Naïve Bayes

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Naive Bayes is a simple yet powerful **probabilistic machine learning algorithm** based on applying **Bayes' theorem** with a strong (naive) assumption of **independence between features**.

Bayes Theorem

Bayes' theorem describes how to **update our beliefs (probabilities)** about a hypothesis when new evidence is observed.

Likelihood

Prior probability (Initial belief)

Posterior probabity.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Evidence

- → P(A|B) = Posterior Probability
- → P(B|A) = Likelihood
- → P(A) = Prior Probability (Hypothesis)
- → P(B) = Marginal Probability (Evidence)

Prior Probability ----> **New Information** -----> **Posterior Probability**

Eg.

P(Disease | Positive Test) = [P(Positive Test | Disease) * P(Disease)] / P(Positive Test)

Disease Diagnosis

Based on historical info, some believes.... flu

① Prior Probability -

$P(\text{Disease}) = 1\% = 0.01$

$P(\text{No Disease}) = 1 - 0.01 = 0.99$

② 90% of time, if people has the disease, they tested positive.

$P(\text{Tested positive} \mid \text{Disease}) = 0.90$

$P(\text{Test Negative} \mid \text{Disease}) = 1 - 0.90 = 0.10$ (false negative) Type II

③ 95% of time, if people doesn't have disease, they tested negative

$P(\text{Neg. Test} \mid \text{No Disease}) = 0.95$

$P(\text{Pos. Test} \mid \text{No Disease}) = 0.05$ (false positive) Type I

Population — 10,000 (Total)

1. Have Disease : 100 (1%)
   - Tested positive = 90 (90%) → True positive ✓
   - Tested negative : 10 (10%) → False negative

2. Have No Disease : 9900 (99%)
   - Tested Negative - 9405 (95%) → True Negative
   - Tested positive — 495 (5%) → False positive ✓

P(Disease | Tested positive)

→ 90/585

$\boxed{P(\text{Disease} / \text{Tes.po}) = 0.1538}$

Total positive = TP + FP

= 90 + 495 = 585

Have disease : 90

Using Bayes Theorem

P(Disease | Positive Test) = [P(Positive Test | Disease) * P(Disease)] / P(Positive Test)

P(Positive Test | Disease) = 0.90
P(Disease) = 0.01
P(Positive Test) = 0.0585

P(Disease | Positive Test) = 0.1538

Naive Bayes

$$
X = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \end{array}
\begin{array}{c}
\begin{array}{ccccc} f_1 & f_2 & f_3 & \cdots & f_m \end{array} \\
\begin{bmatrix}
x_{11} & x_{12} & x_{13} & \cdots & x_{1m} \\
x_{21} & x_{22} & x_{23} & \cdots & x_{2m} \\
\vdots & & & & \\
x_{n1} & x_{n2} & x_{n3} & \cdots & x_{nm}
\end{bmatrix}
\end{array}
$$

$\rightarrow$ Row (Datapoint)

$$
C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix}
$$

Categorical

$$
\boxed{\hat{y} = \underset{c_i \in C}{\operatorname{argmax}}\ P(c_i \mid X)}
$$

$X_i \rightarrow$ $c_1 \rightarrow 0.01$

$c_2 \rightarrow 0.2$

$\boxed{c_3 \rightarrow 0.60}$

$c_4 \rightarrow 0.2$

$\vdots$

$c_n \rightarrow 0.1$