Modelling of biological muscles for Myorobotics emulation

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***Abstract*— Many attempts have been done previously to both emulate muscles in hardware, and to mathematically model them. Such emulations have always been done considering the physical structure of the muscle, not based on the functional response. This work aims to mathematically model he muscle: namely the Golgi Tendon Organ, Spindle and Muscle motor unit, for implementation onto an artificial muscle.**

I. INTRODUCTION

Biomorphic engineering is the field of emulating biology in existing engineering technologies. One major subsection of Biomorphic engineering is Neuromorphic engineering, and Myorobotics. Myorobotics is the field of emulating muscles as artificial muscles, and Neuromorphic engineering aims to mimic neuro-biological structures present in the nervous system as an artificial brain. An artificial muscle has the potential to be actuated and controlled by an artificial brain, but there is a lack of modeling of the muscle which would correspond to a more realistic actuation and control, not to mention making it more biomorphic. Essentially, this work will act as a driver for the myorobotic system to be actuated by a neuromorphic engine.

II. MUSCLE STRUCTURE AND MODEL

The motor unit has primarily three important structures: The Muscle fiber, Golgi tendon organ, and the Spindle. These structures communicate with the brain, obtain motor commands and give feedback.

A. Muscle Fiber

Muscle fiber is a collection of long cylindrical muscle cells, which form the basic unit of muscle actuation.Muscle fibers come in two main types: slow twitch and fast twitch. Slow twitch fibers are characterized by low thresholds, smaller twitch response, and less fatigue. Fast twitch fibers are characterized by high thresholds, larger twitch response, and fatigue faster. A motor unit is a collection of muscle fibers, along with the actuating Axon (or Axons). Motor recruitment is the process of actuating these motor units.
Both the muscle fiber and motor unit can be approximated by the same model[1][3]. This model captures the essence of motor unit behavior, and is shown in Fig. 1. Here, the active force generator $A(t)$ bears a correlation to the tension in the muscle at rest length. In such a condition, for a single spike input, the transient tension developed is known as a "Twitch". The twitch goes to a maxima, and falls down to zero. Continuous stream of twitches results in summation of the force, resulting in a constant tension condition known as 'Tetanus'.

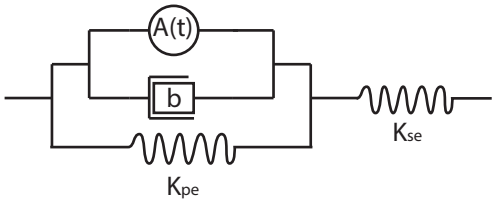

Fig. 1. Hill muscle model

B. Golgi Tendon Organ

The Golgi tendon organ is the Force sensor of a motor unit. It is present in series with the muscle and the tendon, hence it experiences the same force as the muscle. It converts the force input to Spike output, and communicates through the Ib afferent neurons. The mechanism of spike generation can be described as follows: The fibers in the organ are twisted due to the force acting on the organ, and this twist produces spikes in Ib neurons. There is a threshold force, which corresponds to the minimum twist required for causing any increase in spike rate, and also a maximum force above which no change in spike rate is observed. This condition corresponds to the maximum twist in the fibers.
The Golgi tendon organ has been found to be a linear and time invariant system[4]. The organ, for emulation purposes, is considered time invariant, meaning an given input corresponds to a particular output, regardless of when the input is given. The next section discusses the mathematical model and response characteristics in detail.

C. Muscle Spindle

The muscle spindle is the length and velocity sensor of the muscle. It is present in parallel to the muscle to measure the length and velocity changes in the muscle. It contains two afferent outputs- primary and secondary. The primary afferent encodes information on the velocity of the muscle, and the secondary afferent encodes information on the current length of the muscle.
The spindle is an active sensor—for a given $G(t)$, if the muscle follows the same trajectory, the primary afferent firing remains constant- though the secondary afferent is not affected by the $G(t)$. A model similar to McMahon's spindle model (Fig. 2)[1][3] is used, with a non-linear dependence of the damping on the spindle velocity, which models a more accurate behavior[6][7]. The primary afferent firing has been taken as proportional to the length of the series elastic

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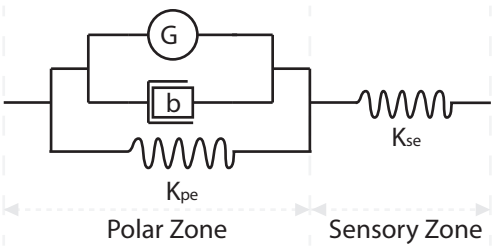

Fig. 2. McMahon Spindle model

element in the so-called sensory zone, and the secondary afferent firing is taken proportional to the length of the so-called parallel elastic element in the polar zone.

