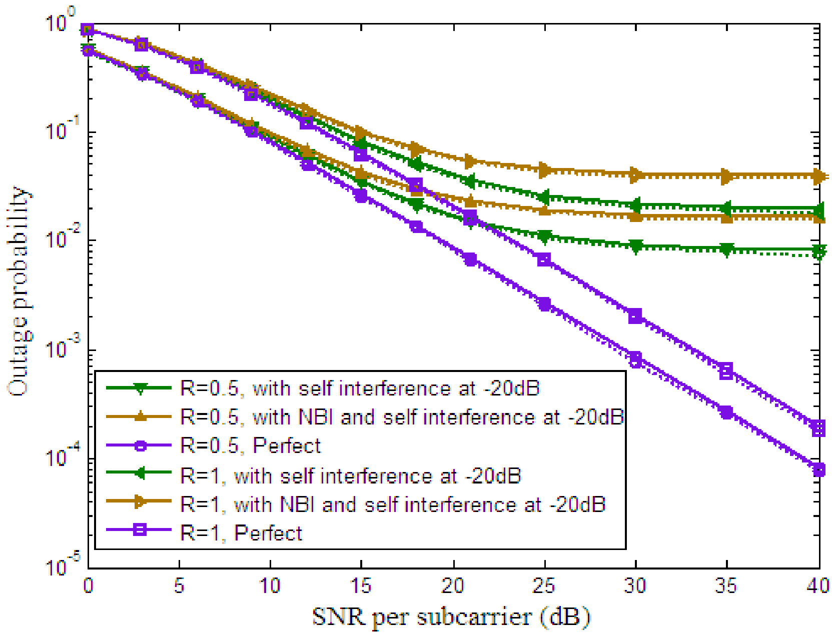

Figure 3.3 Outage performance of the proposed full duplex cognitive AF relay network at 0.5 b/s/Hz and 1 b/s/Hz

The outage performance of the full duplex cognitive AF relay network is shown in Figure 3.3 for the data rates of 0.5 b/s/Hz and 1 b/s/Hz. The analytical and simulation results are closely matching with each other. The simulation results are indicated as dotted lines. At 25dB SNR, the outage probabilities are 0.010 and 0.024 for the data rates of 0.5 b/s/Hz and 1 b/s/Hz respectively when the mean values $\mu_{rr}(k), \mu_{sd}(k)$ of self interference channels $\bar{\boldsymbol{h}}_{rr}$ and $\bar{\boldsymbol{h}}_{sd}$ are at -20dB. In the presence of NBI, the outage probabilities are increased from 0.010 to 0.018 and 0.024 to 0.042 for the data rates of 0.5 b/s/Hz and 1 b/s/Hz respectively when the mean values the mean values of NBI channels $\mu_{br}(k), \mu_{bd}(k)$ are at -20dB. The minimum SNR requirement increases from 21dB to 25dB as the data rate increases from 0.5 b/s/Hz to 1 b/s/Hz, in the absence of both self and narrowband interferences. It is observed that the outage performance is less affected due to self and narrowband interferences at low SNR region as noise is more dominant factor in this region.

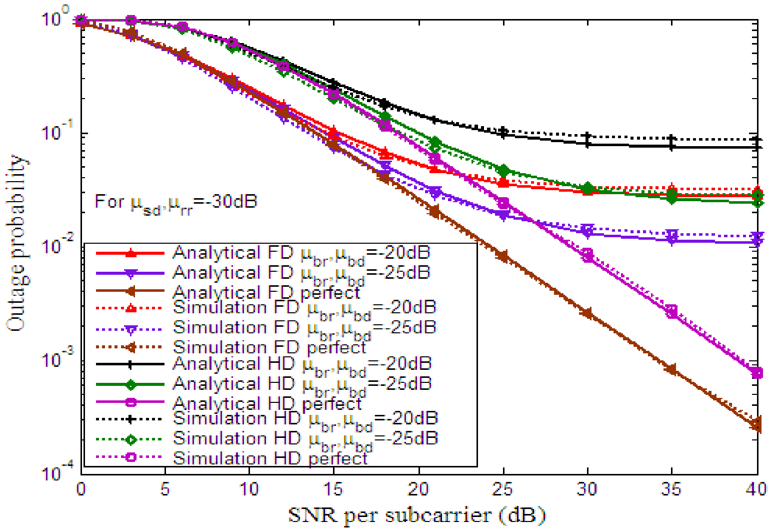

Figure 3.4 Outage Performance of the proposed full duplex cognitive AF relay network is compared with half duplex AF relay

The outage performance of the proposed full duplex cognitive AF relay network at the data rate of $1\,\text{b/s/Hz}$ is compared with half duplex AF relay network in Figure 3.4. It is observed that half duplex mode requires 5dB more SNR compared to full duplex mode while the network is operating without interferences. The SNR gap further increases when the self and /or NBI present in the network. In the absence of NBI, when the network changes from full duplex mode to half duplex mode, the outage probability increases from 0.006 to 0.019 at the SNR of 25dB.

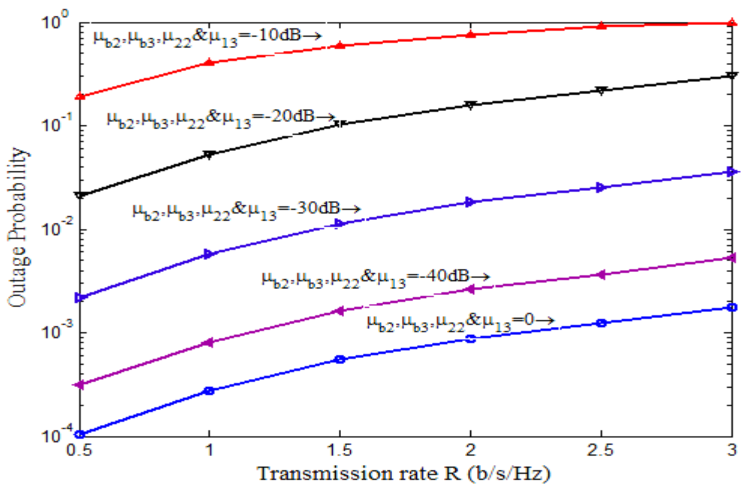

Figure 3.5 Outage performance of cognitive full duplex AF relay network vs transmission rates for the fixed SNR of 40dB

Figure 3.5 shows the end-to-end outage performance of the proposed cognitive full duplex AF relay network for different data rates by fixing the SNR to 40dB. It is observed that, when the mean values μ_{br}, μ_{bd}, μ_{rr} and μ_{sd} of interference channels are below -30dB, the system has better outage performance for all the data rates.

Figure 3.6 shows the average error probability of full duplex cognitive AF relay network. For the mean of the interference channels $\bar{\boldsymbol{h}}_{rr}$, $\bar{\boldsymbol{h}}_{sd}$, $\bar{\boldsymbol{h}}_{br}$ and $\bar{\boldsymbol{h}}_{bd}$ are at -25dB, and the CNR, $\gamma' = \mu_{rd}(k)/\sigma_{zd}^2 = \mu_{sr}(k)/\sigma_{zr}^2$) at 25dB, the average BER is 0.006. When increasing the mean value of the interference channels to -20dB, the average BER becomes 0.016 for the same CNR.

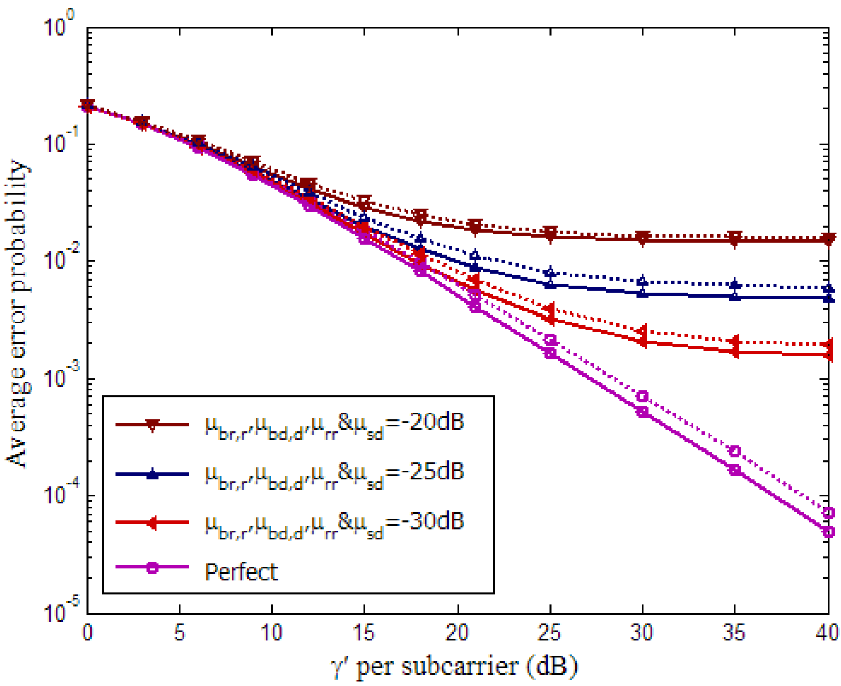

Figure 3.6 Average error probability of full duplex cognitive AF relay network with source power $P_s = 30\text{dBm}$**,** $d_{\min} = 2$ **and** $\bar{N}_e = 1$

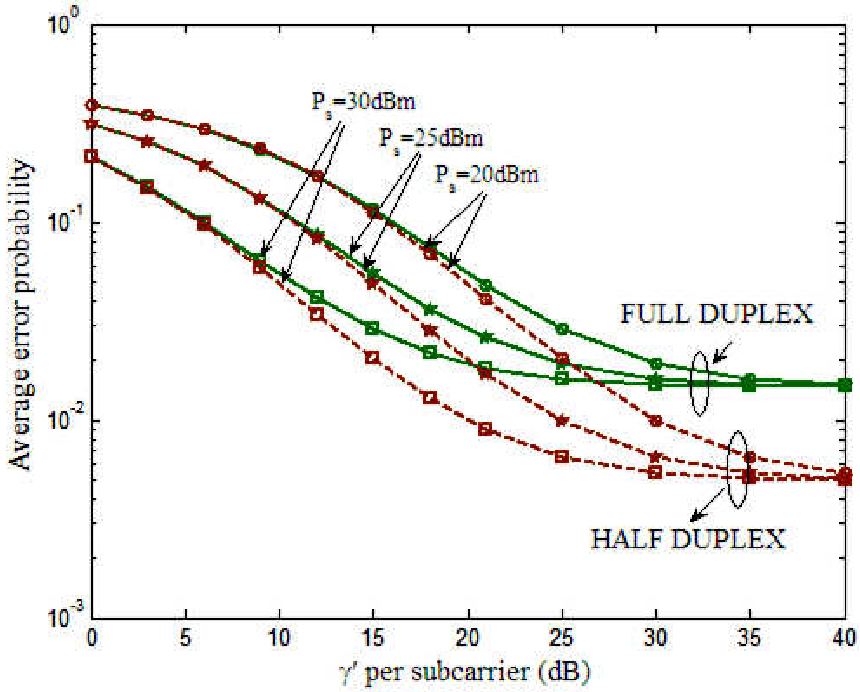

Figure 3.7 Average error probability of full duplex cognitive AF relay network is compared with half duplex mode at $d_{\min} = 2$ and $\bar{N}_e = 1$

Figure 3.7 shows the average BER of the OFDM based cognitive full duplex AF relay network compared with half duplex AF relay network with different source power. The mean of the interference channels $\bar{\boldsymbol{h}}_{rr}$, $\bar{\boldsymbol{h}}_{sd}$, $\bar{\boldsymbol{h}}_{br}$ and $\bar{\boldsymbol{h}}_{bd}$ are fixed at $-20\,\text{dB}$. It is noted that at low CNR region, both modes get the same performance even though source power is increased. However, in the middle of the CNR region, full duplex mode with $P_s = 25\,\text{dBm}$ outperforms the half duplex mode with $P_s = 20\,\text{dBm}$.

3.7 SUMMARY

In this chapter, the performance of OFDM based cognitive full duplex AF relay network is analyzed in the presence of NBI. It is assumed that NBI is affect at relay and destination nodes. Closed form analytical expressions are derived for the outage probability and average error probability of the proposed OFDM based cognitive full duplex AF relay network. Performance of the proposed network is compared with OFDM based full duplex DF relay network and OFDM based half duplex AF relay network. Simulation results are given to validate the analytical derived results. It is concluded that with limited loop interference level, the performance of the proposed network is better than the half duplex AF relay network.

CHAPTER 4

OFDM BASED FULL DUPLEX COGNITIVE RADIO NETWORK WITH RELAY SELECTION IN THE PRESENCE OF NBI

4.1 PREAMBLE

Full duplex transmission in OFDM based CR network improves the spectral efficiency and overcome the bandwidth loss. With limited loop interference, the performance of the OFDM based full duplex cognitive AF relay network is significantly improved in the presence of NBI. However, at high loop interference power, the overall performance of the network is reduced. One way to improve the performance of full duplex relay network is the introduction of multiple relays. In cooperative communications, the number of relay nodes helps the source to forward information to destination node, hence, it virtually forms antenna arrays. Using max-min criterion, the best relay has been has been selected at each subcarrier level. The major advantage of relay selection scheme is high bandwidth efficiency while keeping same diversity order.

Many research works have been reported in wireless relay networks using relay selection scheme. Energy efficiency and spectral efficiency of the DF cooperative relays network is improved using relay ordering and relay selection schemes (Kim et al 2012). Closed form symbol error rate of multiple DF relay network is analyzed under the consideration that the proposed network operates with full diversity (Fareed & Uysal 2009). Outage probability and average error probability of DF cooperative networks with relay selection are analyzed over independent but not identically distributed Nakagami-m fading channels (Duong et al 2009). The outage performance of the cognitive relays network is analyzed with best relay selection over

Rayleigh fading channels (Zhang et al 2013). Although, many of the research proposals are investigated in wireless relay network using relay selection schemes, the performance of OFDM based cooperative CR network using relay selection is not consider so far in the presence of NBI. The objective of this chapter is the proposal of OFDM based full duplex cognitive AF multiple relays network in the presence of NBI. Further, the performance of the proposed network is analyzed over Nakagami-m fading channels.

4.2 SYSTEM MODEL

Consider the OFDM based CR network with multiple full duplex relays shown in Figure 4.1. The proposed network consists of a secondary source node $\mathrm{SU_S}$, a secondary destination node $\mathrm{SU_D}$ and M secondary full duplex relay nodes $\left[\mathrm{SU_{R_1}}, \mathrm{SU_{R_2}}, ...\mathrm{SU_{R_M}}\right]$. The signal is transmitted through full duplex relays using best relay selection scheme.

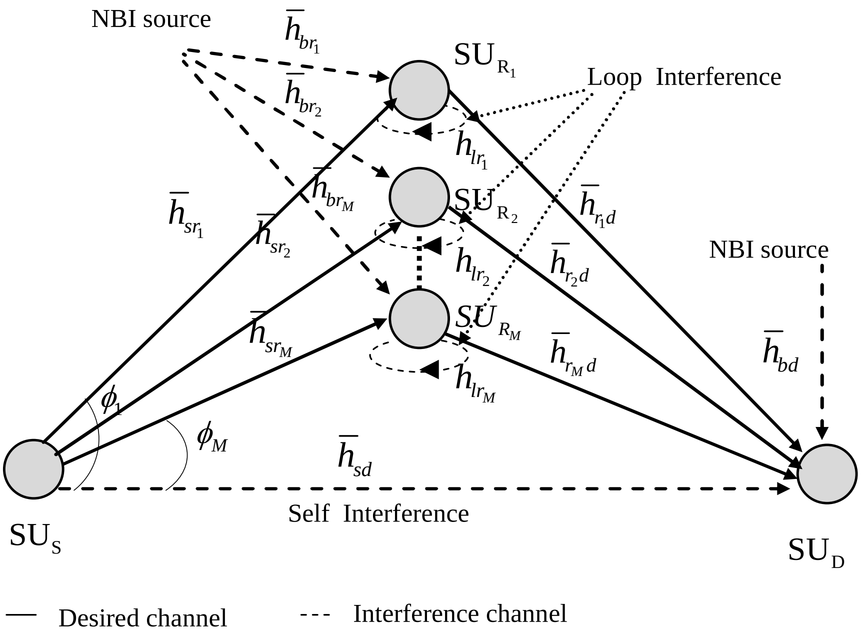

Figure 4.1 OFDM based Cognitive Radio Network with Multiple Full Duplex Relays

4.2.1 Signal Reception at AF Relay Nodes

The $L_{sr_i} \times 1$ CIR vector between the source node $\mathrm{SU_S}$ and i^{th} relay node $\mathrm{SU_{R_i}}$ is defined as $\bar{\boldsymbol{h}}_{sr_i} = \left[\bar{h}_{sr_i}(0), \bar{h}_{sr_i}(1), ..., \bar{h}_{sr_i}(L_{sr_i}-1)\right]^T$, $i \in \{1,2,...,M\}$. The CIR vector of loop/echo interference at i^{th} relay node $\mathrm{SU_{R_i}}$ is defined as $\bar{\boldsymbol{h}}_{lr_i} = \left[\bar{h}_{lr_i}(0), \bar{h}_{lr_i}(1), ..., \bar{h}_{lr_i}(L_{lr_i}-1)\right]^T$. The CIR vector between the NBI source and i^{th} relay node $\mathrm{SU_{R_i}}$ is defined as $\bar{\boldsymbol{h}}_{br_i} = \left[\bar{h}_{br_i}(0), \bar{h}_{br_i}(1), ..., \bar{h}_{br_i}(L_{br_i}-1)\right]^T$. The OFDM signal is assumed to have N subcarriers and uses $v = \max\left(L_{sr_i}, L_{lr_i}, L_{br_i}\right)$ zero padded guard subcarriers to suppress the ISI (Muquet et al 2002). The total number of subcarriers in OFDM signal is $P = N + v$. The $P \times 1$ time-domain receive signal vector at i^{th} relay node $\mathrm{SU_{R_i}}$ in n^{th} OFDM signal duration is expressed as

$$\bar{\boldsymbol{y}}_{sr_i}^{(n)} = \bar{\boldsymbol{H}}_{sr_i} \boldsymbol{F}_{zp} \boldsymbol{x}_s^{(n)} + \bar{\boldsymbol{H}}_{lr_i} \bar{\boldsymbol{x}}_{r_i}^{(n)} + \boldsymbol{A}_{br_i} \bar{\boldsymbol{H}}_{br_i} \bar{\boldsymbol{b}}_{r_i}^{(n)} + \bar{\boldsymbol{z}}_{r_i}^{(n)}, i = 1,....M \tag{4.1}$$

where $\bar{\boldsymbol{H}}_{sr_i}$, $\bar{\boldsymbol{H}}_{lr_i}$ and $\bar{\boldsymbol{H}}_{br_i}$ are $P \times P$ circulant matrices of CIR vectors $\bar{\boldsymbol{h}}_{sr_i}$, $\bar{\boldsymbol{h}}_{lr_i}$ and $\bar{\boldsymbol{h}}_{br_i}$ respectively. $\boldsymbol{F}_{zp}$ is the $P \times N$ zero padded Inverse Discrete Fourier Transform (IDFT) matrix defined as $\boldsymbol{F}_{zp} = \left[\boldsymbol{F}_N \ \ \boldsymbol{0}_{N \times v}\right]^H$. $\boldsymbol{x}_s^{(n)}$ is the $N \times 1$ data vector from source node $\mathrm{SU_S}$ in n^{th} signal duration, $\bar{\boldsymbol{x}}_{r_i}^{(n)} = \mathrm{B}_i \bar{\boldsymbol{y}}_{sr_i}^{(n-1)}$ is a $P \times 1$ loop/echo interference signal vector at i^{th} relay node $\mathrm{SU_{R_i}}$ in $(n-1)^{th}$ signal duration $\bar{\boldsymbol{y}}_{sr_i}^{(n-1)}$ is the time domain OFDM signal at i^{th} relay node in $(n-1)^{th}$ signal duration and $\mathbf{B}_i$ is an circulant amplification matrix defined as $\boldsymbol{F}_P^H \mathbf{E}_i \boldsymbol{F}_P$, where $\mathbf{E}_i = diag\left[\beta_i(1), \beta_i(2), ..., \beta_i(P-1)\right]$ is a $P \times P$ diagonal matrix, $\beta_i(k)$ is an amplification factor at k^{th} subcarrier in i^{th} relay node.

The $\bar{\boldsymbol{b}}_{r_i}^{(n)}$ represents the $P \times 1$ NBI signal vector at i^{th} relay node $\mathrm{SU}_{\mathrm{R_i}}$ in n^{th} signal duration, $\boldsymbol{A}_{br_i}$ is the CFO of NBI signal at i^{th} relay node $\mathrm{SU}_{\mathrm{R_i}}$ represented by a $P \times P$ diagonal matrix $\boldsymbol{A}_{br_i} = diag\left\{1, \exp\left(j2\pi\alpha/P\right), ..., \exp\left(j2\pi\alpha(P-1)/P\right)\right\}$, where α is uniformly distributed random variable over the interval $\left[-0.5, 0.5\right]$ (Batra & Zeidler 2008). $\bar{\boldsymbol{z}}_{r_i}^{(n)}$ is the $P \times 1$ complex white Gaussian noise vector whose elements are independent and identically distributed random variables with zero mean and variance $\sigma_{zr_i}^2$.

Applying P-point DFT, the frequency domain representation of the $P \times 1$ receive signal vector in Equation (4.1) is given by

$$\boldsymbol{y}_{sr_i}^{(n)} = \boldsymbol{F}_p \bar{\boldsymbol{y}}_{sr_i}^{(n)} = \boldsymbol{F}_p \left[\bar{\boldsymbol{H}}_{sr_i} \boldsymbol{F}_{zp} \boldsymbol{x}_s^{(n)} + \bar{\boldsymbol{H}}_{lr_i} \mathrm{B}_i\, \bar{\boldsymbol{y}}_{sr_i}^{(n-1)} + \boldsymbol{A}_{br_i} \bar{\boldsymbol{H}}_{br_i} \bar{\boldsymbol{b}}_{r_i}^{(n)} + \bar{\boldsymbol{z}}_{r_i}^{(n)} \right], i = 1, ...M \qquad (4.2)$$

Since the channel matrices $\bar{\boldsymbol{H}}_{sr_i}$, $\bar{\boldsymbol{H}}_{lr_i}$ and $\bar{\boldsymbol{H}}_{br_i}$ are circulant, it can be decomposed as $\bar{\boldsymbol{H}}_{sr_i} = \boldsymbol{F}_P^H \Lambda_{sr_i} \boldsymbol{F}_P$, $\bar{\boldsymbol{H}}_{lr_i} = \boldsymbol{F}_P^H \Lambda_{lr_i} \boldsymbol{F}_P$ and $\bar{\boldsymbol{H}}_{br_i} = \boldsymbol{F}_P^H \Lambda_{br_i} \boldsymbol{F}_P$ respectively (Gray 2006). Let λ_{sr_i}, λ_{lr_i} and λ_{br_i} are the P -point DFT of CIR vectors $\bar{\boldsymbol{h}}_{sr_i}$, $\bar{\boldsymbol{h}}_{lr_i}$ and $\bar{\boldsymbol{h}}_{br_i}$ respectively. Then, Λ_{sr_i}, Λ_{lr_i} and Λ_{br_i} are $P \times P$ diagonal matrices of λ_{sr_i}, λ_{lr_i} and λ_{br_i} respectively. Applying the decomposition of circulant channel matrices, Equation (4.2) can be simplified as

$$\boldsymbol{y}_{sr_i}^{(n)} = \Lambda_{sr_i} \boldsymbol{V} \boldsymbol{x}_s^{(n)} + \Lambda_{lr_i} \mathrm{E}_i\, \boldsymbol{y}_{sr_i}^{(n-1)} + \boldsymbol{C}_{r_i} \Lambda_{br_i} \boldsymbol{b}_{r_i}^{(n)} + \boldsymbol{z}_{r_i}^{(n)}, i = 1, ...M \qquad (4.3)$$

where $\boldsymbol{V}$ is a $P \times N$ precoding matrix defined as $\boldsymbol{V} = \boldsymbol{F}_P \boldsymbol{F}_{zp}$, $\boldsymbol{C}_{r_i}$ is a $P \times P$ circulant matrix of CFO defined as $\boldsymbol{F}_P \boldsymbol{A}_{br_i} \boldsymbol{F}_P^H$, the combined term $\mathrm{E}_i \boldsymbol{y}_{sr_i}^{(n-1)}$ is the frequency domain loop interference vector, $\boldsymbol{b}_{r_i}^{(n)}$ and $\boldsymbol{z}_{r_i}^{(n)}$ are the frequency domain NBI signal and noise vectors respectively. The $P \times 1$ NBI signal vector

$\boldsymbol{b}_{r_i}^{(n)} = \boldsymbol{F}_P \bar{\boldsymbol{b}}_{r_i}^{(n)}$ is sparse in nature whose k^{th} subcarrier element $\boldsymbol{b}_{r_i}^{(n)}(k) \neq 0$ if $k \in I_{br_i}$. The set I_{br_i} has the indices of the subcarriers which are affected by NBI (Gomaa & Al-Dhahir 2011).

