

Assignment 2 Solutions

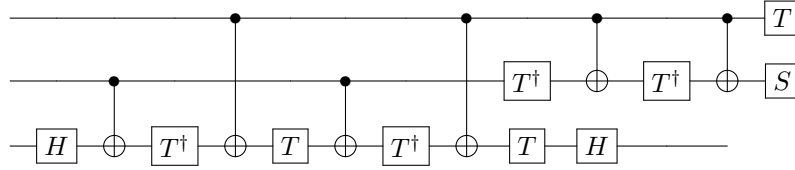
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June 2024

1 Quantum Gates

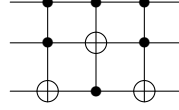
1.1 Question 1

The Toffoli gate can be implemented using Hadamard, CNOT, $\pi/8$ and phase gates in the following manner:



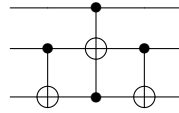
1.2 Question 2

The Fredkin gate can be implemented using three Toffoli gates in the following manner:



1.3 Question 3

The Fredkin gate can be implemented using least number of Toffoli and $\pi/8$ gates in the following manner:



If the first qubit is 0, then the Toffoli just performs the identity, hence the CNOTs cancel leading to an overall identity. If the first qubit is 1, then the Toffoli performs a CNOT on the last 2 qubits, which overall performs the SWAP operation.

Further, by taking $V = \frac{(1-i)(1+iX)}{2}$ which gives $V^2 = X$, we can replace the Toffoli with:

