

PERCENTAGE

%

By- Rahul Agrahari

PERCENTAGE

To determine the percentage, we have to divide the value by the total value and then multiply the resultant to 100.

Percentage formula = (Value/Total value)×100

Example: $2/5 \times 100 = 0.4 \times 100 = 40\%$

QUESTION

Q.1:-What will be the fraction of 4%?

EXPLANATION

Fraction of 4%

$$4 \times (1/100) = 1/25$$

PERCENTAGE CHART

Fractions	Percentage
1/2	50%
1/3	33.33%
1/4	25%
1/5	20%
1/6	16.66%
1/7	14.28%
1/8	12.5%
1/9	11.11%
1/10	10%
1/11	9.09%
1/12	8.33%
1/13	7.69%
1/14	7.14%
1/15	6.66%

FORMULA

Formula

$$X\% \text{ of } Y = \frac{X}{100} \times Y$$

PERCENTAGE TRICKS

To calculate the percentage, we can use the given below tricks.

$$x\% \text{ of } y = y\% \text{ of } x$$

Example- Prove that 10% of 30 is equal to 30% of 10.

Solution- $10\% \text{ of } 30 = 3$

$30\% \text{ of } 10 = 3$

There fore they are equal i.e. $x\% \text{ of } y = y\% \text{ of } x$ holds true.

QUESTION

Q.2:- A salary is 20% more than that of B salary. By how much % is B salary less than that of A?

EXPLANATION

Let B salary = 100₹

then A salary = 120₹

$$= \frac{100 - 120}{120} = \frac{-20}{120} \times 100$$

$(120) \leftarrow \text{that of } A \right) = -16.67\%$

Negative sign indicate less.

QUESTION

**Q.3:- A salary is 20% less than that of B salary.
By how much % is B salary more than that of
A salary?**

EXPLANATION

Let B salary = 100₹

then A salary = 80₹

$$= \frac{100 - 80}{80} \times 100 = 25\%$$

80 ← that of A

Positive sign indicate increment

QUESTION

Q.4:- If 100 increases by 10% = 110.

Then 110 decreases by how much % to get 100?

EXPLANATION

$$100 \xrightarrow[\downarrow ?]{10\% \uparrow} 110$$

$$\frac{100 - 110}{110} \times 100 = -9.09\%$$

IMPORTANT RELATION

$$100 \xrightarrow[16\cdot6\% \downarrow]{10\% \uparrow} 110$$

$$100 \xrightarrow[16\cdot6\% \downarrow]{20\% \uparrow} 120$$

$$100 \xrightarrow[20\% \downarrow]{25\% \uparrow} 125$$

$$100 \xrightarrow[25\% \downarrow]{33\cdot\bar{3}\% \uparrow} 133\cdot\bar{3}$$

$$100 \xrightarrow[33\cdot\bar{3}\% \downarrow]{50\% \uparrow} 150$$

CONCEPT

- If the price of a commodity increases by $x\%$,
the reduction in consumption so as not to
increase the expenditure is given by;

$$= \left(\frac{x}{100+x} * 100 \right) \%$$

- If the price of a commodity decreases by $x\%$,
the increase in consumption so as not to
decrease the expenditure is given by;

$$= \left(\frac{x}{100-x} * 100 \right) \%$$

QUESTION

Q:- If petrol becomes cheaper by 25%. The percentage by which Ashish can drive his bike more, so that his budget for petrol remains unaltered, is

- (a) 22.2%**
- (b) 25%**
- (c) 33.3%**
- (d) 10.2%**

EXPLANATION

100 Decreases by 25% = 75

$[(100 - 75)/75] \times 100 = 33.33\%$

Percentage Increase and Decrease

The percentage increase is equal to the subtraction of original number from a new number, divided by the original number and multiplied by 100.

$$\% \text{ Increase} = [(\text{New number} - \text{Original number})/\text{Original number}] \times 100$$

Increase in number = New number – original number

Similarly, percentage decrease is equal to subtraction of new number from original number, divided by original number and multiplied by 100.

$$\% \text{ Decrease} = [(\text{Original number} - \text{New number})/\text{Original number}] \times 100$$

Where decrease in number = Original number – New number

So basically if the answer is negative then there is percentage decrease.

