# Transactions Letters

## Adaptive Power Loading for OFDM-Based Cognitive Radio Systems with Statistical Interference Constraint

Gaurav Bansal, *Member, IEEE*, Md. Jahangir Hossain, *Member, IEEE*, and Vijay K. Bhargava, *Fellow, IEEE*

***Abstract*—In this letter, we develop an optimal power allocation algorithm for the orthogonal frequency division multiplexing (OFDM)-based cognitive radio (CR) systems with different statistical interference constraints imposed by different primary users (PUs). Given the fact that the interference constraints are met in a statistical manner, the CR transmitter does not require the instantaneous channel quality feedback from the PU receivers. A suboptimal algorithm with reduced complexity has been proposed and the performance has been investigated. Presented numerical results show that with our proposed optimal power allocation algorithm CR user can achieve significantly higher transmission capacity for given statistical interference constraints and a given power budget compared to the classical power allocation algorithms namely, uniform and water-filling power allocation algorithms. The suboptimal algorithm outperforms both water-filling algorithm and uniform power loading algorithm. The proposed suboptimal algorithm give an option of using a low complexity power allocation algorithm where complexity is an issue with a certain amount of transmission rate degradation.**

***Index Terms*—Orthogonal frequency division, power allocation, cognitive radio, dynamic spectrum access, rate adaptation, interference management, convex optimization.**

### I. Introduction

THE utilization of valuable radio spectrum resource can be improved significantly by using the cognitive radio (CR) technology [1]. Orthogonal frequency division multiplexing (OFDM), because of its flexibility in allocating the spectrum, has been recognized as an air interface technology for CR systems [2]. Implementation details and some of the advantages of OFDM-based CR systems have been discussed, for example, in [3]. Because of the coexistence of CR and primary users in side-by-side bands, mutual interference between these users is the limiting factor in order to achieve a good performance for CR systems [4]. Use of the classical power allocation algorithms e.g., well known water-filling algorithm for CR systems may result in higher interference to the primary user (PU) receivers. In [5], we propose a power loading algorithm that maximized the downlink transmission rate of a CR user while keeping the total interference introduced to different PU receivers below a specified threshold. A distributed algorithm for optimal resource allocation in orthogonal frequency division multiplexing access (OFDMA)-based CR systems has been proposed in [6]. Several other resource allocation schemes for OFDM-based CR systems have been proposed in [7], [8], [9], [10].

Most of the above mentioned works assumed that the instantaneous channel qualities from the CR transmitter to both CR and PU receivers are known perfectly at the CR transmitter. In general, when perfect channel state information (CSI) is available at the CR transmitter, it can be exploited to increase the transmission rate of CR users [11]. Further, it is shown in [12] that if the channel fading gain between the PU transmitter and the PU receiver along with the channel gains from the CR transmitter to CR and PU receivers is known (at the CR transmitter), a superior power-control policy can be designed. However, it may not be a practical assumption that the perfect CSI is known at the CR transmitter. In particular although the fading gains among the CR transmitter and its receiver can be known at the CR transmitter via feedback channel, it is difficult, if not impossible, to estimate the instantaneous channel fading gains between the CR transmitter and the PU receivers. For some scenarios, the CR transmitter can have information about the statistics of the channel fading gain among the CR transmitter and the PU receivers. For example, in [13], authors have argued that from the pilot signals transmitted by a PU receiver, the mean value of the random channel fading gain between a PU receiver and the CR transmitter can be estimated. In a recent work in [14], resource allocation algorithms have been proposed for multiple input multiple output (MIMO) OFDM-based CR systems where imperfect CSI is assumed at the CR transmitter. However, the authors have assumed a bounded channel uncertainty model, where the channel estimation error is bounded by some threshold. The developed power allocation algorithm always considered the maximum channel estimation error irrespective of the actual channel estimation error that can lead to a conservative design. In this letter, we use a probabilistic

Manuscript received March 16, 2010; revised October 26, 2010 and January 31, 2011, accepted accepted February 1, 2011. The associate editor coordinating the review of this paper and approving it for publication was A. Nallanathan.

