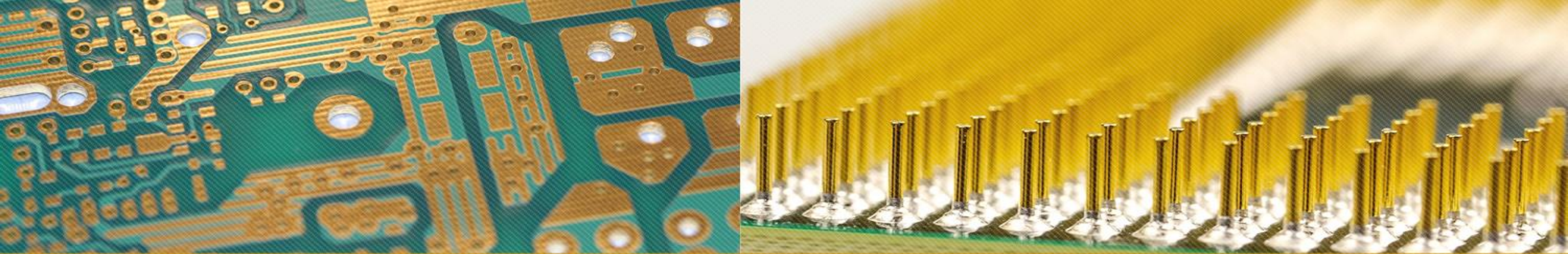
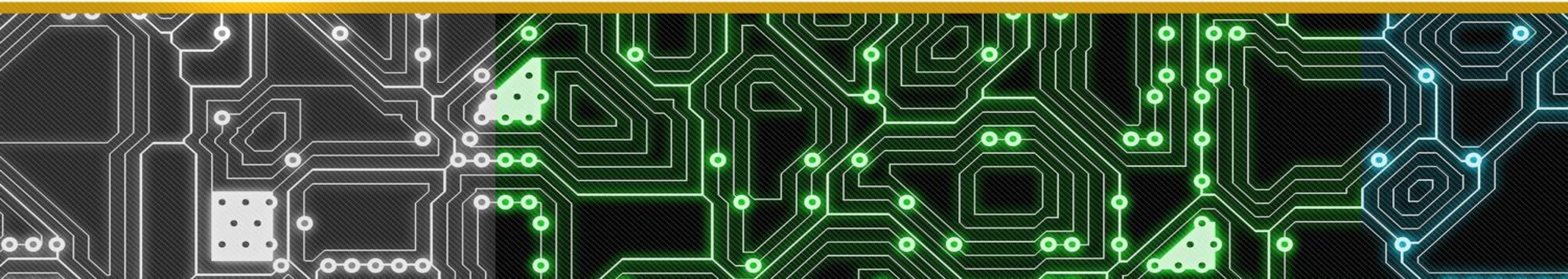
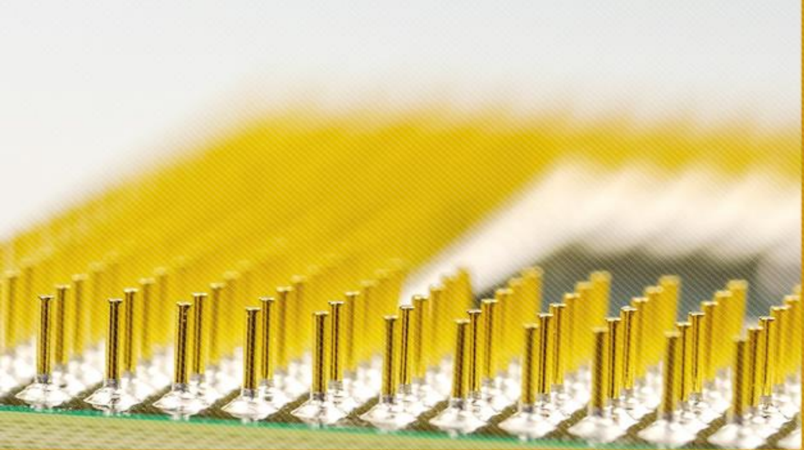
In the name of **GOD**