The amplification factor $\beta_i(k)$ at k^{th} subcarrier is chosen such that i^{th} relay node transmit power $P_{r_i}(k)$ is same as the source node transmit power P_s. It is derived as follows. By substituting $\bar{\boldsymbol{y}}_{sr_i}^{(n-1)}$ in $\bar{\boldsymbol{x}}_{r_i}^{(n)}$ and eliminating the residual zero padded interference, the transmit signal at k^{th} subcarrier in n^{th} signal duration is expressed, in an iterative manner, as

$$
\begin{aligned}
x_{r_i}^{(n)}(k) = \beta_i(k) \sum_{j=1}^{n} \left(\lambda_{Ir_i}(k) \beta_i(k) \right)^{j-1} \Big[\lambda_{sr_i}(k) v(k) x_s^{(n-j)}(k) \\
+ \sum_{q_2=0, q_2 \in I_{br_i}}^{P-1} \lambda_{br_i}(q_2) c_{r_i}\left(k, (q_2 - k)_P\right) b_{r_i}^{(n-j)}(q_2) \\
+ z_{r_i}^{(n-j)}(k) \Big], i = 1, \ldots M, k = 0, \ldots P-1
\end{aligned}
\tag{4.4}
$$

For large values of n, the average transmit power at k^{th} subcarrier is computed as

$$
\begin{aligned}
P_{r_i}(k) = E\left(\left| x_{r_i}^{(n)}(k) \right|^2 \right) \leq \beta_i^2(k) \left(\frac{1}{1 - \left| \lambda_{Ir_i}(k) \right|^2 \beta_i^2(k)} \right) \\
\left[\left| \lambda_{sr_i}(k) \right|^2 \left| v(k) \right|^2 P_s + \sum_{q_2=0, q_2 \in I_{br_i}}^{P-1} \left| \lambda_{br_i}(q_2) \right|^2 \varsigma_{r_i} \sigma_{br_i}^2 + \sigma_{zr_i}^2 \right]
\end{aligned}
\tag{4.5}
$$

where $E(.)$ is the expectation operator, $P_s = E\left(\left| x_s^{(n-j)}(k) \right|^2 \right)$ is the average transmit power at each symbol , $\sigma_{zr_i}^2 = E\left(\left| z_{r_i}^{(n-j)}(k) \right|^2 \right)$ is the noise variance. The second term in Equation (4.5) is obtained by applying Cauchy – Schwartz inequality in the second term of Equation (4.4). It is written as

$$E\left(\left|\sum_{q_2=0,q_2\in I_{br_i}}^{P-1}\lambda_{br_i}(q_2)c_{r_i}\left(k,(q_2-k)_P\right)b_{r_i}^{(n-j)}(q_2)\right|^2\right)\leq\sum_{q_2=0,q_2\in I_{br_i}}^{P-1}\left|\lambda_{br_i}(q_2)\right|^2\varsigma_{r_i}\sigma_{br_i}^2 \tag{4.6}$$

where $\sigma_{br_i}^2=E\left(\sum_{q_2=0,q_2\in I_{br_i}}^{P-1}\left|b_{r_i}^{(n-j)}(q_2)\right|^2\right)$ and $\varsigma_{r_i}=\sum_{q_2=0,q_2\in I_{br_i}}^{P-1}\left|c_{r_i}\left(k,(q_2-k)_P\right)\right|^2$ represents the IPPS of NBI signal and squared-norm of CFO respectively. From Equation (4.5), it is clear that the condition $\beta_i^2(k)<\left(1/\left|\lambda_{lr_i}(k)\right|^2\right)$ is to be satisfied in order to guarantee finite i^{th} relay node transmit power $P_{r_i}(k)$. As $|v(k)|^2\approx 1$, the amplification factor $\beta_i^2(k)$ is written as

$$\beta_i(k)\geq\sqrt{\frac{P_{r_i}(k)}{\left[\left|\lambda_{sr_i}(k)\right|^2P_s+\left|\lambda_{lr_i}(k)\right|^2P_{r_i}(k)+\sum_{q_2=0,q_2\in I_{br_i}}^{P-1}\left|\lambda_{br_i}(q_2)\right|^2\varsigma_{r_i}\sigma_{br_i}^2+\sigma_{zr_i}^2\right]}} \tag{4.7}$$

4.2.1.1 SINR analysis at AF relay nodes

Using Equation (4.3), the receive signal $\boldsymbol{y}_{sr_i}^{(n)}$ in n^{th} signal duration at k^{th} subcarrier can expressed as

$$\begin{aligned}y_{sr_i}^{(n)}(k)&=\lambda_{sr_i}(k)v(k)x_s^{(n)}(k)+\lambda_{sr_i}(k)\sum_{q_1=0,q_1\neq k}^{N-1}v(k,q_1)x_s^{(n)}(q_1)\\&+\lambda_{lr_i}(k)x_{r_i}^{(n)}(k)+\sum_{q_2=0,q_2\in I_{br_i}}^{P-1}\lambda_{br_i}(q_2)c_{r_i}\left(k,(q_2-k)_P\right)b_{r_i}^{(n)}(q_2)\\&+z_{r_i}^{(n)}(k);\quad i=1,\ldots,M,k=0,1,\ldots,P-1\end{aligned} \tag{4.8}$$

In this expression, the first term is the desired signal, the second, third and fourth terms denote the residual zero padded interference, loop interference and NBI respectively, fifth term is noise at k^{th} subcarrier. Since the residual zero padded interference is less, it is approximated to zero. Assuming that the Channel Frequency Response (CFR) vectors $\boldsymbol{\lambda}_{sr_i}$, $\boldsymbol{\lambda}_{lr_i}$ and $\boldsymbol{\lambda}_{br_i}$ are known, the instantaneous SINR of the $\boldsymbol{i}^{th}$ relay receive signal $\boldsymbol{y}_{sr_i}^{(n)}$ at k^{th} subcarrier can be expressed as

$$\Gamma_{sr_i}^{FD}(k) = \frac{\left|\lambda_{sr_i}(k)\right|^2 \left|v(k,k)\right|^2 E\left(\left|x_s^{(n)}(k)\right|^2\right)}{\left[\left|\lambda_{lr_i}(k)\right|^2 E\left(\left|x_{r_i}^{(n)}(k)\right|^2\right) + E\left(\left|\sum_{q_2=0, q_2\in I_{br_i}}^{P-1} \lambda_{br_i}(q_2) c_{r_i}\left(k,(q_2-k)_P\right) b_{r_i}^{(n)}(q_2)\right|^2\right) + E\left(\left|z_{r_i}^{(n)}(k)\right|^2\right)\right]}, i=1,....M, k=0,1,...,P-1 \tag{4.9}$$

By substituting Equation (4.6) in Equation (4.9), the lower bound of $\Gamma_{sr_i}^{FD}(k)$ is derived as

$$\Gamma_{sr_i}^{FD}(k) \geq \frac{\left|\lambda_{sr_i}(k)\right|^2 \mathrm{P}_s}{\left|\lambda_{lr_i}(k)\right|^2 P_{r_i} + \sum_{q_2=0, q_2\in I_{br_i}}^{P-1} \left|\lambda_{br_i}(q_2)\right|^2 \varsigma_{r_i} \sigma_{br_i}^2 + \sigma_{zr_i}^2} \tag{4.10}$$

4.2.2 Signal Reception at Destination Node from i^{th} AF Relay Node

The $L_{r_id} \times 1$ CIR vector between relay node $\mathrm{SU_{R_i}}$ and destination node $\mathrm{SU_D}$, $L_{sd} \times 1$ CIR vector between source node $\mathrm{SU_S}$ and destination node $\mathrm{SU_D}$ and $L_{bd} \times 1$ CIR vector between NBI and destination node $\mathrm{SU_D}$ are denoted by $\bar{\boldsymbol{h}}_{r_id}$, $\bar{\boldsymbol{h}}_{sd}$ and $\bar{\boldsymbol{h}}_{bd}$ respectively. The OFDM signal from $\boldsymbol{i}^{th}$ relay node (assume that all other relay nodes are idle) is received at destination node as given by

$$\bar{\boldsymbol{y}}_{r_id}^{(n)} = \bar{\boldsymbol{H}}_{r_id} \bar{\boldsymbol{x}}_{r_i}^{(n)} + \bar{\boldsymbol{H}}_{sd} \boldsymbol{F}_{zp} \boldsymbol{x}_s^{(n)} + \boldsymbol{A}_{bd}^{(n)} \bar{\boldsymbol{H}}_{bd} \bar{\boldsymbol{b}}_d^{(n)} + \bar{\boldsymbol{z}}_d^{(n)}; i=1,...,M \tag{4.11}$$

where $\bar{\boldsymbol{H}}_{r_id}$, $\bar{\boldsymbol{H}}_{sd}$ and $\bar{\boldsymbol{H}}_{bd}$ are $P \times P$ circulant matrices of CIR vectors $\bar{\boldsymbol{h}}_{r_id}$, $\bar{\boldsymbol{h}}_{sd}$ and $\bar{\boldsymbol{h}}_{bd}$ respectively. $\bar{\boldsymbol{x}}_{r_i}^{(n)}$ is a $N \times 1$ time domain i^{th} relay transmit signal vector at n^{th} signal duration, $\boldsymbol{x}_s^{(n)}$ is the $N \times 1$ frequency domain transmit signal vector from source node which acts as an interference in full duplex mode, $\bar{\boldsymbol{b}}_d^{(n)}$ is the $P \times 1$ NBI signal vector at destination node $\mathrm{SU_D}$ at i^{th} relay transmission. $\bar{\boldsymbol{z}}_d^{(n)}$ is the $P \times 1$ noise vector at destination node, $\boldsymbol{A}_{bd}^{(n)}$ denote the CFO of the NBI signal at destination node $\mathrm{SU_D}$. The frequency domain representation of the receive signal $\bar{\boldsymbol{y}}_{r_i d}^{(n)}$ is given by

$$\boldsymbol{y}_{r_i d}^{(n)} = \boldsymbol{F}_p \bar{\boldsymbol{y}}_{r_i d}^{(n)} = \boldsymbol{F}_p \left[\bar{\boldsymbol{H}}_{r_i d} \bar{\boldsymbol{x}}_{r_i}^{(n)} + \bar{\boldsymbol{H}}_{sd} \boldsymbol{F}_{zp} \boldsymbol{x}_s^{(n)} + \boldsymbol{A}_{bd}^{(n)} \bar{\boldsymbol{H}}_{bd} \bar{\boldsymbol{b}}_d^{(n)} + \bar{\boldsymbol{z}}_d^{(n)} \right] \tag{4.12}$$

where $\bar{\boldsymbol{H}}_{r_i d}$, $\bar{\boldsymbol{H}}_{sd}$ and $\bar{\boldsymbol{H}}_{bd}$ matrices are decomposed as $\bar{\boldsymbol{H}}_{r_i d} = \boldsymbol{F}_P^H \Lambda_{r_i d} \boldsymbol{F}_P$, $\bar{\boldsymbol{H}}_{sd} = \boldsymbol{F}_P^H \Lambda_{sd} \boldsymbol{F}_P$ and $\bar{\boldsymbol{H}}_{bd} = \boldsymbol{F}_P^H \Lambda_{bd} \boldsymbol{F}_P$ respectively. $\Lambda_{r_i d}$, Λ_{sd} and Λ_{bd} are $P \times P$ diagonal matrices. Now, Equation (4.12) can be simply written as

$$\boldsymbol{y}_{r_i d}^{(n)} = \Lambda_{r_i d} \boldsymbol{x}_{r_i}^{(n)} + \Lambda_{sd} \boldsymbol{V} \boldsymbol{x}_s^{(n)} + \boldsymbol{C}_{bd} \Lambda_{bd} \boldsymbol{b}_d^{(n)} + \boldsymbol{z}_d^{(n)} \tag{4.13}$$

where $\boldsymbol{C}_{bd} = \boldsymbol{F}_P \boldsymbol{A}_{bd}^{(n)} \boldsymbol{F}_P^H$ is a circulant matrix of CFO at destination node $\mathrm{SU_D}$, $\boldsymbol{V}\boldsymbol{x}_s^{(n)}$ is the self interference at destination node $\mathrm{SU_D}$, $\boldsymbol{b}_d^{(n)}$ and $\boldsymbol{z}_d^{(n)}$ are the frequency domain NBI signal and noise vectors respectively.

4.2.2.1 SINR analysis at destination node

From Equation (4.13), the receive signal $\boldsymbol{y}_{r_i d}^{(n)}$ of destination node at k^{th} subcarrier can be written as

$$\begin{aligned} y_{r_i d}^{(n)}(k) &= \lambda_{r_i d}(k) x_{r_i}^{(n)}(k) + \lambda_{sd}(k) v(k) x_s^{(n)}(k) \\ &\quad + \sum_{q_3=0, q_3 \in I_{bd}}^{P-1} \lambda_{bd}(q_3) c_{bd}\left(k, (q_3 - k)_P\right) b_d^{(n)}(q_3) \\ &\quad + z_d^{(n)}(k); i = 1, \ldots, M, k = 0, 1, \ldots, P-1 \end{aligned} \tag{4.14}$$

With the assumption that CFR vectors $\lambda_{r_i d}$, λ_{sd} and λ_{bd} are known, the instantaneous SINR of the receive signal $\boldsymbol{y}_{r_i d}^{(n)}$ at k^{th} subcarrier is given as

$$\Gamma_{r_i d}^{FD}(k) = \frac{\left|\lambda_{r_i d}(k)\right|^2 E\left(\left|x_{r_i}^{(n)}(k)\right|^2\right)}{\begin{aligned}&\left[\left|\lambda_{sd}(k)\right|^2 \left|v(k)\right|^2 E\left(\left|x_s^{(n)}(k)\right|^2\right)\right.\\ &+E\left(\left|\sum_{q_3=0, q_3\in I_{bd}}^{P-1} \lambda_{bd}(q_3) c_{bd}\left(k,(q_3-k)_P\right) b_d^{(n)}(q_3)\right|^2\right)\\ &\left.+E\left(\left|z_d^{(n)}(k)\right|^2\right)\right]\end{aligned}} ; i=1,\ldots,M, k=0,1,\ldots,P-1 \tag{4.15}$$

From Equation (4.15), the lower bound of $\Gamma_{r_i d}^{FD}(k)$ is obtained as

$$\Gamma_{r_i d}^{FD}(k) \geq \frac{\left|\lambda_{r_i d}(k)\right|^2 \mathrm{P}_{r_i}}{\left|\lambda_{sd}(k)\right|^2 \mathrm{P}_s + \sum_{q_3=0, q_3\in I_{bd}}^{P-1} \left|\lambda_{bd}(q_3)\right|^2 \varsigma_d \sigma_{bd}^2 + \sigma_{zd}^2} \tag{4.16}$$

where $\varsigma_d = \sum_{q_3=0, q_3\in I_{bd}}^{P-1} \left|c_{bd}\left(k,(q_3-k)_P\right)\right|^2$ is the squared-norm of the CFO and $\sigma_{bd}^2 = E\left(\sum_{q_3=0, q_3\in I_{bd}}^{P-1} \left|b_d^{(n)}(q_3)\right|^2\right)$ is the IPPS of NBI at destination node $\mathrm{SU_D}$, $\sigma_{zd}^2 = E\left(\left|z_d^{(n)}(k)\right|^2\right)$ is the noise variance at destination node $\mathrm{SU_D}$.

4.2.3 Best Relay Selection Per-Subcarrier

The overall SINR of full duplex cognitive AF relay network through i^{th} relay node can be expressed as (Rabiei et al 2011)

$$\Gamma_{sr_i d}^{FD}(k) = \frac{\Gamma_{sr_i}^{FD}(k)\Gamma_{r_i d}^{FD}(k)}{\Gamma_{sr_i}^{FD}(k) + \Gamma_{r_i d}^{FD}(k) + 1}; i=1,2,\ldots,M; k=0,1,\ldots,P-1 \tag{4.17}$$

In AF relay network, the maximum data rate depends on the minimum of SINR of $\Gamma_{sr_i}^{FD}(k)$ and $\Gamma_{r_i d}^{FD}(k)$. Hence, the i^{th} relay SINR $\Gamma_{sr_i d}^{FD}(k)$ can be upper bounded by

$$\Gamma_{sr_id}^{FD}(k)=\frac{\Gamma_{sr_i}^{FD}(k)\Gamma_{r_id}^{FD}(k)}{\Gamma_{sr_i}^{FD}(k)+\Gamma_{r_id}^{FD}(k)+1}\leq \min\left(\Gamma_{sr_i}^{FD}(k),\Gamma_{r_id}^{FD}(k)\right) \tag{4.18}$$

The best relay is selected at k^{th} subcarrier based on the max-min criterion (Ding& Uysal 2009) as given by

$$r_{best,k}=\max_{i\in(1,2,..,M)}\min\left(\Gamma_{sr_i}^{FD}(k),\Gamma_{r_id}^{FD}(k)\right) \tag{4.19}$$

where $r_{best,k}$ is the best relay at k^{th} subcarrier. The instantaneous SINR of the best relay at k^{th} subcarrier can be expressed as

$$\Gamma_{srd}^{best}(k)=\min\left(\Gamma_{sr_{best}}^{FD}(k),\Gamma_{r_{best}d}^{FD}(k)\right) \tag{4.20}$$

where $\Gamma_{srd}^{best}(k)$ is the best relay SINR in AF relay network, $\Gamma_{sr_{best}}^{FD}(k)$ is the source to best relay SINR, $\Gamma_{r_{best}d}^{FD}(k)$ is the best relay to destination SINR. In this method, different relays might be selected for different subcarriers and the selected relay forwards the symbol of k^{th} subcarrier to the destination node. For instance, subcarrier 1 may choose relay 1 and subcarrier 2 choose relay M . At the destination node, all the subcarriers are collected (Dai et al 2007).

4.2.4 DF Relay Mode

In this mode, selected relay decode the received signal and transmit the re-encoded signal to the destination node while receiving the next source signal.

The $P\times 1$ frequency domain receive signal vector at destination node $\mathrm{SU_D}$ in n^{th} signal duration is given by

$$\boldsymbol{y}_{r_id}^{(n),DF}=\Lambda_{r_id}\boldsymbol{x}_{r_i}^{(n)}+\Lambda_{sd}\boldsymbol{V}\boldsymbol{x}_s^{(n)}+\boldsymbol{C}_{bd}\,\Lambda_{bd}\boldsymbol{b}_d^{(n)}+\boldsymbol{z}_d^{(n)} \tag{4.21}$$

where Λ_{r_id} , Λ_{sd} and Λ_{bd} are $P\times P$ diagonal matrices, $\boldsymbol{C}_{bd}$ is a circulant matrix of CFO at destination node $\mathrm{SU_D}$, $\boldsymbol{V}\boldsymbol{x}_s^{(n)}$ is the self interference at destination node $\mathrm{SU_D}$, $\boldsymbol{b}_d^{(n)}$ and $\boldsymbol{z}_d^{(n)}$ are the frequency domain NBI signal and noise vectors respectively.

4.2.4.1 SINR analysis at DF relay node

From Equation (4.21), the receive signal $\boldsymbol{y}_{r_i d}^{(n),DF}$ of destination node at k^{th} subcarrier can be written as

$$\begin{aligned} y_{r_i d}^{(n),DF}(k) = {} & \lambda_{r_i d}(k) x_{r_i}^{(n)}(k) + \lambda_{sd}(k) v(k) x_s^{(n)}(k) \\ & + \sum_{q_3=0, q_3 \in I_{bd}}^{P-1} \lambda_{bd}(q_3) c_{bd}\left(k, (q_3 - k)_P\right) b_d^{(n)}(q_3) \\ & + z_d^{(n)}(k); i = 1, \ldots, M, k = 0, 1, \ldots, P-1 \end{aligned} \tag{4.22}$$

With the assumption that CFR vectors $\lambda_{r_i d}$, λ_{sd} and λ_{bd} are known, the instantaneous SINR of the receive signal $\boldsymbol{y}_{r_i d}^{(n),DF}$ at k^{th} subcarrier is given as

$$\Gamma_{r_i d}^{FD-DF}(k) \geq \frac{\left|\lambda_{r_i d}(k)\right|^2 \mathrm{P}_{r_i}}{\left|\lambda_{sd}(k)\right|^2 \mathrm{P}_s + \sum_{q_3=0, q_3 \in I_{bd}}^{P-1} \left|\lambda_{bd}(q_3)\right|^2 \varsigma_d \sigma_{bd}^2 + \sigma_{zd}^2} \tag{4.23}$$

4.3 OUTAGE ANALYSIS OF OFDM BASED COGNITIVE RADIO NETWORK WITH RELAY SELECTION

In this section, the outage performance of the full duplex cognitive AF multiple relays network, half duplex cognitive AF multiple relays network and half duplex cognitive DF multiple relays network are analyzed at each subcarrier using relay selection per subcarrier, for the given data rate of R b/s/Hz.