III. Software implementation

The above mentioned models were discretized and implemented in C++. The Muscle fiber and Spindle, being differential equations, were easily discretized into difference equations. The Golgi tendon organ, being a transfer function, was discretized using Tustin's bilinear approximation, which converts the transfer function from 's-domain' to 'z-domain'.

$$s = \frac{z-1}{z+1}\frac{1}{T}$$

As a general information, 's' domain corresponds to a space of continuous functions, and it represents the 'Laplace transform variable'. The 'z', on the other hand represents the space of discrete time functions, and it also represents the 'z transform variable'. T here corresponds to the sampling time of the function.

The implementation of individual modules of the muscle shall be discussed next. For the initial validation of the discretized model, the parameters were taken from text, and some test data taken from a Myorobotics system. The data fed to the three facets of the muscle were then compared with trend shown by biological muscles (obtained from text). The models showed a trend similar to the biological muscles.

A. Muscle fiber

Fig. 1 shows the state equation of the Hill model:

$$\begin{aligned}\dot{T}(t) = \frac{k_{se}k_{pe}}{b}(x(t)-x_L) + k_{se}\dot{x}(t) + \\ \frac{k_{se}}{b}A(t)\otimes S(t) - \frac{k_{se}+k_{pe}}{b}T(t)\end{aligned} \tag{1}$$

where:
$T(t)$ is the Tension in the muscle
k_{se} is the series elasticity
k_{pe} is the parallel elasticity
b is the damping of the muscle
$x(t)$ is the length of the muscle
x_L is the rest length of the muscle
$A(t)$ is the active force generator, or Impulse response,
$S(t)$ is the Spike train input to the system, and $\otimes$ represents the convolution operator.

The convolution was done in a windowed fashion. Depending on the fall of the twitch response, the window-size was determined, and the spikes were pushed through a queue for convolution.The discretized tension equation implemented was:

$$\begin{aligned}T(x_i, t+\Delta t) = [\frac{k_{se}k_{pe}}{b}(x_i - x_L) + k_{se}\dot{x}|_{x=x_i}]\Delta t + \\ \frac{k_{se}}{b}A(t)\otimes I(t)\Delta t + [1 - \frac{k_{se}+k_{pe}}{b}\Delta t]T(x_i, t)\end{aligned} \tag{2}$$

The activation $A(t)$ used is of the form

$$A(t) = A_0(e^{-t/\tau_1} - e^{-t/\tau_2})$$

This form would model the tetanus twitch, showing the essential properties of a steep rise in the tension, and fall to zero value. Hence, this formulation of twitch has been used. Summarizing, the algorithm for finding the Muscle tension goes as:

- Initialize Spike train $I(t)$ as a queue with zeros.
- Push spike onto spike train.
- $A(t)$ = Decided by the user, based on Twitch fiber requirements.
- Response at time $(= t)$: $\Sigma I(t) \times A(t_{now} - windowsize)$
- With this, we can plug in the convolution term, and given length, Tension can be computed.

B. Golgi Tendon Organ

As mentioned previously, the Golgi tendon organ (GTO) gives a force sensory feedback to the nervous system. It has been shown to be a linear and time invariant system in most cases[4], and has been assumed to be in this work too. It has been found that the transfer function relating Force input to Spike rate output is best modeled as[4]:

$$H(s) = K[1 + \frac{As}{s+a} + \frac{Bs}{s+b}] \tag{3}$$

where: K is the gain of the system
a, b are respectively the larger and smaller time constants
A, B are the corresponding amplitudes of the exponential components.
s is the Laplace variable.