G. Bansal is with Toyota InfoTechnology Center Co. Ltd., Mountain View, CA, USA (e-mail: gbansal@us.toyota-itc.com). He was with the University of British Columbia, Vancouver, BC, Canada.

Md. J. Hossain is with the University of South Australia, Mawson Lakes, SA, Australia (e-mail: jahangir.hossain@unisa.edu.au).

V. K. Bhargava is with the University of British Columbia, Vancouver, BC, Canada (e-mail: vijayb@ece.ubc.ca).

Digital Object Identifier 10.1109/TWC.2011.072011.100397

interference constraint model for OFDM-based CR systems.

There are three key contributions in this letter compared to the work in [5]. First, the proposed power allocation algorithm requires the knowledge of channel fading statistics[1] instead of instantaneous channel fading gain among the CR transmitter and PU receivers. To this regard, we use a probabilistic model of interference threshold constraint where interference can be guaranteed in a statistical manner. Second, the considered co-existence scenario in this letter is more generalized than the one presented in [5]. In particular, in the system model presented in this letter, different PU receivers can impose different interference constraints to the CR system. As such it provisions for different quality of transmission for different PUs. Third, the presented model also considers that the CR transmitter has a maximum transmit power limit. This power limitation allows us to investigate two scenarios in achievable transmission rate of the CR user; one is power limited scenario and other is interference limited scenario.

We also propose a suboptimal power allocation algorithm that reduces operational complexity at the expense of certain amount of transmission rate of the CR user. Presented simulation results demonstrate the strength of our proposed algorithms compared to the classical algorithms for the CR scenario. Specifically, the presented simulation results show that the optimal algorithm can load more power into CR OFDM subcarriers' in order to achieve higher transmission rate while keeping the interferences below given thresholds with a certain probability. The suboptimal algorithm has same complexity as the water-filling algorithm but achieves higher transmission rate than the water-filling algorithm. It also achieves higher transmission rate than the uniform power allocation algorithm. The suboptimal algorithm trade-off complexity with the transmission rate and gives an alternative of using a low complexity power loading algorithm where complexity is an issue.

The organization of this letter is as follows. While Section II describes the system model, Section III presents the problem formulation and the optimal solution for power allocation over CR OFDM subcarriers. In Section IV, we propose low-complexity suboptimal and two classical algorithms that are used for conventional OFDM systems. Selected numerical results and complexity analysis of various algorithms under consideration are presented in Section V. Finally, Section VI concludes the letter.

## II. System Model

In the frequency domain, we consider that a CR user and $L$ PUs co-exist in side-by-side bands as depicted in Fig. 1 (similar co-existence model is used in [5]). We assume that the bandwidth available to the CR user is divided into $N$ subcarriers with an intercarrier spacing of $\Delta f$. The $L$ PUs are assumed to be occupying frequency bands of bandwidth $B_1$, $B_2$,$\cdots$,$B_L$, respectively. In spatial domain, different PUs and the CR user may be located in different geographical locations as shown in Fig. 2. In this figure, for clarity we

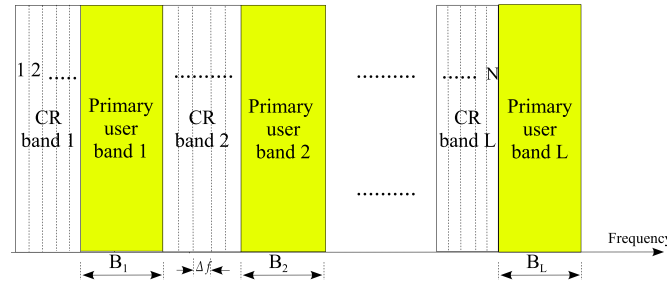


Fig. 1. Co-existence of a CR user and $L$ PU users in the frequency domain.

