

## ASSIGNMENT-1 (Due on Sep 7, 2018)

- COORDINATE SYSTEMS:** Proxima Centauri ( $\alpha$ -Centauri C) is the closest star to Sun and is part of a triple system. It has the epoch 1950 coordinates  $(\alpha, \delta) = (14^{\text{h}} 26.3^{\text{m}}, -62^{\text{d}} 29^{\text{m}})$  while the centre of the system is located at  $(\alpha, \delta) = (14^{\text{h}} 36.2^{\text{m}}, -60^{\text{d}} 36^{\text{m}})$ .
  - What is the angular separation of Proxima Centauri from the centre of the triple star system?
  - If the distance to the Proxima Centauri is  $4 \times 10^{18}$  cm, how far is the star from the centre of the triple system?
  - Precess the coordinates of Proxima Centauri to epoch 1990.
  - The propermotion of Proxima Centauri is  $3.84'' \text{ yr}^{-1}$  with a position angle of  $282^\circ$ . Calculate the change in  $\alpha$  and  $\delta$  due to propermotion between 1950 and 1990.
  - Which effect makes the largest contribution to changes in the coordinates of Proxima Centauri? Precession or Propermotion?
- MAGNITUDE TO FLUX CONVERSION:** Calculate the fluxes for three objects corresponding to the following magnitudes from SDSS (in AB system). you can use <https://www.sdss.org/dr12/algorithms/magnitudes/> for details.

Band	Wavelength (Å)	Object 1 m(dm)	Object 2 m(dm)	Object 3 m(dm)
u	3543	20.19(0.08)	19.58(0.04)	19.95(0.03)
g	4770	18.21(0.01)	18.66(0.01)	19.42(0.01)
r	6231	17.20(0.01)	18.15(0.01)	19.40(0.01)
i	7625	16.73(0.01)	17.83(0.01)	19.09(0.01)
z	9134	16.35(0.01)	17.72(0.01)	18.93(0.04)

Coordinate of the object 1:  $21^{\text{h}}59^{\text{m}}33.62^{\text{s}}+00^{\text{d}}54^{\text{m}}49.7^{\text{s}}$ ; 2:  $21^{\text{h}}57^{\text{m}}59.04^{\text{s}}+00^{\text{d}}57^{\text{m}}25.6^{\text{s}}$ ; and 3:  $21^{\text{h}}57^{\text{m}}35.60^{\text{s}}+00^{\text{d}}58^{\text{m}}16.6^{\text{s}}$ . Our plot your fluxes on the spectra of these objects you can download from <http://skyserver.sdss.org/dr14/en/tools/explore/summary.aspx>. You can take necessary help for this part from Soumak. First object is an elliptical galaxy, second one is a star forming galaxy and the third one is a high-z quasar.

- TRIGONOMETRIC PARALLAX:** Let us consider the stellar mass of our Galaxy (assumed to be a disk of radius 10 kpc and height 100 pc and made up of stars like Sun) is  $10^{10} M_{\odot}$ . If  $0.005''$  is our limit for measuring the trigonometric parallax then for how many stars you will be able to measure the distance from us using trigonometric parallax method? (Hint: stellar mass: total mass of stars in a galaxy).
- MAGNITUDES:** The faintest galaxies observed by the Hubble Space Telescope have apparent magnitudes around 30. Suppose these galaxies are  $\approx 3$  giga-parsecs away ( $10^9$  parsecs). Assuming every star in these galaxies emits about the same amount of light as the Sun, how many stars would these galaxies contain? (Hint: Absolute magnitude of Sun is 5).
- KEPLER PROBLEM:** Consider a Sun-like star, which is orbited by a planet with a period of 80 years. If the separation of the star and the planet appears to be 20 arc seconds, how far away is the star from us?