This is a continuous transfer function which cannot be implemented on any controller. Hence, it was discretized using Tustin's approximation to obtain an expression of the form:

$$\begin{aligned}d_0 r(t_i) = K[l_0 F(t_i) + l_1 F(t_{i-1}) + l_2 F(t_{i-2})] - \\ d_1 r(t_{i-1}) - d_2 r(t_{i-2})\end{aligned} \tag{4}$$

where the coefficients relate to the original transfer function as:
$l_0 = 4(1 + A + B) + 2T(a + b + aB + Ab) + abT^2$
$l_1 = 2(abT^2 - 4(1 + A + B))$
$l_2 = 4(1 + A + B) - 2T(a + b + Ab + aB) + abT^2$
$d_0 = 4 + T(2a + 2b + abT)$
$d_1 = 2(abT^2 - 4)$
$d_2 = T(abT - 2a - 2b) + 4$, and T is the sampling time of the approximation.

The threshold and saturation have been implemented based on the physical action in a GTO. In a GTO, the nerves are twisted by the input force, and the spike rate is proportional to this twist. During actuation, a minimum force is required to produce an appreciable twist, and hence a change from baseline firing, whereas there is a maximum twist which can be given to the nerves, above which no change in spike rate is seen.Hence, the saturation and threshold have been set similarly.

C. Muscle Spindle

Spindle is the most complicated of all the three sub modules of the muscle. It contains 2 inputs, and 2 outputs.As mentioned previously, the Primary afferent firing rate has been modeled to be proportional to the length of series elastic element, and the Secondary afferent has been modeled to be proportional to the length of parallel elastic element. The equations corresponding to the muscle spindle are as follows:

$$\dot{T}(t) = \frac{k_{se}k_{pe}}{b}(x(t) - x_L) + k_{se}\dot{x}(t) + \frac{k_{se}}{b}G(t) - \frac{k_{se}+k_{pe}}{b}T(t) \tag{5}$$

, where:
T is the Tension in the spindle
k_{se} is the series elasticity
k_{pe} is the parallel elasticity
b is the damping of the spindle
x is the length of the spindle
x_L is the rest length of the spindle
$G(t)$ specifies the gamma activation, and :

$$b = \beta_0 + \beta_1 \dot{x}^a$$

where β_1 and β_0 are some constants, and a is generally taken to be 0.6 for the best fit[6]. The afferent firing rates are given by:

$$S_{Ia} = \frac{\alpha_1 T}{k_{se}}, S_{II} = \alpha_2(x - \frac{T}{k_{se}})$$

where $\alpha_i$are scaling factors. These equations were discretized as before, and the resulting equations for the afferent firing rates were obtained as:

$$S_{Ia}(t+\Delta t) = [\frac{\alpha_1 k_{pe}}{b}(x - x_L) + \alpha_1 \dot{x}|_{x=x_i}]\Delta t + \frac{\alpha_1}{b}G(t)\Delta t + [1 - \frac{k_{se}+k_{pe}}{b}\Delta t]S_{Ia}(t) \tag{6}$$

$$S_{II}(t+\Delta t) = [\frac{\alpha_2 k_{pe}}{b}(x - x_L)]\Delta t - \frac{\alpha_2}{b}G(t)\Delta t + [1 - \frac{k_{se}+k_{pe}}{b}\Delta t]S_{II}(t) \tag{7}$$

Implementation given these equations is straightforward, since our functions are very simple difference equations. The $G(t)$ is a reference signal given by the CNS, which encodes the path to be taken by the muscle with no change in muscle tension. It can be found to be:

$$G(t) = G(0) - b\dot{x}_{desired}(t) - k_{pe}(x_{desired}(t) - x(0))$$

If the muscle follows the exact trajectory as that desired, S_{Ia} don't change their firing rate. Discrepancies cause a change in the primary afferent firing rate. The secondary don't show as much change, though. The spindle becomes an active sensor due to the action of $G(t)$, or the gamma co-activation. So to summarize the spindle can be used in the following ways:

- Use as passive sensor: Have $G(t) = 0$, and use the spike rate as a measure of muscle length and velocity. or,
- Use as an active sensor: Define $G(t)$ as shown previously, and the primary afferent gives error in trajectory, along with the absolute length from secondary.