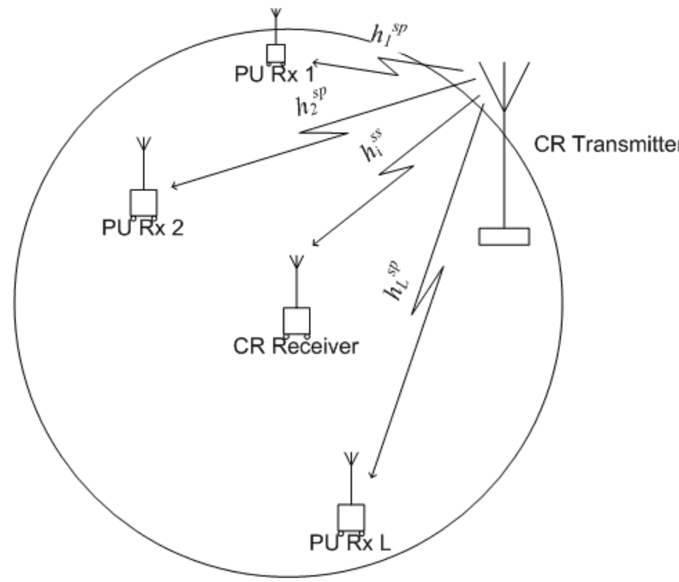


Fig. 2. Co-existence of a CR user and $L$ PU users in the spatial domain.

have shown only the $l$th PU transmitter and receiver pair. However, there should be total $L$ PU transmitter and receiver pairs. The channel quality between the CR transmitter and the CR receiver can vary in both time and frequency domain due to fading. The channel quality between the CR transmitter and the PU receivers' can also vary due to fading. For convenience, we denote the channel fading gain between the CR transmitter and the CR receiver in the $i$th $(i = 1, 2, \cdots, N)$ CR subcarrier by $h_i^{ss}$. The channel fading gain between the CR transmitter and the $l$th $(l = 1, 2, \cdots, L)$ PU receiver is denoted by $h_l^{sp}$. We assume that a given PU band experiences frequency flat fading. Further, we assume that the instantaneous values of the channel gains $h_i^{ss}$ are known at the CR transmitter. However, it is assumed that instead of instantaneous values of the channel fading gains $h_l^{sp}$, the distribution type and the corresponding distribution parameters of the random fading gains $h_l^{sp}$ are known at the CR transmitter.

For ideal modulation and coding scheme, the transmission rate of the CR user in $i$th subcarrier, $C_i$ is connected via Shannon capacity formula and can be expressed as [15]

$$C_i = \Delta f \log_2 \left( 1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^{L} J_i^{(l)}} \right), \tag{1}$$

where $P_i$ is the transmit power in $i$th CR subcarrier, $\sigma^2$ is the additive white Gaussian noise (AWGN) variance and $J_i^{(l)}$

[1]We assume a non-line-of-sight (NLOS) propagation environment. So that the random amplitude fading gain can be modeled as Rayleigh distributed and by statistics we refer to both Rayleigh distribution and its mean value.

denotes the interference introduced by $l$th PU transmitter in the $i$th subcarrier of CR user. The interference values $J_i^{(l)}$ can vary randomly in time domain and we assume that the CR receiver can perfectly estimate the total interference value i.e., $\sum_{l=1}^{L} J_i^{(l)}$ which is made available at the CR transmitter via a feedback channel. The interference introduced by $i$th CR subcarrier transmission in the $l$th PU receiver can be expressed as [4], [5]

$$I_i^{(l)} = P_i \left|h_l^{sp}\right|^2 T_s \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df, \quad (2)$$

where $T_s$ is the symbol duration and $d_{il}$ represents the spectral distance between the $i$th CR subcarrier and $l$th PU band.

## III. Problem Formulation and Optimal Power Allocation

Our design goal is to find power values for each subcarrier, $P_i$ $(i = 1, 2, \cdots, N)$ for given instantaneous fading gains $h_i^{ss}$, given fading statistics of $h_l^{sp}$ and the total transmit power budget $P_T$. As such the total transmission rate of the CR user, $C$ is maximized while the probability that the interference introduced to $l$th $(l = 1, 2, \cdots, L)$ PU band is kept below the threshold $I_{th}^{(l)}$ $(l = 1, 2, \cdots, L)$, respectively, with the probability value $a$ or above. Mathematically, the problem in our hand can be formulated as a constrained optimization problem as follows