4.3.1 AF Relay Mode

The end-to-end outage performance of the full duplex cognitive AF multiple relays network in k^{th} subcarrier with best relay selection out of M relays at the data rate of R b/s/Hz is expressed as (Torabi 2009)

$$P_{out,k}^{FD-AF-BR}\left(R\right)=\left[\Pr\left(\log\left(1+\Gamma_{sr_id}^{FD}\left(k\right)\right)<R\right)\right]^{M} \quad (4.24)$$

For the data rate of R b/s/Hz and the threshold SINR, $\gamma = 2^{R}-1$, the CDF of the overall SINR $\Gamma_{sr_id}^{FD}\left(k\right)\leq\min\left(\Gamma_{sr_i}^{FD}(k),\Gamma_{r_id}^{FD}(k)\right)$ can be written as

$$F_{\Gamma_{sr_id}^{FD}(k)}\left(\gamma\right)=1-\left[\left(1-F_{\Gamma_{sr_i}^{FD}(k)}\left(\gamma\right)\right)\left(1-F_{\Gamma_{r_id}^{FD}(k)}\left(\gamma\right)\right)\right] \quad (4.25)$$

where $F_{\Gamma_{sr_i}^{FD}(k)}\left(\gamma\right)$ and $F_{\Gamma_{r_id}^{FD}(k)}\left(\gamma\right)$ are the CDFs of $\Gamma_{sr_i}^{FD}\left(k\right)$ and $\Gamma_{r_id}^{FD}\left(k\right)$ respectively. Using Equation (4.10), the CDF of $\Gamma_{sr_i}^{FD}(k)$ can be expressed as

$$F_{\Gamma_{sr_i}^{FD}(k)}\left(\gamma\right)=P\left(\frac{X\left(k\right)}{Y\left(k\right)+\varsigma_{r_i}Z\left(k\right)+b}<\gamma\right) \quad (4.26)$$

where $X\left(k\right)=\left|\lambda_{sr_i}\left(k\right)\right|^{2}$, $Y\left(k\right)=\left|\lambda_{lr_i}\left(k\right)\right|^{2}$, $Z\left(k\right)=\sum_{q_2=0,q_2\in I_{br_i}}^{P-1}\left|\lambda_{br_i}\left(q_2\right)\right|^{2}$ and $b=\frac{1}{SNR}$. Now, the CDF of $\Gamma_{sr_i}^{FD}(k)$ conditioned on the random variables $Y\left(k\right)$ and $Z\left(k\right)$ can be written as

$$\begin{aligned}F_{\Gamma_{sr_i}^{FD}(k)}\left(\gamma\right)=&\int_0^\infty\int_0^\infty P\left(X\left(k\right)<\left(\gamma\right)\left(y\left(k\right)+\varsigma_{r_i}z\left(k\right)+b\right)\Big|_{Y(k)=y(k),Z(k)=z(k)}\right)\\&\times f_{Y(k)}\left(y\left(k\right)\right)f_{Z(k)}\left(z\left(k\right)\right)dy\left(k\right)dz\left(k\right)\end{aligned} \quad (4.27)$$

The random variables $X\left(k\right)$ and $Y\left(k\right)$ are exponentially distributed and $Z\left(k\right)$ is chi-square distributed with $2I_{br_i}$ degrees of freedom (Proakis & Salehi 2008). The PDFs of $X\left(k\right)$ and $Y\left(k\right)$ are defined as

$$f_{X(k)}\left(x\left(k\right)\right)=\left(1/\mu_{sr_i}\left(k\right)\right)\exp\left(-x\left(k\right)/\mu_{sr_i}\left(k\right)\right), x\left(k\right)\geq 0 \quad (4.28)$$

$$f_{Y(k)}\left(y\left(k\right)\right)=\left(1/\mu_{lr_i}\left(k\right)\right)\exp\left(-y\left(k\right)/\mu_{lr_i}\left(k\right)\right), y\left(k\right)\geq 0 \quad (4.29)$$

The means of the random variables $X(k)$, $Y(k)$ are $\mu_{sr_i}(k) = 2\sigma_x^2$ and $\mu_{lr_i}(k) = 2\sigma_y^2$ respectively. The PDF of $Z(k)$ is given by

$$f_{Z(k)}\left(z(k)\right) = \left(z(k)^{I_{br_i}-1} \Big/ \left(\mu_{br_i}(k)/I_{br_i}\right)^{I_{br_i}} \Gamma\left(I_{br_i}\right)\right) \times \exp\left(-z(k) \Big/ \left(\mu_{br_i}(k)/I_{br_i}\right)\right), z(k) \geq 0 \tag{4.30}$$

By substituting the PDF of $X(k)$ from Equation (4.28) in Equation (4.27) and then integrating, the CDF of $\Gamma_{sr_i}(k)$ is written as

$$F_{\Gamma_{sr_i}^{FD}(k)}(\gamma) = \int_0^\infty \int_0^\infty \left[1 - \exp\left(-\frac{\left(y(k) + \varsigma_{r_i} z(k) + b\right)\gamma}{\mu_{sr_i}(k)}\right)\right] \times f_{Y(k)}\left(y(k)\right) f_{Z(k)}\left(z(k)\right) dy(k)\, dz(k) \tag{4.31}$$

Then, substituting the PDF of $Y(k)$ and $Z(k)$ from Equation (4.29) and Equation (4.30) in Equation (4.31) and then integrating, the CDF of $\Gamma_{sr_i}(k)$ is determined as

$$F_{\Gamma_{sr_i}^{FD}(k)}(\gamma) = 1 - \exp\left(-\gamma / \mu_{sr_i}(k) SNR\right) \frac{\left[1 + \left(\gamma\varsigma_{r_i} / I_{br_i} SIR_{r_i}^{nbi}(k)\right)\right]^{-I_{br_i}}}{\left[\left(\gamma / SIR_{r_i}^{loop}(k)\right) + 1\right]} \tag{4.32}$$

where $SIR_{r_i}^{nbi}(k) = \mu_{sr_i}(k)/\mu_{br_i}(k)$ and $SIR_{r_i}^{loop}(k) = \mu_{sr_i}(k)/\mu_{lr_i}(k)$ are Signal to Interference Ratios. Applying the approximation $\left(1 + (x/r)\right)^{-r} \approx \exp(-x)$ for the last term of the numerator of Equation (4.32), the CDF of $\Gamma_{sr_i}(k)$ can be written as

$$F_{\Gamma_{sr_i}^{FD}(k)}(\gamma) = 1 - \frac{\exp\left[-\gamma\left\{\left(1/\mu_{sr_i}(k) SNR\right) + \left(\varsigma_{r_i} / SIR_{r_i}^{nbi}(k)\right)\right\}\right]}{\left[\left(\gamma / SIR_{r_i}^{loop}(k)\right) + 1\right]} \tag{4.33}$$

Let $\lambda_{r_i}^{a_1}(k) = \left\{\left(1/\mu_{sr_i}(k)SNR\right) + \left(\varsigma_{r_i}/SIR_{r_i}^{nbi}(k)\right)\right\}$ and $\lambda_{r_i}^{a_2}(k) = 1/SIR_{r_i}^{loop}(k)$, Equation (4.33) is simplified as

$$F_{\Gamma_{sr_i}^{FD}(k)}(\gamma) = 1 - \exp\left(-\lambda_{r_i}^{a_1}(k)\gamma\right) / \left(\lambda_{r_i}^{a_2}(k)\gamma + 1\right) \tag{4.34}$$

Similarly, using Equation (4.16), the CDF of $\Gamma_{r_i d}^{FD}(k)$ is determined as

$$F_{\Gamma_{r_i d}^{FD}(k)}(\gamma) = 1 - \exp\left(-\lambda_{d_i}^{a_1}(k)\gamma\right) / \left(\lambda_{d_i}^{a_2}(k)\gamma + 1\right) \tag{4.35}$$

where $\lambda_{d_i}^{a_1}(k) = \left\{\left(1/\mu_{r_i d}(k)SNR\right) + \left(\varsigma_d/SIR_d^{nbi}(k)\right)\right\}$ and $\lambda_{d_i}^{a_2}(k) = 1/SIR_d^{self}(k)$, $SIR_d^{nbi}(k) = \mu_{r_i d}(k)/\mu_{bd}(k)$ and $SIR_d^{self}(k) = \mu_{r_i d}(k)/\mu_{sd}(k)$.

By substituting CDF of $\Gamma_{sr_i}^{FD}(k)$ and $\Gamma_{r_i d}^{FD}(k)$ from Equation (4.34) and Equation (4.35) in Equation (4.25), the CDF of $\Gamma_{sr_i d}^{FD}(k)$ is obtained as

$$\begin{aligned} F_{\Gamma_{sr_i d}^{FD}(k)}(\gamma) = 1 - \Big[&\left(1 - \left(1 - \exp\left(-\lambda_{r_i}^{a_1}(k)\gamma\right)\left(1 - \lambda_{r_i}^{a_2}(k)\gamma\right)\right)\right) \\ &\left(1 - \left(1 - \exp\left(-\lambda_{d_i}^{a_1}(k)\gamma\right)\left(1 - \lambda_{d_i}^{a_2}(k)\gamma\right)\right)\right)\Big] \end{aligned} \tag{4.36}$$

where $\lambda_{r_i}^{a_1}(k)$, $\lambda_{r_i}^{a_2}(k)$ $\lambda_{d_i}^{a_1}(k)$ and $\lambda_{d_i}^{a_2}(k)$ are relatively normalized signal powers over noise and interferences. After simplification, Equation (4.36) becomes

$$\begin{aligned} F_{\Gamma_{sr_i d}^{FD}(k)}(\gamma) = 1 - &\exp\left(-\left(\lambda_{r_i}^{a_1}(k) + \lambda_{d_i}^{a_1}(k)\right)\gamma\right) \\ &\times\left(1 - \lambda_{r_i}^{a_2}(k)\gamma - \lambda_{d_i}^{a_2}(k)\gamma + \lambda_{r_i}^{a_2}(k)\lambda_{d_i}^{a_2}(k)\gamma^2\right) \end{aligned} \tag{4.37}$$

Therefore, the lower bound outage probability of the full duplex cognitive AF multiple relay network is determined as

$$P_{out,k}^{FD-AF-BR}(\gamma) \geq \Big[1-\exp\Big(-\big(\lambda_{r_i}^{a_1}(k)+\lambda_{d_i}^{a_1}(k)\big)\gamma\Big)$$
$$\big(1-\lambda_{r_i}^{a_2}(k)\gamma-\lambda_{d_i}^{a_2}(k)\gamma+\lambda_{r_i}^{a_2}(k)\lambda_{d_i}^{a_2}(k)\gamma^2\big)\Big]^M \quad (4.38)$$

By substituting $\lambda_{r_i}^{a_1}(k)$, $\lambda_{r_i}^{a_2}(k)$, $\lambda_{d_i}^{a_1}(k)$ and $\lambda_{d_i}^{a_2}(k)$ in Equation (4.38), it becomes

$$P_{out,k}^{FD-AF-BR}(R) \geq \left[1-\exp\left(-\left(2^R-1\right)\left(\frac{1}{\mu_{sr_i}(k)SNR}+\frac{\varsigma_{r_i}\mu_{br_i}(k)}{\mu_{sr_i}(k)}+\frac{1}{\mu_{r_id}(k)SNR}+\frac{\varsigma_d\mu_{bd}(k)}{\mu_{r_id}(k)}\right)\right)\right. \quad (4.39)$$
$$\left.\times\left(1-\left(2^R-1\right)\left(\frac{\mu_{lr_i}(k)}{\mu_{sr_i}(k)}+\frac{\mu_{sd}(k)}{\mu_{r_id}(k)}-\frac{\mu_{lr_i}(k)\mu_{sd}(k)}{\mu_{sr_i}(k)\mu_{r_id}(k)}\left(2^R-1\right)\right)\right)\right]^M$$

4.3.2 AF Relay Mode over Nakagami-m Fading

Using Equation (4.3), $\Gamma_{sr_i}^{FD}(k)$ can be written as

$$\Gamma_{sr_i,1}^{FD}(k)=\frac{SNRX(k)}{INR\varsigma_{r_i,1}Y(k)+1}\approx\frac{SNRX(k)}{INR\varsigma_{r_i,1}Y(k)} \text{ at high INR} \quad (4.40)$$

where $X(k)=\left|\lambda_{sr_i,1}(k)\right|^2$,in this, $\left|\lambda_{sr_i,1}(k)\right|$ follows Nakagami- m distribution with parameters m_{sr_i} and Ω_{sr_i}/m_{sr_i} , $X(k)$ follows a gamma distribution with shape and scale parameters $\alpha_{sr_i}>0$ and $\beta_{sr_i}>0$ mapped to m_{sr_i} and Ω_{sr_i}/m_{sr_i} (Proakis & Salehi 2008), $Y(k)=\sum_{q_2=0,q_2\in I_{br_i},P}^{P}\left|\lambda_{br_i}(q_2)\right|^2$, in this, P^{th} term is added by the loop interference and $Y(k)$ follows the independent gamma variates with parameters α_{br_i,q_2} and β_{br_i,q_2} respectively (Moschopoulos 1985).

The PDF of $\left|\lambda_{sr_i,1}(k)\right|$, $X(k)$ and $Y(k)$ are expressed as

$$f_{\left|\lambda_{sr_i,1}(k)\right|}(\gamma)=\left(2/\Gamma\left(m_{sr_i}\right)\right)\left(m_{sr_i}/\Omega_{sr_i}\right)^{m_{sr_i}}\gamma^{2m_{sr_i}-1}\exp\left(-m_{sr_i}\gamma^2/\Omega_{sr_i}\right) \quad (4.41)$$

$$f_{X(k)}(\gamma) = \left(1/\Gamma\left(\alpha_{sr_i}\right)\right)\beta_{sr_i}^{-\alpha_{sr_i}} x^{\alpha_{sr_i}-1} \exp\left(-x/\beta_{sr_i}\right) \tag{4.42}$$

$$f_{Y(k)}(\gamma) = \left[C_1 \beta_{br_i,1}^{-\rho_1} / \Gamma(\rho_1)\right] y^{\rho_1 - 1} \exp\left(-y(1-b_1)/\beta_{br_i,1}\right) \tag{4.43}$$

Let the index of the NBI terms be 1, 2, 3 and loop interference index is 4 , then the interference parameters are written as

$\rho_1 = \sum_{q_2=1}^{4} \alpha_{br_i,q_2}, > 0$, $C_1 = \prod_{q_2=1}^{4}\left(\beta_{br_i,1}/\beta_{br_i,q_2}\right)^{\alpha_{br_i,q_2}}$, $\beta_{br_i,1}$ can be defined as $\beta_{br_i,1} = \min_{q_2} \beta_{br_i,q_2}$ and $b_1 = \max_{2 \le q_2 \le 4}\left(1 - \left(\beta_{br_i,1}/\beta_{br_i,q_2}\right)\right)$.

Using Equation (4.40), the CDF of $\Gamma_{sr_i,1}^{FD}(k)$ is expressed as

$$F_{\Gamma_{sr_i,1}^{FD}(k)}(\gamma) = P\left(SNRX(k)/INR\varsigma_{r_i,1}Y(k) < \gamma\right) \tag{4.44}$$

Assume $\varsigma_{r_i,1} = 1$, the CDF of $\Gamma_{sr_i,1}^{FD}(k)$ can be simplified as

$$F_{\Gamma_{sr_i,1}^{FD}(k)}(\gamma) = \int_0^{\infty} \int_0^{\frac{yINR}{SNR}} f_X(x) f_Y(y) dxdy \tag{4.45}$$

By substituting Equation (4.42) and Equation (4.43) in Equation (4.45) and then integrating, it becomes

$$F_{\Gamma_{sr_i,1}^{FD}(k)}(\gamma) = \frac{C_1 \beta_{br_i,1}^{-\rho_1}}{\Gamma(\rho_1)\Gamma\left(\alpha_{sr_i}\right)} \frac{A_1^{\alpha_{sr_i}} \Gamma\left(\rho_1 + \alpha_{sr_i}\right)}{\alpha_{sr_i} B_1^{\rho_1 + \alpha_{sr_i}}} {}_2F_1\left(1, \rho_1 + \alpha_{sr_i}; \alpha_{sr_i} + 1; \left(A_1/B_1\right)\right) \tag{4.46}$$

where $A_1 = \left(\dfrac{\alpha_{sr_i}\gamma INR}{\beta_{sr_i} SNR}\right)$, $B_1 = \left(\left(\dfrac{\alpha_{sr_i}\gamma INR}{\beta_{sr_i} SNR}\right) + \dfrac{(1-b_1)}{\beta_{br_i,1}}\right)$ and ${}_2F_1(a,b;c;x)$ is Gauss hypergeometric function (Gradshteyn & Ryzhik 1980).

Similarly, the CDF of $\Gamma_{r_i d,1}^{FD}(k)$ is obtained as

$$F_{\Gamma^{FD}_{r_i d,1}(k)}(\gamma)=\frac{C_2\beta_{bd,1}^{-\rho_2}}{\Gamma(\rho_2)\Gamma(\alpha_{r_i d})}\frac{A_2^{\alpha_{r_i d}}\Gamma(\rho_2+\alpha_{r_i d})}{\alpha_{r_i d}B_2^{\rho_2+\alpha_{r_i d}}}\,{}_2F_1\left(1,\rho_2+\alpha_{r_i d};\alpha_{r_i d}+1;\left(A_2/B_2\right)\right) \quad (4.47)$$

where $A_2=\left(\dfrac{\alpha_{r_i d}\gamma INR}{\beta_{r_i d}SNR}\right)$, $B_2=\left(\left(\dfrac{\alpha_{r_i d}\gamma INR}{\beta_{r_i d}SNR}\right)+\dfrac{(1-b_2)}{\beta_{bd,1}}\right)$, $\rho_2=\sum_{q_3=1}^{4}\alpha_{bd,q_3},>0$,

$C_2=\prod_{q_3=1}^{4}\left(\beta_{bd,1}/\beta_{bd,q_3}\right)^{\alpha_{bd,q_3}}$ and $b_2=\max_{2\le q_3\le 4}\left(1-\left(\beta_{bd,1}/\beta_{bd,q_3}\right)\right)$. By substituting Equation (4.46) and Equation (4.47) in Equation (4.25), the end-to-end outage probability of the proposed network is obtained. Without interferences, the CDF of $\Gamma^{FD}_{sr_i,1}(k)$ becomes

$$F_{\Gamma^{FD}_{sr_i,1}(k)}(\gamma)=P\left(SNRX(k)<\gamma\right) \quad (4.48)$$

By substituting the PDF of $X(k)$ and then integrating, it becomes

$$\begin{aligned}F_{\Gamma^{FD}_{sr_i,1}(k)}(\gamma)&=\left(1/\Gamma\left(\alpha_{sr_i}\right)\right)\alpha_{sr_i}^{-1}\left(\alpha_{sr_i}\gamma/\beta_{sr_i}SNR\right)^{\alpha_{sr_i}}\\&\times{}_1F_1\left(\alpha_{sr_i};1+\alpha_{sr_i};-\left(\alpha_{sr_i}\gamma/\beta_{sr_i}SNR\right)\right)\end{aligned} \quad (4.49)$$

Similarly, the CDF of $\Gamma^{FD}_{r_i d,1}(k)$ is

$$\begin{aligned}F_{\Gamma^{FD}_{r_i d,1}(k)}(\gamma)&=\left(1/\Gamma\left(\alpha_{r_i d}\right)\right)\alpha_{r_i d}^{-1}\left(\alpha_{r_i d}\gamma/\beta_{r_i d}SNR\right)^{\alpha_{r_i d}}\\&\times{}_1F_1\left(\alpha_{r_i d};1+\alpha_{r_i d};-\left(\alpha_{r_i d}\gamma/\beta_{r_i d}SNR\right)\right)\end{aligned} \quad (4.50)$$

where ${}_1F_1(a,b;x)$ is the confluent hypergeometric function.

4.3.3 DF Relay Mode

With the assumption that all the channels are independent and identically distributed and using the best relay selection per subcarrier, the end-to-end outage probability of the full duplex cognitive DF multiple relays network at k^{th} subcarrier for the data rate of R b/s/Hz can be expressed as

$$P_{out,k}^{FD-DF-BR}\left(R\right)=\left[\Pr\left(\min\left(\log\left(1+\Gamma_{sr_i}^{FD}\left(k\right)\right),\log\left(1+\Gamma_{r_id}^{FD}\left(k\right)\right)\right)<R\right)\right]^M \tag{4.51}$$

From (Laneman et al 2004), Equation (4.51) can be re-written as

$$\begin{aligned}P_{out,k}^{FD-DF-BR}\left(R\right)=&\Big[\Pr\left(\Gamma_{sr_i}^{FD}\left(k\right)<2^R-1\right)\\&+\left\{1-\Pr\left(\Gamma_{sr_i}^{FD}\left(k\right)<2^R-1\right)\right\}\Pr\left(\Gamma_{r_id}^{FD}\left(k\right)<2^R-1\right)\Big]^M\end{aligned} \tag{4.52}$$

By substituting the CDF of $\Gamma_{sr_i}\left(k\right)$ and $\Gamma_{r_id}^{FD}\left(k\right)$ in Equation (4.52), the upper bound outage probability of the full duplex cognitive DF multiple relays network can be determined as

$$\begin{aligned}P_{out,k}^{FD-DF-BR}\left(R\right)\le&\Big[1-\exp\left(-\left(\lambda_{r_i}^{a_1}\left(k\right)+\lambda_{d_i}^{a_1}\left(k\right)\right)\gamma\right)\\&\left(1-\lambda_{r_i}^{a_2}\left(k\right)\gamma-\lambda_{d_i}^{a_2}\left(k\right)\gamma+\lambda_{r_i}^{a_2}\left(k\right)\lambda_{d_i}^{a_2}\left(k\right)\gamma^2\right)\Big]^M\end{aligned} \tag{4.53}$$

Since in the full duplex mode, the direct link (from source to destination) acts as a self interference, the maximum average mutual information of the AF multiple relays network and DF multiple relays network using proactive DF relaying scheme provides the same outage probability.

4.3.4 Half Duplex AF Relay Mode

The outage probability of the half duplex cognitive AF multiple relays network using the best relay selection at each subcarrier can be expressed as

$$P_{out,k}^{HD-AF-BR}\left(R\right)=\Pr\left(\Gamma_{AF}\left(k\right)<2^{2R}-1\right)=\int_0^{2^{2R}-1} f_{\Gamma_{AF}(k)}\left(\gamma_2\right)d\gamma_2 \tag{4.54}$$

where the threshold SINR $\gamma_2=2^{2R}-1$ and $\Gamma_{AF}\left(k\right)$ is the overall SINR of the network. It is given by

$$\Gamma_{AF}\left(k\right)=\Gamma_{srd,2}^{best}\left(k\right)+\Gamma_{sd,1}^{HD}\left(k\right) \tag{4.55}$$

where $\Gamma_{srd,2}^{best}\left(k\right)$ is the best relay SINR at second time slot, $\Gamma_{sd,1}^{HD}\left(k\right)$ is the source to destination SINR at first time slot. The SINR $\Gamma_{srd,2}^{best}\left(k\right)$ can be expressed as

$$\Gamma_{srd,2}^{best}\left(k\right)=\min\left(\Gamma_{sr_{best,1}}^{FD}\left(k\right),\Gamma_{r_{best,2}d}^{FD}\left(k\right)\right) \tag{4.56}$$

where $\Gamma_{sr_{best,1}}^{FD}\left(k\right)$ is the source to best relay SINR in first time slot, $\Gamma_{r_{best,2}d}^{FD}\left(k\right)$ is the best relay to destination SINR at second time slot.