Percentage Difference Formula

- The percentage difference is:
The **difference** between two values divided by
the **average** of the two values. Shown as
a percentage difference.
- As if 2 numbers are given as X and Y. Then
percentage difference can be calculated as
$$= [(X-Y)/(\{X+Y\}/2)] * 100$$

EXAMPLE

Q. Juice costs \$4 in one shop and \$6 in another shop, what is the percentage difference?

SOLUTION

$$= [(X-Y)/(\{X+Y\}/2}] * 100$$

$$= [(6 - 4)/(\{6+4\}/2] \times 100 = 40\%$$

CONCEPT

If any resultant quantity can be written as

$$R = a \times b$$

\uparrow \uparrow
 $x\% (\uparrow)$ $y\% (\uparrow)$

If a increases by $x\% (\uparrow)$
and b " " $y\% (\uparrow)$

Change in resultant quantity .

$$\boxed{\Delta R\% = x + y + \frac{xy}{100}}$$

Note: - This formula is only applicable when the number is product of two things .

EXAMPLE

$$\text{Exkunse} = \text{Priu} \times \text{Qty}$$

If Priu increase by 20%
and Qty increase by 10%
then total change in expenses

$$\Delta E = P \times Q$$

↑

↑

20%

10%

$$\Delta E\% = 20 + 10 + \frac{20 \times 10}{100}$$

$$\Delta E\% = 32\%$$

EXAMPLE

$$E = P \times Q$$

$\uparrow \rightarrow \text{Increase}$
 $\downarrow \rightarrow \text{Decrease}$

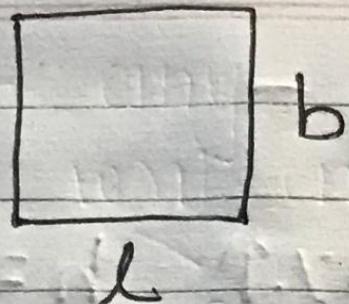
$\Delta E\% = +(\Delta C) - (\Delta R) + \frac{(+\Delta C)(-\Delta R)}{100}$

$$\Delta E\% = +(\Delta C) - (\Delta R) + \frac{(+\Delta C)(-\Delta R)}{100}$$

$$\Delta E\% = \frac{\Delta C - \Delta R}{100}$$

QUESTION

Q.



IF $l = 20 \text{ cm}$ $b = 10 \text{ cm}$

Change in Area?

$$A = 20 + 10 + \frac{20 \times 10}{100} = 32 \text{ cm}^2$$

CONCEPT

Q. $\lambda = 20 \text{ yr.} (\uparrow)$ $b = 10 \text{ yr.} (\uparrow)$ $h = 10 \text{ yr.} (\downarrow)$
 $\Delta V = ?$

$$V = \boxed{\lambda \times b \times h}$$

$$V = \frac{A \times h}{\uparrow \quad \uparrow}\\ \quad \quad \quad 32 \text{ yr.} \uparrow \quad 10 \text{ yr.} \downarrow$$

$$= 32 - 10 = \frac{32 \times 10}{100}$$

$$= 22 - 3.2 = 18.8 \text{ yr.}$$

+ve sign indicate increment

ALTERNATE METHOD

Alternate Method: —

$$l = 20\gamma \cdot (↑) \quad b = 10\gamma \cdot T \quad h = 10\gamma \cdot ↓$$

Let $v_i = l, b_i = b, h_i = h$

then $l_f = 1.2l, b_f = 1.1b, h_f = 0.9h$

$$v_i = l b h$$

$$v_f = 1.2 \times 1.1 \times 0.9 l b h = 1.188 l b h$$

$$\Delta V\% = \frac{v_f - v_i}{v_i} \times 100$$

$$= \frac{1.188 l b h - l b h}{l b h} \times 100$$

$$= 18.8\%$$

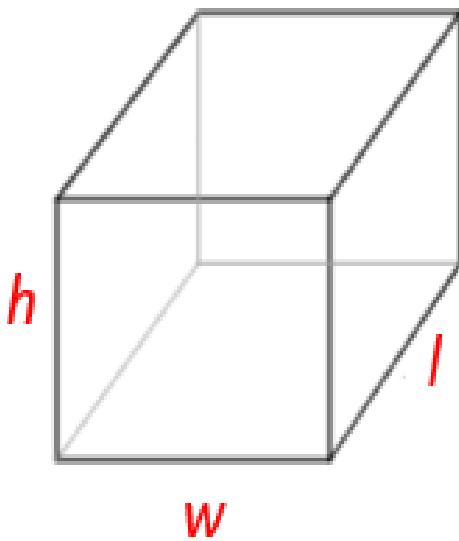
QUESTION- CAT

The length, breadth and height of a cuboid increases by 10%, 10% and 20% respectively. Find the percentage increase in the cuboid's volume.

