VIT

Vellore Institute of Technology

Continuous Assessment Test (CAT)- I– September 2022

Programme	: B.Tech.		
Course Title	: Complex Variables and Linear Algebra	Semester	: Fall 2022-2023
		Code	: BMAT201L
Faculty	: Dr. M. Dhivya, Dr. N. Durga, Dr. M. Prasannalakshmi,	Slot	: A1+TA1+TAA1
	Dr. C. Rajivganthi, Dr. Ashish Kumar, Dr. Prosenjit,	Class Nbr	: CH2022231001157,
	Dr. Ankit Kumar, Dr. Surath Ghosh, Dr. Sandip Saha,		60, 62, 64,67, 70,
	Dr. Kamalesh, Dr. P. Sushmitha, Dr. Amitkumar Rahul		72, 74, 76, 80, 82
Duration	: 1 ½ Hours		
		Max. Marks	: 50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks

1. Prove that the function $u = e^x(x \cos y - y \sin y)$ is harmonic. Also find a function v such that $f(z) = u + iv$ is analytic and express $f(z)$ in terms of z. **[10]**

2. Show that the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ satisfies the Cauchy Riemann equations at origin but not analytic at $z = 0$. **[10]**

3. In a two-dimensional fluid flow, if $\psi(x, y) = x^2 - y^2 - 3x - 2y + 2xy$ can represent the stream function, find the velocity potential and complex potential. **[5]**

4. a) Find the points for which the following function is not conformal $f(z) = z^2 + \frac{1}{z^2}$

   b) Find the image of the circle $|z| = 2$ under the transformation $w=(\sqrt{2}\, e^{\frac{i\pi}{4}})z$. **[5]**

5. Find the image of the wedge $|z| \leq \frac{1}{2}, \frac{-\pi}{8} < arg(z) < \frac{\pi}{8}$

   under the mappings (i) $w = z^2$ and (ii) $w = iz$. Sketch the region of images. **[10]**

6. Determine the bilinear transformation which maps the points $z = 0, -i, 2i$ into the points $w = 5i, \infty, \frac{-i}{3}$ respectively. List the invariant points of this transformation. Sketch the image of $|z - i| < 1$ under the obtained transformation. **[10]**

$$\Leftrightarrow \Leftrightarrow \Leftrightarrow$$

$w = iz$

$w = i(x + iy)$

$= xi - y$.

$\frac{w}{i} = z$

$|v - iu| \leq \frac{1}{2}$

$v^2 + u^2 \leq \frac{1}{2}, \quad ze^\delta - \int e^\delta$.