A MULTI-OBJECTIVE PROBLEM FOR PRACTICAL PORTFOLIO OPTIMIZATION

Zahra Eftekhari & Milad Samimifar





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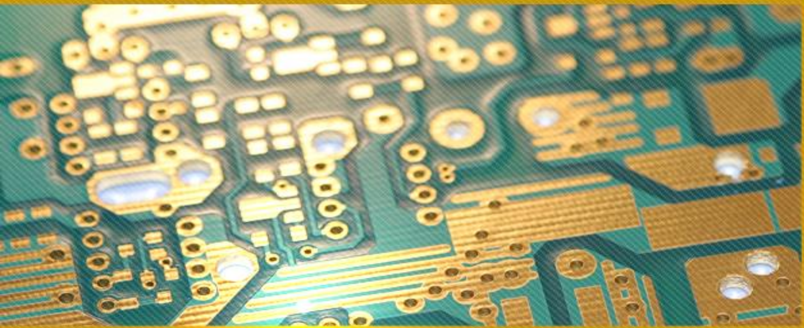

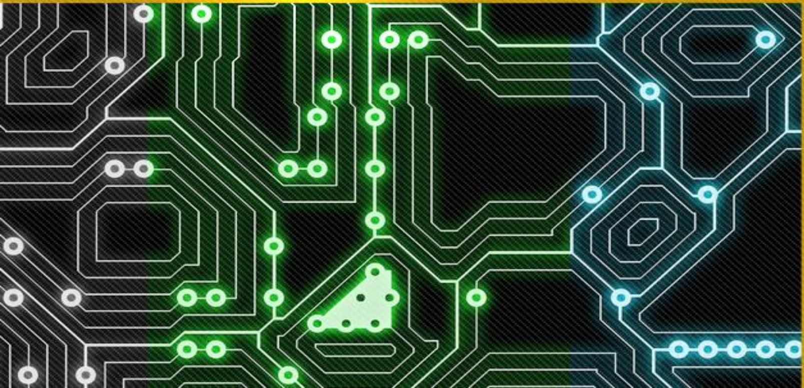

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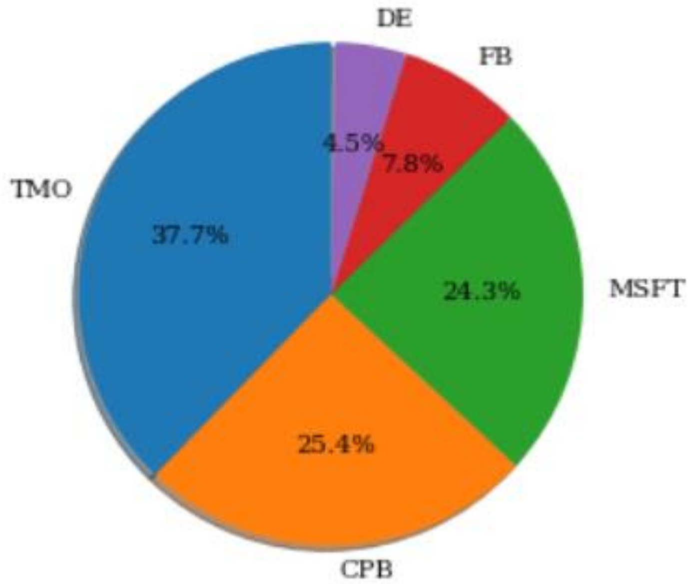
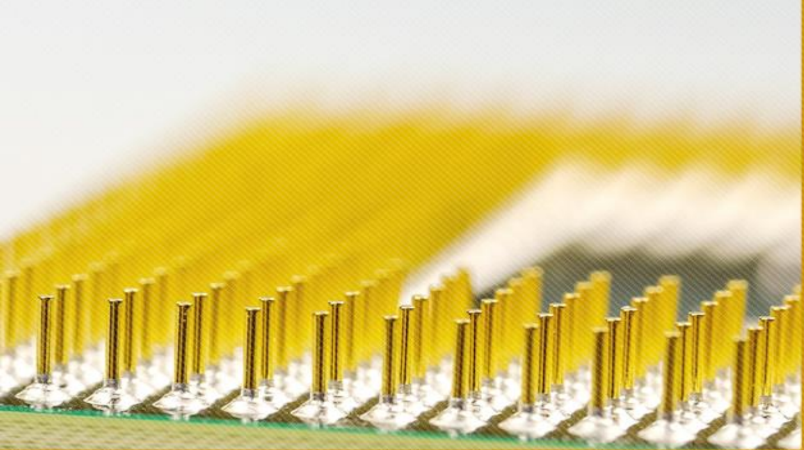
Introduction

