Astrotech 1: Linear Polarization measurement of a lab source

Vishal Upendran, Mousumi Mahato, Shyam Sunder, Rajalakshmi Rajendran, Khaiminthan Vaiphei

February 16, 2019

1 Objective

To measure the linear polarization of an LED source. The experiment entails to find the two parameters which define linear polarization - p and θ, which are in turn obtained from the normalized Stokes parameters q and u. Aperture photometry is to be performed, and the parameters are to be found over different runs.

2 Theory

The Stokes parameters I, Q, U and V measure various properties of the polarization ellipse. They are defined as :

1. $I = I_0 + I_{90} = I_{45} + I_{135}$, corresponds to the total intensity of the beam, with the subscripts representing the components of polarization.
2. $Q = I_0 - I_{90}$ is a measure of linear polarization at 0°and 90°. It is 1 for a ray polarized along 0°and -1 for a ray polarized along 90°.
3. $Q = I_{45} - I_{135}$ is a measure of linear polarization at 45°and 135°. It is 1 for a ray polarized along 45°and -1 for a ray polarized along 135°.
4. The Stokes V parameter is a measure of circular polarization of the wave, and is not relevant to our work.

The normalized Stokes parameters and the polarization parameters are:

1. Normalized Stokes $q = \frac{Q}{I}$.
2. Normalized Stokes $u = \frac{U}{I}$.
3. Degree of polarization $p = \sqrt{u^2 + q^2}$.
4. Angle of polarization $\theta = 0.5 \tan^{-1} \frac{q}{u}$.

3 Experimental Setup

3.0.1 Materials required

1. An LED source with a 100 micron pin hole.

2. Collimator lens, f=15cm.

3. One Polarizer.

4. A Half wave plate with markings.

5. A Walloston Prism with a mount.

6. CCD setup with an appropriate software for capturing and storing the data.

7. An optical bench to perform the experiment.

8. Kimwipes to clean the glasses, if needed.

9. A torch to perform the HWP rotation in the dark.

The setup schematic is shown in Fig. 1. The setup essentially contains an LED source+pinhole to genenerate a point source. This is passed through a Collimator lens to obtain a plane parallel wavefront. A Polarizer is used to obtain a polarized wave. A Half-Wave Plate (HWP) is then used for rotating the light wave. The primary component which separates out the two polarizations is the Wollaston Prism (WP). And finally, a CCD is used to capture the images of the two polarizations.

3.1 Wollaston Prism

A Wollaston prism [1] is an optical device made of two orthogonal prisms of a Bi-refringent material. These prisms are cemented at their base, and the the setup has different refractive indices for different polarizations. It is thus used to separate out the two components of polarization in this experiment.