- (a) 40%
- (b) 42%
- (c) 45.2%
- (d) 48.1%

EXPLANATION

Volume of Cuboid



$$V = lwh$$

$$V = l \times b \times h$$

$$V = A \times h$$

$$A = l \times b$$

$$\Delta A\% = 10 + 10 + \frac{10 \times 10}{100} = 21\%$$

$$V = A \times h$$

$$\Delta V\% = 21 + 20 + \frac{21 \times 20}{100}$$

$$\Delta V\% = 45.2\%$$

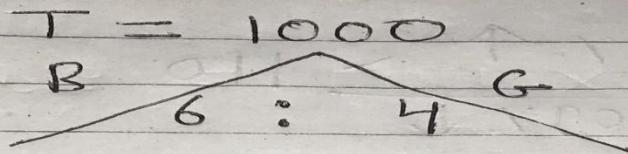
QUESTION

1000 students appeared for an examination. The boys and girls ratio is 6 : 4. 60% of boys and 40% of girls passed the exam. The % of candidates failed the exam is

- (a) 62%
- (c) 36%

- (b) 48%
- (d) 24%

EXPLANATION



$$B = \frac{6}{10} \times 100\% \\ = 600$$

$$G = \frac{4}{10} \times 100\% \\ = 400$$

$$\text{Passed boys} = 600 \times \frac{60}{100\%} = 360$$

$$\text{Passed Girls} = 400 \times \frac{40}{100\%} = 160$$

$$\text{Total passed Students} = 520$$
$$\text{Failed Students} = 480$$

Percentage failed students

$$= \frac{480}{1000} \times 100\% = \underline{\underline{48\%}}$$

GATE- 2012

Q. The data given in the following table summarizes the monthly budget of an average household

Category	Amount
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Others	1800

The approximate percentage of the monthly budget NOT spent on savings is

- (a) 10% (b) 14% (c) 81% (d) 86%

EXPLANATION

Total Monthly budget = 10500

Spent on savings = 1500

Not spent on savings = 9000

$$\begin{aligned}\text{Percentage} &= \frac{9000}{10500} \times 100 \\ &= 85.7142 \approx 86\%.\end{aligned}$$

QUESTION

Maruti is priced 20% more than Nano but 25% less than Santro. What percentage of Santro's price is Nano's price?

- (a) 50%
- (b) 62.5%
- (c) 37.5%
- (d) 66.66%

EXPLANATION

$$1 - P_M = 1.2 P_N - [i]$$

$$P_M = 0.75 P_S - [ii]$$

From eqn one and two

$$P_S = \frac{1.2}{0.75} P_N = \cancel{1.6 P_N}$$

$$P_N = \frac{0.75}{1.2} P_S = 0.625 P_S$$

$$= 62.5\%$$

Ans
=

QUESTION

A building worth Rs. 5,00,000 was constructed on a land worth Rs. 3,00,000 Co-incidentally that year, the value of land appreciated by $x\%$ and the value of the building depreciated by the same $x\%$. Another co-incidence was that at the end of the year, the value of the land was same as the value of the building. Find the value of x .

- (a) 16.66%
 - (b) 20%
 - (c) 25%
 - (d) 33.33%

EXPLANATION

$$B_c = 500000$$

$$L_c = 300000$$

Value of land appreciated by = $x\%$.

Value of the building depreciated = $y\%$.

$$L_c \left[1 + \frac{x}{100} \right] = B_c \left[1 - \frac{y}{100} \right]$$

$$300000 \left[1 + \frac{x}{100} \right] = 500000 \left[1 - \frac{y}{100} \right]$$

$$x = 25\%$$

QUESTION

At the start of a year, the land and the building a company owned was together valued at Rs. 10,00,000. In the year the value of land appreciates by 20%, whereas the value of a building depreciates by 10%. If at the end of the year, the land and building was together valued at Rs. 10,20,000, find the value of the land alone at the start of the year.