- **Portfolio Optimization**





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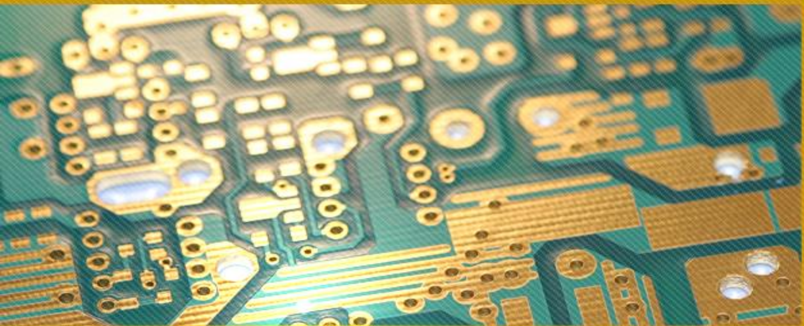

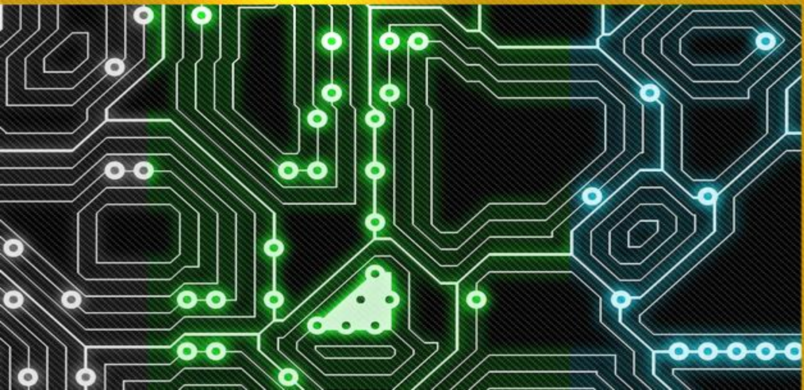

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Model

- **Return on n assets:**

	Period 1	Period 2	$\cdots$	Period m
Asset 1	r_{11}	r_{12}	$\cdots$	r_{1m}
Asset 2	r_{21}	r_{22}	$\cdots$	r_{2m}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Asset n	r_{n1}	r_{n2}	$\cdots$	r_{nm}

Mean-Variance model

- **Markowitz model**

Maximize $R = \sum_{i=1}^{n} \overline{r_i} y_i = \bar{r}^T y$

Minimize $V = y^T Q y$

Subject to $1^T y = 1$

$0 \leq y_i$ for $i = 1,2,\ldots,n$

$$Q = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

$$\sigma_{ik} = \frac{\sum_{j=1}^{m} (r_{ij} - \bar{r}_i)(r_{kj} - \bar{r}_k)}{m}$$

Multi Objective Optimization Problem

$$\begin{array}{rlr} \text{Minimize/Maximize} & f_s(\mathbf{x}) & s = 1, 2, ..., S \\ \text{subject to} & g_j(\mathbf{x}) \geq 0 & j = 1, 2, ..., J \\ & h_c(\mathbf{x}) = 0 & c = 1, 2, ..., C \\ & x_i^{min} \leq x_i \leq x_i^{max} & i = 1, 2, ..., n \end{array}$$

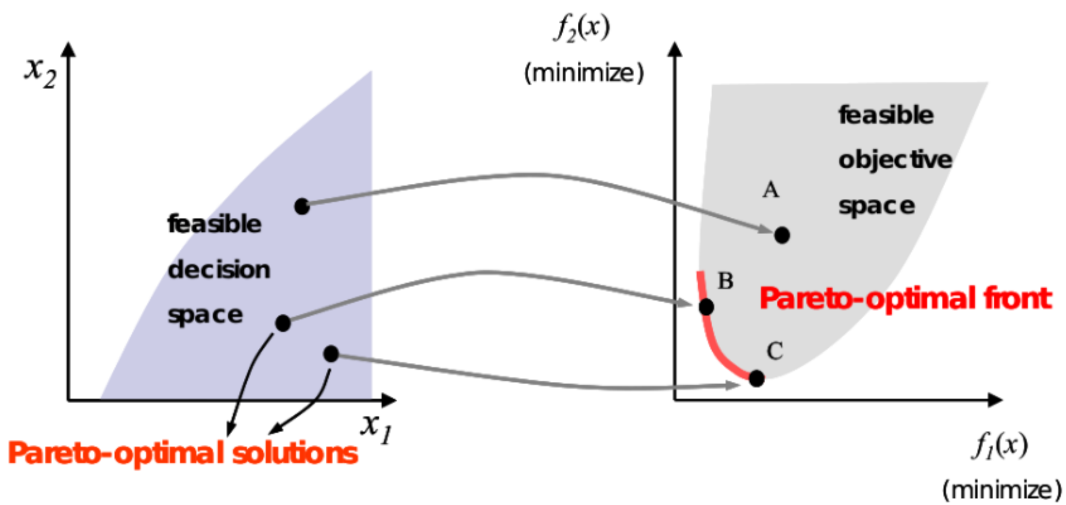
- How to solve this problems?