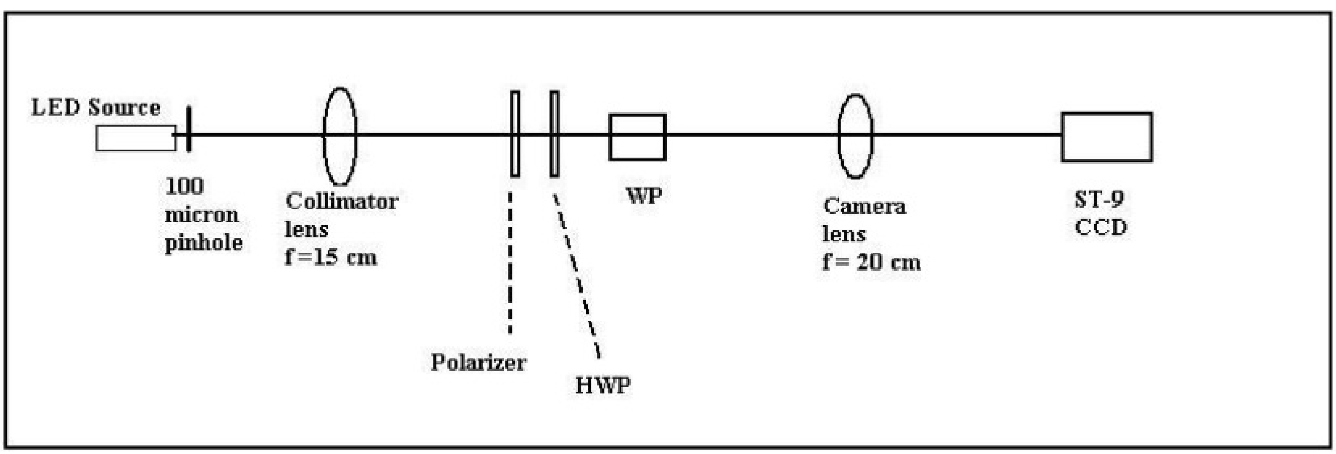


Figure 1: Schematic of the experimental setup

3.2 Half Wave Plate

Measurement of light along 0°, 45°, 90°and 135°are required. In such a case, it is not feasible to move either the light source or the Polarizer. Hence, a HWP is used.
A HWP rotates a light wave by 2ϕ if it is rotated by ϕ. Thus, by rotating the HWP by 22.5°, we can obtain I_{45} and I_{135}. Similarly, by rotating the HWP by 45°, we obtain I_0 and I_{90}, but their locations are swapped in the image. Another rotation to 67.5°gives us I_{45} and I_{135}, but with their locations swapped.

Why are so many measurements required in the first place? There might be systematic errors due to the instrument itself, which must be compensated for by a correction factor.
The nomenclature of the final beams obtained are I_o for the beam at 0°on the CCD, and I_e for the beam at 90°on the CCD. Thus, the correction factor is defined as:

$$K = \left(\frac{I_o\,(0°) \times I_o\,(22.5°) \times I_o\,(45°) \times I_o\,(67.5°)}{I_e\,(0°) \times I_e\,(22.5°) \times I_e\,(45°) \times I_e\,(67.5°)} \right)^{0.25} \tag{1}$$

The correction is then applied as:

$$K = \frac{I_o}{I_e} \tag{2}$$

This multiplication is done for all I_e.

4 Experimental Procedure

1. Make sure all background lights are off, and switch on the LED source. Obtain a point source, and then a collimated beam of light.
2. Take exposure of the image for orientations of the HWP at 0°, 22.5°, 45°and 67.5°. This is to perform the K-correction.
3. Perform the experiment multiple times.

Experiment was performed 5 times for the 4 orientations of HWP. Each image was an exposure of 0.12 sec, and a delay of 5 sec was given to take the image. The dark current was subtracted automatically by the CCD, and the final images were given as FIT files.

5 Precautions

1. The optical setup is not mounted rigidly, and hence there must not be any perturbations to the optical bench.
2. The optics must be handled only by a minimum area of contact.
3. Any and all stray light must be eliminated, even from a laptop for it will interfere with the CCD images.
4. While analyzing the data, care must be taken to subtract the sky, and then proceed to calculations.

6 Data Analysis

6.1 Procedure

Astropy [2]+Photutils [3] was used to perform photometry.

1. The general procedure is to find out the two beams, subtract sky from them, and then calculate the Stokes parameters, polarization fraction and angle of polarization.

2. To find out the two beams, we need to find the centers and the Full Width at Half Maximum (FWHM) of the beams. The centers are found at the maximum intensity in the two halves of the images. The FWHM, however is found at the point along a section of the maximum of the image, where the intensity value is closest to the half maximum. A nearest neighbour algorithm using L2 norm was used for this.

3. Our 'image' was defined within a circle of radius 3FWHM (which corresponds to $\approx 6\sigma$), and the annulus for taking sky value taken from 4FWHM to 6FWHM.

4. Photutils was then used to obtain the corrected intensity value inside our image by correcting for the background.

5. The CCD has a gain of 2.8 electrons per ADU. Thus, this was multiplied to obtain the correct electron counts.

6. K-correction was done by using Eq. 1 to I_e.

7. q, p, u and θ values were found using their definitions, and are reported in Sec. 6.2.

8. Poisson noise alone was assumed, and dark current was neglected. Error propagation was done using *Tensorflow* [4], which perform Automatic Differentiation, thus giving us our Jacobians and presenting an easy way to cacluate the errors.