- (a) Rs. 7,00,000 (b) Rs. 6,00,000
(c) Rs. 5,00,000 (d) Rs. 4,00,000

EXPLANTION

$$L + B = 1000000 \quad \text{--- [i]}$$

$$1.2L + 0.9B = 1020000 \quad \text{--- [ii]}$$

from eqn (i)

$$B = (1000000 - L)$$

$$1.2L + 0.9(1000000 - L) = 1020000$$

$$1.2L + 900000 - 0.9L = 1020000$$

$$\boxed{L = 400000}$$

Ans

UPSC

Q:-A person bought an article and sold it at a loss of 10%. If he has bought at 20% less and sold it for Rs 55 more, he would be gained 40%. Find C.P.

(a) Rs 250

(b) Rs 225

(c) Rs 275

(d) Rs 300

EXPLANATION

Date :
Page :

CP_2

\downarrow

Ans:- $0.8CP_1 \leftarrow 20\% \downarrow (CP), \frac{10\% \downarrow}{(S.P)_1} = 0.9CP_1 = (0.9CP_1 + SS)$

$40\% \uparrow$

$0.8CP_1 \times 1.40 = 0.9CP_1 + SS$

$1.12CP_1 = 0.9CP_1 + SS$

$CP_1 = 250$] Ans//

UPSC

Q:-A person sold an article at 10% profit if he had bought it 10% less price and sold it at 3Rs. More (than present S.P.) he would have got 25% profit (On his new investment). What was the (actual) C.P. of the article.

EXPLANATION

$$\text{Ans: } 0.9CP_1 \xleftarrow[10\% \downarrow]{(CP_1)} SP_1 = 1.1(CP_1) \xrightarrow[10\% \uparrow]{SP_2 \downarrow} (1.1(CP_1 + 3))$$

$$25\% \uparrow$$

$$0.9CP_1 \times 1.25 = 1.1(CP_1 + 3)$$

$$1.125CP_1 = 1.1(CP_1 + 3)$$

$$CP_1 = 120$$

GATE-2012

Q.8:-There are two candidates P and Q in an election. During the campaign 40% of the voters promised to vote for P and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. suppose, P lost by 2 votes, then what was the total number of voters?

EXPLANATION

Ans:- Let total voters x

$$P = x \left(\frac{40}{100} \right) - \left[\left(\frac{x \times 40}{100} \right) \times \frac{15}{100} \right] + \left[\left(\frac{x \times 60}{100} \right) \times \frac{25}{100} \right]$$

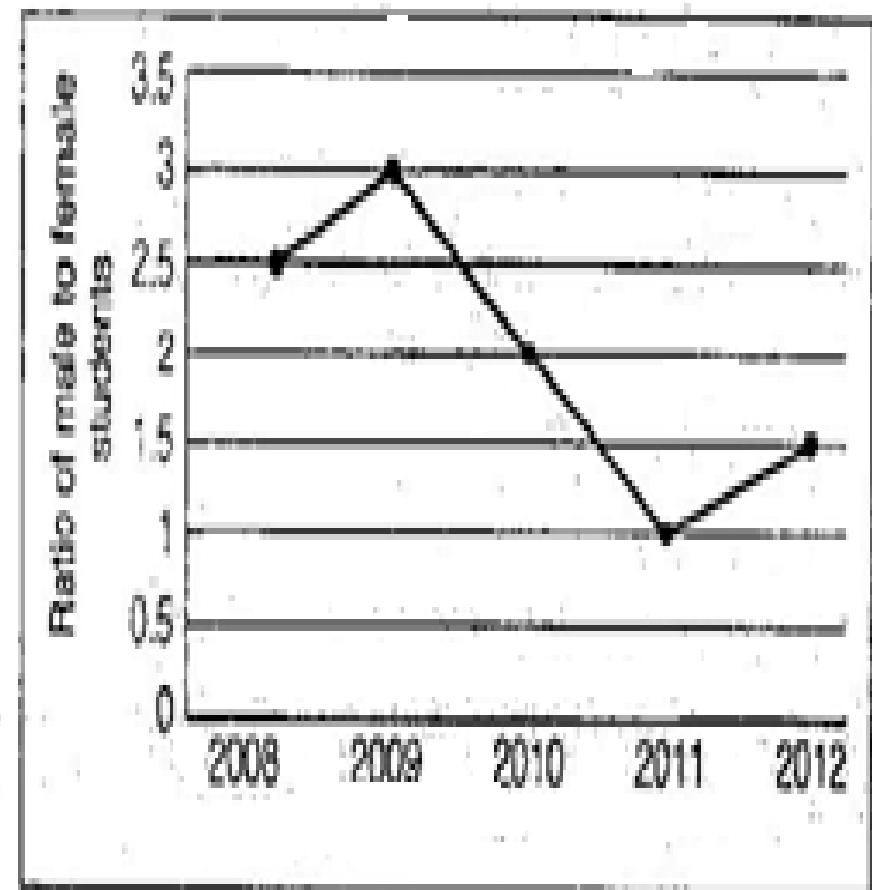
$$Q = x \left(\frac{60}{100} \right) + \left[\left(\frac{x \times 40}{100} \right) \times \frac{15}{100} \right] - \left[\left(\frac{x \times 60}{100} \right) \times \frac{25}{100} \right]$$