6. **HYDROSTATIC EQUILIBRIUM:** Is hydrostatic equilibrium possible for constant density isothermal gas? Suppose the density profile is as given below then how the pressure as a function of  $r$  should behave so that one can have hydrostatic equilibrium.

$$\rho(r) = \frac{\rho(r_c)}{1 + (r/r_c)^2}$$

7. **JEANS MASS:** Calculate the Jeans mass and Jeans length of a spherical cloud of Hydrogen gas having density  $1 \text{ cm}^{-3}$  and temperature  $10^4 \text{ K}$ . If one completely ionise this cloud without changing the temperature what will happen to the cloud?
8. **TWO BODY RELAXATION:** Estimate the two-body relaxation time for stars in (i) our Galaxy ( $R = 15 \text{ kpc}$ ,  $v = 200 \text{ km/s}$ ,  $N = 10^{11}$ ) assumed to be spherical in shape and in a Globular cluster ( $R = 5 \text{ pc}$ ,  $v = 30 \text{ km/s}$ ,  $N=10^6$ ). Which of these systems can be considered collisionless and why? (Remember  $R$  = radius,  $v$  = rms velocity of the stars and  $N$  total number of stars).
9. **SCHWARZSCHILD RADIUS:** Estimate the gravitational radius of the Sun. To what density should the solar matter be squeezed before it becomes a black hole? If the mass of the object is instead  $10^8$  times the mass of the Sun, to what density does it have to be squeezed before it becomes a black hole?
10. **GRAVITATIONAL LENSING:** Show that if the distance  $D_s$  to the source is fixed, then the area  $\pi D_L^2 \theta_E^2$  inside the Einstein radius is largest when the lens is midway between the source and the observer:  $D_s = 2 D_L$ . If the objects that might act as lenses are uniformly spread in space, a source is most likely to fall within the Einstein radius of a lens that is about halfway between it and the observer.
11. **GRAVITATIONAL MICROLENSING:** If the lens is a  $1 M_\odot$  object at distance  $D_L$  from us, find the Einstein radius of a star at a distance  $D_s = 2 D_L$ . As such Einstein rings are very small in size, gravitational lensing by compact objects in the halo of a galaxy is often referred to as gravitational microlensing.
12. **GRAVITATIONAL AMPLIFICATION:** Let the source be stationary, while the lens moves with proper motion  $\mu$  milliarcsec per year; at time  $t = 0$  they are closest on the sky, separated by an angle  $\beta_0 \ll \theta_E$ . Show that at small separation, the sum of the images is brighter than the source by roughly a factor

$$\frac{F(\text{image})}{F(\text{sources})} \sim \frac{\theta_E/\beta_0}{\sqrt{1 + (\mu t/\beta_0)^2}}$$

13. **HII Regions:** A star emits  $10^{48}$  photons per second. The star is embedded in an ISM (assume to be made only of Hydrogen atoms) of number density  $1 \text{ cm}^{-3}$ . Estimate the radius of the Strömrgren sphere using basic assumptions discussed in the class. It is well known that  $2/3$  of hydrogen recombinations will end up in  $\text{Ly } \alpha$  (i.e. line with a rest wavelength of  $1215 \text{ \AA}$ ). Estimate the  $\text{Ly } \alpha$  luminosity from this H II region ignoring any optical depth effects.
14. **Collisional ionisation of hydrogen:** A hydrogen gas of density  $10 \text{ cm}^{-3}$  is in collisional ionisation equilibrium at a temperature of  $10^3 \text{ K}$ . Estimate the fraction of neutral hydrogen in this gas.

15. **Understanding Redshifts:** Consider a QSO at  $z = 3$  whose intrinsic spectrum can be approximated with a powerlaw,

$$L_{\lambda} = 10^{43} \left( \frac{\lambda}{3000} \right)^{-1}$$

If our line of sight to this QSO is intercepted by a HI cloud having a column density of  $7 \times 10^{16} \text{ cm}^{-2}$  how the spectrum received in earth will look like. Consider only the continuum absorption. HINT: Hydrogen ionising cross-section as a function of frequency is

$$\sigma(\nu) = 6.8 \times 10^{-18} \left( \frac{\nu}{\nu_0} \right)^{-3}$$

with  $\nu_0$  being the rest frequency corresponding to 13.6 eV (or wavelength 912 Å).