In half duplex mode, the receive signal $\boldsymbol{y}_{sd,1}^{(n)}$ at k^{th}subcarrier can be written as

$$\begin{aligned} y_{sd,1}^{(n)}\left(k\right)&=\lambda_{sd,1}\left(k\right)v\left(k\right)x_{s}^{(n)}\left(k\right)\\ &+\sum_{q_4=0,q_4\in I_{bd,1}}^{P-1}\lambda_{bd,1}\left(q_4\right)c_{d,1}\left(k,\left(q_4-k\right)_P\right)b_{d,1}^{(n)}\left(q_4\right)+z_{d,1}^{(n)}\left(k\right) \end{aligned} \tag{4.57}$$

By assuming that the CFR vectors $\lambda_{sd,1}$ and $\lambda_{bd,1}$ are known, the instantaneous SINR of the receive signal $\boldsymbol{y}_{sd,1}^{(n)}$ at k^{th} subcarrier is expressed as

$$\Gamma_{sd,1}^{HD}\left(k\right)\geq\frac{\left|\lambda_{sd,1}\left(k\right)\right|^{2}\mathrm{P}_{s}}{\sum_{q_4=0,q_4\in I_{bd,1}}^{P-1}\left|\lambda_{bd,1}\left(q_4\right)\right|^{2}\varsigma_{d,1}\sigma_{bd,1}^{2}+\sigma_{zd,1}^{2}} \tag{4.58}$$

Using Equation (4.34), the CDF of $\Gamma_{sd,1}^{HD}\left(k\right)$can be written as

$$F_{\Gamma_{sd,1}^{HD}(k)}\left(\gamma_2\right)=1-\exp\left(-\lambda_{d,1}\left(k\right)\gamma_2\right) \tag{4.59}$$

Differentiating Equation (4.59) with respect to γ_2 , the PDF of $\Gamma_{sd,1}^{HD}\left(k\right)$ is determined as

$$\frac{dF_{\Gamma_{sd,1}^{HD}(k)}\left(\gamma_2\right)}{d\gamma_2}=f_{\Gamma_{sd,1}^{HD}(k)}\left(\gamma_2\right)=\lambda_{d,1}\left(k\right)\exp\left(-\lambda_{d,1}\left(k\right)\gamma_2\right) \tag{4.60}$$

The PDF of $\Gamma_{AF}(k)$ is computed using Moment Generating Function (MGF). It can be expressed as

$$f_{\Gamma_{AF}(k)}(\gamma_2) = L^{-1}\left(\mathrm{M}_{\Gamma_{AF}(k)}(s)\right) \tag{4.61}$$

where L^{-1} is inverse Laplace transform operator, $\mathrm{M}_{\Gamma_{AF}(k)}(s)$ is MGF of overall SINR and is given by

$$\mathrm{M}_{\Gamma_{AF}(k)}(s) = \mathrm{M}_{\Gamma_{sd,1}^{HD}(k)}(s)\mathrm{M}_{\Gamma_{srd,2}^{best}(k)}(s) \tag{4.62}$$

where $\mathrm{M}_{\Gamma_{sd,1}^{HD}(k)}(s)$ and $\mathrm{M}_{\Gamma_{srd,2}^{best}(k)}(s)$ are the MGF of SINR $\Gamma_{sd,1}^{HD}(k)$ and $\Gamma_{srd,2}^{best}(k)$ respectively. The MGF $\mathrm{M}_{\Gamma_{srd,2}^{best}(k)}(s)$ can be computed as follows. In half duplex mode, the best relay per subcarrier can be selected as

$$\Gamma_{srd,2}^{best-HD}(k) = \max_{i\in[1,2,\ldots M]} \Gamma_{sr_id,2}^{HD}(k) \tag{4.63}$$

where $\Gamma_{sr_id,2}^{HD}(k)$ is the i^{th} relay node SINR of the half duplex AF relay network. It can be written as

$$\Gamma_{sr_id,2}^{HD}(k) \le \min\left(\Gamma_{sr_i,1}^{HD}(k), \Gamma_{r_id,2}^{HD}(k)\right) \tag{4.64}$$

The CDF of $\Gamma_{sr_i,1}^{HD}(k)$ and $\Gamma_{r_id,2}^{HD}(k)$ are same as the CDF of $\Gamma_{sr_i}^{FD}(k)$ and $\Gamma_{r_id}^{FD}(k)$ except the self/loop interference. Using Equation (4.34) and Equation (4.35), it can be expressed as follows

$$F_{\Gamma_{sr_i,1}^{HD}(k)}(\gamma_2) = 1-\exp\left(-\lambda_{r_i}^{a_1}(k)\gamma_2\right) \tag{4.65}$$

$$F_{\Gamma_{r_id,2}^{HD}(k)}(\gamma_2) = 1-\exp\left(-\lambda_{d_i}^{a_1}(k)\gamma_2\right) \tag{4.66}$$

Using Equation (4.65) and Equation (4.66), the CDF of $\Gamma_{sr_i d,2}^{HD}(k)$ is computed as

$$F_{\Gamma_{sr_i d,2}^{HD}(k)}(\gamma_2) = 1 - \exp\left(-\left(\lambda_{r_i}^{a_1}(k) + \lambda_{d_i}^{a_1}(k)\right)\gamma_2\right) \tag{4.67}$$

Let $\lambda_{sr_i d,2}(k) = \lambda_{r_i}^{a_1}(k) + \lambda_{d_i}^{a_1}(k)$, then (4.64) can be reduces to

$$F_{\Gamma_{sr_i d,2}^{HD}(k)}(\gamma_2) = 1 - \exp\left(-\lambda_{sr_i d,2}(k)\gamma_2\right) \tag{4.68}$$

The CDF of the best relay SINR $\Gamma_{srd,2}^{best-HD}(k)$ is computed as

$$F_{\Gamma_{srd,2}^{best,HD}(k)}(\gamma_2) = \left[F_{\Gamma_{sr_i d,2}^{HD}(k)}(\gamma_2)\right]^M = \left[1 - \exp\left(-\lambda_{sr_i d,2}(k)\gamma_2\right)\right]^M \tag{4.69}$$

Differentiating Equation (4.69) by γ_2, the PDF of $\Gamma_{srd,2}^{best-HD}(k)$ is determined as

$$f_{\Gamma_{srd,2}^{best,HD}(k)}(\gamma_2) = \frac{M}{\lambda_{sr_i d,2}(k)} \exp\left(-\lambda_{sr_i d,2}(k)\gamma_2\right)\left(1 - \exp\left(-\lambda_{sr_i d,2}(k)\gamma_2\right)\right)^{M-1} \tag{4.70}$$

The MGF of $\Gamma_{srd,2}^{best,HD}(k)$ is defined as

$$\mathrm{M}_{\Gamma_{srd,2}^{best,HD}(k)}(s) = E\left(\exp(-\gamma_2 s)\right) = \int_0^{\infty} \exp(-\gamma_2 s)\, f_{\Gamma_{srd,2}^{best,HD}(k)}(\gamma_2) d\gamma_2 \tag{4.71}$$

Substituting Equation (4.70) in Equation (4.71) , the MGF of $\Gamma_{srd,2}^{best,HD}(k)$ is determined as

$$\mathrm{M}_{\Gamma_{srd,2}^{best,HD}(k)}(s) = \sum_{l=1}^{M} \binom{M}{l} \frac{l(-1)^{l-1}}{\left(l + \lambda_{sr_i d,2}(k)s\right)} \tag{4.72}$$

The MGF of the $\Gamma_{sd,1}^{HD}(k)$ is determined as

$$\mathrm{M}_{\Gamma_{sd,1}^{HD}(k)}(s) = \left(1 + \lambda_{d,1}(k)s\right)^{-1} \tag{4.73}$$

Finally, the MGF of the overall SINR $\Gamma_{AF}(k)$ is derived as

$$\begin{aligned}\mathrm{M}_{\Gamma^{AF}(k)}(s) &= \mathrm{M}_{\Gamma^{HD}_{sd,1}(k)}(s)\mathrm{M}_{\Gamma^{best}_{srd,2}(k)}(s) \\ &= \sum_{l=1}^{M}\binom{M}{l}\frac{l(-1)^{l-1}}{\left(l+\lambda_{sr_id,2}(k)s\right)\left(1+\lambda_{d,1}(k)s\right)}\end{aligned} \tag{4.74}$$

By taking inverse Laplace transform, the PDF of $\Gamma_{AF}(k)$ is obtained as

$$f_{\Gamma_{AF}(k)}(\gamma_2) = \sum_{l=1}^{M}\binom{M}{l}\frac{l(-1)^{l-1}}{\left(\lambda_{sr_id,2}(k)-l\,\lambda_{d,1}(k)\right)}\left[\exp\left(\frac{-l\gamma_2}{\lambda_{sr_id,2}(k)}\right)-\exp\left(\frac{-\gamma_2}{\lambda_{d,1}(k)}\right)\right] \tag{4.75}$$

By substituting the PDF $f_{\Gamma_{AF}(k)}(\gamma)$ in Equation (4.54) and then integrating, the outage probability of the half duplex cognitive AF multiple relays network can be determined as

$$\begin{aligned}P^{HD-AF-BR}_{out,k}(R) \geq 1 + &\sum_{l=1}^{M}\binom{M}{l}\frac{l(-1)^{l-1}}{\left(\lambda_{sr_id,2}(k)-l\,\lambda_{d,1}(k)\right)} \\ &\times\left[\lambda_{d,1}(k)\exp\left(\frac{-\left(2^{2R}-1\right)}{\lambda_{d,1}(k)}\right)-\frac{\lambda_{sr_id,2}(k)}{l}\exp\left(\frac{-l\left(2^{2R}-1\right)}{\lambda_{sr_id,2}(k)}\right)\right]\end{aligned} \tag{4.76}$$

4.3.5 Half Duplex DF Relay Mode

The end-to-end upper bound outage probability of the half duplex cognitive DF multiple relays network at k^{th} subcarrier for the data rate of R b/s/Hz is given by (Suraweera 2006)

$$P^{HD-DF-BR}_{out,k}(R) \leq \sum_{i=0}^{M}\Pr\left(I_{DF} < R \,\middle\|\, C\right| = i\right)\Pr\left(|C| = i\right) \tag{4.77}$$

where $\Pr\left(\left|C\right|=i\right)$ is the probability of i links between source and relay nodes have the SINR greater than the threshold SINR γ_2, $i=0$ indicates that direct link between source and destination node. The probability $\Pr\left(\left|C\right|=i\right)$ is defined as

$$\Pr\left(\left|C\right|=i\right)=\binom{M}{i}\Pr\left(\Gamma_{sr_i,1}^{HD}(k)>\gamma_2\right)^i\left[1-\Pr\left(\Gamma_{sr_i,1}^{HD}(k)>\gamma_2\right)\right]^{M-i},i\in[1,..M] \quad (4.78)$$

The probability $\Pr\left(I_{DF}<R\left|\left|C\right|=i\right.\right)$ can be expressed as

$$\Pr\left(I_{DF}<R\left|\left|C\right|=1,...,i\right.\right)=\Pr\left(\Gamma_{DF_0}+\Gamma_{DF}^{best,HD}\Big|_{\left|C\right|=1,...,i}<2^{2R}-1\right) \quad (4.79)$$

where I_{DF} is the average mutual information of half duplex DF multiple relays network . Γ_{DF_0} and $\Gamma_{DF}^{best,HD}$ are the SINR from source node to destination node and best relay node to destination node respectively. They are written as

$$\Gamma_{DF_0}=\Gamma_{sd,1}^{HD}(k) \quad (4.80)$$

$$\Gamma_{DF}^{best,HD}=\max_{i\in[1,2,..,I]}\Gamma_{DF_i} \quad (4.81)$$

where I is the set of cooperating relays for which $\Gamma_{sr_i,1}^{HD}(k)>\gamma_2$, Γ_{DF_i} is the i^{th} relay node to destination node SINR and it can be expressed as

$$\Gamma_{DF_i}=\Gamma_{r_id}^{HD}(k);i\in[1,2,...,I] \quad (4.82)$$

The CDF of Γ_{DF_0} with parameter mapping $\lambda_{DF_0}(k)=\lambda_{d,1}(k)$ is expressed as

$$F_{\Gamma_{DF_0}}(\gamma_2)=1-\exp\left(-\lambda_{DF_0}(k)\gamma_2\right) \quad (4.83)$$

Similarly, the CDF of Γ_{DF_i} with parameter mapping $\lambda_{DF_i}(k)=\lambda_{d_i}^{a_1}(k)$ can be written as

$$F_{\Gamma_{DF_i}}(\gamma_2) = 1 - \exp\left(-\left(\lambda_{DF_i}(k)\right)\gamma_2\right); \quad i \in [1, 2, ..., I] \tag{4.84}$$

Hence, the CDF of $\Gamma_{DF}^{best,HD}$ can be obtained as

$$F_{\Gamma_{DF}^{best,HD}}(\gamma_2) = \left[1 - \exp\left(-\left(\lambda_{DF_i}(k)\right)\gamma_2\right)\right]^I \tag{4.85}$$

Using Equation (4.83) and Equation (4.85), the probability $\Pr\left(I_{DF} < R \| C| = 1, ..., i\right)$ can be derived as

$$\Pr\left(I_{DF} < R \| C| = 1, ..., i\right) \le \left[1 - \exp\left(-\lambda_{DF_i}(k)\gamma_2\right)\right]^{I+1} \tag{4.86}$$

Substituting Equation (4.78) and Equation (4.86) in Equation (4.77), the overall outage probability of the half duplex cognitive DF multiple relays network can be obtained.

4.4 RESULTS AND DISCUSSION

In this section, the outage performance of the proposed full duplex cognitive AF multiple relays network is analyzed in the presence of NBI and compared with half duplex AF, half duplex DF multiple relays network. The numerical parameters of proposed full duplex cognitive AF multiple relays network are given in Table 4.1

Table 4.1 Numerical parameters of the proposed full duplex cognitive AF multiple relays network

Symbol	Parameters	Value
N	Number of subcarriers in OFDM signal	64
M	Number of relay nodes	3
v	Guard sequence length	20
L	Number of channel taps	16
I_{br_i}, I_{bd}	Number of subcarriers with NBI	3
β	Carrier frequency offset parameter of NBI signal	0.5
R	Data rates (in bits/s/Hz)	0.5 and 1
$\mu_{sr_i}(k), \mu_{r_id}(k)$	Mean values of the signal channels	0 dB

For the data rate of 1 b/s/Hz at each subcarrier of OFDM signal, the outage performance of the proposed full duplex cognitive AF multiple relays network is shown in Figure 4.2. The simulation results are indicated in dotted lines. In this figure, FDR denote the full duplex relay. The mean values $\mu_{lr_i}(k)$, $\mu_{sd}(k)$, $\mu_{br_i}(k)$ and $\mu_{bd}(k)$ of the interference CFR scalars λ_{lr_i}, λ_{sd}, λ_{br_i} and λ_{bd} are set at $-25\mathrm{dB}$. It is observed that when the number of relay nodes (M) increases from 2 to 3, the SNR requirement decreases from 20 dB to 14 dB due to the selection diversity gain at the relay nodes, for the outage probability of 10^{-3}. In the absence of NBI, the SNR requirement further decreases from 18 dB to 13 dB for the same outage probability.

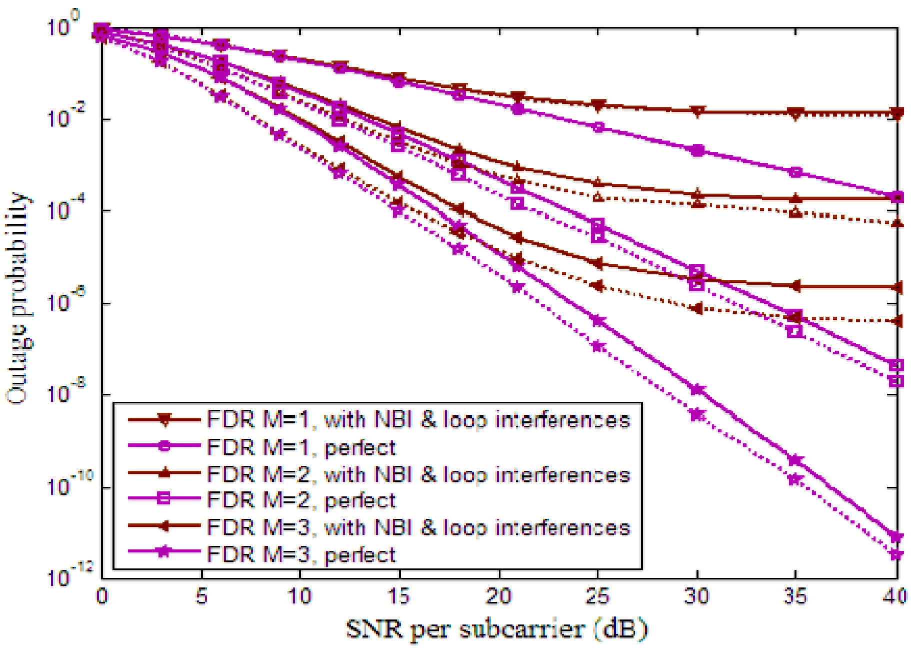

Figure 4.2 Outage performance of the proposed full duplex cognitive AF multiple relays network at 1 b/s/Hz

In Figure 4.3, the outage performance of the half duplex cognitive AF multiple relays network is compared with half duplex DF multiple relays network at each subcarrier in the presence of NBI. The data rate is fixed at 1b/s/Hz and the dotted lines are indicating simulations. In this figure, HDR denote the half duplex relay. The gap between the analytical and simulations of the half duplex AF multiple relays network are due lower bounded outage probability. The mean values $\mu_{br_i}(k)$ and $\mu_{bd}(k)$ of the interference CFR scalars λ_{br_i} and λ_{bd} are set at -25dB. The direct path channel λ_{sd} act as an interference in full duplex mode and act as a desired channel ($\lambda_{sd,1}$) in half duplex mode where the mean value $\mu_{sd,1}(k)$ is -2dB. At $M = 3$, the SNR requirement for the half duplex AF multiple relays network is 12 dB compared to 15 dB for half duplex DF multiple relays network, at the outage probability of 10^{-3}. Further, it observed that when M is decreasing, the outage becomes higher in the half duplex DF multiple relays network.

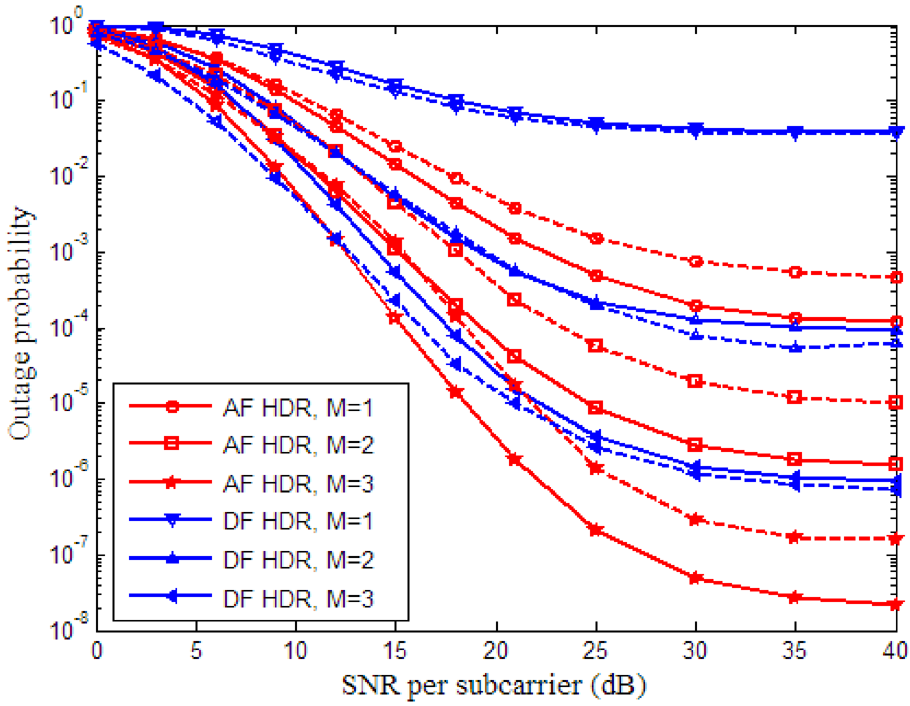

Figure 4.3 Outage performance of the half duplex AF multiple relays network is compared with half duplex DF multiple relays network

In Figure 4.4, the outage performance of the proposed network is compared with half duplex AF multiple relays network and half duplex DF multiple relays network at each subcarrier in the presence of NBI. The data rate is fixed at 1 b/s/Hz. The mean values of the interference channels are same as in previous Figure From this figure, it is observed that for $M = 3$ and at low SNR regime, the performance of the proposed network is better than the half duplex AF and half duplex DF relaying strategies in the presence of loop and narrowband interferences.

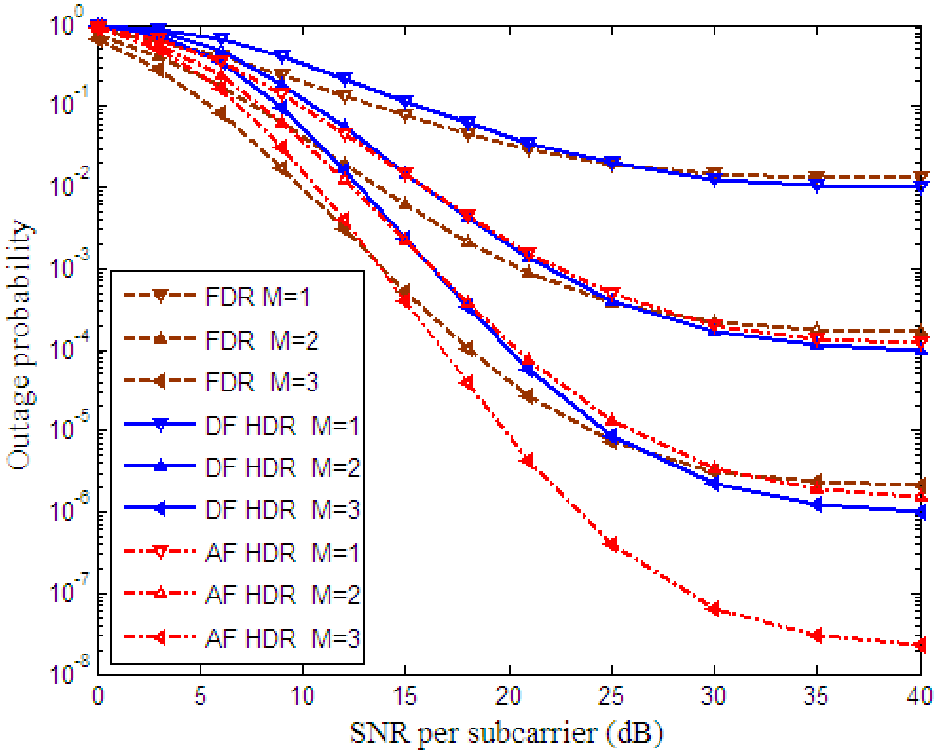

Figure 4.4 Outage performance of the proposed network is compared with half duplex AF and half duplex DF multiple relays network

Figure 4.5 shows the outage performance of the half duplex cognitive AF multiple relays network compared with half duplex DF multiple relays networks in the absence of NBI, at the data rate of 1 b/s/Hz. Simulations are indicated in dotted lines. At $M = 3$, the SNR requirement of the half duplex AF multiple relays network is 12 dB compared to 14 dB for half duplex DF multiple relays network at the outage probability of 10^{-3}. Further, for the outage probability of 10^{-3} and $M = 2$, SNR requirements of the half duplex AF multiple relays network and half duplex DF multiple relays network are 15 dB and 18 dB respectively.