$$Q - P = 2$$

$$\left[0.6x + 0.06x - 0.15x \right] - \left[0.4x - 0.06x + 0.15x \right] = 2$$
$$x = 100$$

GATE-2015

Q.9:-The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?



EXPLANATION

Ans:-

$$\left(\frac{M}{F} \right)_{2008} = 2.5$$

$$\left(\frac{M}{F} \right)_{2009} = 3$$

$$F_{2008} = x \\ F_{2009} = 2x$$

$$M_{2008} = 2.5x$$

$$M_{2009} = 3(2x)$$

$$\% = \frac{6x - 2.5x}{2.5x} \times 100$$

$$= 140 \% \quad \underline{\text{Ans}}$$

GATE-2019

Q.10:-A retaining wall with measurements 30 m x 12 m x 6 m was constructed with bricks of dimensions 8 cm x 6 cm x 6 cm If 60% of the wall consists of bricks, the number of bricks used for the construction is _____ lakhs.

- (a) 30
- (b) 40
- (c) 75
- (d) 45

SOLUTION

Number of bricks = x

$$\Rightarrow 30 \times 12 \times 6 \times 10^6 \times 0.6 = 8 \times 6 \times 6 \times x$$

$$x = 4.5 \times 10^6 = 45 \times 10^5 = 45 \text{ lakhs bricks}$$

GATE-2019

Q.11 :-Population of state X increased by $x\%$ and the population of state Y increased by $y\%$ from 2001 to 2011. Assume that x is greater than y . Let P be the ratio of the population of state X to state Y in a given year. The percentage increase in P from 2001 to 2011

- (a) X/Y
- (b) $100(X - Y)/(100+Y)$
- (c) $(X - Y)$
- (d) $100 (X - Y)/(100+X)$

EXPLANATION

ii Ans:- Let a, b be initial population

$$\frac{a}{b} = P \text{ (Ratio earlier)}$$

$$\frac{a \left(1 + \frac{R}{100}\right)}{b \left(1 + \frac{C}{100}\right)} = P' \text{ (New Ratio)}$$

so required % change

$$\begin{aligned} \frac{P' - P}{P} \times 100 &= \frac{\frac{a}{b} \left(\frac{100+R}{100+C}\right) - \frac{a}{b}}{\frac{a}{b}} \times 100 \\ &= \frac{100(R-C)}{100+R} \end{aligned}$$

GATE-2019

Q.12:-The radius as well as the height of a circular cone increases by 10%. The percentage increase in its volume is _____.