Multi Objective Optimization Problem

Solution x1 dominates x2 if:

- solution x1 is no worse than x2 in all objectives
- and solution x1 is strictly better than x2 in at least one objective

Pareto-optimal: The non-dominated set of the entire feasible decision space is called

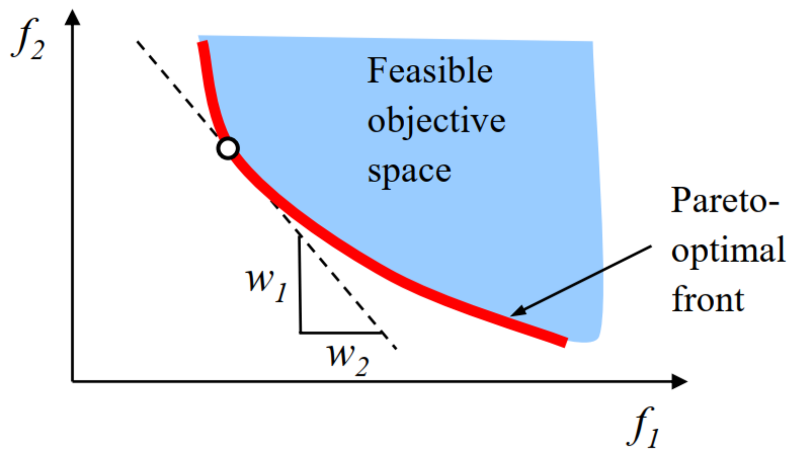


Solving Multi Objective Optimization Problem

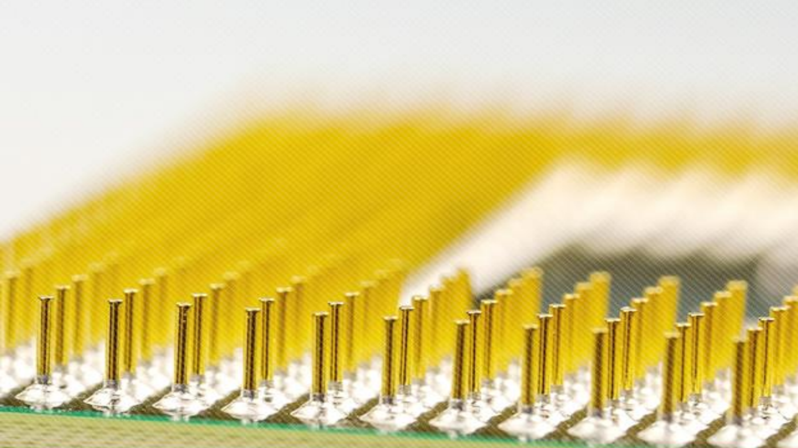
- **Weighted Sum Method**

Simplest way

$$\begin{array}{lll} \text{Minimize} & \sum_{s=1}^{S} w_s f_s(\mathbf{x}) & \\ \text{subject to} & g_j(\mathbf{x}) \geq 0 & j = 1, 2, ..., J \\ & h_c(\mathbf{x}) = 0 & c = 1, 2, ..., C \\ & x_i^{min} \leq x_i \leq x_i^{max} & i = 1, 2, ..., n \end{array}$$