$$C = \max_{P_i} \quad \Delta f \sum_{i=1}^{N} \log_2 \left(1 + \frac{\left|h_i^{ss}\right|^2 P_i}{\sigma^2 + \sum_{l=1}^{L} J_i^{(l)}}\right), \quad (3)$$

subject to:

$$\begin{aligned} \mathrm{Pr.} \left(\sum_{i=1}^{N} I_i^{(l)}(d_{il}, P_i) \le I_{th}^{(l)}\right) \quad &\ge \quad a, \quad \forall l, && (4) \\ P_i \quad &\ge \quad 0, \quad \forall i, && (5) \\ \sum_{i=1}^{N} P_i \quad &\le \quad P_T, && (6) \end{aligned}$$

where Pr. denotes the probability.

Now the probabilistic interference constraint in Eq. (4) can be written as

$$\mathrm{Pr.} \left(\left|h_l^{sp}\right|^2 \sum_{i=1}^{N} K_i^{(l)} P_i \le I_{th}^{(l)}\right) \ge a, \quad \forall l, \quad (7)$$

where $K_i^{(l)} = T_s \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df$. Since $|h_l^{sp}|$ is assumed to be Rayleigh distributed with a known parameter $\lambda_l$, the distribution of $|h_l^{sp}|^2$ corresponds to an exponential distribution with the parameter $\lambda_l^2$. The constraint in Eq. (7) can be evaluated in closed-form for the Rayleigh fading case as follows

$$1 - e^{-\frac{I_{th}^{(l)}}{2\lambda_l^2 \sum_{i=1}^{N} P_i K_i^{(l)}}} \ge a, \quad \forall l. \quad (8)$$

After some mathematical manipulations, Eq. (8) can be written as

$$\sum_{i=1}^{N} P_i K_i^{(l)} \le \frac{I_{th}^{(l)}}{2\lambda_l^2 (-\ln(1-a))}, \quad \forall l. \quad (9)$$

*Theorem 1:* The power profile for which the total transmission rate in Eq. (3) is maximized for the given constraints in Eqs. (5), (6), and (9) can be written as

$$P_i^* = \left[w_i - \frac{\sigma^2 + \sum_{l=1}^{L} J_i^{(l)}}{\left|h_i^{ss}\right|^2}\right]^+ \quad \forall i, \quad (10)$$

where $w_i = \frac{1}{\beta + \sum_{l=1}^{L} \gamma_l K_i^{(l)}}$ and $\beta$, and $\gamma_l$ are deterministic Lagrange parameters.

*Proof:* The proof is given in the Appendix A. ■

Solving for $L + 1$ Lagrange parameters ($\beta$, and $\gamma_l, l = 1, 2, \cdots, L$) can be computationally complex. The Newton's method can be used to find the Lagrange parameters in a quadratic complexity [16]. The interior point method can also be used to maximize the total transmission rate in Eq. (3), given the constraints in Eqs. (5), (6), and (9). The complexity using interior point method would be $O(N^3)$. As the complexity of the proposed algorithm can be quite high, in what follows, we propose a low-complexity suboptimal power loading algorithm.

## IV. Suboptimal and Classical Power Loading Schemes

In this section, we propose a low complexity suboptimal algorithm and also describe classical algorithms namely; uniform loading and water-filling algorithms which are used for conventional OFDM systems. It is important to mention that the classical algorithms are also suboptimal in the context of CR systems as they do not take interference constraints into account.