- (a) 17.1
- (b) 21.0
- (c) 33.1
- (d) 72.8

EXPLANATION

12 Ans:- Volume of a circular cone

$$= \frac{1}{3} \pi r^2 h$$

$$V_i = \frac{1}{3} \pi r_1^2 h_1$$

$$\begin{aligned} V_f &= \frac{1}{3} \pi (1.1r_1)^2 \times 1.1h_1 \\ &= 1.331 (\frac{1}{3} \pi r_1^2 h_1) \end{aligned}$$

$$\begin{aligned} \therefore \text{Change in Volume} &= \frac{V_f - V_i}{V_i} \times 100 \\ &= 33.1\% \end{aligned}$$

GATE-2019

Q.13:-In a country of 1400 million population 70% own mobile phones. Among the mobile phone owners, only 294 million access the Internet. Among these Internet users, only half buy goods from e-commerce portals. What is the percentage of these buyers in the country?

EXPLANATION

13 Ans:- Number of internet users = 294
Half of which use e-commerce

$$\text{Portal} = \frac{294}{2} = 147$$

Percentage of e-commerce portals
in the country (out of 1400 Million)

$$= \frac{147}{1400} \times 100 = 10.5\%$$

SAIL

**Q.14:-It is given that 5% increase in X always means 3% increase in Y and 5% increase in Y always implies 2.5% increase in Z.
If Y is increased by 30%, then Z^2 should be increased by**

- A) 32.25%
- B) 60%
- C) 69%
- D) 90%

EXPLANATION

5% (increase) → X means 3% (increase) → Y

5% (increase) → Y means 2.5% (increase) → Z

Then Y is increased by 30%

30% Y → 15% Z

Means that Z goes up from 1.5 then Z^2 will go up $(1.15)^2 = 1.3225$

Hence increases of 32.25%.

Option A is correct.

GATE-2019

Q.15:-Fiscal deficit was 4% of the GDP in 2015, and that increased to 5% in 2016. If the GDP increased by 10% from 2015 to 2016, then percentage increase in the actual fiscal deficit is _____

- (a) 25.00
- (b) 35.70
- (c) 10.00
- (d) 37.50

EXPLANATION

Let GDP in 2015 = 100

GDP in 2016 = $100 + 10 = 110$

Deficit in 2015 = 4

Deficit in 2016 = 5% of 110 = 5.5

Percentage increase in fiscal deficit

$$= [(5.5 - 4)/4] \times 100 = 37.5 \%$$

GATE-2019

Q.16:-Mola is a digital platform for taxis in a city. It offers three type of rides-Pool, Mini and Prime. The table below presents the number of rides for the past four months. The platform earns one US dollar per ride. What is the percentage share of revenue contributed by prime to the total revenues of Mola, for the entire duration?

TYPE	MONTH	MONTH	MONTH	MONTH
	January	February	March	April
Pool	170	320	215	190
Mini	110	220	180	70
Prime	75	180	120	90

- (a) 38.74
- (b) 23.97
- (c) 25.86
- (d) 16.24

EXPLANATION

Revenue by pool = $170 + 320 + 215 + 190 = 895$

Revenue by mini = $110 + 220 + 180 + 70 = 580$

Revenue by prime = $75 + 180 + 120 + 90 = 465$

Total revenue by Mola = $895 + 580 + 465 = 1940$

Percentage of prime = $(465 \div 1940) \times 100 = \underline{23.97}$

CAT

Q.5:-The production of coal in 2006 is 10% more than the production of coal in 2005. If the share of Lignite, a type of coal, in the total coal production was 25% and 20% in the years 2005 and 2006, by what percentage has the production of lignite changed in 2006 compared to the year 2005?

- (a) 5%
- (b) - 5%
- (c) 12%
- (d) - 12%

EXPLANATION

5 Ans. - Let production of coal in 2005 = x
then production in 2006 ($10\% \uparrow$)
 $= 1.1x$

Production of Lignite in 2005 = $x \times 0.25$
in 2006 = $1.1x \times 0.2$

$$\Delta\% = \frac{0.22x - 0.25x}{0.25x} \times 100$$

$$= \frac{-0.03x}{0.25x} \times 100$$

$$= -12\% \quad \text{Ans} //$$

PERCENTAGE

Q.1:-A student took five papers in an examination, where the full marks were the same for each paper. His marks in these papers were in the proportion of 6 : 7 : 8 : 9 : 10. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is:

