VIT
Vellore Institute of Technology

Continuous Assessment Test (CAT)- II– October 2022

Programme	: B.Tech.	Semester	: Fall 2022-2023
Course Title	: Complex Variables and Linear Algebra	Code	: BMAT201L
		Slot	: A1+TA1+TAA1
Faculty	: Dr. M. Dhivya, Dr. N. Durga, Dr. M. Prasannalakshmi, Dr. C. Rajivganthi, Dr. Ashish Kumar, Dr. Prosenjit, Dr. Ankit Kumar, Dr. Surath Ghosh, Dr. Sandip Saha, Dr. Kamalesh, Dr. P. Sushmitha, Dr. Amitkumar Rahul	Class Nbr	: CH2022231001157, 60, 62, 64,67, 70, 72, 74, 76, 80, 82
Duration	: 1 ½ Hours	Max. Marks : 50	

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks

1. a) A complex function $f(z)$ is defined as follows

$$f(z) = \begin{cases} Re(z), & x \geq 5 \\ Im(z+1), & x < 5 \end{cases}$$

Evaluate $\int_C f(z)\, dz$, where C is the curve $\qquad$ [5+5]

$$y = \begin{cases} -x^2, & from -2-4i \text{ to } 0 \\ 0, & from\ 0\ to\ 5 \\ x, & from\ 5\ to\ 10+10i \end{cases}$$

b) Expand the function $f(z) = \frac{z-1}{z^2}$ in a Laurent series valid for i) $|z-1|>1$ ii) $|z-1|<1$.

2. a) Evaluate $\int_C \frac{z^2 e^{2z+1} dz}{(z+i)^2(z^2-9)}$, where C is $|z+1|=3$.

b) Classify the singularity z=0 for $f(z) = \frac{1}{1-e^{z^2}}$ $\qquad$ [7+3]

3.

Evaluate $\int_0^{2\pi} \frac{1}{1-2a\cos\theta+a^2}\, d\theta$, $|a|\neq1$ $\qquad$ [10]

4. a) Check whether the given set of vectors $S = \left\{ \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 5 & 4 \\ 0 & 5 \end{pmatrix} \right\}$ is linearly

independent or not. $\qquad$ [5+5]

b) Find a basis for the vector space $W = \{(x_1, x_2, x_3, x_4) \in R^4 | x_1 + x_2 - 2x_3 + x_4 = 0, 2x_1 - x_2 + x_3 = 0, 4x_1 + x_2 - 3x_3 + 2x_4 = 0\}$. What is the dimension of W?

5. a) Let V be the set of all polynomials of degree at most 2. Is $W = \{p(x) \in V : p''(x) = 0\}$ a subspace of V?

b) Check whether $W = \{A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} : A^2 = \alpha\ I\}$ is a subspace of set of all 2×2 matrices. $\qquad$ [5+5]