### A. Proposed Suboptimal Algorithm

The power has to be assigned to $N$ CR subcarriers such that the transmission rate of CR user can be maximized while all $L+1$ (total power constraint from Eq. (6) and $L$ interference constraints from Eq. (9)) constraints are to be satisfied. The complexity of the optimal algorithm comes from the fact that these $L+1$ constraints have to be met simultaneously. In order to reduce such complexity, we follow a two step procedure as follows. First, we keep only one of the $L + 1$ constraints and find power allocation suboptimally in each subcarrier. Without loss of generality, let us denote that the allocated power in the $i$th subcarrier due to the $l$th constraint by $P_i^{(l)}$. In order to satisfy $l$th interference constraint given in Eq. (9), power is allocated according to ladder profile as in [5]. It is based on the heuristics that if a subcarrier is closer to a PU band, it introduces more interference. Hence, less power should be allocated to that particular subcarrier. In particular, we propose to allocate power in each CR subcarrier such that the allocated power is inversely proportional to the factor $K_i^{(l)}$ that depends on the spectral distance between $i$th CR subcarrier and the $l$th PU band. The allocated power in the $i$th subcarrier because of the $l$th interference constraint is written as

$$P_i^{(l)} = P / K_i^{(l)}, \quad \forall l, \quad (11)$$

where $P$ can be calculated by assuming strict equality in the $l$th interference constraint in Eq. (9). Using Eq. (11), in this equality constraint

$$\sum_{i=1}^{N} P_i K_i^{(l)} = \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))}, \tag{12}$$

we can write

$$P = \frac{I_{th}^{(l)}}{2N\lambda_l^2 \ln\frac{1}{(1-a)}}. \tag{13}$$

Using Eq. (11) and (13), we can calculate $P_i^{(l)}$ as

$$P_i^{(l)} = \frac{I_{th}^{(l)}}{2K_i^{(l)} N\lambda_l^2 \ln\frac{1}{(1-a)}}, \quad \forall l. \tag{14}$$

Now we need to calculate power values $P_i^{(L+1)}$ due to the total power constraint. In order to meet the total power constraint, we use the standard water-filling algorithm [15] to distribute total power $P_T$ among $N$ CR subcarriers. According to the water-filling algorithm with a total power constraint $P_T$, the power values can be written as

$$P_i^{(L+1)} = \max\left\{0, \frac{1}{\alpha} - \frac{\sigma^2 + \sum_{l=1}^{L} J_i^{(l)}}{|h_i^{ss}|^2}\right\} \quad \forall i, \tag{15}$$

where the Lagrange constant $\alpha$ can be calculated from the following equation

$$\sum_{i=1}^{N} \max\left\{0, \frac{1}{\alpha} - \frac{\sigma^2 + \sum_{l=1}^{L} J_i^{(l)}}{|h_i^{ss}|^2}\right\} = P_T. \tag{16}$$

Now the final power value for $i$th subcarrier according to the suboptimal algorithm is set to the minimum of power values in Eq. (14) and (15) i.e.,

$$P_i^{\text{subopt}} = \min\{P_i^{(1)}, P_i^{(2)}, \ldots\ldots, P_i^{(L+1)}\} \qquad \forall i. \tag{17}$$

With this suboptimal algorithm for a given subcarrier by selecting the minimum power value from $L+1$ power values, all constraints are satisfied. However it may happen that none of these constraints is met strictly. Hence finally, in order to maximize the transmission rate, the power value is scaled until one of the constraints is met strictly.

### B. Uniform Loading Algorithm

In uniform power loading algorithm, which is used in the conventional OFDM systems due to its reduced complexity, equal amount of power is allocated in each subcarrier such that all $L+1$ constraints in Eqs. (6) and (9) can be satisfied. By assuming equal power in each subcarrier and solving Eq. (9) to satisfy the strict equality on $l$th interference constraint ($I_{th}^{(l)}$), the corresponding power for $i$th subcarrier can be written as

$$P_i^{(l)} = \frac{I_{th}^{(l)}}{2\sum_{i=1}^{N} K_i^{(l)} \lambda_l^2 \ln\frac{1}{(1-a)}} \quad \forall l. \tag{18}$$

The power allocation for constraint in Eq. (6) can be written as

$$P_i^{(L+1)} = P_T/N. \tag{19}$$

The final power allocation to each subcarrier is done according to Eq. (17). It should be noted that at least one of these $L+1$ constraints will be met strictly and hence, scaling of power value is not required.