- (a) 1**
- (b) 3**
- (c) 4**
- (d) 5**

EXPLANATION

1 Ans: - 6 : 7 : 8 : 9 : 10

$$\frac{6x + 7x + 8x + 9x + 10x}{5} = 60$$

$$40x = 300$$
$$x = \frac{300}{40}$$

$$1^{\text{st}} \text{ baker} = 6x = 6 \times \frac{300}{40} = 45\% < 50\%$$

$$2^{\text{nd}} \text{ baker} = 7x = 7 \times \frac{300}{40} = 52.5\% > 50\%$$

$$3^{\text{rd}} \text{ baker} = 8x = 8 \times \frac{300}{40} = 60\% > 50\%$$

$$4^{\text{th}} \text{ baker} = 9x = 9 \times \frac{300}{40} = 67.5\% > 50\%$$

$$5^{\text{th}} \text{ baker} = 10x = 10 \times \frac{300}{40} = 75\% > 50\%$$

In 4 bakers got more than 50% Marks.

CAT

Q.2:- One bacteria splits into eight bacteria of the next generation. But due to environment only 50% of any generation can produce the next generation . If the seventh generation number is 4096 million, what is the number in first generation?

- (a) 1 million**
- (b) 2 million**
- (c) 4 million**
- (d) 8 million**

EXPLANATION

Ans. - Let in 1st generation bacteria = xc

Second generation bacteria = $(xc \times 8 \times 0.5)$

3rd " " " = $(xc \times 8 \times 0.5)^2$

4th " " " = $(xc \times 8 \times 0.5)^3$

5th " " " = $(xc \times 8 \times 0.5)^4$

$(xc \times 8 \times 0.5)^6 = 4096$ Million

$xc \times 8 \times 0.5 = (4096)^{1/6} = (4^6)^{1/6} = 4$

$xc = 1$ Million

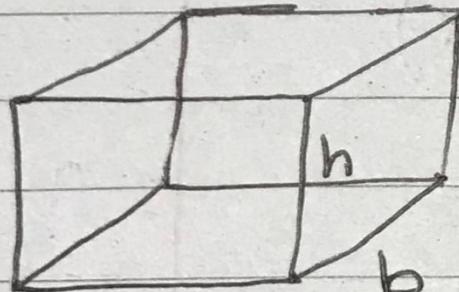
CAT

Q.3:- The length, breadth and height of a cube increases by 10%, 10% and 20% respectively. Find the percentage increase in the total surface area of cube.

- (a) 10%
- (b) 20%
- (c) 22.22%
- (d) 28.33%

EXPLANATION

3Ans:- Surface Area of cube = $2(lb + bh + hl)$



Let initial

$$l = 10, b = 10, h = 10$$

then final

$$l+1l = 11, l+1b = 11, l+2h = 12$$

$$(SA)_i = 2[10^2 + 10^2 + 10^2] = 2 \times 3 \times 10^2 = 600$$

$$(SA)_f = 2[11 \times 11 + 11 \times 12 + 12 \times 11] = 770$$

$$\therefore = \frac{770 - 600}{600} \times 100 = 28.33\%$$

PERCENTAGE

Q.4:- During a heating process, the surface area of a sphere increases by 44%. Find the percentage change in the sphere's radius and volume

- (a) 11%, 48%
- (b) 20%, 72.8%
- (c) 22%, 66%
- (d) 25%, 62.8%

EXPLANATION

$$\text{Ans.} - (\text{S.A})_{\text{sphere}} = 4\pi r^2$$

$$(\text{S.A})_i = 4\pi r^2$$

$$(\text{S.A})_f = 4\pi r_i^2$$

$$\% = \frac{4\pi r_i^2 - 4\pi r^2}{4\pi r^2} \times 100$$

$$0.44 = \frac{r_i^2}{r^2} - 1$$

$$\frac{r_i^2}{r^2} = 1.44 = 1.2^2$$

$$\frac{r_i}{r} = 1.2 \Rightarrow r_i = 1.2 r$$

r_i in ~~is~~ changing 20% (\uparrow)

$$V_i = \frac{4}{3}\pi r^3$$

$$V_f = \frac{4}{3}\pi r_i^3 = \frac{4}{3}\pi \times (1.2r)^3$$

$$V_f = \frac{4}{3}\pi \times 1.728 r^3$$

$$\Delta V \% = \frac{V_f - V_i}{V_i} = \frac{1.728 r^3 - r^3}{r^3}$$
$$= 72.8\%$$

Thank
you



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