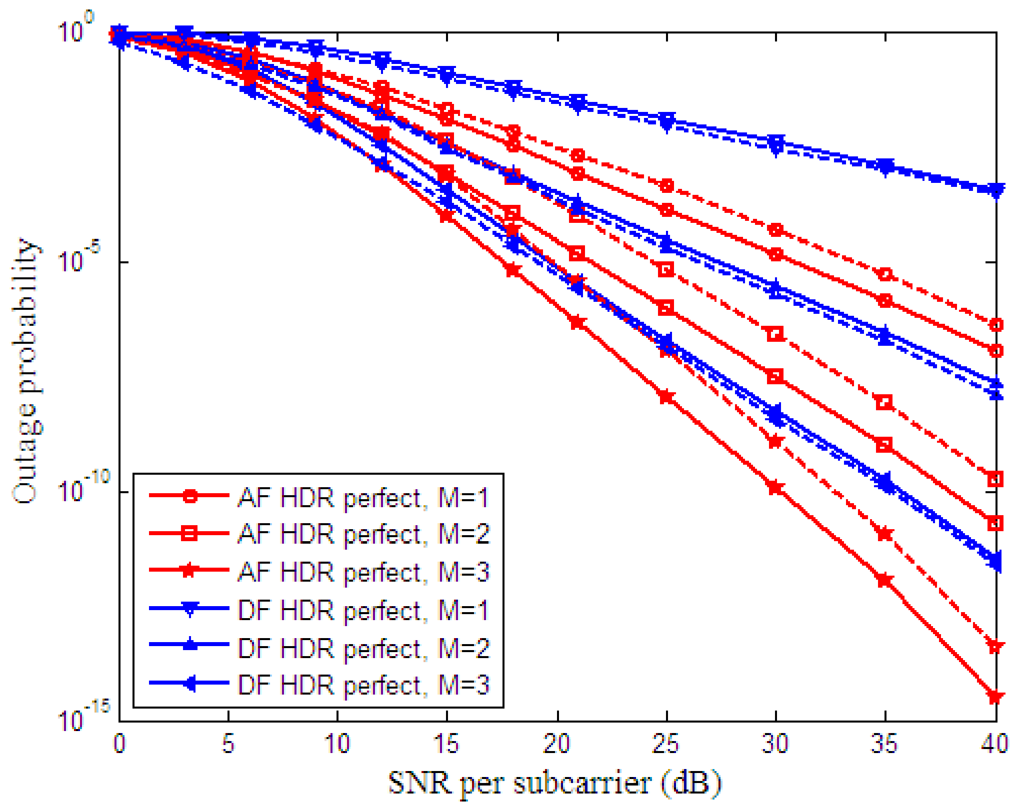

Figure 4.5 Outage performance of the proposed network is compared with half duplex AF and half duplex DF multiple relays network without interferences

In Figure 4.6, the outage performance of the full duplex cognitive AF single relay network is compared with half duplex AF and half duplex DF single relay network at the data rate of 0.5 b/s/Hz. Simulations are indicated in dotted lines. The mean of the interference CFR scalars are same as in Figure 4.3. It is observed that SNR requirement of the half duplex AF single relay network is 11 dB and for the full duplex AF single relay network is 20 dB at the outage probability of 10^{-2}. It indicates that in a single relay model, half duplex AF relay network provides the better performance than other network models due to the additional diversity gain from the direct link and the amplification factor.

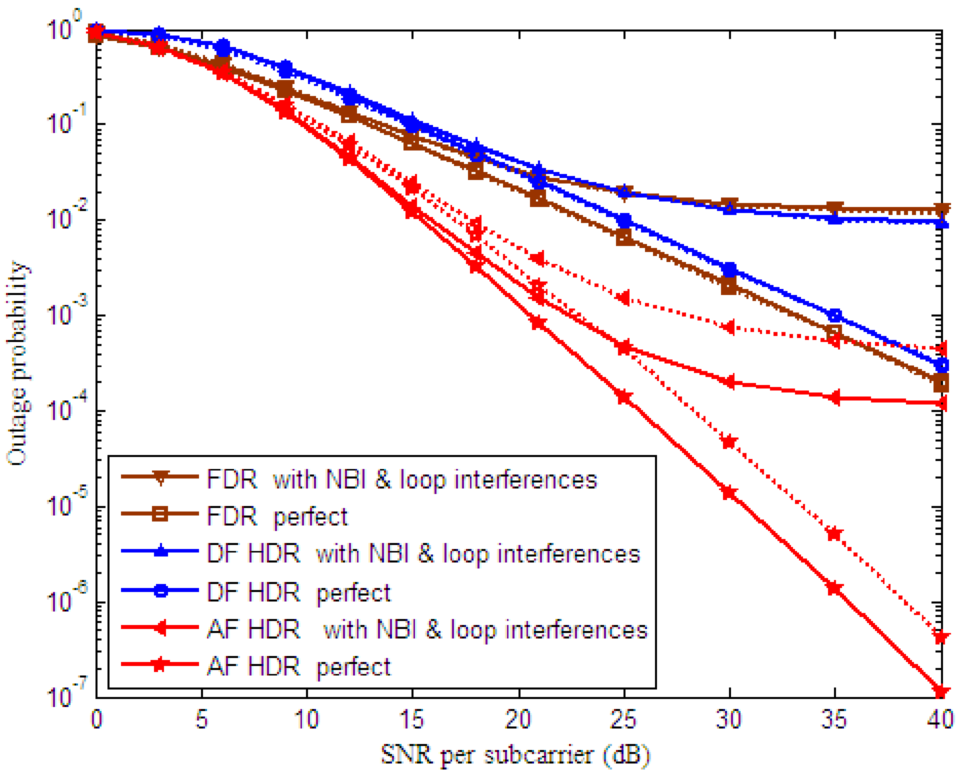

Figure 4.6 Comparison of the outage performance of full duplex cognitive AF single relay network, half duplex AF and half duplex DF single relay network

The outage performance of the proposed full duplex cognitive AF multiple relays network is shown in Figure 4.7 for the data rates varying from 0.5 b/s/Hz to 3 b/s/Hz at each subcarrier and SNR of 30 dB. The number of cooperating relay nodes is 3. For the NBI and loop interference is at -10dB, the outage probability is 10^{-2}, at the data rate of 0.5 b/s/Hz. When the interference decreases to -20 dB, the proposed network supports the data rate up to 2.5 b/s/Hz, at the outage probability of 10^{-2}. In the absence of NBI and loop interference, the network outage probability is decreases to 10^{-5}, at the data rate of 3 b/s/Hz

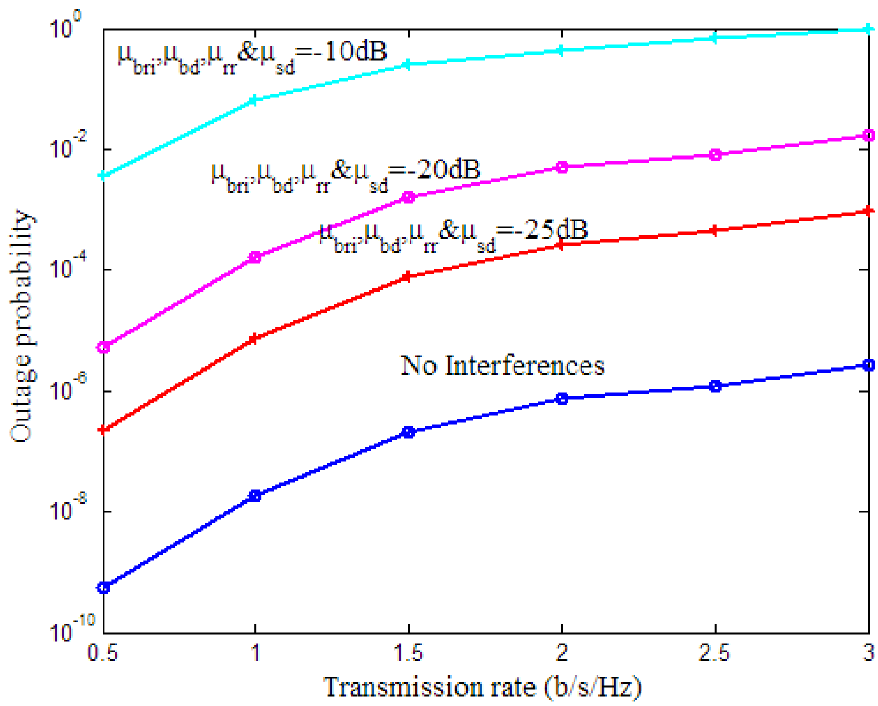

Figure 4.7 Outage performance of the proposed full duplex cognitive AF multiple relays network at various data rate

The outage performance of the proposed full duplex cognitive AF multiple relays network at the data rate of $1\,\text{b/s/Hz}$ is shown in Figure 4.8. The desired link shape parameters α_{sr_i} and $\alpha_{r_i d}$ is $0\,\text{dB}$ and scale parameters β_{sr_i} and $\beta_{r_i d}$ is $10\,\text{dB}$. The shape parameters α_{br_i,q_2} and α_{bd,q_2} of the interference link is $-10\,\text{dB}$ and scale parameters $\beta_{br_i,1}$ and $\beta_{bd,1}$ are $-11\,\text{dB}$ and $-12\,\text{dB}$ respectively, β_{br_i,q_2} and β_{bd,q_2} is $-10\,\text{dB}$ for all q_2, INR is $7\,\text{dB}$. It is assumed that INR value is same for all NBI signals and loop interference. At 25dB SNR, when M is increases from 1 to 2, the outage probability decreases from 0.0026 to 8.03×10^{-6}. It indicates that using the selection relay per subcarrier scheme, the outage performance of the network is highly improved. Further, increasing the M value to 3, the outage probability reduces to 2.59×10^{-8}. It is observed that the outage of the perfect network confirm the NBI effect in the proposed network.

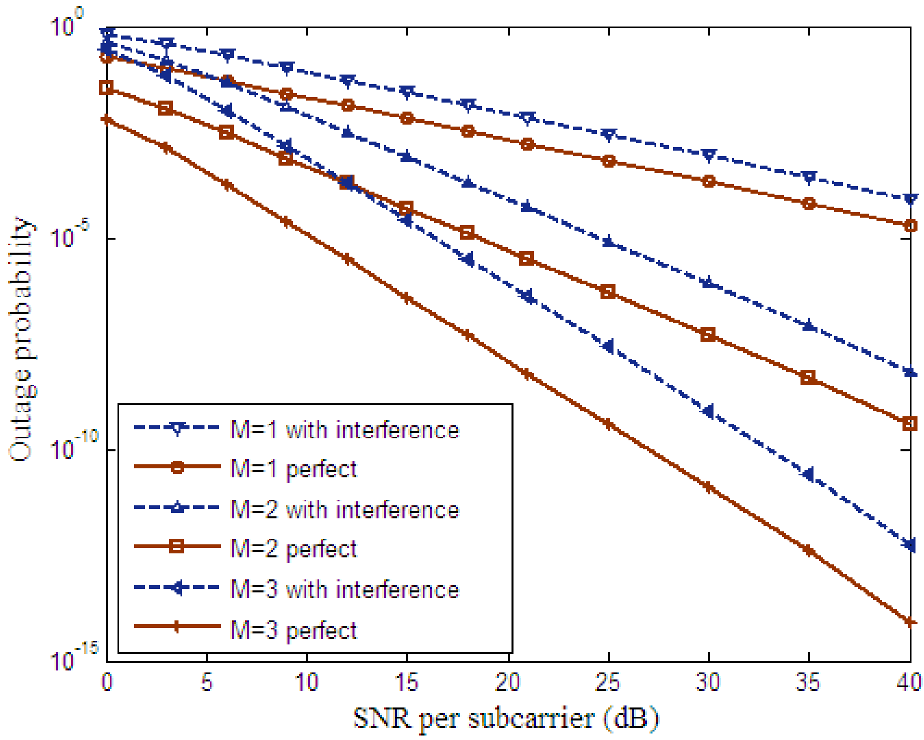

Figure 4.8 Outage performance of the proposed full duplex AF multiple relays network with different M values

The outage performance of the proposed full duplex cognitive AF multiple relays network with various m and R values is shown in Figure 4.9. The desired link shape parameters α_{sr_i} and α_{r_id} is $0\,\text{dB}$ and fading gain parameters Ω_{sr_i} and Ω_{r_id} is $10\,\text{dB}$. The interference link shape parameters α_{br_i,q_2} and α_{bd,q_2} are $-10\,\text{dB}$ and scale parameters $\beta_{br_i,1}$ and $\beta_{bd,1}$ are $-11\,\text{dB}$ and $-12\,\text{dB}$ respectively, β_{br_i,q_2} and β_{bd,q_2} are $-10\,\text{dB}$ for all q_2, INR is $7\,\text{dB}$. At $25\,\text{dB}$ SNR and the data rate varies from $0.5\,\text{b/s/Hz}$ to $1\,\text{b/s/Hz}$, the outage probability increases from 0.196 to 0.302 for $m = 0.5$ and from 0.001 to 0.002 for $m = 1$. It means that the outage of the network is also reduced by increasing the m value.

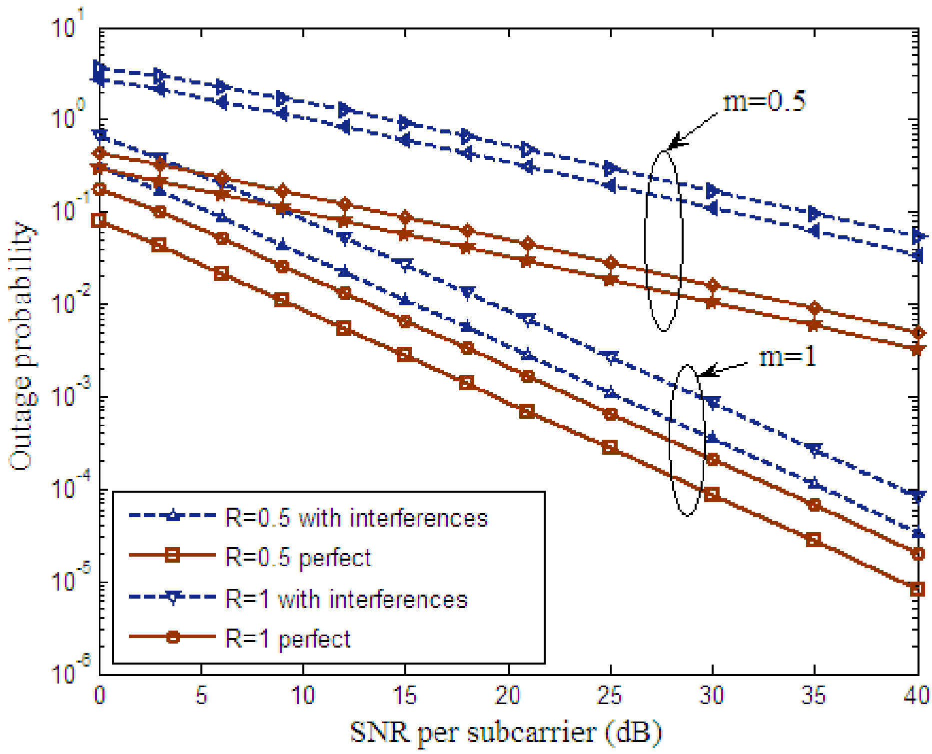

Figure 4.9 Outage performance of the proposed full duplex AF multiple relays network with different m and R values

The outage performance of the proposed full duplex cognitive AF multiple relays network for the various m values is shown in Figure 4.10. The data rate R is 1 b/s/Hz. The shape and scale parameters are same as the previous Figure. Figure shows that when increasing the m values, the outage probability of the network is significantly decreased. At $m=1$ (Rayleigh fading) and SNR of 25dB, the outage probability of the network is 0.002. At fading parameter K of 1.4 and the value of $m=(K+1)^2/(2K+1)=1.5$ (Rician fading), the outage probability of the network is 2.21×10^{-5}.

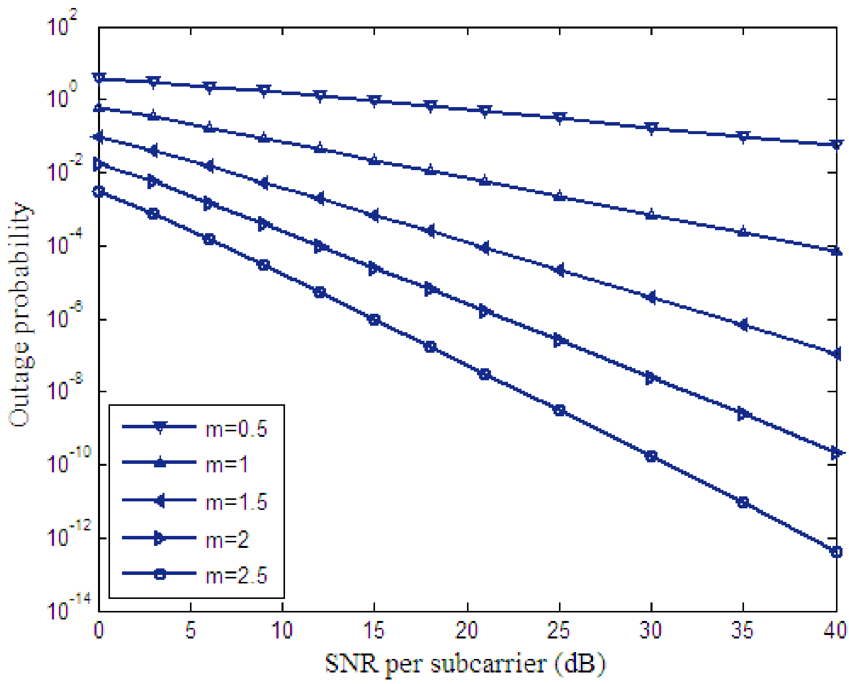

Figure 4.10 Outage Performance of the proposed full duplex AF multiple relays network with different m values

4.5 SUMMARY

In this chapter, the performance of OFDM based cognitive full duplex AF multiple relays network is analyzed in the presence of NBI. Best relay selection scheme is applied based on max-min criterion. Closed form analytical expression is derived for the outage probability of the proposed OFDM based cognitive full duplex AF multiple relays network in the presence of NBI. Performance of the proposed network is compared with OFDM based half duplex AF multiple relays network. Simulation results are given for proposed network, half duplex AF multiple relays network and half duplex DF multiple relays network and simulation results validate the derivation of the proposed OFDM based full duplex AF multiple relays network. It is concluded that the outage performance of the proposed OFDM based cognitive AF multiple relays network better than the half duplex AF multiple relays network at low SNR region.

CHAPTER 5

OFDM BASED MULTI-ANTENNAS FULL DUPLEX RELAY NETWORK IN THE PRESENCE OF NBI

5.1 PREAMBLE

Relay selection in OFDM based CR network improves the bandwidth efficiency in the presence of NBI. Another way to develop the performance of OFDM based CR network in the presence of NBI is that introduction of multiple antennas at relay and destination nodes. Multi-antennas technology constitutes a breakthrough in wireless systems which help to achieve the significant performance gains such as array gain, spatial diversity gain and spatial multiplexing gain. Another important benefit of multi-antennas is mitigation of interference using array gain which increases the tolerance to noise and interference power thereby improves the output SINR (Biglieri et al 2007). A new approach, optimum combining is proposed to suppress the NBI in OFDM based CR network.

Many of the works are proposed for wireless systems employing optimum combining. The performance of multi-antennas relays using optimum combining with joint relay and antenna selection is analyzed in the presence of co-channel interference (Suraweera & Beaulieu 2014). Using Eigen domain interference suppressing algorithms, the NBI is mitigated through exploiting the characteristics of the received signal in time and spatial domain (Liu et al 2014). Optimum combing is employed in digital mobile radio environments to reduce the power of the interference signals at the receiver (Winter 1984). Although, many of the research works are proposed with optimal weights based combining in wireless networks, the effect of NBI in OFDM based CR network through optimum combining is not yet to be considered. In this chapter, OFDM based multi-antennas full duplex DF relay network is proposed for the suppression of NBI.

5.2 SYSTEM MODEL

Consider an OFDM based full duplex multi-antennas DF relay network shown in Figure 5.1. In this network, source node SU_s has single antenna and destination node SU_d has M_D antennas. Full duplex DF relay node SU_r uses M_R receive antennas and uses single antenna for transmitting data to the destination node SU_d at the same time slot (Yue & Zhang 2006). It is assumed that there is no direct link between source node SU_s and destination node SU_d due to severe fading conditions. The $L_{sr_i} \times 1$ CIR vector between the source node SU_s to i^{th} receive antenna of the relay node SU_r is denoted as $\bar{\boldsymbol{h}}_{sr_i}$. The $L_{lr_i} \times 1$ CIR vector of loop/ echo interference at i^{th} receive antenna of the relay node SU_r is denoted as $\bar{\boldsymbol{h}}_{lr_i}$. The $L_{br_i} \times 1$CIR vector between the NBI node and i^{th} receive antenna of the relay node SU_r is denoted as $\bar{\boldsymbol{h}}_{br_i}$. $\bar{\boldsymbol{H}}_{sr_i}$, $\bar{\boldsymbol{H}}_{lr_i}$ and $\bar{\boldsymbol{H}}_{br_i}$ are $N \times N$ circulant matrices of CIR vectors $\bar{\boldsymbol{h}}_{sr_i}$, $\bar{\boldsymbol{h}}_{lr_i}$ and $\bar{\boldsymbol{h}}_{br_i}$ respectively (Gray 2006). The OFDM signal is assumed to have N subcarriers and use $v_1 \geq \max\left\{L_{sr_i}, L_{lr_i}, L_{br_i}\right\}$ guard subcarriers to suppress ISI. The total number of subcarriers in an OFDM signal is denoted by $P = N + v$.

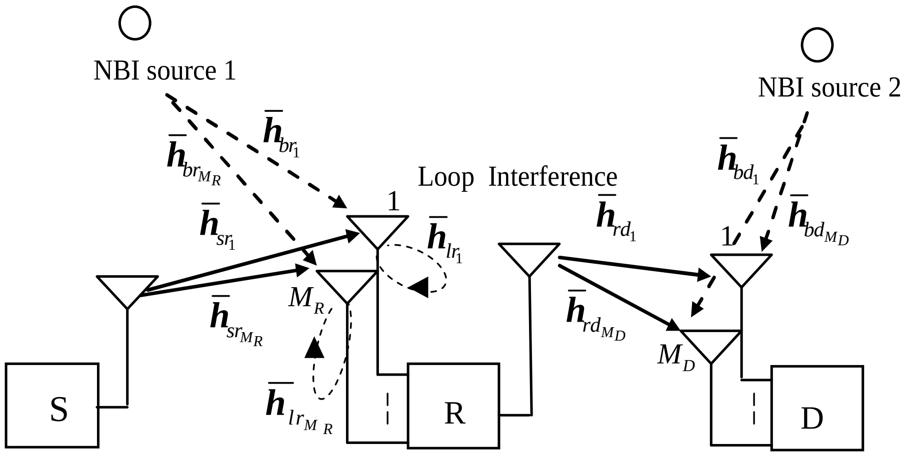

Figure 5.1 Full duplex Multi-antennas DF relay network

5.2.1 Receive Signal at Relay node

After removing the cyclic prefix, the $N \times 1$ time-domain receive signal vector $\bar{\boldsymbol{y}}_{sr_i}^{(n)}$ at i^{th} receive antenna of the relay node $\mathrm{SU_r}$ in n^{th} signal duration is given by

$$\bar{\boldsymbol{y}}_{sr_i}^{(n)} = \bar{\boldsymbol{H}}_{sr_i} \boldsymbol{F}_N^H \boldsymbol{x}_s^{(n)} + \bar{\boldsymbol{H}}_{lr_i} \bar{\boldsymbol{x}}_{r_i}^{(n)} + \boldsymbol{A}_{br}^{(n)} \bar{\boldsymbol{H}}_{br_i} \bar{\boldsymbol{b}}_r^{(n)} + \bar{\boldsymbol{z}}_{r_i}^{(n)}; \quad 1 \le i \le M_R \tag{5.1}$$

where $\boldsymbol{F}_N^H$ denote $N \times N$ IDFT matrix, $\boldsymbol{x}_s^{(n)}$ is $N \times 1$ data vector from source node $\mathrm{SU_s}$ in n^{th} signal duration. $\bar{\boldsymbol{x}}_{r_i}^{(n)}$ is $N \times 1$ loop/echo interference signal vector at i^{th} receive antenna of the relay node $\mathrm{SU_r}$ in $(n-1)^{th}$ signal duration. The $\bar{\boldsymbol{b}}_r^{(n)}$ represents the $N \times 1$ NBI signal vector, $\boldsymbol{A}_{br}^{(n)}$ is the CFO for NBI signal represented by a $N \times N$ diagonal matrix $\boldsymbol{A}_{br}^{(n)} = diag\ \{1,$ $\exp(j2\pi\alpha/N)$, ..., $\exp(j2\pi\alpha(N-1)/N)\}$, where α is uniformly distributed random variable over the interval $[-0.5, 0.5]$, $\bar{\boldsymbol{z}}_{r_i}^{(n)}$ is $N \times 1$ complex white Gaussian noise vector whose elements are independent and identically distributed with zero mean.