6.2 Results

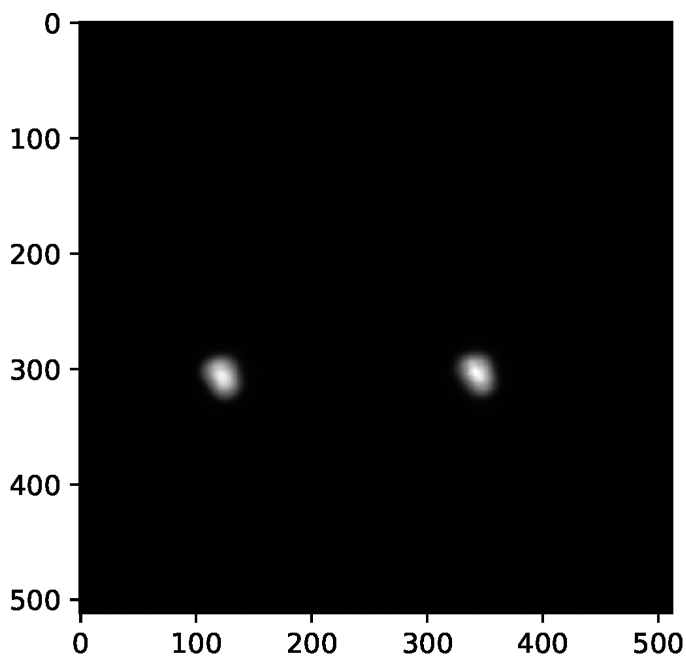
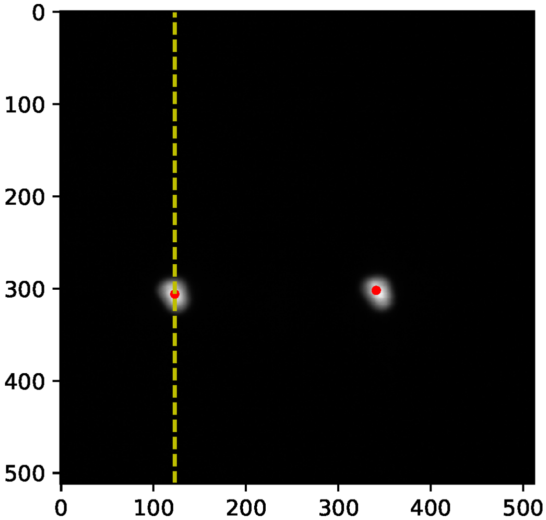
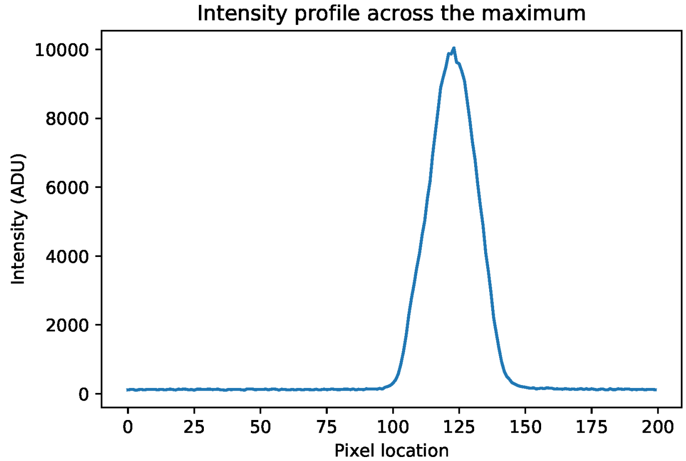


Figure 2: A representative image of the two polarized beams. The left side (to the viewer) is the I_o beam and the right side is the I_e beam. This is run 1, corresponding to a HWP orientation of 0°.

A sample file from one of the runs is shown in Fig. 2. The maximum from the image, and the corresponding intensity profile are shown in Fig. 3. The annulus made, and the bounding circle for the beam are shown in Fig. 4.



(a) Depicting the maximum finding algorithm. The centers of the two beams have been marked with a red dot.



(b) Intensity variation along the yellow line marked in the picture aside.

Figure 3: The maximum finding algorithm along with the intensity profile. The FWHM is taken from this figure, and used for generating our discs and annuli.

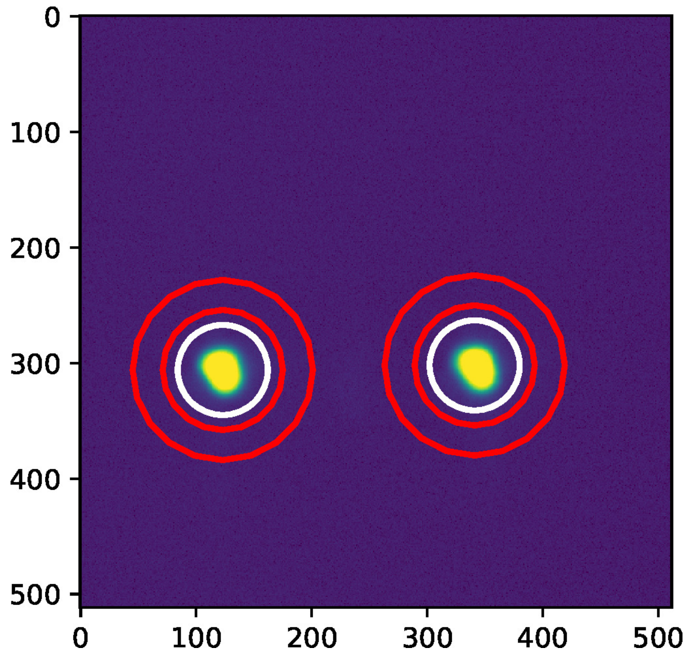


Figure 4: Annulus (red) and the circular disc corresponding to 3xFWHM (white).