IV. Results

A. Muscle Fiber

The muscle fiber behavior has been show in Fig. 3 through 6. Fig. 3 shows the twitch behaviour of a biological muscle, and Fig. 4 shows the twitch behaviour of our model. Fig. 5 shows the tetanus of a biological muscle, and Fig. 6 shows the tetanus of our model.

The model captures the essence of the muscle fiber, which

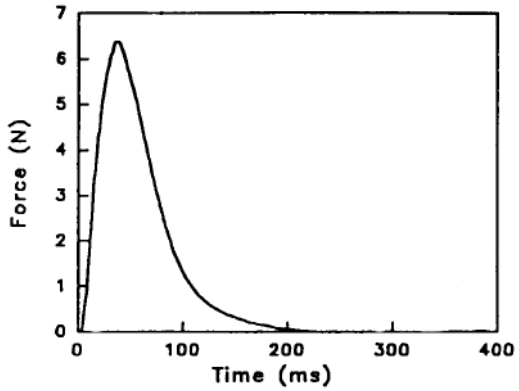

Fig. 3. The Twitch response of a biological muscle, adapted from [2], edited.

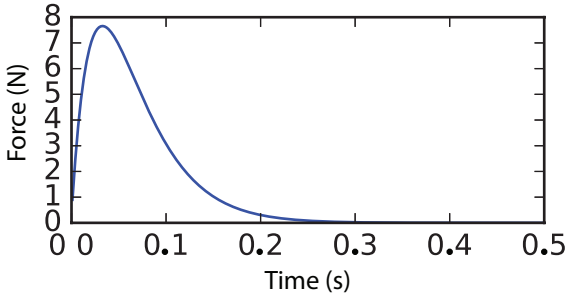

Fig. 4. The Twitch response of our model

is the most important takeaway from these results.

B. Golgi Tendon Organ

The GTO model was derived from the response of the biological tissues, and not based on the physical structure: though, we did enforce the physical structure by threshold

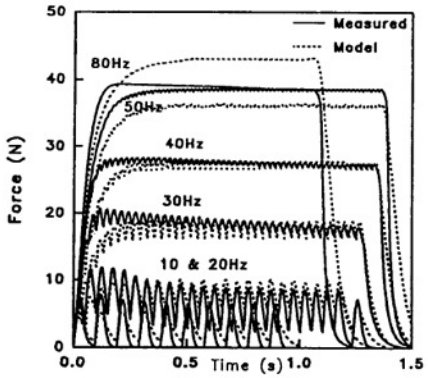

Fig. 5. Tetanus of a biological muscle [2]

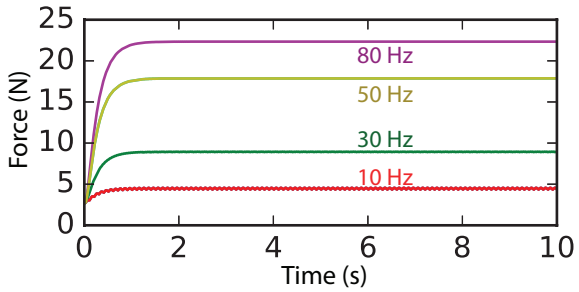

Fig. 6. Tetanus of our model

and saturation. Fig. 7 shows the behavior of the biological GTO to a ramp input: fast ramp and slow ramp, and Fig. 8 shows the response of our model to a similar input.

It can be seen that the model and the biological system

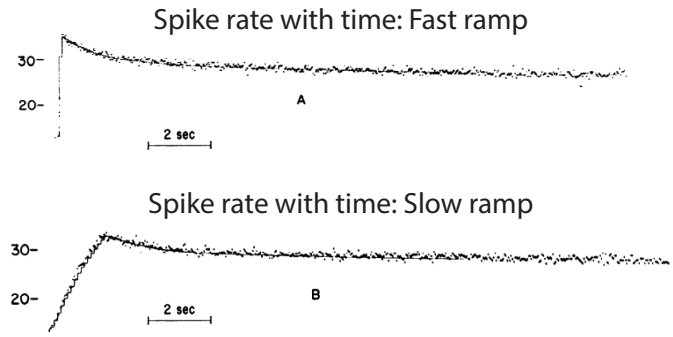

Fig. 7. The response of biological GTO[4]

correlate very well. Here again, the trend of Spike rate is requested to be observed, rather than the actual values, which differ according to the system parameters.

