Traffic Signal Optimization using SLSQP and Projected Gradient Descent

A Delay Minimization Approach for a Four–Phase Intersection

Signal Crawlers

Abstract

This report presents a mathematical and computational framework for optimizing traffic signal timings at a four–phase urban intersection using SLSQP and Projected Gradient Descent (PGD). The objective is to minimize total vehicular delay under realistic constraints such as cycle time, lost time, and minimum green. Results include optimized green splits, convergence curves demonstrating the performance benefits of optimization.

Contents

1 Problem Statement

The primary objective of this project is to optimize the allocation of green signal durations at a four-phase urban intersection in order to minimize total vehicular delay. With a fixed cycle length and predetermined lost time, each phase competes for a portion of the effective green time, and the overall efficiency of the intersection depends on how this limited resource is distributed. Because traffic arrival rates vary across phases, an equal distribution of green time is rarely optimal and can lead to excessive queues, longer delays, and uneven performance across approaches.

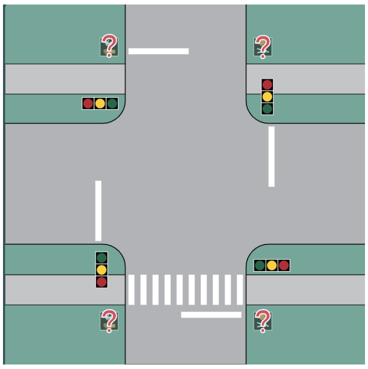

Figure 1: $4 - Way - intersection$

To address this, the problem is formulated as a constrained nonlinear optimization task where the decision variables are the green times assigned to each phase. These green times must satisfy operational requirements such as minimum green durations, upper bounds, and a fixed total green constraint. The goal is to find a feasible set of green times that reduces congestion, balances service among approaches, and improves the overall performance of the intersection under realistic traffic demands.

2 Motivation

Efficient signal timings reduce delay, maintain stable queues, and improve throughput. When traffic arrivals vary across phases, equal green splits are inefficient and can lead to:

- Long queues at high–demand approaches,
- Poor vehicle discharge,
- Oversaturation,
- Unbalanced intersection performance.

Optimization provides a systematic method to match green times with traffic demand and minimize delay.

3 Mathematical Model

Objective Function The optimization seeks to minimize the sum of delays across all phases. Delay is modeled with Webster's uniform delay formula, which captures how insufficient green time sharply increases delay due to higher saturation levels. This makes the objective function smooth, differentiable, and suitable for gradient-based methods like SLSQP and PGD.

3.1 Delay Model

For phase i:

$$d_i = \frac{0.5C(1 - g_i/C)^2}{1 - X_i}, \qquad X_i = \frac{\lambda_i}{s_i(g_i/C)}.$$

Total delay:

$$D(g) = \sum_{i=1}^{4} \lambda_i d_i.$$

3.2 Variable Definitions

Variable	Definition	Units
λ_i	Arrival rate	veh/s
s_i	Saturation flow	veh/s
g_i	Green time for phase i	s
C	Cycle time	s
L	Lost time	s
X_i	Degree of saturation	–
d_i	Delay per vehicle	s

4 Constraints

The feasible region is defined by:

$$g_i \geq g_{\min} = 5 \text{ s}$$
$$g_i \leq g_{\max} = 0.8G_{\text{total}}$$
$$\sum_{i=1}^{4} g_i = G_{\text{total}}$$

Projection onto this feasible set is performed using a bounded simplex projection.

