Data 605 HW 7

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Q1 :- Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y.

Ans :- We are given that Y denotes the minimum of the Xis. Suppose each independent random variable Xi has k possibilities.

Suppose that each Xi has k possibilities: 1, 2, . . . , k. Then, the total possible number of assignments for the entire collection of random variables X1, X2, . . . , Xn is (k)^n . This will form the denominator for our probability distribution function.

The number of ways of getting Y = 1 is k^n - (k - 1)^n / k^n, since k^n represents the total number of options and (k-1)^n represents all of the options where none of the Xi's are equal to 1.

When X = 1:
P(X=1) = k^n - (k-1)^n / k^n

Similarly When X= 2 & 3:
P(X=2) = (k-2+1)^n - (k-2)^n / k^n
P(X=3) = (k-3+1)^n - (k-3)^n / k^n

Generalization this for (X=m):
P(X=m) = (k-m+1)^n - (k-m)^n * k^n

Q2:-Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

a) What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

```
p_fail <- 1/10
n= 8
p_notfail <- 1-p_fail

prob_geom  <- 1-pgeom(n-1, p_fail)
prob_geom
```

```
## [1] 0.4304672
```

```
#Expected value
expec_val <- 1/p_fail
expec_val
```

```
## [1] 10
```

```
#Standard Deviation
SD <- sqrt(p_notfail/(p_fail^2))
SD
```

```
## [1] 9.486833
```

b) What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

```
n <- 8
lambda <- 1/10

p_expo <- pexp(n, lambda, lower.tail=FALSE)
round(p_expo,2)
```

```
## [1] 0.45
```

```
#Expected value = 1/lambda
expec_val <- 1/lambda
expec_val
```

```
## [1] 10
```

```
#SD = 1/??^2
SD <- sqrt(1/lambda^2)
SD
```

```
## [1] 10
```

c) :- What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

(n Choose k) p^k * (1-p)^(n???k)

```
n <- 8
p <- 1/10
q <- (1-p)
k <- 0



p_binomial <- dbinom(k, n, p)
p_binomial
```

```
## [1] 0.4304672
```

```
#Expected value
exp_val <- n * p
exp_val
```

```
## [1] 0.8
```

```
# Standard deviation
sd_bino <- sqrt(n*p*q)
sd_bino
```

```
## [1] 0.8485281
```

d) What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

```
#Since average number of failures in every 10 years is 1 so average number of failures in 8 years will
```

```r
lambda <- 8/10
k <- 0

ppois(0,lambda = .8 )
```

```
## [1] 0.449329
```

```r
#Expected value
exp_val <- 8/10
exp_val
```

```
## [1] 0.8
```

```r
# Standard deviation
SD <- sqrt(8/10)
SD
```

```
## [1] 0.8944272
```