The raw quantities are shown next - i.e, the computed intensities, with background subtracted are tabulated first. Please note that from here on we shall be summarizing all the data available.
The values for I_o are given in Table. 1, and for the corrected I_e are given in Table. 2. The parameters q, u, p and θ are computed only for 0° and 22.5° positions of the HWP, since the correction from remaining orientations have been taken into account. The final obtained normalized Stokes parameters are shown in Table. 3.

Run number	$I_o(0°)$	$I_o(22.5°)$	$I_o(45°)$	$I_o(67.5°)$
1	1.576e+7±3.970e+3	1.666e+7±4.082e+3	1.681e+7±4.1e+3	1.726e+7±4.155e+3
2	1.653e+7±4.065e+3	1.733e+7±4.162e+3	1.748e+7±4.181e+3	1.668e+7±4.084e+3
3	1.678e+7±4.096e+3	1.654e+7±4.067e+3	1.634e+7±4.042e+3	1.550e+7±3.937e+3
4	1.563e+7±3.954e+3	1.534e+7±3.916e+3	1.542e+7±3.927e+3	1.6e+7±4.e+3
5	1.571e+7±3.964e+3	1.563e+7±3.953e+3	1.614e+7±4.018e+3	1.594e+7±3.992e+3

Table 1: Tabulation of all the computed I_o values

Run number	K	$I_e(0°)$	$I_e(22.5°)$	$I_e(45°)$	$I_e(67.5°)$
1	1.0078	1.58e+7±3.975e+3	1.654e+7±4.066e+3	1.683e+7±4.103e+3	1.733e+7±4.163e+3
2	1.0186	1.673e+7±4.090e+3	1.715e+7±4.141e+3	1.741e+7±4.172e+3	1.672e+7±4.089e+3
3	1.0129	1.696e+7±4.118e+3	1.649e+7±4.061e+3	1.625e+7±4.032e+3	1.546e+7±3.932e+3
4	1.0148	1.556e+7±3.945e+3	1.525e+7±3.905e+3	1.558e+7±3.947e+3	1.599e+7±3.999e+3
5	1.014	1.582e+7±3.978e+3	1.565e+7±3.956e+3	1.597e+7±3.996e+3	1.597e+7±3.996e+3

Table 2: Tabulation of all the computed I_e values

Run number	q	u	p	θ
1	1.1388e-3±1.7800e-4	-3.8342e-3±1.7356e-4	3.9997e-3±1.7392e-4	-6.4104e-1±2.2207e-2
2	6.044e-3±1.7340e-4	-5.0584e-3±1.7031e-4	7.8814e-3±1.7213e-4	-3.4843e-1±1.0886e-2
3	5.2972e-3±1.7217e-4	-1.5050e-3±1.7400e-4	5.5069e-3±1.7231e-4	-1.3841e-1±1.5786e-2
4	-2.2314e-3±1.7905e-4	-2.8223e-3±1.8081e-4	3.5978e-3±1.8014e-4	4.5090e-1±2.4978e-2
5	3.6397e-3±1.7807e-4	6.4757e-4±1.7880e-4	3.6968e-3±1.781e-4	8.8038e-2±2.4180e-2

Table 3: Tabulation of all the parameters values

We do not produce the expression for errors propagated from the counts to the Stokes parameters, since it is taken care of by *Tensorflow*, a package for Deep learning, which used Automatic differentiation (or Chain rule) to compute the gradients.

7 Conclusion

We have thus measured the linear polarization of the source using a HWP and a WP. However, we can make some observations:

1. The intensity fluctuation over the runs is much larger than the Poisson noise, so it can be concluded that the total light permitted by the setup while rotating the HWP somehow changed with each run. While we were carful while performing the experiment, there could have been certain off-axis motion of the setup which might result in this error. However, we are not very sure about the exact source of error.

2. Due to the intensity fluctuations, there are fluctuations in the computed Stokes parameters too. The variation in these parameters over different runs is approximately 4 times larger than the average σ of the measurement.

3. For all the runs, it can be seen that the p value is very small = the incoming beam is almost unpolarized, and the angle of polarization also has a slight variation.

4. The values have all been computed after taking the CCD gain into account. Dark current has not been included here.

Thus, a linear polarization measurement for the given LED source was performed.

References

[1] Wikipedia contributors. Wollaston prism — Wikipedia, the free encyclopedia, 2018. [Online; accessed 16-February-2019].