For the GTO, we had some force data from a myorobotics system available, hence we experimented what would be results of feeding this force into our model. The result is shown in Fig. 8. The threshold force was set to be 10N, and the saturation was set to 200N. The response of the model is shown in Fig. 9. The simple model captures the essence of the GTO, which is the takeaway from the simulation.

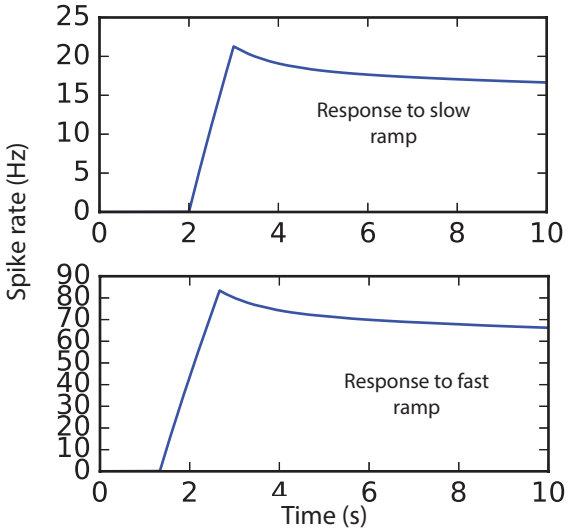

Fig. 8. The Golgi tendon organ response of our model

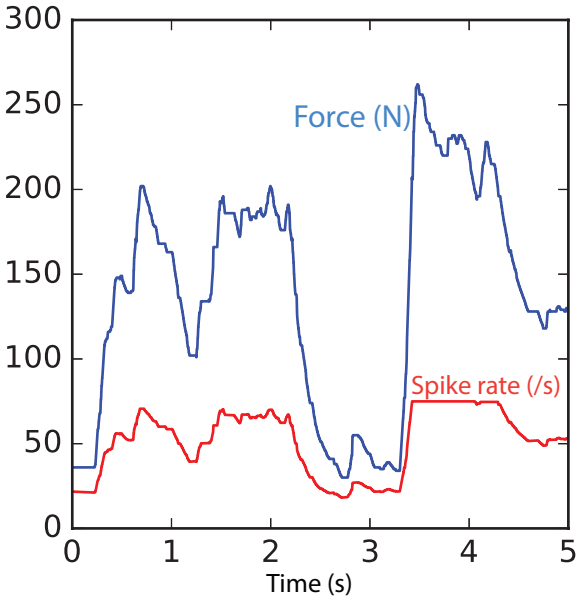

Fig. 9. GTO response from sensor data

C. Muscle Spindle

The properties of the muscle spindle discussed earlier are shown in Fig. 10. These plots correspond to the spindle without any γ activation, which is important to show the actual response of the afferents to given inputs. The model response is shown in Fig. 11

As can be seen from the figures, the model response is very close to the spindle response from the text. The defining features are the rapid increase, overshoot and fall of the Primary afferent, and the steady increase and steady state value for the Secondary afferent.

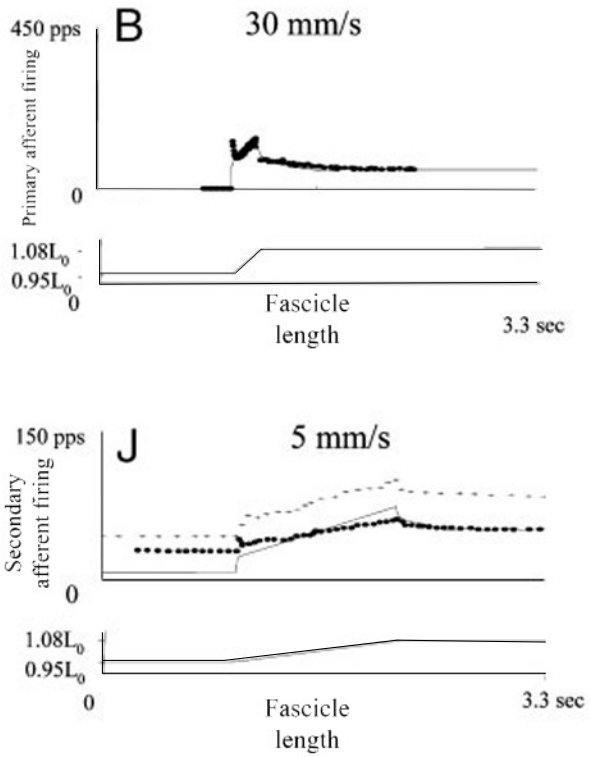

Fig. 10. Afferent response of a biological spindle [7]