The frequency domain representation of the receive signal vector $\bar{\boldsymbol{y}}_{sr_i}^{(n)}$ is obtained by taking N-point DFT of (5.1)

$$\boldsymbol{y}_{sr_i}^{(n)} = \boldsymbol{F}_N \bar{\boldsymbol{y}}_{sr_i}^{(n)} = \Lambda_{sr_i} \boldsymbol{x}^{(n)} + \Lambda_{lr_i} \boldsymbol{x}_{r_i}^{(n)} + \boldsymbol{C}_{br} \Lambda_{br_i} \boldsymbol{b}_r^{(n)} + \boldsymbol{z}_{r_i}^{(n)}; \quad 1 \le i \le M_R \tag{5.2}$$

where Λ_{sr_i}, Λ_{lr_i} and Λ_{br_i} are the frequency domain diagonal channel matrices obtained using the decomposition of the time domain channel matrices $\bar{\boldsymbol{H}}_{sr_i} = \boldsymbol{F}_N^H \Lambda_{sr_i} \boldsymbol{F}_N$, $\bar{\boldsymbol{H}}_{lr_i} = \boldsymbol{F}_N^H \Lambda_{lr_i} \boldsymbol{F}_N$ and $\bar{\boldsymbol{H}}_{br_i} = \boldsymbol{F}_P^H \Lambda_{br_i} \boldsymbol{F}_P$ respectively. The elements of the diagonal matrices are N-point DFT of CIR vectors $\bar{\boldsymbol{h}}_{sr_i}$, $\bar{\boldsymbol{h}}_{lr_i}$ and $\bar{\boldsymbol{h}}_{br_i}$ respectively. $\boldsymbol{C}_{br}$ is $N \times N$ circulant matrix of CFO defined as $\boldsymbol{F}_N \boldsymbol{A}_{br}^{(n)} \boldsymbol{F}_N^H$, $\boldsymbol{x}_{r_i}^{(n-1)}$ is the loop interference in frequency domain, $\boldsymbol{b}_r^{(n)}$ and $\boldsymbol{z}_{r_i}^{(n)}$ are the NBI signal and noise vectors in frequency domain respectively. The $N \times 1$ NBI signal vector at relay node $\mathrm{SU_r}$ is defined as $\boldsymbol{b}_r^{(n)} = \boldsymbol{F}_N \bar{\boldsymbol{b}}_r^{(n)}$ which is sparse in nature. The k^{th} subcarrier element of $\boldsymbol{b}_r^{(n)}(k) \ne 0$ if $k \in I_{br}$, where I_{br} is a set of indices of non-zero elements.

It is assumed that the NBI and CFO of NBI are quasi-static over one OFDM signal. Hence, the $M_R \times 1$ receive signal vector at k^{th} subcarrier in relay node $\mathrm{SU_r}$ is written as

$$\boldsymbol{y}_{sr,k}^{(n)} = \lambda_{sr} x^{(n)}(k) + \lambda_{lr} x_r^{(n)}(k) + \sum_{j_1=1,\, j_1 \in I_{br}}^{N_{br}} \lambda_{br_{j_1}} c_{br_{j_1}} b_{r_{j_1}}^{(n)} + \boldsymbol{z}_r^{(n)}; \; 0 \le k \le N-1 \tag{5.3}$$

where $\lambda_{sr} = \left[\lambda_{sr_1}, \lambda_{sr_2}, \ldots, \lambda_{sr_{M_R}}\right]^T$, λ_{lr} and λ_{br} are $M_R \times 1$ column vector whose elements are independent identically distributed complex Gaussian random variables with zero mean and covariance matrix $\Sigma = E\left(\lambda_{xy} \lambda_{xy}^H\right)$,

$xy \in \{sr, lr, br\}$. In this, first term is the desired signal, second term is the loop interference, third term is the NBI signal and fourth term is noise.

5.2.2 Receive Signal at Destination Node

The relay node $\mathrm{SU_r}$ forwards the re-encoded OFDM signal to the destination node $\mathrm{SU_d}$. The $L_{rd_l} \times 1$ CIR vector between the relay node $\mathrm{SU_r}$ and l^{th} receive antenna of destination node $\mathrm{SU_d}$ is denoted as $\bar{\boldsymbol{h}}_{rd_l}$. The $L_{bd_l} \times 1$ CIR vector between NBI source and l^{th} receive antenna of destination node $\mathrm{SU_d}$ is denoted as $\bar{\boldsymbol{h}}_{bd_l}$. The OFDM signal uses $v_2 \geq \max\{L_{rd_l}, L_{bd_l}\}$ guard subcarriers. After removing the cyclic prefix, the $N \times 1$ time domain receive signal vector in l^{th} receive antenna and n^{th} signal duration at destination node $\mathrm{SU_d}$ is written as

$$\bar{\boldsymbol{y}}_{rd_l}^{(n)} = \bar{\boldsymbol{H}}_{rd_l} \bar{\boldsymbol{x}}_r^{(n)} + \boldsymbol{A}_{bd} \bar{\boldsymbol{H}}_{bd_l} \bar{\boldsymbol{b}}_d^{(n)} + \bar{\boldsymbol{z}}_{d_l}^{(n)}; \quad 1 \leq l \leq M_D \tag{5.4}$$

By taking N-point DFT, the $N \times 1$ frequency domain receive signal vector in l^{th} receive antenna of destination node $\mathrm{SU_d}$ is given by

$$\boldsymbol{y}_{rd_l}^{(n)} = \Lambda_{rd_l} \boldsymbol{x}_r^{(n)} + \boldsymbol{C}_{bd} \Lambda_{bd_l} \boldsymbol{b}_d^{(n)} + \boldsymbol{z}_{d_l}^{(n)}; \quad 1 \leq l \leq M_D \tag{5.5}$$

where Λ_{rd_l} and Λ_{bd_l} are the frequency domain diagonal channel matrices. $\boldsymbol{C}_{bd}$ is $N \times N$ circulant matrix of CFO defined as $\boldsymbol{F}_N \boldsymbol{A}_{bd} \boldsymbol{F}_N^H$.

The $M_D \times 1$ receive signal vector $\boldsymbol{y}_{rd}^{(n)}(k)$ at k^{th} subcarrier in destination node $\mathrm{SU_d}$ is given by

$$\boldsymbol{y}_{rd,k}^{(n)} = \lambda_{rd} x_r^{(n)}(k) + \sum_{j_2=1, j_2 \in I_{bd}}^{N_{bd}} \lambda_{bd_{j_2}} c_{bd_{j_2}} b_{d_{j_2}}^{(n)} + \boldsymbol{z}_d^{(n)}; 0 \leq k \leq N-1 \tag{5.6}$$

5.3 NBI SUPPRESSION USING OPTIMUM COMBINING

In this section, the process of combining multiple antennas received signal at each subcarrier of OFDM signal using the techniques of optimum combining and maximal ratio combining in relay and destination nodes are described.

5.3.1 Optimum Combining at Relay Node

The receive signals from M_R antennas are optimally combined at each subcarrier. The optimal combining weight vector at k^{th} subcarrier in the relay node $\mathrm{SU_r}$ is defined as (Shah & Hoimovich 1998)

$$\boldsymbol{w}_{OC,k}^{R} = \boldsymbol{R}_1^{-1}\lambda_{sr} \tag{5.7}$$

where λ_{sr} is $M_R \times 1$ column vector, $\boldsymbol{R}_1$ is the covariance matrix of the NBI and loop interference in the received signal at relay node $\mathrm{SU_r}$. Using Equation (5.3), it is determined as

$$\boldsymbol{R}_1 = E\left(\left|\lambda_{lr} x_r^{(n)}(k) + \sum_{j_1=1,\, j_1 \in I_{br}}^{N_{br}} \lambda_{br_{j_1}} c_{br_{j_1}} b_{r_{j_1}}^{(n)}\right|^2\right) \tag{5.8}$$

Assuming that the channel state information and CFO are perfectly known at the receiver, Equation (5.8) can be written as

$$\boldsymbol{R}_1 = \lambda_{lr}\lambda_{lr}^{H} E\left(\left|x_r^{(n)}(k)\right|^2\right) + \sum_{j_1=1, j_1\in I_{br}}^{N_{br}} \lambda_{br_{j_1}} \lambda_{br_{j_1}}^{H} \sum_{j_1=1, j_1\in I_{br}}^{N_{br}} \left|c_{br_{j_1}}\right|^2 \sum_{j_1=1, j_1\in I_{br}}^{N_{br}} E\left(\left|b_{r_{j_1}}^{(n)}\right|^2\right) \tag{5.9}$$

After simplification, Equation (5.9) becomes

$$\boldsymbol{R}_1 = \lambda_{lr}\lambda_{lr}^{H} P_r + \sum_{j_1=1,\, j_1 \in I_{br}}^{N_{br}} \lambda_{br_{j_1}} \lambda_{br_{j_1}}^{H} N_{br} \varsigma_{br} P_{\mathrm{int}} \tag{5.10}$$

where $P_r = E\left(\left|x_r^{(n)}\right|^2\right)$ is relay transmit power, $N_{br} = \mathrm{length}\left(\mathrm{I_{br}}\right)$ is number of interference affected subcarriers of the OFDM signal at relay node,

$\varsigma_{br} = \sum_{j_1=1, j_1 \in I_{br}}^{N_{br}} \left| c_{br_{j_1}} \right|^2$ is the squared norm of the CFO at relay node and the IPPS of NBI at relay node is denoted as $N_{br} P_{\text{int}} = \sum_{j_1=1, j_1 \in I_{br}}^{N_{br}} E\left(\left| b_{r_{j_1}}^{(n)} \right|^2 \right)$.

At high loop interference power, $P_r \approx P_{\text{int}}$, then Equation (5.10) can be written as

$$\boldsymbol{R}_1 \approx \left(\lambda_{lr} \lambda_{lr}^H + N_{br} \varsigma_{br} \sum_{j_1=1, j_1 \in I_{br}}^{N_{br}} \lambda_{br_{j_1}} \lambda_{br_{j_1}}^H \right) P_{\text{int}} \tag{5.11}$$

Now, Equation (5.11) can be rewritten as

$$\boldsymbol{R}_1 \approx \left(\left(N_{br} + 1 \right) \varsigma_{br} \sum_{j_1=1, j_1 \in I_{br}}^{N_{br}+1} \lambda_{br_{j_1}} \lambda_{br_{j_1}}^H \right) P_{\text{int}} = \left(N_{br} + 1 \right) \varsigma_{br} \boldsymbol{R}_{11} P_{\text{int}} \tag{5.12}$$

where $\boldsymbol{R}_{11} = \sum_{j_1=1, j_1 \in I_{br}}^{N_{br}} \lambda_{br_{j_1}} \lambda_{br_{j_1}}^H$ is the covariance matrix of the loop interference and NBI. By substituting Equation (5.12) in Equation (5.7), the relay node optimum combiner weight vector can be obtained as

$$\boldsymbol{w}_{OC,k}^R = \frac{\boldsymbol{R}_{11}^{-1} \lambda_{sr}}{\left(N_{br} + 1 \right) \varsigma_{br} P_{\text{int}}} \tag{5.13}$$

Using Equation (5.13), the scalar optimum combiner output signal $\boldsymbol{y}_{sr,k}^{OC}$ at relay node SU_r is obtained as

$$y_{sr,k}^{OC} = \left(\boldsymbol{w}_{OC,k}^{R} \right)^{\boldsymbol{H}} \boldsymbol{y}_{sr,k} \tag{5.14}$$

By substituting Equation (5.3) and Equation (5.13) in Equation (5.14), the optimum combiner output at relay node SU_r is expressed as

$$y_{sr,k}^{OC}=\frac{\boldsymbol{R}_{11}^{-1}\left|\lambda_{sr}\right|^{2}x^{(n)}(k)}{\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}}+\frac{\boldsymbol{R}_{11}^{-1}\lambda_{sr}^{H}\lambda_{lr}x_{r}^{(n)}(k)}{\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}}+\frac{\boldsymbol{R}_{11}^{-1}\lambda_{sr}^{H}\sum_{j_1=1,\,j_1\in I_{br}}^{N_{br}}\lambda_{br_{j_1}}c_{br_{j_1}}b_{r_{j_1}}^{(n)}}{\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}}+\frac{\boldsymbol{R}_{11}^{-1}\lambda_{sr}^{H}\boldsymbol{z}_{r}^{(n)}}{\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}} \tag{5.15}$$

From Equation (5.15), the output SINR at k^{th} subcarrier in relay node is expressed as

$$\Gamma_{sr,OC}^{(n)}(k)=\frac{\boldsymbol{R}_{11}^{-2}\left|\lambda_{sr}\right|^{4}E\left(\left|x^{(n)}(k)\right|^{2}\right)\Big/\left(\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}\right)^{2}}{\left(\left(\boldsymbol{R}_{11}^{-1}\left|\lambda_{sr}\right|^{2}\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}+\boldsymbol{R}_{11}^{-2}\left|\lambda_{sr}\right|^{2}\sigma_{zr}^{2}\right)\Big/\left(\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}\right)^{2}\right)} \tag{5.16}$$

Multiplying both numerator and denominator of Equation (5.16) by $\boldsymbol{R}_{11}\left(\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}\right)^{2}\Big/\left|\lambda_{sr}(k,k)\right|^{2}$, it can be simplified as

$$\Gamma_{sr,OC}^{(n)}(k)=\frac{\left|\lambda_{sr}\right|^{2}\boldsymbol{R}_{11}^{-1}P_{s}}{\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}+\boldsymbol{R}_{11}^{-1}\sigma_{zr}^{2}} \tag{5.17}$$

where $P_{s}=E\left(\left|x_{s}^{(n)}\right|^{2}\right)$ is source transmit power. At high interference power, noise variance σ_{zr}^{2} is very less and negligible. Hence, the maximum output Signal to Interference Ratio (SIR) at relay node $\mathrm{SU_r}$ using optimum combining is written as

$$\Gamma_{sr,OC}^{(n)}(k)=\frac{\lambda_{sr}^{H}\boldsymbol{R}_{11}^{-1}\lambda_{sr}P_{s}}{\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}} \tag{5.18}$$

Let $\lambda_{1}(k)=\lambda_{sr}^{H}\boldsymbol{R}_{11}^{-1}\lambda_{sr}$, then (5.18) becomes

$$\Gamma_{sr,OC}^{(n)}(k)=\lambda_{1}(k)\left(P_{s}\Big/\left(N_{br}+1\right)\varsigma_{br}P_{\text{int}}\right) \tag{5.19}$$

where $\lambda_{1}(k)$ is a scalar random variable follows the null distribution of the Hotelling's T^{2} statistic (Muirhead 2005).

5.3.2 Optimum Combining at Destination Node

The optimum combining weight vector at destination node is defined as

$$\boldsymbol{w}_{OC,k}^{D} = \boldsymbol{R}_2^{-1}\lambda_{rd} \tag{5.20}$$

where λ_{rd} is $M_D \times 1$ column vector, $\boldsymbol{R}_2$ is the covariance matrix of the NBI term in the received signal at destination node $\mathrm{SU_d}$. Using Equation (5.6), it is determined as

$$\boldsymbol{R}_2 = E\left(\left|\sum_{j_2=1,\, j_2\in I_{bd}}^{N_{bd}} \lambda_{bd_{j_2}} c_{bd_{j_2}} b_{d_{j_2}}^{(n)}\right|^2\right) \tag{5.21}$$

With the knowledge of channel state information and CFO, Equation (5.21) can be written as

$$\boldsymbol{R}_2 = \sum_{j_2=1, j_2\in I_{bd}}^{N_{bd}} \lambda_{bd_{j_2}} \lambda_{bd_{j_2}}^{H} \sum_{j_2=1, j_2\in I_{bd}}^{N_{bd}} \left|c_{bd_{j_2}}\right|^2 \sum_{j_2=1, j_2\in I_{bd}}^{N_{bd}} E\left(\left|b_{d_{j_2}}^{(n)}\right|^2\right) \tag{5.22}$$

Now, the covariance matrix of the NBI term can be determined as

$$\boldsymbol{R}_2 = \sum_{j_2=1,\, j_2\in I_{bd}}^{N_{bd}} \lambda_{bd_{j_2}} \lambda_{bd_{j_2}}^{H} N_{bd}\varsigma_{bd} P_{\text{int}} = \boldsymbol{R}_{22} N_{bd}\varsigma_{bd} P_{\text{int}} \tag{5.23}$$

where $\varsigma_{bd} = \sum_{j_2=1,\, j_2\in I_{bd}}^{N_{bd}} \left|c_{bd_{j_2}}\right|^2$ is the squared norm of CFO, $N_{bd} = \text{length}\left(\mathrm{I}_{bd}\right)$ is number of interference affected subcarriers of the OFDM signal at destination node, $N_{bd}P_{\text{int}} = \sum_{j_2=1,\, j_2\in I_{bd}}^{N_{bd}} E\left(\left|b_{d_{j_2}}^{(n)}\right|^2\right)$ is IPPS at destination node and $\boldsymbol{R}_{22} = \sum_{j_2=1,\, j_2\in I_{bd}}^{N_{bd}} \lambda_{bd_{j_2}} \lambda_{bd_{j_2}}^{H}$ is the covariance matrix. By substituting Equation (5.23) in Equation (5.20), the optimum combining weight vector at destination node $\mathrm{SU_d}$ is obtained as

$$\boldsymbol{w}_{OC,k}^{D} = \frac{\boldsymbol{R}_{22}^{-1}\lambda_{rd}}{N_{bd}\varsigma_{bd}P_{\text{int}}} \tag{5.24}$$

The scalar optimum combiner output $y_{rd,OC}^{(n)}(k)$ at destination node SU_d is expressed as

$$y_{rd,k}^{OC} = \left(\boldsymbol{w}_{OC,k}^{D}\right)^{H} \boldsymbol{y}_{rd,k} \tag{5.25}$$

By substituting Equation (5.6) and Equation (5.24) in Equation (5.25), the optimum combiner output at destination node SU_d is written as

$$y_{rd,k}^{OC} = \frac{\boldsymbol{R}_{22}^{-1}\left|\lambda_{rd}\right|^{2} x^{(n)}(k)}{N_{bd}\varsigma_{bd}P_{\text{int}}} + \frac{\boldsymbol{R}_{22}^{-1}\lambda_{rd}^{H}\sum\limits_{j_2=1,\, j_2\in I_{bd}}^{N_{bd}} \lambda_{bd_{j_2}} c_{bd_{j_2}} b_{d_{j_2}}^{(n)}}{N_{bd}\varsigma_{bd}P_{\text{int}}} + \frac{\boldsymbol{R}_{22}^{-1}\lambda_{rd}^{H}\boldsymbol{z}_{d}^{(n)}}{N_{bd}\varsigma_{bd}P_{\text{int}}} \tag{5.26}$$

From Equation (5.26), the output SINR at destination node is expressed as

$$y_{rd,k}^{OC} = \frac{\boldsymbol{R}_{22}^{-2}\left|\lambda_{rd}\right|^{4} E\left(\left|x^{(n)}(k)\right|^{2}\right) \Big/ \left(N_{bd}\varsigma_{bd}P_{\text{int}}\right)^{2}}{\left(\left(\boldsymbol{R}_{22}^{-1}\left|\lambda_{rd}\right|^{2} N_{bd}\varsigma_{bd}P_{\text{int}} + \boldsymbol{R}_{22}^{-2}\left|\lambda_{rd}\right|^{2}\sigma_{zd}^{2}\right) \Big/ \left(N_{bd}\varsigma_{bd}P_{\text{int}}\right)^{2}\right)} \tag{5.27}$$

Multiply both numerator and denominator by $\boldsymbol{R}_{22}\left(N_{bd}\varsigma_{bd}P_{\text{int}}\right)^{2} \big/ \left|\lambda_{rd}\right|^{2}$, Equation (5.27) can be simplified as

$$\Gamma_{rd,OC}^{(n)}(k) = \frac{\left|\lambda_{rd}\right|^{2}\boldsymbol{R}_{22}^{-1}P_{r}}{N_{bd}\varsigma_{bd}P_{\text{int}} + \boldsymbol{R}_{22}^{-1}\sigma_{zd}^{2}} \tag{5.28}$$

At high interference power, noise variance σ_{zd}^{2} is very less and negligible. Hence, the maximum output SIR at destination node is written as

$$\Gamma_{rd,OC}^{(n)}(k) = \frac{\lambda_{rd}^{H}\boldsymbol{R}_{22}^{-1}\lambda_{rd}P_{r}}{N_{bd}\varsigma_{bd}P_{\text{int}}} \tag{5.29}$$

Let $\lambda_2(k) = \lambda_{rd}^{H}\boldsymbol{R}_{22}^{-1}\lambda_{rd}$, then Equation (5.29) becomes

$$\Gamma_{rd,OC}^{(n)}(k) = \lambda_2(k)\left(P_r / N_{bd}\varsigma_{bd}P_{\text{int}}\right) \tag{5.30}$$

5.3.3 Maximal Ratio Combining at Relay and Destination Nodes

The Maximal Ratio Combining weight vector at relay node $\mathrm{SU_r}$ is defined as (Shah & Hoimovich 2000)

$$\boldsymbol{w}_{MRC,k}^{R} = \boldsymbol{\lambda}_{sr} \tag{5.31}$$

The combined output signal after processing maximal ratio combining at relay node $\mathrm{SU_r}$ is expressed as

$$y_{sr,k}^{MRC} = \left(\boldsymbol{w}_{MRC,k}^{R}\right)^{H} \boldsymbol{y}_{sr}^{(n)} \tag{5.32}$$

By substituting Equation (5.31) and Equation (5.3) in Equation (5.32), the combined output signal in scalar at relay node $\mathrm{SU_r}$ is expressed as

$$y_{sr,k}^{MRC} = \left|\lambda_{sr}\right|^{2} x^{(n)}(k) + \lambda_{sr}^{H} \lambda_{lr} x_{r}^{(n)}(k) + \lambda_{sr}^{H} \sum_{j_1=1,\, j_1 \in I_{br}}^{N_{br}} \lambda_{br_{j_1}} c_{br_{j_1}} b_{r_{j_1}}^{(n)} + \lambda_{sr}^{H} \boldsymbol{z}_{r}^{(n)}$$

(5.33)

From Equation (5.33), the output SINR using maximal ratio combining at relay node $\mathrm{SU_r}$ can be expressed as

$$\Gamma_{sr,MRC}^{(n)}(k) = \frac{\left|\lambda_{sr}\right|^{4} P_{s}}{\left|\lambda_{sr}\right|^{2}\left|\lambda_{lr}\right|^{2} P_{r} + \left|\lambda_{sr}\right|^{2} \sum_{j_1=1,\, j_1 \in I_{br}}^{N_{br}} \left|\lambda_{br_{j_1}}\right|^{2} N_{br} \varsigma_{br} P_{\mathrm{int}} + \left|\lambda_{sr}\right|^{2} \sigma_{zr}^{2}}$$

(5.34)

At high loop interference power, $P_r \approx P_{\mathrm{int}}$, Equation (5.34) can be written as

$$\Gamma_{sr,MRC}^{(n)}(k) = \frac{\left|\lambda_{sr}\right|^{4} P_{s}}{\left(\left|\lambda_{sr}\right|^{2}\left|\lambda_{lr}\right|^{2} + N_{br} \varsigma_{br} \left|\lambda_{sr}\right|^{2} \sum_{j_1=1,\, j_1 \in I_{br}}^{N_{br}} \left|\lambda_{br_{j_1}}\right|^{2}\right) P_{\mathrm{int}} + \left|\lambda_{sr}\right|^{2} \sigma_{zr}^{2}}$$

(5.35)

Now, output SINR using maximal ratio combining at relay node can be written as

$$\Gamma_{sr,MRC}^{(n)}(k)=\frac{\left|\lambda_{sr}\right|^{4}E\left(\left|x^{(n)}(k)\right|^{2}\right)}{\left((N_{br}+1)\varsigma_{br}\left|\lambda_{sr}\right|^{2}\sum_{j_1=1,\,j_1\in I_{br}}^{N_{br}+1}\left|\lambda_{br_{j_1}}\right|^{2}\right)P_{\mathrm{int}}+\left|\lambda_{sr}\right|^{2}\sigma_{zr}^{2}} \tag{5.36}$$

Let $\lambda_{m_1}(k,j_1)=\lambda_{sr}^{H}\lambda_{br_{j_1}}/\left|\lambda_{sr}\right|$ is complex Gaussian random variable conditioned on λ_{sr}. Then, Equation (5.36) can be simplified as

$$\Gamma_{sr,MRC}^{(n)}(k)=\frac{\left|\lambda_{sr}\right|^{2}P_{s}}{\sum_{j_1=1,\,j_1\in I_{br}}^{N_{br}+1}\left|\lambda_{m_1}(k,j_1)\right|^{2}(N_{br}+1)\varsigma_{br}P_{\mathrm{int}}+\left|\lambda_{sr}\right|^{2}\sigma_{zr}^{2}} \tag{5.37}$$

At high interference power, noise variance σ_{zr}^{2} is very less and negligible. Hence, the maximum output SIR using maximal ratio combining at relay node is determined as

$$\Gamma_{sr,MRC}^{(n)}(k)=\frac{\sum_{i=1}^{M_R}\left|\lambda_{sr}(k,i)\right|^{2}P_{s}}{\sum_{j_1=1,\,j_1\in I_{br}}^{N_{br}+1}\left|\lambda_{m_1}(k,j_1)\right|^{2}(N_{br}+1)\varsigma_{br}P_{\mathrm{int}}} \tag{5.38}$$

Similarly, the output SIR using maximal ratio combining at the destination node can be written as

$$\Gamma_{rd,MRC}^{(n)}(k)=\frac{\sum_{l=1}^{M_D}\left|\lambda_{rd}(k,l)\right|^{2}P_{r}}{\sum_{j_2=1,\,j_2\in I_{bd}}^{N_{bd}}\left|\lambda_{m_2}(k,j_2)\right|^{2}N_{bd}\varsigma_{bd}P_{\mathrm{int}}} \tag{5.39}$$

where $\lambda_{m_2}(k,j_2)=\lambda_{rd}^{H}\lambda_{bd_{j_2}}/\left|\lambda_{rd}\right|$ is complex Gaussian random variable conditioned on λ_{rd}.