- **Evolutionary algorithms: NSGA-II, SMS-EMOA**



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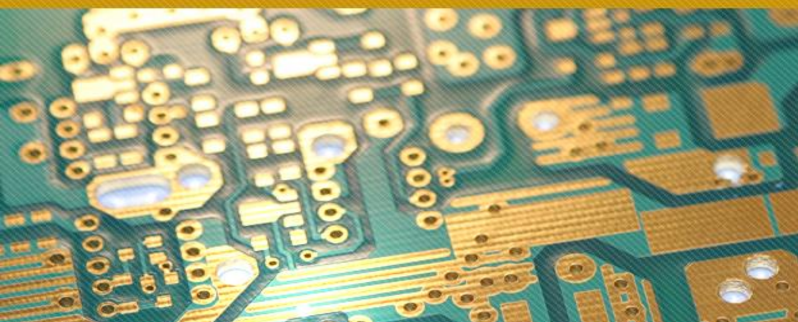

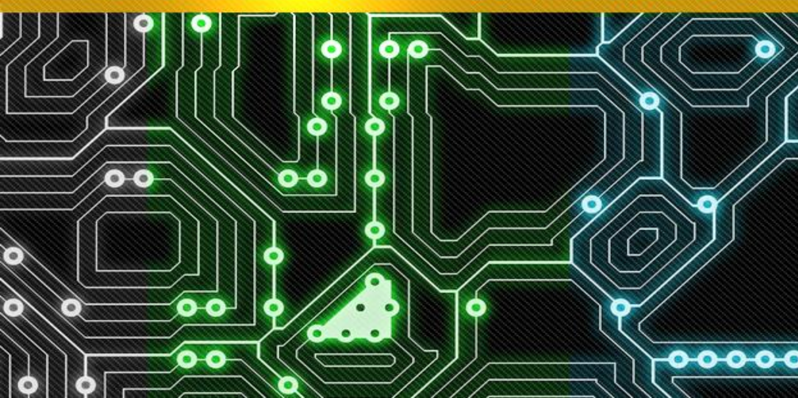

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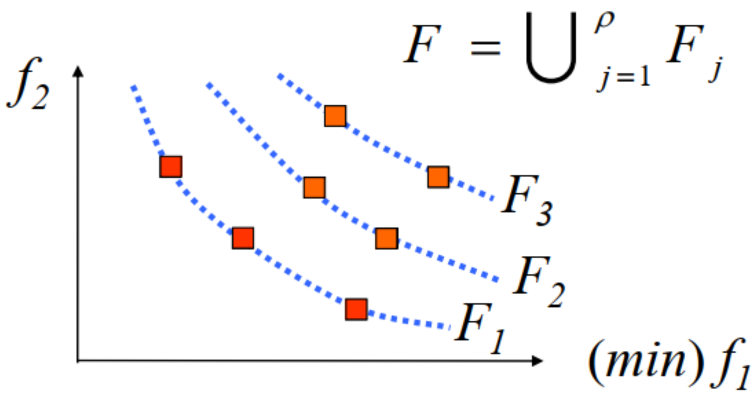
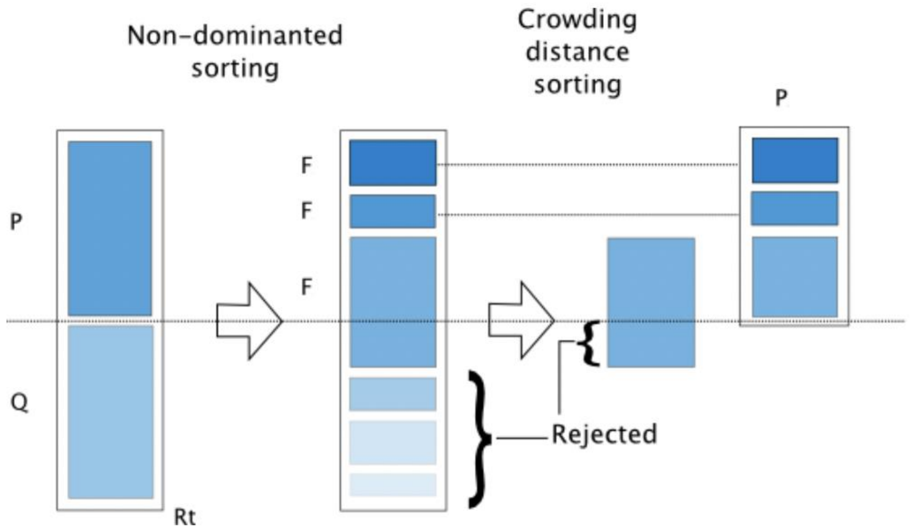
Simulation