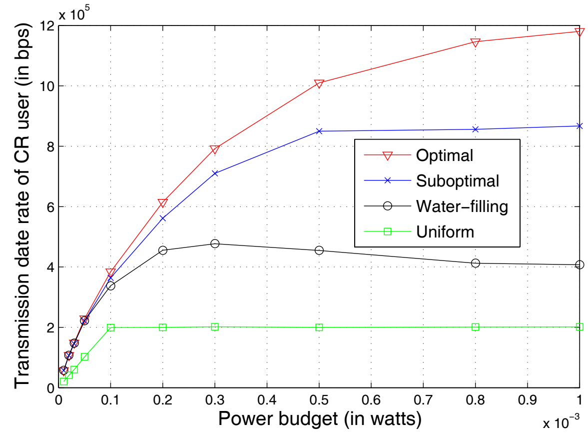


Fig. 3. Maximum transmitted data rate vs. power budget ($P_T$) for CR users.

### C. Water-Filling Power Loading Algorithm

In water-filling algorithm, which is optimal power allocation algorithm in conventional OFDM system, we use the total power allocated by uniform loading algorithm as the power constraint. The power value for $i$th subcarrier, denoted by $P_i^{(WF)}$, are obtained using the standard water-filling algorithm as mentioned in Eq (15) and (16) considering the total power constraint equal to the total power allocated by uniform loading algorithm. The power values will satisfy the total power constraint given in Eq. (6), however it is checked that if the power values satisfy the interference constraints specified in Eq. (9). If a particular interference constraint is not satisfied, the power value in each subcarrier $P_i^{(WF)}$ is reduced such that the all interference constraints are satisfied. Also, if none of these interference constraints is met strictly, the power value $P_i^{(WF)}$ is increased until one of these interference constraints is met strictly.

## V. Numerical Results and Complexity of Algorithms

In this section we present a numerical example where we assume that there are three PU bands ($L$=3), and there are twelve OFDM subcarriers ($N = 12$) for the CR user. The values of $T_s$, $\Delta f$, $B_1$, $B_2$, and $B_3$ have been assigned to be $4\mu$ seconds, 0.3125 MHz, 1 MHz, 2 MHz, and 5 MHz, respectively. AWGN variance, ($\sigma^2$) is assumed to be equal to $10^{-8}$W and the channel fading gains are assumed to follow the Rayleigh distribution. The average channel power gains for $|h_i^{ss}|^2$, $|h_1^{sp}|^2$, $|h_2^{sp}|^2$, and $|h_3^{sp}|^2$ are assumed to be -10dB, -5dB, -7dB, and -10dB, respectively. The values of $J_i^{(l)}$ are generated randomly with an average value of $1\times10^{-6}$W. The values of $I_{th}^{(1)}$, and $I_{th}^{(3)}$ have been assumed to be $1\times10^{-6}$W, and $5\times10^{-6}$W, respectively. Average transmitted data rates for different algorithms under consideration are obtained from $100,000$ independent simulation runs.

In Fig. 3, we plot the achievable maximum transmission rate for the CR user versus the total power budget for various algorithms. The value of $I_{th}^{(2)}$ has been fixed to $2\times10^{-6}$ W, and the value of $a$ has been considered to be 0.95. From this figure, we observe that the optimal algorithm is able to achieve the highest transmission rate for a given power budget.

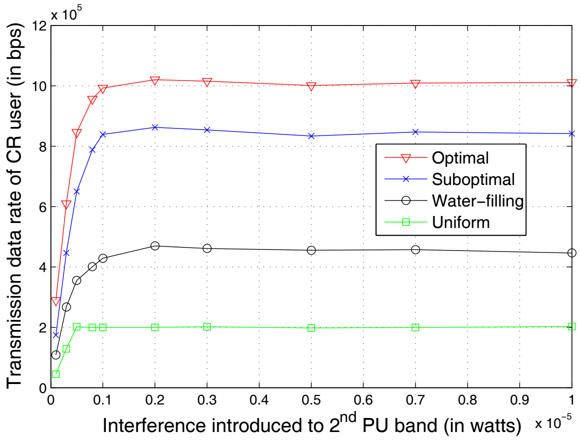


Fig. 4. Transmission data rate of the CR user vs. interference threshold for $2^{nd}$ PU band, $I_{th}^{(2)}$.