5.4 OUTAGE ANALYSIS OF OFDM BASED MULTI-ANTENNAS FULL DUPLEX RELAY NETWORK

In this section, the outage performance of the full duplex multiple antennas relay network with optimum combiner is analyzed at subcarrier level.

5.4.1 End-to-End Outage Probability Using Optimum Combining

The overall outage probability of the proposed network using optimum combining at k^{th} subcarrier for the given data rate of R b/s/Hz is defined as

$$P_{out,k}^{FD-DF-OC}\left(R\right)=\Pr\left(\min\left(\log\left(1+\Gamma_{sr,OC}^{(n)}\left(k\right)\right),\log\left(1+\Gamma_{rd,OC}^{(n)}\left(k\right)\right)\right)<R\right) \tag{5.40}$$

where $\min\left(\log\left(1+\Gamma_{sr,OC}^{(n)}\left(k\right)\right),\log\left(1+\Gamma_{rd,OC}^{(n)}\left(k\right)\right)\right)$ is maximum average mutual information of the DF relay network. Now, Equation (5.40) can be written as

$$\begin{aligned}P_{out,k}^{FD-DF-OC}\left(R\right)&=\Pr\left(\Gamma_{sr,OC}^{(n)}\left(k\right)<2^{R}-1\right)\\&+\left[1-\Pr\left(\Gamma_{sr,OC}^{(n)}\left(k\right)<2^{R}-1\right)\right]\Pr\left(\Gamma_{rd,OC}^{(n)}\left(k\right)<2^{R}-1\right)\end{aligned} \tag{5.41}$$

where $\Pr\left(\Gamma_{sr,OC}^{(n)}\left(k\right)<2^{R}-1\right)$ is the CDF of source to relay Signal to Interference Ratio (SIR) and $\Pr\left(\Gamma_{rd,OC}^{(n)}\left(k\right)<2^{R}-1\right)$ is the CDF of relay to destination SIR. In source to relay SIR, $\Gamma_{sr,OC}^{(n)}\left(k\right)$, λ_{sr} is $M_R\times 1$ column vector whose elements are complex Gaussian random variables with zero mean and covariance matrix, $\Sigma=\lambda_{sr}\lambda_{sr}^{H}$, the interference covariance matrix, $\boldsymbol{R}_{11}$ follows the complex Wishart distribution $CW_{M_R}\left(\Sigma,N_{br}\right)$, with parameters $\Sigma=\mathrm{I}_{M_R}$ and N_{br} degrees of freedom. From (Shah & Hoimovich 1998), the PDF of source to relay SIR can be obtained as

$$f_{\Gamma^{(n)}_{sr,OC}(k)}(\gamma) = \frac{\Gamma(N_{br}+2)}{\Gamma(M_R)\Gamma(N_{br}-M_R+2)}(P_1)^{N_{br}+2-M_R}\frac{\gamma^{M_R-1}}{(P_1+\gamma)^{N_{br}+2}} \qquad (5.42)$$

where $P_1 = ((N_{br}+1)P_{\text{int}}/P_s)$, $\gamma = 2^R - 1$ is the threshold SIR and $\Gamma(\cdot)$ is the Gamma function. Using Equation (5.42), the CDF of $\Gamma^{(n)}_{sr,OC}(k)$ can be written as

$$\begin{aligned} F_{\Gamma^{(n)}_{sr,OC}(k)}(\gamma) &= \int_0^{\gamma} f_{\Gamma^{(n)}_{sr,OC}(k)}(\gamma)\,d\gamma \\ &= \frac{\Gamma(N_{br}+2)}{\Gamma(M_R)\Gamma(N_{br}-M_R+2)}(P_1)^{N_{br}+2-M_R}\int_0^{\gamma}\frac{\gamma^{M_R-1}}{(P_1+\gamma)^{N_{br}+2}}d\gamma \end{aligned} \qquad (5.43)$$

By evaluating the integral, source to relay CDF can be obtained as (Shah & Hoimovich1998)

$$\begin{aligned} F_{\Gamma^{(n)}_{sr,OC}(k)}(\gamma) &= \frac{\Gamma(N_{br}+2)}{\Gamma(M_R+1)\Gamma(N_{br}+2-M_R)}(P_1)^{M_R} \\ &\times {}_2F_1\left(N_{br}+2, M_R; M_R+1; -\frac{\gamma}{P_1}\right) \end{aligned} \qquad (5.44)$$

Similarly, the CDF of the $\Gamma^{(n)}_{rd,OC}(k)$ is determined as

$$\begin{aligned} F_{\Gamma^{(n)}_{rd,OC}(k)}(\gamma) &= \frac{\Gamma(N_{bd}+1)}{\Gamma(M_D+1)\Gamma(N_{bd}+1-M_D)}(P_2)^{M_D} \\ &\times {}_2F_1\left(N_{bd}+1, M_D; M_D+1; -\frac{\gamma}{P_2}\right) \end{aligned} \qquad (5.45)$$

where $P_2 = (N_{bd}P_{\text{int}}/P_s)$. By substituting CDF from Equation (5.44) and Equation (5.45) in Equation (5.41), the overall outage probability of the proposed network using optimum combining is obtained.

5.4.2 End-to-End Outage Probability Using Maximal Ratio Combining

Let, the source to relay SIR using maximal ratio combining as

$$\Gamma_{sr,MRC}^{(n)}(k) = XP_s / Y(N_{br}+1)\varsigma_{br}P_{\text{int}} \tag{5.46}$$

where $X = \sum_{i=1}^{M_R} \left|\lambda_{sr}(k,i)\right|^2$ follows central chi-square distribution with $2M_R$ degrees of freedom and $Y = \sum_{j_1=1,\, j_1 \in I_{br}}^{N_{br}+1} \left|\lambda_{m_1}(k, j_1)\right|^2$ also follows central chi-square distribution with $2(N_{br}+1)$ degrees of freedom. The ratio of two independent central chi-square random variables follows central F distribution (James 1964). Hence, the PDF of the source to relay SIR, $\Gamma_{sr,MRC}^{(n)}(k)$ using maximal ratio combining is determined as (James 1964)

$$f_{\Gamma_{sr,MRC}^{(n)}(k)}(\gamma) = \frac{\Gamma(N_{br}+1+M_R)}{\Gamma(N_{br}+1)\Gamma(M_R)}(P_1)^{N_{br}+1}\frac{\gamma^{M_R-1}}{(P_1+\gamma)^{N_{br}+1+M_R}} \tag{5.47}$$

Using Equation (5.43), the corresponding CDF can be obtained as

$$\begin{aligned} F_{\Gamma_{sr,MRC}^{(n)}(k)}(\gamma) &= \frac{\Gamma(N_{br}+1+M_R)}{\Gamma(N_{br}+1)\Gamma(M_R)}(P_{ratio})^{N_{br}+1} \\ &\times {}_2F_1\left(N_{br}+1+M_R, M_R; M_R+1; -\frac{\gamma}{P_{ratio}}\right) \end{aligned} \tag{5.48}$$

Similarly, the CDF of $\Gamma_{rd,MRC}^{(n)}(k)$ can be determined as

$$\begin{aligned} F_{\Gamma_{rd,MRC}^{(n)}(k)}(\gamma) &= \frac{\Gamma(N_{bd}+M_D)}{\Gamma(N_{bd})\Gamma(M_D)}(P_{ratio})^{N_{bd}} \\ &\times {}_2F_1\left(N_{bd}+M_D, M_D; M_D+1; -\frac{\gamma}{P_{ratio}}\right) \end{aligned} \tag{5.49}$$

By substituting CDF from Equation (5.48) and Equation (5.49) in Equation (5.41), the overall outage probability of the network using maximal ratio combining is obtained.

5.5 RESULTS AND DISCUSSION

In this section, the outage performance of the proposed full duplex multi-antennas relay network is analyzed in the presence of NBI. The numerical parameters of the proposed full duplex multi-antennas DF relay network are given in Table 5.1.

Table 5.1 Numerical parameters of the proposed full duplex multi-antennas DF relay network

Symbol	Parameter	Value
N_{br}, N_{bd}	Number of interference affected subcarriers at relay and destination node	6, 7
M_R, M_D	Number of receiving antennas in the relay and destination node	5, 6
β	Carrier frequency offset parameter of NBI signal	0.5
R	Data rates (in bits/s/Hz)	0.5 and 1

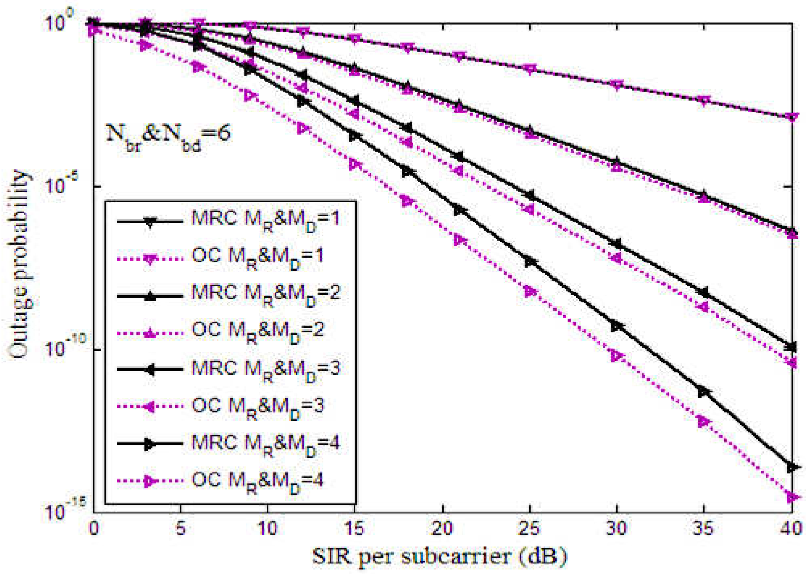

Figure 5.2 Outage performance of the proposed full duplex multi-antennas DF relay network with various numbers of antennas

The overall outage performance of the proposed full duplex multi-antennas DF relay network at the data rate of 1 b/s/Hz is shown in Figure 5.2. The number of interference affected subcarriers N_{br} at relay node and N_{bd} at destination node are 6. The interference power $N_{br}P_{\text{int}}$ and $N_{bd}P_{\text{int}}$ value is fixed at 0 dB. In this figure, OC denote optimum combining and MRC denote maximal ratio combining. The proposed network with OC is compared with MRC. Figure shows that when increasing the M_R and M_D, the performance of the network with OC is significantly improved than the network with MRC. For the outage probability of 10^{-5} and M_R, M_D of 2, the minimum SNR requirement of the network with OC and the network with MRC are almost same. For the same outage probability and M_R, M_D of 3, the minimum SNR requirement of the network with OC is 23 dB compared to 24 dB in MRC. Further increasing M_R and M_D to 4, SNR requirement of the network with OC is reduces from 23 dB to 17 dB compared to 19 dB in network with MRC

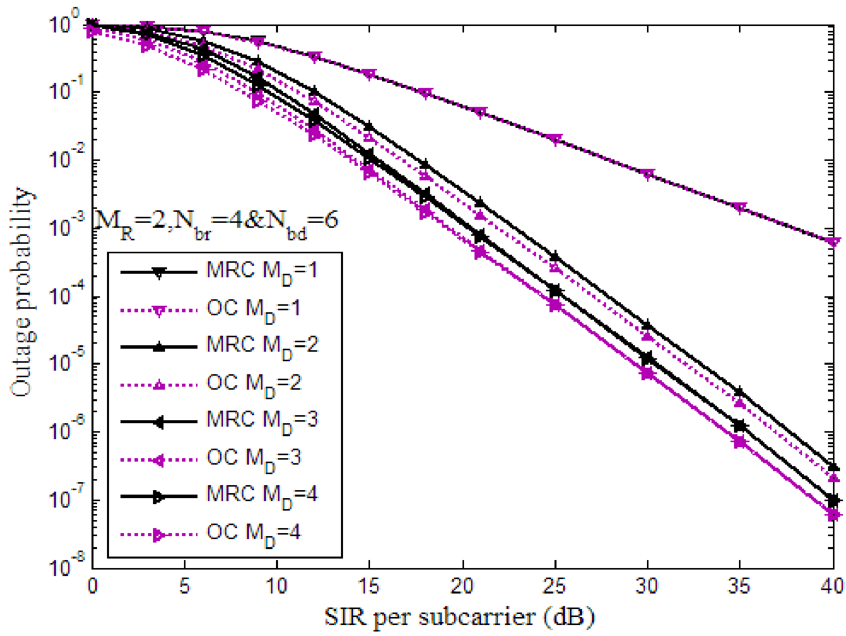

Figure 5.3 Outage performance of the proposed full duplex multi-antennas DF relay network for $M_R = 2$, $N_{br} = 4$, $N_{bd} = 6$

The outage performance of the proposed full duplex multi-antennas DF relay network with various M_D antennas is shown in Figure 5.3. The data rate is fixed at $1\ \mathrm{b/s/Hz}$. The number of antennas M_R at relay node is 2 and the number of interference affected subcarriers N_{br} at relay node and N_{bd} at destination node are 4 and 6 respectively. The interference power is same as in Fig.2. For the outage probability of 10^{-3} and M_R at 2, M_D at 3, the minimum SNR requirement of the network with OC is $18\,\mathrm{dB}$ compared to $19\,\mathrm{dB}$ for the network with MRC. For the same outage probability and the value of M_D at 4, the minimum SNR requirement of the network with OC and the network with MRC is same as M_D at 3. It indicates that the source to relay node outage due to M_R and N_{br} values affects the overall outage probability of the network.

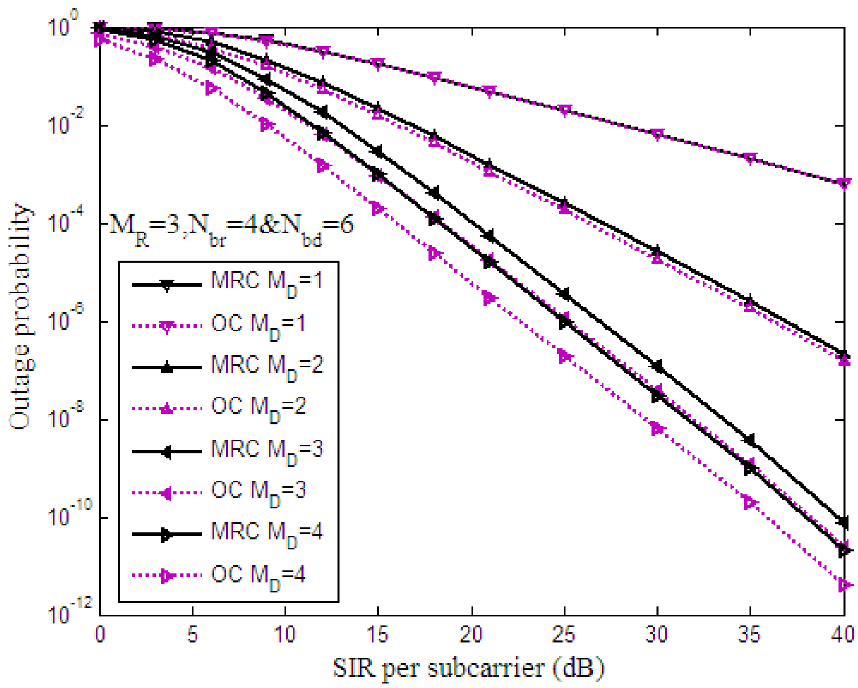

Figure 5.4 Outage performance of the proposed multi-antennas DF relay network for $M_R = 3, N_{br} = 4,\ N_{bd} = 6$

The outage performance of the proposed multi-antennas DF relay network with various M_D values is shown in Figure 5.4. The data rate is fixed at $1\,\text{b/s/Hz}$. The number of antennas M_R at relay node is 3 and the number of interference affected subcarriers N_{br} at relay node and N_{bd} at destination node are 4 and 6 respectively For the outage probability of 10^{-3} and M_R, M_D at 3, the minimum SNR requirement of the network with OC is $15\,\text{dB}$ compared to $17\,\text{dB}$ for the network with MRC. For the same outage probability and the value of M_D at 4, the minimum SNR requirement of the network with OC is $12\,\text{dB}$ compared to $15\,\text{dB}$ for the network with MRC. It indicates that when increasing the M_D, the effect of source to relay node outage is reduced in the overall outage probability of the network.

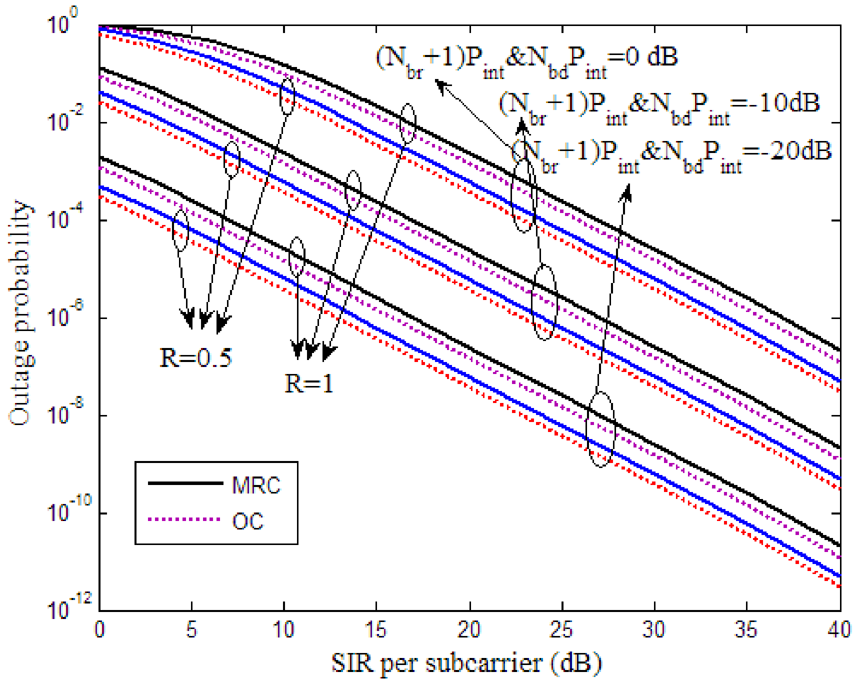

Figure 5.5 Outage performance of the proposed multi-antennas DF relay network with various interference power and R values

The outage performance of the proposed multi-antennas DF relay network with various interference power and R values is shown in Figure 5.5. The number of antennas M_R and M_D is 3 and the number of interference N_{br} and N_{bd} per subcarrier is 4. It is observed that the outage of the network is increased by increasing the interference power values and R values. At $25\,\text{dB}$ SNR and data rate of 1 b/s/Hz, when increasing the interference value from $-20\,\text{dB}$ to $-10\,\text{dB}$, the outage probability of the network with OC increase from 1.4×10^{-8} to 1.4×10^{-6} and from 2.4×10^{-8} to 2.4×10^{-6} in MRC. For the same SIR and interference values and the data rate of $0.5\,\text{b/s/Hz}$, the outage probability of the network with OC increase from 3.6×10^{-9} to 3.6×10^{-7} and from 6.12×10^{-9} to 6.12×10^{-7} in MRC.

5.6 SUMMARY

In this chapter, the outage performance of OFDM based multi-antennas full duplex DF relay network is analyzed in the presence of NBI. Multiple antennas are considered at relay and destination nodes. Closed form analytical expressions are derived for outage probability of the proposed OFDM based multi-antennas full duplex DF relay network using hyper-geometric functions. Performance of the proposed network with optimum combining is compared with OFDM based multi-antennas full duplex DF relay network with maximal ratio combining. From this analysis, it is concluded that the outage performance of the OFDM based multi-antennas DF relay network using optimum combining is outperforms the OFDM based multi-antennas DF relay network using maximal ratio combining.

CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 CONCLUSION

In this thesis, the effect of NBI in OFDM signal is minimized using cognitive half duplex AF relay network which maximizes the overall SINR by exploiting the spatial diversity and frequency diversity at each subcarrier. NBI is modeled as a sparse vector in frequency domain and assumed as quasi-static over an OFDM symbol. Analytical expressions for SINR from source to relay node, relay to destination node and source to destination node are derived. Given the channel state information, the desired signal power and interference power are modeled as exponential and Gamma distributions respectively. Using this statistical modeling, closed form analytical expressions are derived for the end-to-end outage probability and average error probability of the proposed half duplex CR network over Rayleigh fading environment. Since maximal ratio combining provides diversity gain in the proposed half duplex AF relay network, the minimum SNR requirement is reduced by $5\mathrm{dB}$ compared to the conventional non-cooperative system.

The second major contribution of this thesis is the use of full duplex relay in the place of half duplex relay in the CR network. The performance of OFDM based cognitive full duplex AF relay network is analyzed in the presence of NBI over Rayleigh fading channel. Closed form analytical expressions for outage probability and average error probability are derived for the proposed OFDM based cognitive full duplex AF relay network. In spite of the self interference at full duplex relay, the outage performance of the proposed network is better than half duplex network at the low SNR regime.