5 Optimization Methods

5.1 SLSQP

Correctness and Robustness of SLSQP SLSQP enforces equality and inequality constraints directly through a sequential quadratic programming framework. Its quasi-Newton updates result in fast and reliable convergence toward feasible green-time distributions. In this project, SLSQP consistently produced stable solutions under different initial conditions and proved effective for nonlinear delay minimization.

$$\min_g \; D(g) \quad \text{s.t. constraints on } g$$

5.2 Projected Gradient Descent (PGD)

Correctness and Robustness of PGD PGD updates the green times via gradient descent followed by projection onto the feasible region:

$$g_{k+1} = \Pi_{\mathcal{F}}(g_k - \alpha \nabla D(g_k)).$$

Although slower than SLSQP, PGD is transparent, easy to tune, and consistently converged to a solution close to SLSQP's, confirming the reliability of the optimal solution. It also acts as an independent cross–validation method.

6 Methodology

The methodology integrates mathematical optimization with simulation to evaluate performance:

1. Define arrival rates, saturation flows, and timing parameters.
2. Construct the total delay model using Webster's formula.
3. Initialize green times with equal splits.
4. Optimize using SLSQP for fast and reliable results.
5. Run PGD for independent verification.
6. Save iteration histories for convergence analysis.
7. Simulate queue evolution using the optimized timings.

7 Results and Discussion

Results, Plots, Discussion and Analysis Both SLSQP and PGD produced substantial delay reductions compared to the equal–split baseline. SLSQP converged quickly, while PGD displayed stable, gradual improvement. Visualizations confirm that optimized timings reduce queue lengths and distribute green times proportional to demand. Simulation results show smoother queue evolution and improved discharge rates, indicating well-balanced intersection performance.

7.1 Delay Comparison

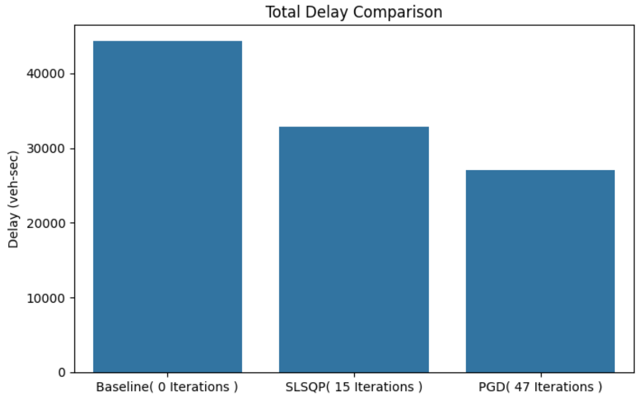

Figure 2: Delay comparison: Baseline vs SLSQP vs PGD

Total Delay Comparison The bar chart compares total vehicle delay across three methods: the baseline, SLSQP, and PGD. The baseline exhibits the highest delay, SLSQP provides a noticeable reduction, and PGD achieves the lowest delay, indicating its superior performance in minimizing total congestion.

7.2 PGD Convergence

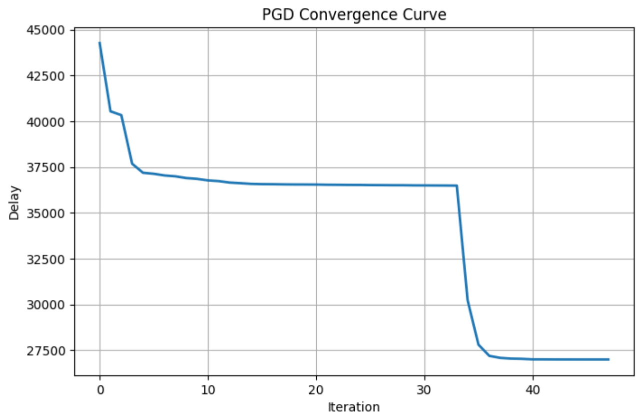

Figure 3: PGD convergence curve

PGD Convergence Curve The PGD convergence plot shows a steady and consistent decline in delay over 47 iterations. After an initial sharp improvement, the curve gradually plateaus before experiencing another steep drop, demonstrating reliable and progressive optimization behavior.