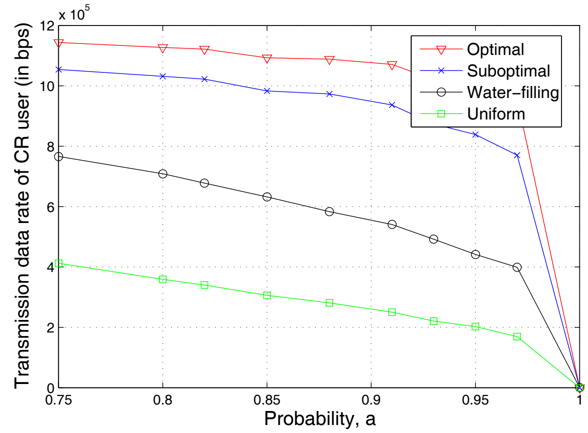


Fig. 5. Transmission rate of the CR user vs. probability, $a$ with which instantaneous interference introduced to the PU band remains below interference threshold, $I_{th}$.

Further, suboptimal algorithm outperforms both water-filling and uniform power loading algorithms. It should be noted that as we increase the power budget for CR user, the interference constraint becomes dominant and the transmission rate of CR user does not increase as the power budget increases. This is expected as in this region the CR system operates in an interference limited scenario.

In Fig. 4, we plot the achievable transmission data rate for the CR user versus interference threshold for second PU band, ($I_{th}^{(2)}$) for all the algorithms under consideration. The value of total transmit power, $P_T$ has been assumed to be $5\times10^{-4}$W. Again, we observe that the optimal algorithm achieves higher transmission rate than that of other algorithms. The suboptimal algorithm performs better than other suboptimal and classical algorithms. Further, water-filling algorithm achieves higher transmission rate than the uniform algorithm. The transmission rate versus interference threshold curve saturates after a certain value of $I_{th}^{(2)}$. The reason is that although $I_{th}^{(2)}$ is relaxed by increasing its value, other constraints ($I_{th}^{(1)}$, $I_{th}^{(3)}$, and $P_T$) becomes dominant.

In Fig. 5, we plot achievable transmission rate for the CR user versus probability $a$. The values of $P_T$, and $I_{th}^{(2)}$, is assumed to be $5\times10^{-4}$ W, and $2\times10^{-6}$ W, respectively. As expected, we observe that the optimal algorithm performs better than the other algorithms. Also, suboptimal algorithm outperforms the water-filling algorithm, which performs better than the uniform power loading algorithm. It is observed from Fig. 5 that as expected as the value of $a$ increases, the achievable transmission rate of CR user decreases for a given power budget and interference thresholds.

TABLE I
COMPLEXITY OF DIFFERENT ALGORITHMS

| Algorithm | Complexity |
|---|---|
| Proposed optimal | $O(N^3)$ |
| Proposed suboptimal | $O(LN) + O(N\log(N))$ |
| Uniform loading | $O(LN)$ |
| Water-filling | $O(LN) + O(N\log(N))$ |

Complexities of various algorithms under consideration are presented in Table I. We can observe that the complexity of the proposed optimal algorithm is higher compared to other algorithms. Our proposed suboptimal algorithm which performs better than the water-filling algorithm has a similar complexity as the water-flling algorithm. The uniform loading algorithm has the lowest complexity but it also performs wose as compared to all other schemes.

## VI. CONCLUSION

In this letter, we have developed an optimal power allocation algorithm for the orthogonal frequency division multiplexing (OFDM)-based CR system. As such the transmission rate of the CR user is maximized for a given power budget and different probabilistic interference constraints imposed by different PU receivers. Instead of instantaneous channel fading gains between the PU receivers and the CR transmitter, the developed optimal power allocation algorithm requires the fading statistics and corresponding parameters to be known at the CR transmitter. We have also proposed and investigated performance of a low complexity suboptimal power allocation algorithm. Presented selected numerical results have shown that our proposed optimal power allocation can achieve significantly higher transmission rate for CR user compared to the classical power allocation algorithms namely, the uniform and water-filling power allocation algorithms that are used for the conventional OFDM-based system. The proposed low complexity suboptimal algorithm achieved better performance than both uniform and water-filling power loading algorithms.