The third major contribution of this thesis is the use of multiple full duplex relays in the proposed full duplex CR network. A relay selection scheme based on max-min SINR criterion is applied to choose the best relay at each subcarrier. The performance of OFDM based cognitive full duplex AF multiple relays network is analyzed in the presence of NBI over Nakagami-m fading channel environment. The PDF of the interference term in the SINR between the source to relay and relay to destination are modeled as Moschopoulos multi-variate Gaussian. Using this, closed form analytical expression is derived for the outage probability of the proposed OFDM based cognitive full duplex AF multiple relays network in the presence of NBI. At low SNR regime, it is observed that, the performance of the proposed AF multiple relays network is better than the half duplex AF and half duplex DF multiple relays network in the presence of loop and narrowband interferences.

The fourth major contribution of the thesis is the use of multiple antennas at full duplex DF relay node and destination node. The use of multiple antennas improves the received signal strength through array gain by coherent combining and diversity gain by combining the number of independent signal fading paths. As the loop interference and NBI dominate noise, the concept of optimum combining, instead of maximum ratio combining, is applied at relay and destination nodes to minimize the effect of interference. The PDF of SIR between source to relay node and relay to destination node are modeled as Hotelling's T^2 distribution. Closed form analytical expressions are derived for end-to-end outage probability of the proposed OFDM based multi-antennas full duplex DF relay network using hypergeometric functions. Numerical results show that the outage performance of the OFDM based multiple antennas relay network using optimum combining achieves the better performance than the network with maximal ratio combining. Further increasing the number of antennas M_R at relay node and number of antennas M_D at destination node to 4, SNR requirement of the network with optimal combining is reduced from $23\,\mathrm{dB}$ to $17\,\mathrm{dB}$ compared to $19\,\mathrm{dB}$ in network with maximal ratio combining.

6.2 FUTURE WORK

A sustainable future of wireless network must be both spectral efficient and energy efficient. Hence, innovative technologies and cellular topologies that can meet with the pace of mobile data explosion are to be developed in an energy-efficient and sustainable manner. In this aspect, relaying and user cooperation have recently emerged as potential candidate technologies for future wireless applications and standards. Hence, the future wireless network would be heterogeneous and characterized by a small-cell infrastructure relying on inexpensive and low power base stations in order to achieve high data rates. Further, relays would play major role in improving coverage and minimizing energy consumption with high reliability and reduced packet transmissions/retransmissions in wireless networks using distributed diversity. The outage performance analysis for the various types of proposed relay based networks would be very much beneficial for the future wireless communications.

The derived analytical expressions for outage and BER performance of the proposed CR networks can be extended for analyzing the performance of multi-source and multi-destination nodes in the presence of NBI.

REFERENCES

1. Aalo, VA & Zhang, J 2000, 'Performance of antenna array systems with optimum combining in a Rayleigh fading environment', IEEE Communications Letters, Vol. 4, no. 4, pp. 125-127.

2. Adinoyi, A & Yanikomeroglu, H 2007, 'Cooperative relaying in multi-antenna fixed relay networks', IEEE Transactions on Wireless Communications, Vol. 6, no. 2, pp. 533-544.

3. Adinoyi, A, Fan, Y, Yanikomeroglu, H, Poor, HV & Al-Shaalan, F 2009, 'Performance of selection relaying and cooperative diversity', IEEE Transactions on Wireless Communications, Vol. 8, no. 12, pp. 5790-5795.

4. Alian, EMH & Mitran, P 2013, 'A phase adjustment approach for interference reduction in OFDM-based cognitive radios', IEEE Transaction on wireless Communications, Vol. 12, no. 9, pp. 4668-4679.

5. Aloqlah, MS & Badarneh, OS 2013, 'Performance analysis of dual-hop systems with decode-and-forward relays over generalized fading channels', Journal of Communication, Vol. 8, no. 7, pp. 407-413.

6. Bansal, G, Hossain, MJ & Bhargava, VK 2008, 'Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems', IEEE Transactions on Wireless Communications, Vol. 7, no. 11, pp. 4710-4718.

7. Baranwal, TK, Michalopoulos, DS & Schober, R 2013, 'Outage analysis of multihop full duplex relaying' , IEEE Communication Letters, Vol. 17, no. 1, pp. 63-66.

8. Batra, A, Lingam, S & Balakrishnan, J 2006, 'Multi-band OFDM: A cognitive radio for UWB', IEEE International Symposium on Circuits and Systems, Island of Kos, pp. 4094-4097.

9. Batra, A & Zeidler, JR 2008, 'Narrowband interference mitigation in OFDM systems', IEEE Military Communications Conference, San Diego, CA, pp. 1-7.

10. Batra, A & Zeidler, JR 2009, 'Narrowband interference mitigation in BICM OFDM systems', IEEE International Conference on ASSP, Taipei, pp. 2605-2608.
11. Baum, CW & Pursley, MB 1992, 'Bayesian methods for erasure insertion in frequency-hop communication systems with partial band interference', IEEE Transaction on Communications, Vol. 40, no. 7, pp. 1231-1238.
12. Biglieri, E, Calderbank, R, Constantinides, A, Goldsmith, A, Paulraj, A & Poor, HV 2007, 'MIMO wireless communications', Cambridge University Press, UK.
13. Bingham, JAC 1990, 'Multicarrier modulation for data transmission: An idea whose time has come', IEEE Communication Magazine, Vol. 28, no. 5, pp. 5-14.
14. Chi, DW & Das, P 2009, 'Effects of narrowband interference and nonlinear amplifier in companded OFDM with application to 802.11n WLAN', IEEE Military Communications Conference, Boston, MA, pp. 1-8.
15. Coulson, AJ 2004, 'Narrowband interference in pilot symbol assisted OFDM systems', IEEE Transactions on Wireless Communications, Vol. 3, no. 6, pp. 2277-2287.
16. Coulson, AJ 2006, 'Bit error rate performance of OFDM in narrowband interference with excision filtering', IEEE Transaction on Wireless Communications, Vol. 5, no. 9, pp. 2484-2492.
17. Darsena, D, Gelli, G, Paura, L & Verde, F 2007, 'A constrained maximum- SINR NBI-resistant receiver for OFDM systems', IEEE Transaction on Signal Processing, Vol. 55, no. 6, pp. 3032-3047.
18. Datsikas, CK, Sagias, NC, Lazarakis, FI & Thombras, GS 2008, 'Outage analysis of decode and forward relaying over Nakagami-m fading channels' , IEEE Signal Processing Letters, Vol. 15, pp. 41-44.
19. Ding, Y, & Uysal, M 2009, 'Amplify-and-forward cooperative OFDM with multiple-relays: Performance analysis and relay selection methods' , IEEE Transactions on Wireless Communications, Vol. 8, no. 10, pp. 4963-4968.
20. Duong, TQ, Bao, VNQ & Zepernick, HJ 2009, 'On the performance of selection decode-and-forward relay networks over Nakagami-m fading channels' , IEEE Communication Letters, Vol. 13, no. 3, pp. 172-174.

21. Ekin, S, Abdallah, MM, Qaraqe, KA & Serpedin, E 2012, 'Random subcarrier allocation in OFDM-based cognitive radio networks ', IEEE Transactions on Signal Processing, Vol. 60, no. 9, pp. 4758-4774.

22. Fan, L, Lei, X, Hu, RS & Shang, S 2013, 'Distributed two-way switch and stay combining with a single amplify-and-forward relay', IEEE Wireless Communications Letters, Vol. 2, no. 4, pp. 379-382.

23. Fareed, MM & Uysal, M 2009, 'On relay selection for decode-and-forward relaying', IEEE Transactions on Wireless Communications, Vol. 8, no. 7, pp. 3341-3346.

24. Gomaa, A & Al-Dhahir, N 2011, 'A Sparsity-aware approach for NBI estimation in MIMO-OFDM', IEEE Transaction on wireless Communications, Vol. 10, no. 6, pp. 1854-1862.

25. Gradshteyn, IS & Ryzhik, IM 1980, 'Table of integrals, series and products', San Diego: CA, Academic Press.

26. Gray, RM 2006, 'Toeplitz and circulant matrices: A review', Now Publishers Inc.

27. Hasna, MO & Alouini, MS 2004, 'Harmonic mean and end-to-end performance of transmission systems with relays', IEEE Transactions on Communications, Vol. 52, no. 1, pp. 130-135.

28. Hassan, MH & Hossain, MJ 2013, 'Cooperative beamforming for cognitive radio systems with asynchronous interference to primary user', IEEE Transaction on Wireless Communications, Vol. 12, no. 11, pp. 5468-5479.

29. Haykin, S 2005, 'Cognitive Radio: Brain-Empowered Wireless Communications' , IEEE Journal on Selected Areas of Communications, Vol. 23, no. 2, pp. 201-220.

30. Hu, J & Beaulieu, NC 2007, 'Performance analysis of decode-and-forward relaying with selection combining', IEEE Communication Letters, Vol. 11, no. 6, pp. 489-491.

31. Ikki, SS & Ahmed, MH 2010, 'Performance analysis of adaptive decode-and-forward cooperative diversity networks with best-relay selection', IEEE Transactions on Communications, Vol. 58, no. 1, pp. 68-72.

32. James, AT 1964, ‘Distributions of matrix variates and latent roots derived from normal samples’, Annals Mathematics of Statistics, Vol. 35, pp. 475–501.

33. Janaswamy, R 2000, ‘Radio wave propagation and smart antenna for wireless communications’, Kluwer Academic Publishers, Boston, MA.

34. Ju, M, Kim, IM, & Kim, DI 2012, ‘Joint relay selection and relay ordering for DF-based cooperative relay networks’, IEEE Transactions on Communications, Vol. 60, no. 4, pp. 908-915.

35. Kang, X, Garg, HK, Liang, YC & Zhang, R 2010, ‘Optimal power allocation for OFDM-based cognitive radio with new primary transmission protection criteria’, IEEE Transactions on Wireless Communications, Vol. 9, no. 6, pp. 2066-2075.

36. Karagiannidis, GK, Tsiftsis, TA, Mallik, RK, Sagias, NC & Kotsopoulos, SA 2005, ‘Closed-form bounds for multihop relayed communications in Nakagami-m fading’, Proceeding of IEEE WTC, Chengdu, pp. 269- 273.

37. Kim, SJ, Wang, X & Madihian, M 2008, ‘Optimal resource allocation in multi-hop OFDMA wireless networks with cooperative relay’, IEEE Transactions on Wireless Communications, Vol. 7, no. 5, pp. 1833-1838.

38. Krikidis, I, Charalambous, T & Thompson, JS 2012, ‘Buffer-aided relay selection for cooperative diversity systems without delay constraints’, IEEE Transactions on Wireless Communications, Vol. 11, no. 5, pp. 1957-1967.

39. Kwon, T, Lin, S, Choi, S & Hong, D 2010, ‘Optimal duplex mode for DF relay in terms of the outage probability’, IEEE Transactions on Vehicular Technology, Vol. 59, no. 7, pp. 3628-3634.

40. Laneman, JN, Tse, DNC, Wornell, GW 2004, ‘Cooperative diversity in wireless networks: efficient protocols and outage behavior’, IEEE Transaction on Information Theory, Vol. 50, no. 12, pp. 3062-3080.

41. Lee, J, Wang, H, Andrews, JG & Hong, D 2011, ‘Outage probability of cognitive relay networks with interference constraints’, IEEE Transactions on Wireless Communications, Vol. 10, no. 2, pp. 390-395.

42. Lee, KC, Li, CP, Wang, TY & Li, HJ 2014, 'Performance analysis of dual-hop amplify and forward systems with multiple antennas and cochannel interference', IEEE Transactions on Wireless Communication, Vol. 13, no. 6, pp. 3070-3087.

43. Li, Q, Li, KH & Teh, KC 2011, 'Achieving optimal diversity-multiplexing tradeoff for full-duplex MIMO multihop relay networks', IEEE Transactions on Information Theory, Vol. 57, no. 1, pp. 303-316.

44. Liu, KJR, Sadek, AK, Su, W & Kwasinski, A 2009, 'Cooperative communications and networking', Cambridge University Press, UK.

45. Mahmoud, HA, Yücek, T & Arslan, H 2009, 'OFDM for cognitive radio: Merits and challenges', IEEE Wireless Communications, Vol. 16, no. 2, pp. 1536-1284.

46. Maichalernnukul, K, Kaiser, T & Zheng, F 2011, 'Performance of ultra wideband cooperative relay systems in the presence of narrowband interference' , IEEE International Conference on UWB, Bologna, pp. 405-409.

47. Marey, M & Steendom, H 2007, 'Analysis of the narrowband interference effect on OFDM timing synchronization', IEEE Transactions on Signal Processing, Vol. 55, no. 9, pp. 4558-4566.

48. Michalopoulos, DS & Karagiannidis, GK 2008, 'Performance analysis of single relay selection in Rayleigh fading', IEEE Transactions on Wireless Communications, Vol. 7, no. 10, pp. 3718-3724.

49. Michalopoulos, DS, Lioumpas, AS, Karagiannidis, GK & Schober, R 2010, 'Selective cooperative relaying over time-varying channels', IEEE Transactions on Communications, Vol. 58, no. 8, pp. 2402-2412.

50. Moschopoulos, PG 1985, 'The distribution of the sum of independent Gamma random variables', Annals Institute of Statistics and Mathematics, vol. 37, pp. 541-544.

51. Muirhead, RJ 2005, 'Aspects of multivariate statistical theory', John Wiley & Sons.

52. Müller, A & Speidel, J 2007, 'Exact symbol error probability of M-PSK for multihop transmission with regenerative relays', IEEE Communication Letters, Vol. 11, no. 12, pp. 952-954.

53. Muquet, B, Wang, Z & Giannakis, G 2002, ‘Cyclic prefixing or zero padding for wireless multicarrier transmissions? ’, IEEE Transactions on Communication, Vol. 50, pp. 2136-2148.

54. Nilsson, R, Sjoberg, F & LeBlanc, J 2003, ‘A rank-reduced LMMSE canceller for narrowband interference suppression in OFDM-based systems’, IEEE Transaction on Communications, Vol. 51, no. 12, pp. 2126-2140.

55. Oestges, C, & Clerckx, B 2007, ‘MIMO wireless communications: From real world propagation to space time code design’, Elsevier, UK.

56. Park, JC, Wang, JS, Kang, HG, Yoon, S, Song, I & Kim, YH 2011, ‘Detection of variable subband nulling mode for OFDM-based cognitive radio in narrowband interference channels’, IEEE Transactions on Wireless Communications, Vol. 10, no. 3, pp. 782-791.

57. Paulraj, AJ & Kailath, T 1994, ‘Increasing capacity in wireless broadcast systems using distributed transmission/directional reception’, U.S. Patent, no. 5, pp. 345-599.

58. Popovski, P 2007, ‘Opportunistic interference cancellation in cognitive radio systems’, IEEE International Conference on Dynamic Spectrum Access Networking, Dublin, pp. 472-475.

59. Proakis, JG & Salehi, M 2008, ‘Digital Communication’, Mc-Graw Hill, NY.

60. Rabiei, P, Namgoong, W & Al-Dhahir, N 2011, ‘On the performance of OFDM-based amplify-and-forward relay networks in the presence of phase noise’, IEEE Transactions on Communication, Vol. 59, pp. 1548-1466.

61. Redfern, AJ 2002, ‘Receiver window design for multicarrier communication systems’, IEEE Journal on Selected Areas of Communications, Vol. 20, no. 5, pp. 1029-1036.

62. Riihonen, T, Werner, S, & Wichman, R 2011, ‘Transmit power optimization for multiantenna decode-and-forward relays with loopback self-interference from full-duplex operation’, IEEE Proceeding of ASILOMAR, Pacific Grove, CA, pp. 1408-1412.

63. Ropokis, GA, Rontogiannis, AA & Berberidis, K 2011, ‘BER performance analysis of cooperative DaF relay networks and a new optimal DaF strategy’, IEEE Transactions on Wireless Communications, Vol. 10, no. 4, pp. 1044-1049.

64. Sanguinetti, L, Morelli, M & Poor, HV 2010, 'Robust EM-based detection of BICM-OFDM transmissions in the presence of narrowband interference', European Wireless conference, Lucca, pp. 696-700.

65. Seddik, KG, Sadek, AK, Su, W & Liu, KJR 2007, 'Outage analysis and optimal power allocation for multinode relay networks', IEEE Signal Processing Letters, Vol. 14, no. 6, pp. 377-380.

66. Selvaraj, MD & Mallik, RK 2010, 'Scaled selection combining based cooperative diversity system with decode and forward relaying' , IEEE Transactions on Vehicular Technology, Vol. 59, no. 9, pp. 4388-4399.

67. Shah, A, & Hoimovich, AM 1998, 'Performance analysis of optimum combining in wireless communications with Rayleigh fading and cochannel interference', IEEE Transactions on Vehicular Technology, Vol. 46, no. 4, pp. 473-479.

68. Shah, A, & Hoimovich, AM 2000, 'Performance analysis of maximal ratio combining and comparison with optimum combining for mobile radio communications with cochannel interference', IEEE Transactions on Vehicular Technology, Vol. 49, no. 4, pp. 1454-1463.

69. Shi, K, Zhou, Y, Kelleci, B, Fischer, TW, Serpedin, E & Karşilayan, Aİ, 2007, 'Impacts of narrowband interference on OFDM-UWB receivers: Analysis and mitigation', IEEE Transactions on Signal Processing, Vol. 55, no. 3, pp. 1118-1128.

70. Simon, MK & Alouini, MS 2000, 'Digital communications over fading channels: A unified approach to performance analysis', Wiley, NewYork.

71. Suraweera, HA, Smith, PJ & Armstrong, J 2006, 'Outage probability of cooperative relay networks in Nakagami-m fading channels', IEEE Communication Letters, Vol. 10, no. 12, pp. 834-836.

72. Suraweera, N & Beaulieu, NC 2014, 'Optimum combining with joint relay and antenna selection for multiple-antenna relays in the presence of co-channel interference', IEEE Communication Letters, Vol. 18, no. 8, pp. 1459-1462.

73. Telatar, E 1999, 'Capacity of multi-antenna Gaussian channels. Eur. Trans. Telecomm', Vol. 10, no. 6, pp. 585-596.

74. Torabi, M, Ajib, W & Haccoun, D 2009, 'Performance analysis of amplify-and-forward cooperative networks with relay selection over Rayleigh fading channels', IEEE 69th Vehicular Technology Conference, (VTC), Barcelona, pp. 1-5.

75. Wyglinski, AM, Nekovee, M & Hou, YT 2010, 'Cognitive radio communications and networks', Elsevier, USA.

76. Xu, W, Zhang, J, Zhang, P & Tellambura, C 2006, 'Outage probability of cooperative relay networks in Nakagami-m fading channels', IEEE Communications Letters, Vol. 16, no. 8, pp. 1252-1255.

77. Xu, W & Liu, X 2010, 'On optimum selection relaying protocols in cooperative wireless networks', IEEE Transactions on Wireless Communications, Vol. 58, no. 1, pp. 52-57.

78. Xu, T, Ge, J & Ding, H 2014, 'An efficient distributed link selection scheme for AF-based cognitive selection relaying networks', IEEE Communications Letters, Vol. 18, no. 2, pp. 253-256.

79. Yamaguchi, H 2004, 'Active interference cancellation technique for MB-OFDM cognitive radio', Proceeding of 34th European Microwave Conference, Amsterdam, Netherland, pp. 1105-1108.

80. Yang, H 2005, 'A road to future broadband wireless access: MIMO-OFDM-based air interface', IEEE Communication Magazine, Vol. 43, no. 1, pp. 53-60.

81. Yan, Z, Zhang, X & Wang, W 2011, 'Exact outage performance of cognitive relay networks with maximum transmit power limits', IEEE Communications Letters, Vol. 15, no. 12, pp. 1317-1319.

82. Yue, DW & Zhang, QT 2006, 'Outage performance analysis of correlated MIMO optimum combining systems with and without co-channel interference', Proceeding of IEEE WTC, Chengdu, pp. 269-273.

83. Yue, D, Wang, X & Xu, F 2006, 'Performance analysis for optimum combining of Rayleigh fading signals with correlated Rayleigh interferers and noise', IEEE Communications Letters, Vol. 13, no. 5, pp. 269-272.

84. Yuksel, M & Erkip, E 2007, 'Multi-antenna cooperative wireless systems: A diversity-multiplexing tradeoff perspective', IEEE Transactions on Information Theory, Vol. 53, no. 10, pp. 3371-3392.

85. Yu, H, Lee, IH, & Stüber, GL 2012, 'Outage probability of decode-and-forward cooperative relaying systems with co-channel interference', IEEE Transactions on Wireless Communications, Vol. 11, no. 1, pp. 266-274.

86. Zhang, Q, Jia, J & Zhang, J 2009, 'Cooperative relay to improve diversity in cognitive radio networks', IEEE Communication magazine, Vol. 47, no. 2, pp. 111-117.

87. Zhang, X, Tao, M, Jiao, W & Sum Ng, C 2009, 'End-to-end outage minimization in OFDM based linear relay networks', IEEE Transactions on Communications, Vol. 57, no. 10, pp. 3034-3044.

88. Zhang, X, Yan, Z, Gao, Y & Wang, W 2013, 'On the study of outage performance for cognitive relay networks (CRN) with the Nth best-relay selection in Rayleigh-fading channels', IEEE Wireless Communication Letters, Vol. 2, no. 1, pp. 110-113.

89. Zhao, Y, Adve, R & Lim, TJ 2007, 'Improving amplify-and-forward relay networks: Optimal power allocation versus selection', IEEE Transactions on Wireless Communications, Vol. 6, no. 8, pp. 3114-3123.

90. Zhu, Y, Xin, Y & Kam, P 2008, 'Outage probability of Rician fading relay channels', IEEE Transactions on Vehicular Technology, Vol. 57, no. 4, pp. 2648-2652.

91. Zou, Y, Champagne, B, Zhu, WP & Hanzo, L 2015, 'Relay-selection improves the security-reliability trade-off in cognitive radio systems', IEEE Transactions on Communications, Vol. 63, no. 1, pp. 215-228.

LIST OF PUBLICATIONS

International Journals

1. **Rajkumar, S,** Senthilkumaran, VN & Thiruvengadam, SJ 2015, 'Outage analysis of OFDM based Cognitive AF relay network in the presence of narrowband interference', ETRI Journal, Vol. 37, no. 3, pp. 460-470(Print ISSN: 1225-6463; E ISSN: 2333-7326 – As indicated in Annexure I of refereed journal list given in Anna University Portal version 2014.1). Impact Factor: 0.945.

2. **Rajkumar, S** & Thiruvengadam, SJ 2015, 'Outage analysis of full duplex cognitive network with relay selection per subcarrier over Nakagami-*m* fading channel', International Journal of Applied Engineering Research, Vol. 10, no. 6, pp. 5249-5254. (ISSN: 0973 4562, - As indicated in Annexure II of refereed journal list given in Anna University Portal version 2014.1).

3. **Rajkumar, S** & Thiruvengadam, SJ 2015, 'Average error probability of OFDM based cognitive decode and forward relay network with narrowband interference', Published in International Journal of Applied Engineering Research, Vol. 10, no. 17, pp. 13352-13357. (ISSN: 0973 4562, - As indicated in Annexure II of refereed journal list given in Anna University Portal version 2014.1).

National / International Conferences

1. **Rajkumar, S** & Thiruvengadam, SJ 2014, 'Outage analysis of OFDM based full duplex cognitive DF relay network in the presence of narrowband interference', Proceedings of IEEE National Conference on Communication Signal Processing and Networking, Palakkad, Kerala, 10-12 Oct-2014.

2. **Rajkumar, S** & Thiruvengadam, SJ 2015, 'Performance analysis of OFDM based full duplex cognitive AF relay network in the presence of narrowband interference', Proceedings in IEEE International Conference on Signal Processing Informatics Communication and Energy Systems, NIT, Calicut, Kerala.