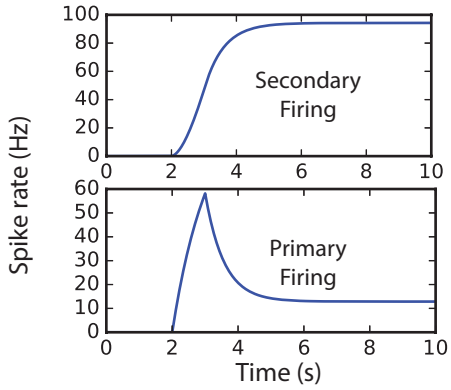

Fig. 11. Afferent response from our model

V. Parameter estimation protocol

From the previous section, it can be seen that our model captures the essence of the biological tissue functionally. To use the model, we will need to fix the parameters for the model. We propose three broad classes of fixing these parameters, and then we will proceed to explain which would be the best for each of the module:

- Control engineering method: The parameters are fixed by the user. The user will have to define basic properties of the system like peak overshoot, steady state value, rise time/fall time, etc. This method of determining parameters will be best where the transfer function of the tissue is known.
- Data from literature: We assume the artificial muscle needs to emulate a particular muscle of some animal: for example, Cat Soleus muscle. We take the corresponding parameters (if available) from the literature, and implement our model.
- Infer from Artificial muscle: The physical system on which we implement these models require to have some structural similarities with the biological muscle and tissues, i.e the experiments proposed in the next section should be feasible on them. In such a system, a combination of the control engineering method, data from the physical system, and inferences from the literature can be drawn to determine the independent parameters of our model.

As mentioned previously, the physical system will need to satisfy certain conditions before behaving like an artificial muscle. Some of the expected behavior of such a physical would be:

- The physical system emulating the muscle fiber should be decomposable into a hill type model; i.e it must have a series and parallel elastic component, a damping with the parallel elastic component, and an active force generator. Generally, any physical system with a spring at the actuator can be broken down into an elastic and damping component.
- The physical system emulating a muscle spindle should be an active sensor for the gamma activation to be present. The system can be taken as the muscle fiber itself, as the spindle is located in parallel to the muscle fibers in a biological muscle. This condition may not apply if there is an ensemble of spindles present, in which case the local tension in the spindle will be different from the fibers.
- The physical system emulating the GTO can be a very simple force sensor. There are no particular system requirements, other than the known range of operation, which will fix the threshold and saturation of the model.

A. Golgi Tendon Organ

GTO is by far the easiest to determine the parameters. Since it already has a transfer function, it is easier to use

the Control System approach to determine the parameters (refer Eq. 4). The procedure is as follows:

- Give a step input to the system. The output will be of the form $r(t) = kF_g[1 + Ae^{-at} + Be^{-bt}]$.
- Decide the Saturation and Threshold force. Threshold force will correspond to a baseline firing rate. Saturation will be used for determining the maximum firing rate.
- Find K at the steady state of system given a step input at threshold. $K = \frac{r_{th,ss}}{F_{th}}$, where $r_{th,ss}$ is the firing rate at steady state, and F_{th} is the threshold force.
- Find the maximum overshoot (at $t = 0$ sec) for saturation force to get one relation between A and B. Fix the other based on trend from text. One relation would be $r(o) = KF_s(1 + A + B)$
- Decide upon how fast or slow the sensor should come to steady state- i.e, have a trend from the text. Also, fix up on the fall rate at t=0 sec for the given response. Any step value can be taken. This would be used to solve for a,b. One of the expressions should be $\frac{dr}{dt}|_{t=0} = KF_0(-aA - bB)$
- Thus, all the parameters have been determined.

