$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$t(A) = \lambda\vec{v} = A\vec{v}$$

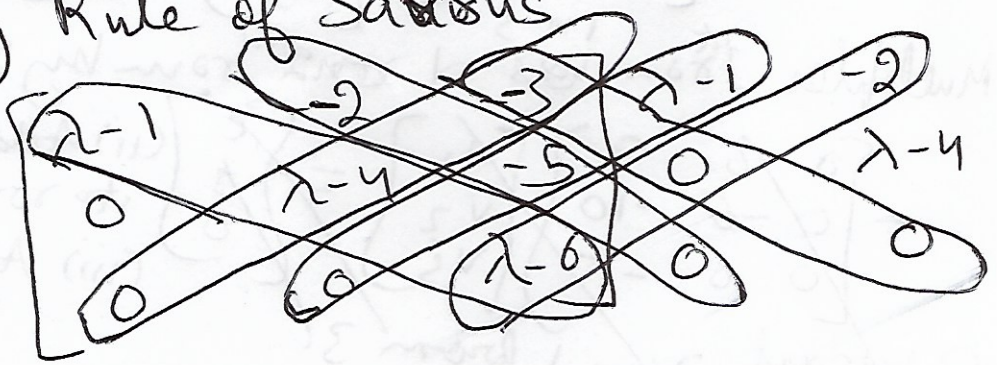
$$\det(\lambda I_n - A) = 0$$

$$\det\left(\lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-4 & -5 \\ 0 & 0 & \lambda-6 \end{bmatrix}\right) = 0$$

Using Rule of Sarrus



$$(\lambda-1)(\lambda-4)(\lambda-6) + 0 + 0$$

$$- (0) - (0) - 0 = 0$$

$$(\lambda^2 - 4\lambda - \lambda + 4)(\lambda-6) = 0$$

$$\lambda^3 - 6\lambda^2 - 4\lambda^2 + 24\lambda - \lambda^2 + 6\lambda + 4\lambda - 24 = 0$$

$$(\lambda^2 - 5\lambda + 4)(\lambda-6) = 0$$

$$\lambda^3 - 6\lambda^2 - 5\lambda^2 + 30\lambda + 4\lambda - 24 = 0.$$

(2) $\lambda^3 - 11\lambda^2 + 30\lambda - 24 = 0 \Rightarrow$ Characteristic Polynomial

$\lambda^3(\lambda - 11) + 2(17\lambda - 12) = 0.$

$(\lambda - 1)(\lambda - 6)(\lambda - 4) = 0$

$\lambda = 1 \; ; \; \lambda = 6 \; ; \; \lambda = 4$

Eigen Values

$\lambda = 1.$ $\quad det(\lambda I - A) = 0 \Leftrightarrow (\lambda I_n - A)\vec{v} = 0$

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \right) \vec{v} = 0$$

$$\begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(i) Multiply 1st row by -3 & 2nd row by 2 & add the

$$\begin{bmatrix} 0 & 6 & 9 \\ 0 & -6 & -10 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(ii) Add -5 times row 2 to row 3

(iii) Add $-3/2$ times row 2 to row 1

Subtract row 1 from 3

$$\begin{bmatrix} 0 & 6 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_2 = 0 \quad v_3 = 0 \quad v_1 = 1.$

$\therefore \Sigma_{\lambda=1} = 1, 0, 0.$

When $\lambda = 6$.

$$\left(6\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - \begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}\right)\vec{v} = 0.$$

$$\left(\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix} - \begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}\right)\vec{v} = 0$$

$$\begin{bmatrix}5&-2&-3\\0&2&-5\\0&0&0\end{bmatrix}\vec{v} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}5&-2&-3\\0&2&-5\\0&0&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

(i) Divide Row 1 by 5,

(ii) divide row 2 by 2 & multiple by $\frac{2}{5}$ & add row 1.

$$\begin{bmatrix}1&0&-8/5\\0&1&-5/2\\0&0&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$v_1 - 8/5 v_3 = 0$$

$$v_2 - 5/2 v_3 = 0$$

Lets assume if $v_3 = 1$.

Then eigenVector of $\lambda = 6$

$= (8/5, 5/2, 1)$ if $z = 1$.

When $\lambda = 4$.

$$\left(4\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}\right)\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left(\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}-\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}\right)\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}3&-2&-3\\0&0&-5\\0&0&-2\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

(i) Divide row 1 by 3 & row 2 by 5.

(ii) 2 times row 2 plus row 3

(iii) Row 2 plus row 1.

$$\begin{bmatrix}1&-2/3&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$v_1 - 2/3\,v_2 = 0$$

$$v_3 = 0.$$

$$v_1 = 2/3\,v_2.$$

Eigen vector $= 2/3, 1, 0$ if $v_2 = 1$.