HW8

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Company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out?

```
n  <- 1000
lbulbs <- 100

# The distribution of the minimum value of n independent expontially distributed variables with mean ??

b <- n/lbulbs
b
```

```
## [1] 10
```

Assume that X1 and X2 are independent random variables, each having an exponential density with parameter lambda. Show that Z = X1-X2 has density .

$$fZ(z) = (\frac{1}{2})\lambda e^{\lambda - |z|}$$

Ans :-

Both X1 and X2 are evaluated on the interval

$$0 \leq x < \infty.$$

$$1) When \quad X2 \geq X1$$

$$\int_0^\infty fX^{(x)} fY^{x-z} dx$$

$$\int_0^\infty \lambda e^{-\lambda x} \lambda e^{-\lambda(x???z)} dx$$

$$\lambda e^{\lambda z} \int_0^\infty \lambda e^{-2\lambda x} dx$$

$$\lambda e^{\lambda z}(\frac{-1}{2} e^{-2\lambda x})$$

$$fZ(z) = \frac{\lambda}{2} e^{\lambda z}$$

$$2) When \quad X1 \geq X2$$

$$fZ(z) = \frac{\lambda}{2} e^{-\lambda z}$$

$$fZ(z) = \frac{\lambda}{2} e^{-\lambda |z|}$$

Let X be a continuous random variable with mean ?? = 10 and variance ??2 = 100/3. Using Chebyshev's Inequality, find an upper bound for the following probabilities.

pX = var / e^2

a) P(|X-10|>=2)

```
var  <- 100/3
e <- 2^2
var/e
```

```
## [1] 8.333333
```

b) P(|X-10|>=5)

```
e<- 5^2
var/e
```

```
## [1] 1.333333
```

c) P(|X-10|>=9)

```
e <-9^2
var/e
```

```
## [1] 0.4115226
```

d) P(|X-10|>=20)

```
e <- 20^2
var/e
```

```
## [1] 0.08333333
```