[2] A. M. Price-Whelan, B. M. Sipőcz, H. M. Günther, P. L. Lim, S. M. Crawford, S. Conseil, D. L. Shupe, M. W. Craig, N. Dencheva, A. Ginsburg, J. T. VanderPlas, L. D. Bradley, D. Pérez-Suárez, M. de Val-Borro, (Primary Paper Contributors, T. L. Aldcroft, K. L. Cruz, T. P. Robitaille, E. J. Tollerud, (Astropy Coordination Committee, C. Ardelean, T. Babej, Y. P. Bach, M. Bachetti, A. V. Bakanov, S. P. Bamford, G. Barentsen, P. Barmby, A. Baumbach, K. L. Berry, F. Biscani, M. Boquien, K. A. Bostroem, L. G. Bouma, G. B. Brammer, E. M. Bray, H. Breytenbach, H. Buddelmeijer, D. J. Burke, G. Calderone, J. L. Cano Rodríguez, M. Cara, J. V. M. Cardoso, S. Cheedella, Y. Copin, L. Corrales, D. Crichton, D. D'Avella, C. Deil, É. Depagne, J. P. Dietrich, A. Donath, M. Droettboom, N. Earl, T. Erben, S. Fabbro, L. A. Ferreira, T. Finethy, R. T. Fox, L. H. Garrison, S. L. J. Gibbons, D. A. Goldstein, R. Gommers, J. P. Greco, P. Greenfield, A. M. Groener, F. Grollier, A. Hagen, P. Hirst, D. Homeier, A. J. Horton, G. Hosseinzadeh, L. Hu, J. S. Hunkeler, Ž. Ivezić, A. Jain, T. Jenness, G. Kanarek, S. Kendrew, N. S. Kern, W. E. Kerzendorf, A. Khvalko, J. King, D. Kirkby, A. M. Kulkarni, A. Kumar, A. Lee, D. Lenz, S. P. Littlefair, Z. Ma, D. M. Macleod, M. Mastropietro, C. McCully, S. Montagnac, B. M. Morris, M. Mueller, S. J. Mumford, D. Muna, N. A. Murphy, S. Nelson, G. H. Nguyen, J. P. Ninan, M. Nöthe, S. Ogaz, S. Oh, J. K. Parejko, N. Parley, S. Pascual, R. Patil, A. A. Patil, A. L. Plunkett, J. X. Prochaska, T. Rastogi, V. Reddy Janga, J. Sabater, P. Sakurikar, M. Seifert, L. E. Sherbert, H. Sherwood-Taylor, A. Y. Shih, J. Sick, M. T. Silbiger, S. Singanamalla, L. P. Singer, P. H. Sladen, K. A. Sooley, S. Sornarajah, O. Streicher, P. Teuben, S. W. Thomas, G. R. Tremblay, J. E. H. Turner, V. Terrón, M. H. van Kerkwijk, A. de la Vega, L. L. Watkins, B. A. Weaver, J. B. Whitmore, J. Woillez, V. Zabalza, and (Astropy Contributors. The Astropy Project: Building an Open-science Project and Status of the v2.0 Core Package. , 156:123, September 2018.

[3] Larry Bradley, Brigitta Sipocz, Thomas Robitaille, Erik Tollerud, Z Vincius, Christoph Deil, Kyle Barbary, Hans Moritz Gnther, Mihai Cara, Ivo Busko, Simon Conseil, Michael Droettboom, Azalee Bostroem, E. M. Bray, Lars Andersen Bratholm, Tom Wilson, Matt Craig, Geert Barentsen, Sergio Pascual, Axel Donath, Johnny Greco, Gabriel Perren, P. L. Lim, and Wolfgang Kerzendorf. astropy/photutils: v0.6, January 2019.

[4] Martín Abadi, Ashish Agarwal, Paul Barham, Eugene Brevdo, Zhifeng Chen, Craig Citro, Greg S. Corrado, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Ian Goodfellow, Andrew Harp, Geoffrey Irving, Michael Isard, Yangqing Jia, Rafal Jozefowicz, Lukasz Kaiser, Manjunath Kudlur, Josh Levenberg, Dandelion Mané, Rajat Monga, Sherry Moore, Derek Murray, Chris Olah, Mike Schuster, Jonathon Shlens, Benoit Steiner, Ilya Sutskever, Kunal Talwar, Paul Tucker, Vincent Vanhoucke, Vijay Vasudevan, Fernanda Viégas, Oriol Vinyals, Pete Warden, Martin Wattenberg, Martin Wicke, Yuan Yu, and Xiaoqiang Zheng. TensorFlow: Large-scale machine learning on heterogeneous systems, 2015. Software available from tensorflow.org.