Homework #2

ECGR 5100, Fall 2022
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1. Let A be the set of the letters in the word "trivial"

$$A = \{a, i, l, r, t, v\}$$

and let B be the letters in the word "difficult."

(a) Find $A \cup B$

(b) Find $A - B$

(c) What would the universe of discourse, $\mathcal{U}$, be?

(d) Given $\mathcal{U}$, find $(A' \cup B')$

Soln-1 Given: A = {t, r, i, v, a, l} and B = {d, i, f, c, u, l, t}

(a) $A \cup B$ = {a, c, d, f, i, l, r, t, u, v}

(b) $A - B$ = {r, v, a}

(c) A Universe U is the larger set which gives the context. For our case we can consider U as a universal set of all possible letters in English alphabet.

(d) For given Universe U, $(A' \cup B')$ will contain all the elements that are neither in set A nor in set B as shown below:
$(A' \cup B')$ = {b, e, g, h, j, k, m, n, o, p, q, s, w, x, y, z}

2. A survey of households in the United States reveals that 96% have at least one screen (TV set, computer monitor, projector), 98% have cable service, and 95% have cable service and at least one screen. What, if anything, can you say about the households that have neither cable service nor a screen? Explain your reasoning in a short paragraph ($2 - 5$ sentences).

Soln-2 Given:

- A - At least one screen = 96%
- B - have cable service = 98%
- $A \cap B$: At least one screen and have cable service = 95%
  A intersection B means households have at least one television set and have cable service which is 95% we need to calculate households which have neither television set nor internet service which is $\neg(A \cup B)$.
  We know $A \cup B = A + B - (A \cap B)$
  Therefore,
- $A \cup B = 0.96 + 0.98 - 0.95$
- $A \cup B = 0.99$
  So, $\neg(A \cup B)$ will be $(1 - 0.99)$ which is $0.01 = 1\%$ of households have neither screen nor cable service.

3. Given the relation that you discovered in the process of doing a research project

$$R = \left\{ \langle x, x^2 \rangle \middle| -\infty < x < \infty \right\}$$

(a) Is R a function?

(b) Is R an injection? If not, give an example.

(c) Is R a surjection? If not, give an example.

How would you go about proving your answers above? Have you introduced any assumptions that might change your answers? (For example, if $R \subseteq S \times T$, what are S and T?)

Now, assuming you understand R well now, write a paragraph ($3-5$ sentences) for the average reader with a college (B.S.) degree, what you can say about R and what you prove about whether or not R is a bijection.

Soln-3

(a) For a given Relation R, we can say that R is a Function because every element x in set S has a unique mapping to every element of x^2 in set T.

(b) Given Relation R is not Injection.
For example,
If we have set S = {-3, -2, 3, 4} then set T = {9, 16}
then R = $\{\langle x, x^2 \rangle \big| -\infty < x < \infty\}$ = {(-3,9), (-2, 4), (3, 9), (4, 16)}.
Therefore, we can see that -3 and 3 in set S are mapped to same element 9 in set T, which contradicts the definition of Injection function as Injective function is a function where no two elements from set S should be mapped to same element in set T.

(c) Given Relation R is Surjective.
For example:
If we have set S = {-3, -2, 3, 4} then set T = {9, 16}
then R = $\{\langle x, x^2 \rangle \big| -\infty < x < \infty\}$ = {(-3,9), (-2, 4), (3, 9), (4, 16)}.
We can see that every element x^2 in set T has mapping to every element in set S. Therefore, R is Surjective.

The Given Relation R is not Bijection because for a function to be Bijective it must be both Injective and Surjective. Whereas in our case the given relation is not Injective, therefore we can say that R is not Bijective.

4. Let V be the set of vowels in English

$$V = \{a, e, i, o, u, y\}$$

and the relation R be "u comes before v in the usual alphabetic order or if $u = v$" where $u, v \in V$.

(a) Is V the universe of discourse? Is R reflexive?

(b) Is R antisymmetric? Would your answer change if $\mathcal{U}$ was alphabet a, b, ... z?

(c) Is R transitive? If not, can you prove it?

Given your understanding of this relation, write a paragraph the proves or disproves that R is partial order.

Soln-4 Given: V = {a, e, i, o, u} and relation R = "u comes before v in the usual alphabetic order or if u = v" where $(u, v) \epsilon V$.
With the given definition above we get:
R = {(a, e), (a, i), (a, o), (a, u), (e, i), (e, o), (e, u), (i, o), (i, u), (o, u), (a, a), (e, e), (i, i), (o, o), (u, u)}

(a) Given set V is not the Universe of Discourse. R is a reflexive because R can contain pairs u, v such that u = v.