- **Algorithm:**
- **1. NSGA-II: Non-dominated Sorting Genetic Algorithm**

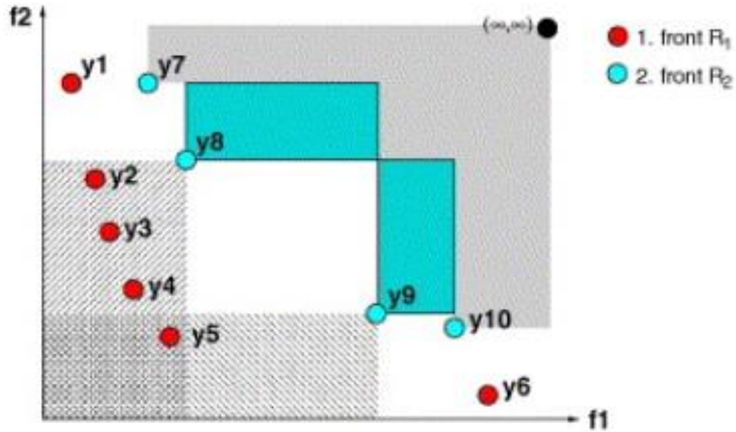
  Based on genetic algorithm





Simulation

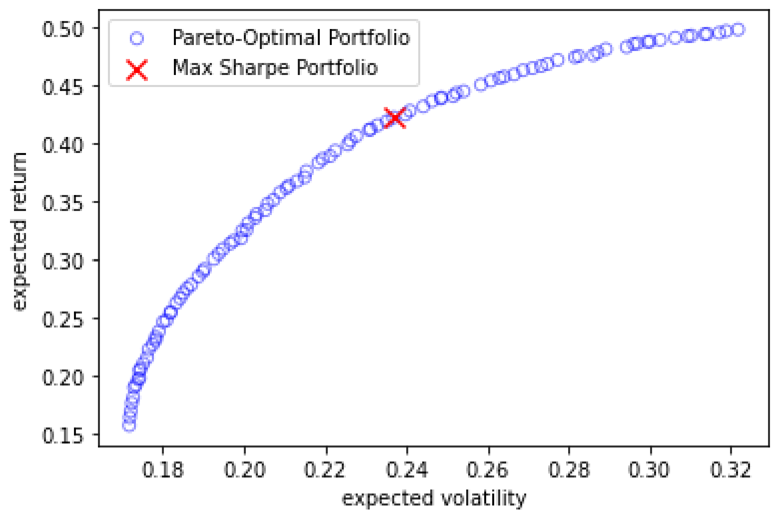
- **Algorithm :**
- **2. SMS-EMOA**



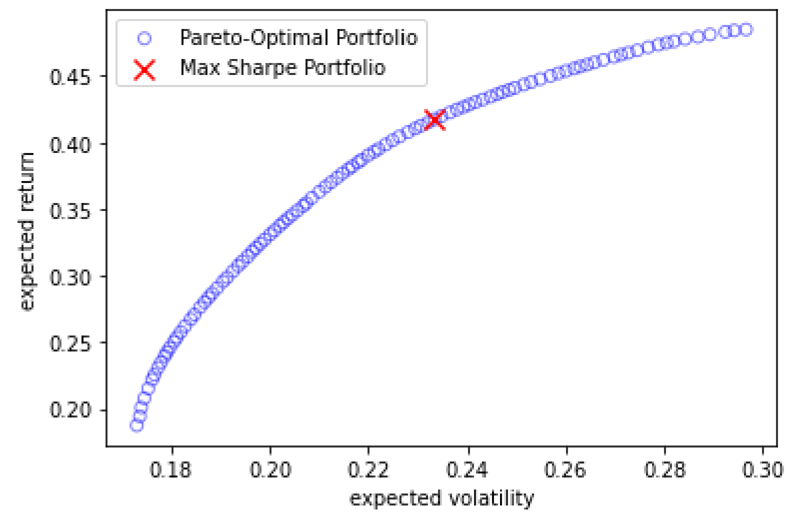
Simulation

- **Data:** Daily profit of 24 shares at USA stock market collected from Jan 2018 to Dec 2020 (Three years)