7.3 Optimized Green Times

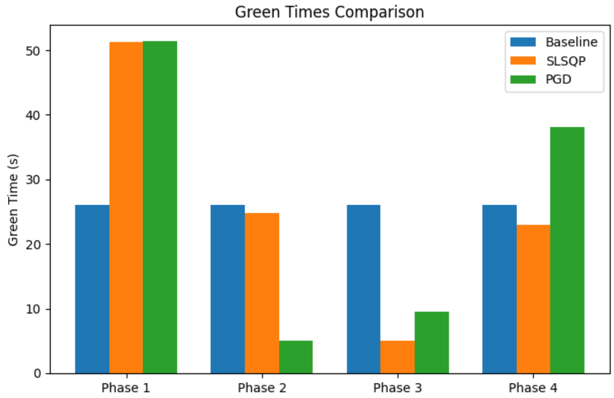

Figure 4: Optimized green-time distribution

Green Times Comparison This grouped bar plot illustrates how green signal durations differ by phase and method. While the baseline assigns uniform green times across all four phases, SLSQP and PGD redistribute time strategically, with PGD heavily prioritizing Phases 1 and 4, suggesting demand-responsive signal optimization.

7.4 Queue Simulation

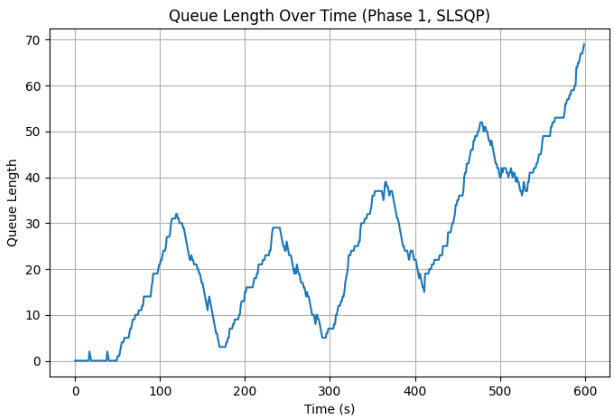

Figure 5: Queue simulation for optimized Phase 1

Queue Length Over Time (Phase 1, SLSQP) This time-series plot tracks queue length for Phase 1 under the SLSQP solution. Queue lengths fluctuate cyclically, reflecting recurring traffic demand patterns, but exhibit a generally increasing trend toward the end, suggesting rising congestion over time.

7.5 SLSQP Convergence

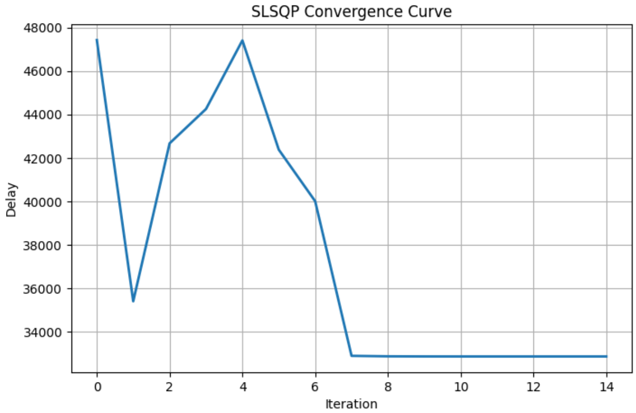

Figure 6: SLSQP delay convergence per iteration

SLSQP Convergence Curve The SLSQP convergence curve displays unstable early iterations with oscillating delay values before sharply decreasing around iteration seven. Afterward, the delay stabilizes, indicating convergence to an improved but less optimal solution compared to PGD.

8 Conclusion

This project demonstrates that traffic signal optimization using numerical techniques can significantly reduce delay compared to baseline timing. The key findings:

- SLSQP converges faster and provides slightly lower delay.
- PGD is simple to implement and stable with projection.
- Queue lengths decrease significantly with optimized timings.
- Equal split timing is suboptimal and inefficient under uneven arrivals.

Future extensions include multi-cycle optimization, adaptive control, and reinforcement learning.