(b) R is antisymmetric because R can contain all pairs of u, v and v, u such that u = v. If Universe of Discourse U contains all alphabets from a to z, the relation R will still be antisymmetric as based on the definition of R it will still contain all the pairs of u, v where u = v. Therefore, our answer will not change.

(c) Given relation R is transitive because for every pair u, v and v, w there exists a pair u, w in the relation R.

From the above solutions we can see that R is Reflexive, Antisymmetric, and Transitive. And for a relation to be partial order it must satisfy these three characteristics. Therefore, we can say that R is partial order relation.

5. Given the following scenario, build up a formal mathematical description of the system.

> A discrete mathematics course contains 1 mathematics major who is a freshman, 12 mathematics majors who are sophomores, 15 computer engineering majors who are sophomores, and 2 mathematics majors who are juniors, 2 computer engineering majors who are juniors, and 1 computer engineering major who is a senior.

Express each of these statements as quantified predicates and *prove* its truth value.

(a) There is a student in the class who is a junior.

(b) Every student in the class is a computer engineer.

(c) There is a student in the class that is neither a sophomore nor a junior.

(d) There is a student in the class that is neither a sophomore nor a junior.

(e) There is a major such that there is a student in the class in every year of study with that major.

<u>Soln-5</u> Let's consider predicate P(s, c, m) as statement that Student **s** who has standing Class **c** and is Majoring in **m**.

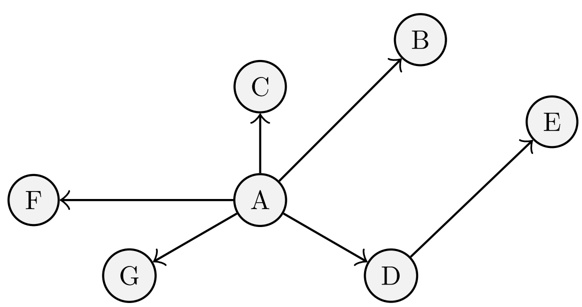
(a) The preposition is $\exists s \exists m P(s, junior, m)$. It is true for given information.

(b) The preposition is $\forall s \exists c P(s, c, computerengineering)$. This is false, because there are some students who have mathematics major.

(c) The preposition is $\exists s \exists c \exists m (P(s, c, m) \land c \neq \ sophmore \land c \neq \ junior)$. This is a true, as there is a freshman majoring in mathematics.

(d) The preposition is $\exists m \exists s \forall y [InClass(s) \land Major(s, m) \land Year(s, y)]$. This is false, as there is no student in computer engineering who is Freshman.

6. Given the following illustration of a graph G, answer these basic questions about G.



(a) Define graph G formally.

(b) Is G an undirected graph?

(c) Is G formally a tree? If yes,

- what is the root of G?
- what is the height of G?
- how many cycles are in G?

<u>Soln-6</u>

(a) The given Graph can be formally defined as G = (V, E), where V stands for Vertices, and E stands Edges.
V = {A, B, C, D, E, F, G}
E = {(A, B), (A, C), (A, D), (D, E), (A, F), (A, G)}

(b) The given graph is not an undirected graph as the relationship between the Vertices are not symmetric.

(c) G is formally a tree as it is acyclic, directed, and connected.

- The root of G is node A.

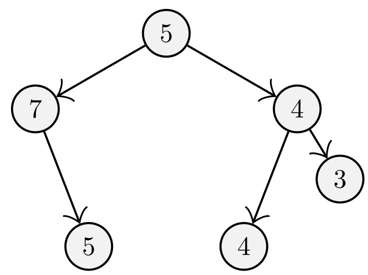
- The height of G is 2.
- There are no cycles in G.

7. Let T be a binary tree that is constructed as follows. Given a node n and a datum x, an `add()` operation will...

    - set n to be a new node with x as the datum and left=right=NULL
    - (recursively) add x to sub-tree n.left if x is greater-than-or-equal-to n's datum
    - (recursively) add x to sub-tree n.right if x is less-than n's datum

    Draw (illustrate) the tree for the inputs: 5,7,4,3,4,5

    Soln-7



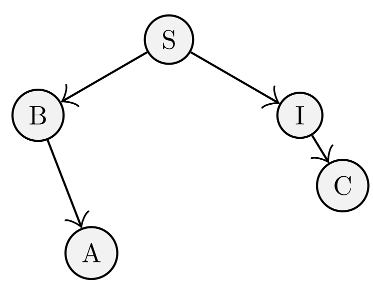
8. A binary tree T visits nodes in the following order using an in-order traversal.

$$B - A - S - I - C$$

    (a) Draw (illustrate) this tree graphically.
    (b) Is there more than one tree that will produce this output?

    Soln-8

    (a) The binary tree for given in-order traversal can be represented as follows:



    (b) Yes, two structureally different trees can have same inorder traversal output. For example, tree shown below produces same output as above on inorder traversal.