- **1. NSGA-II**



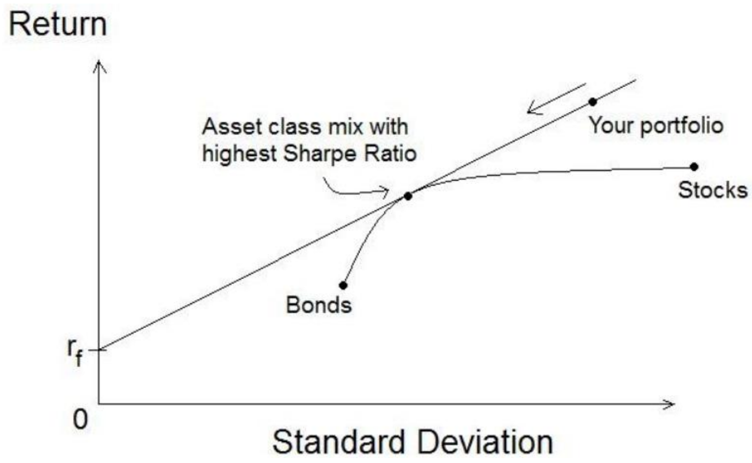
- **2. SMS-EMOA**



Simulation

- **Sharpe ratio:**

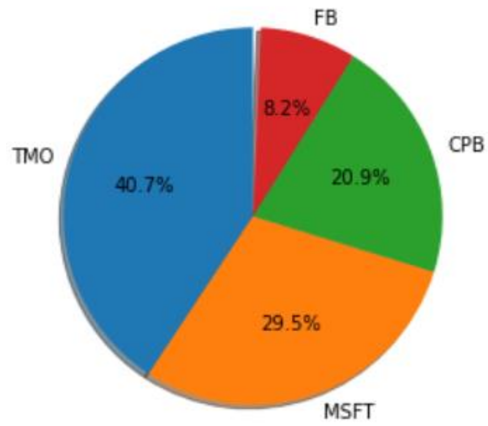
$$S = \left(\frac{R_p - R_f}{\sigma_p}\right)$$

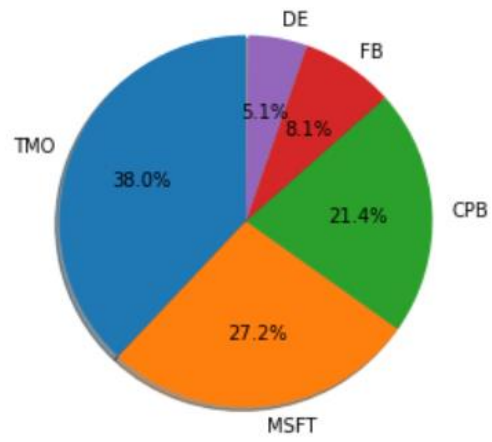


Simulation

	NSGA-II	SMS-EMOA	Equal weights
Time(s)	18.7	7.72	-
Return	0.422	0.417	0.184
Volatility	0.236	0.233	0.220

Weights:





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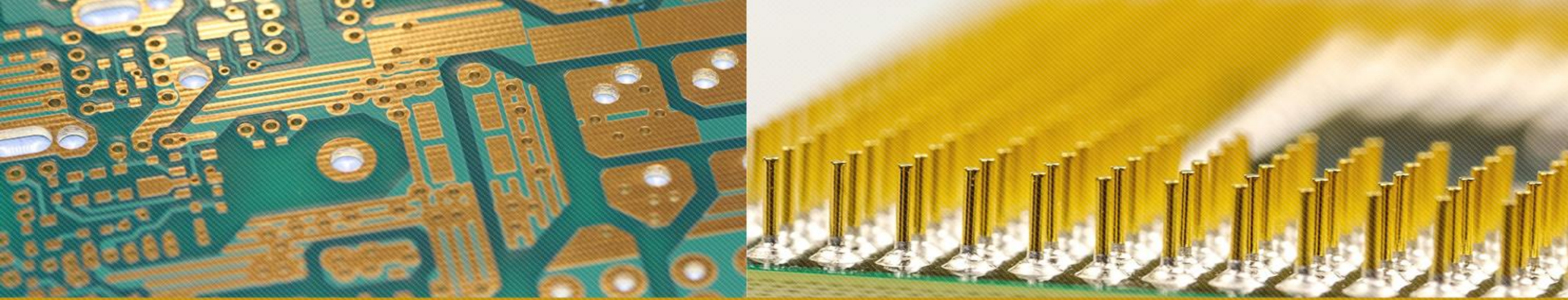

THANK YOU FOR YOUR ATTENTION