B. Muscle Fiber

The parameter estimation here will be more along inference from the system, as the system need to show some characteristics of muscle inherently, without any actuation coming into the picture. The equations being used here are from the Hill model we described in previous section. The procedure is as follows:

- Have a constant tension initially, and the control signal for tension constant. Have a sudden change in length. The increase in tension corresponds to Series elastic element. Hence, $k_{se} = \frac{\Delta T}{\Delta x}$
- Let the tension go to steady state. Compare initial and final steady state tension to obtain parallel elastic element. Hence, $\Delta T(1+\frac{k_{pe}}{k_{se}}) = k_{pe}k_{se}\Delta x$, from which k_{pe} can be obtained.
- The transient response on the stretch represents the damping of the system. From $b = \tau(k_{pe} + k_{se})$, the damping constant can be obtained.
- A(t): Give a single spike input to the system at constant length, preferably at the equilibrium length. The rise and fall in tension will be the twitch. The tension can be represented as difference of two exponentials- as discussed previously. The two time constants can be determined by curve fitting, or any other method. A(t) can be found out as $A(t) = (1 + \frac{k_{pe}}{k_{se}})T_{twitch} + \frac{b}{k_{se}}\dot{T}_{twitch}$.

Thus, all the parameters have been found.

C. Muscle Spindle

The structure of muscle spindle is similar to that of the muscle fiber. Hence, the same set of experiments can be performed for the muscle spindle. But in this case, we will need to account for $G(t)$ instead of A(t). We can either have $G(t)$ or neglect it. Neglecting it will result in our spindle being a passive system, whereas having it will result in our system being an active system. The procedure for both is same, but just a slight change will be present:

- Have a constant tension initially, and the signal for tension constant. Have a sudden change in length. The increase in tension corresponds to Series elastic element. Hence, $k_{se} = \frac{\Delta T}{\Delta x}$
- Let the tension go to steady state. Compare initial and final steady state tension to obtain parallel elastic element. Hence, $\Delta T(1+\frac{k_{pe}}{k_{se}}) = k_{pe}k_{se}\Delta x$, from which k_{pe} can be obtained.
- The transient response on the stretch represents the damping of the system. From $\beta_0 = \tau(k_{pe} + k_{se})$, the damping constant can be obtained.
- To find the velocity coefficient of the damping (β_1), we give the spindle a constant stretch $\dot{x} = c$. In this case, the expression would be $\dot{T} = \frac{k_{pe}k_{se}ct - T(k_{pe}+k_{se})}{\beta_0+\beta_1 c^a} + k_{se}c$. Hence, β_1 can be found.

For the active sensing, have the $G(t)$ as a constant (i.e, maintaining the spindle at constant length), and perform all the above experiments. The $G(t)$ itself for a desired trajectory x_d is given by:
$G(t) = G(0) - b\dot{x}_d - k_{pe}(x_d - x(0))$

VI. Actuation Protocol

With the parameters of our model determined, we will need to define some protocol to be used for the actuation of the physical system using our model. In this section, we assume the relations to be generic functions, which are specific to the physical system/sensor in consideration, and the relations defined by our model. If the sensor/actuator relates to the desired output as

$$\psi(t) = \hat{H}(\phi(t))$$

,
where $\psi(t)$ is the desired output, $\phi(t)$ is the output from the sensor/ input to controller, and $\hat{H}$ maps this input to output for the physical system. Meanwhile, our models determine relations between $\psi(t)$, the desired response, and spike train $S(t)$, which can be generalized as

$$\psi(t) = \hat{L}(S(t))$$

From the above two equations, it can be seen that a simple inversion of mapping can be done to couple the input and desired output from the physical system.

VII. Discussion

This work has taken into account the degree of complexity which can be incorporated into implementing the model onto a microcontroller or any other device, yet wraps up the major characteristics of a biological muscle. One major work which remains to be done is interaction of modules in a ensemble. In such a system, the output of a unit is not only a linear summation of individual outputs, but also

involves correlation of signals. This implementation will help in making the system much more realistic than it is now. Similarly, implementation of fatigue has not been done in this work, as it has not been understood very well, and its implementation is beyond the scope of this work.
A lot of work can be done in this field given our models. One instance would be to let the brain learn the model parameters based on its experiences: which can include power training, stamina training, etc. , and this can make sure one system can be used for multiple purposes. given correct training.

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