## ACKNOWLEDGEMENT

This research was supported by the Natural Sciences and Engineering Research Council (NSERC) under a Canada Graduate Scholarship and a Strategic Project Grant. We would also like to thank anonymous reviewers for their valuable suggestions.

## APPENDIX A
## PROOF OF THEOREM 1

We use the fact that minimization of negative value of a concave function in Eq. (3) is equivalent to its maximization.

Introducing the so-called Lagrange parameters $\gamma_l$, $\mu_i$, and $\beta$ for the inequality constraints in Eqs. (5), (6), and (9), respectively, the Karush-Kuhn-Tucker (KKT) conditions can be written as follows [16]

$$\begin{aligned}
P_i &\geq 0, \quad \forall i,\\
\sum_{i=1}^{N} P_i - P_T &\leq 0,\\
\sum_{i=1}^{N} P_i K_i^{(l)} - \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))} &\leq 0, \quad \forall l\\
\mu_i &\geq 0, \quad \forall i,\\
\mu_i P_i &= 0, \quad \forall i,\\
\beta &\geq 0,\\
\beta\left(\sum_{i=1}^{N} P_i - P_T\right) &= 0,\\
\gamma_l &\geq 0, \quad \forall l,\\
\gamma_l\left(\sum_{i=1}^{N} P_i K_i^{(l)} - \frac{I_{th}^{(l)}}{2\lambda_l^2(-\ln(1-a))}\right) &= 0, \quad \forall l,\\
-\frac{1}{\left(\frac{\sigma^2+\sum_{l=1}^{L} J_i^{(l)}}{\left|h_i^{ss}\right|^2} + P_i\right)} - \mu_i + \beta + \sum_{l=1}^{L} \gamma_l K_i^{(l)} &= 0, \quad \forall i
\end{aligned} \tag{20}$$

Now we can eliminate $\mu_i$, and can write from conditions in Eq. (20) as follows

$$\frac{1}{\left(\frac{\sigma^2+\sum_{l=1}^{L} J_i^{(l)}}{\left|h_i^{ss}\right|^2} + P_i\right)} \leq \beta + \sum_{l=1}^{L} \gamma_l K_i^{(l)}, \quad \forall i, \tag{21}$$

$$P_i\beta - \frac{P_i}{\left(\frac{\sigma^2+\sum_{l=1}^{L} J_i^{(l)}}{\left|h_i^{ss}\right|^2} + P_i\right)} + P_i\sum_{l=1}^{L} \gamma_l K_i^{(l)} = 0, \quad \forall i. \tag{22}$$

If $\beta + \sum_{l=1}^{L} \gamma_l K_i^{(l)} < \frac{1}{\left(\frac{\sigma^2+\sum_{l=1}^{L} J_i^{(l)}}{\left|h_i^{ss}\right|^2}\right)}$, then Eq. (21) can only hold if $P_i^* > 0$, which by solving Eq. (22) gives, $P_i^* = \frac{1}{\beta+\sum_{l=1}^{L}\gamma_l K_i^{(l)}} - \frac{\sigma^2+\sum_{l=1}^{L} J_i^{(l)}}{\left|h_i^{ss}\right|^2}$. On the other hand, if $\beta + \sum_{l=1}^{L} \gamma_l K_i^{(l)} \geq \frac{1}{\left(\frac{\sigma^2+\sum_{l=1}^{L} J_i^{(l)}}{\left|h_i^{ss}\right|^2}\right)}$, $P_i^* > 0$ is impossible, because it would violate Eq. (22). Hence, the optimal power values can be written as

$$P_i^* = \left[w_i - \frac{\sigma^2 + \sum_{l=1}^{L} J_i^{(l)}}{|h_i^{ss}|^2}\right]^+ \quad \forall i, \tag{23}$$

where $w_i = \frac{1}{\beta+\sum_{l=1}^{L}\gamma_l K_i^{(l)}}$. Theorem is now completed.

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