# DATA 605 Discussion 12

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#### **Heart Disease**

Data Context: This database contains 76 attributes, but all published experiments refer to using a subset of 14 of them. In particular, the Cleveland database is the only one that has been used by ML researchers to this date. The "goal" field refers to the presence of heart disease in the patient. It is integer valued from 0 (no presence) to 4.

Downladed from : https://www.kaggle.com/ronitf/heart-disease-uci

The terms choosen are as follows:

Dichotomous Term is sex (1 = male; 0 = female) Quadratic Term: Age Interaction sex vs. cholesterol (quantitative)

#### Read data

```
heart <- read.csv("heart.csv",header=TRUE, sep=",")
head(heart, 10)</pre>
```

```
##
       i..age sex cp trestbps chol fbs restecg thalach exang oldpeak slope ca
                     3
## 1
            63
                  1
                              145
                                    233
                                           1
                                                    0
                                                            150
                                                                     0
                                                                            2.3
                                                                                          0
## 2
            37
                  1
                     2
                              130
                                    250
                                           0
                                                    1
                                                            187
                                                                     0
                                                                            3.5
                                                                                      0
                                                                                          0
## 3
            41
                  0
                     1
                              130
                                    204
                                           0
                                                    0
                                                            172
                                                                     0
                                                                            1.4
                                                                                      2
                                                                                          0
                                                                                      2
## 4
            56
                  1
                     1
                                    236
                                           0
                                                    1
                                                                     0
                                                                            0.8
                                                                                          0
                              120
                                                            178
## 5
            57
                 0
                     0
                              120
                                    354
                                           0
                                                    1
                                                            163
                                                                     1
                                                                            0.6
                                                                                      2
                                                                                          0
## 6
            57
                  1
                     0
                              140
                                    192
                                           0
                                                    1
                                                            148
                                                                     0
                                                                            0.4
                                                                                      1
                                                                                          0
## 7
            56
                 0
                     1
                              140
                                    294
                                           0
                                                    0
                                                            153
                                                                     0
                                                                            1.3
                                                                                      1
                                                                                          0
                                                                                      2
## 8
            44
                  1
                     1
                              120
                                    263
                                           0
                                                    1
                                                            173
                                                                     0
                                                                            0.0
                                                                                          0
## 9
            52
                     2
                                                                            0.5
                                                                                      2
                                                                                          0
                  1
                                    199
                                           1
                                                    1
                                                            162
                                                                     0
                              172
                                                                                      2
            57
                     2
## 10
                  1
                              150
                                    168
                                           0
                                                    1
                                                            174
                                                                     0
                                                                            1.6
                                                                                          0
##
       thal target
## 1
          1
## 2
          2
                   1
          2
##
                   1
## 4
          2
                   1
## 5
          2
                   1
## 6
          1
                   1
## 7
          2
                   1
          3
## 8
                   1
## 9
          3
                   1
          2
## 10
                   1
```

#### Apply Linear Model

```
# Quadratic Term
age2 <- heart$\"i..age^2
# Dichotomous vs. quantative
sx_chl <- heart$sex * heart$chol</pre>
# first model
modl <- lm(thalach ~ sex + i..age
+ age2 + cp + trestbps+fbs+restecg+exang+oldpeak+slope+ca+thal+target+ chol + sx_chl, heart)
summary(modl)
##
## Call:
## lm(formula = thalach ~ sex + i..age + age2 + cp + trestbps +
      fbs + restecg + exang + oldpeak + slope + ca + thal + target +
##
      chol + sx_chl, data = heart)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -59.300 -10.379
                   1.885 11.830
                                   48.304
##
## Coefficients:
##
                Estimate Std. Error t value
                                              Pr(>|t|)
## (Intercept) 164.889971 32.913932 5.010 0.000000954 ***
               0.315713 10.883667 0.029
                                              0.976878
## sex
## ï..age
              -1.319470
                          1.184114 -1.114
                                              0.266078
## age2
              0.004353 0.010980 0.396
                                              0.692068
## ср
                2.124421
                          1.199762 1.771
                                            0.077672 .
## trestbps
                0.124942
                          0.064708
                                     1.931
                                              0.054485
## fbs
               1.960890
                          3.104881
                                    0.632
                                              0.528183
## restecg
               -1.418114
                         2.068313 -0.686 0.493495
## exang
               -9.491782
                         2.626219 -3.614
                                            0.000356 ***
                          1.202700 -0.737
## oldpeak
               -0.886666
                                              0.461585
## slope
               7.556577
                           2.181148 3.464
                                              0.000612 ***
## ca
               -0.452962
                          1.172178 -0.386
                                              0.699466
## thal
               2.165220
                           1.870993
                                    1.157
                                              0.248130
## target
               7.944773
                           3.002587
                                      2.646
                                              0.008595 **
## chol
                0.036010
                          0.030141
                                     1.195
                                              0.233190
## sx chl
                0.003584
                          0.042354
                                     0.085
                                              0.932622
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.27 on 287 degrees of freedom
## Multiple R-squared: 0.3956, Adjusted R-squared: 0.364
## F-statistic: 12.52 on 15 and 287 DF, p-value: < 2.2e-16
```

### Using Backwards Elimination

Removing variable with highest p value - one at a time. Starting with sex:

```
modl <- update(modl, .~. -sex)</pre>
summary(modl)
##
## Call:
## lm(formula = thalach ~ i..age + age2 + cp + trestbps + fbs +
      restecg + exang + oldpeak + slope + ca + thal + target +
##
      chol + sx_chl, data = heart)
##
## Residuals:
     \mathtt{Min}
             1Q Median
                           3Q
                                 Max
## -59.28 -10.35
                 1.90 11.80 48.30
##
## Coefficients:
                Estimate Std. Error t value
                                              Pr(>|t|)
##
## (Intercept) 164.993433 32.663303
                                      5.051 0.00000078 ***
               -1.317503
                                             0.265174
## ï..age
                          1.180117 -1.116
## age2
                0.004334
                           0.010942
                                     0.396
                                              0.692310
## ср
                2.125509
                          1.197094
                                     1.776
                                             0.076862 .
                          0.064578 1.934
## trestbps
               0.124899
                                             0.054084 .
## fbs
               1.969747
                           3.084466 0.639
                                              0.523589
                           2.057077 -0.692
## restecg
               -1.423273
                                              0.489563
               -9.491102
                           2.621555 -3.620
                                              0.000347 ***
## exang
## oldpeak
              -0.882572
                           1.192316 -0.740
                                              0.459772
## slope
               7.567227
                           2.146290
                                     3.526
                                              0.000491 ***
## ca
               -0.454019
                          1.169577 -0.388
                                              0.698161
## thal
                2.168395
                          1.864546
                                     1.163
                                              0.245808
## target
                7.944107
                           2.997286 2.650
                                              0.008483 **
                0.035387
## chol
                           0.021119 1.676
                                              0.094909 .
## sx_chl
                0.004780
                           0.009733
                                      0.491
                                              0.623737
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 18.23 on 288 degrees of freedom
## Multiple R-squared: 0.3956, Adjusted R-squared: 0.3662
## F-statistic: 13.46 on 14 and 288 DF, p-value: < 2.2e-16
Removing ca: number of major vessels
modl <- update(modl, .~. -ca)</pre>
summary(modl)
##
## Call:
## lm(formula = thalach ~ i..age + age2 + cp + trestbps + fbs +
##
      restecg + exang + oldpeak + slope + thal + target + chol +
##
      sx_chl, data = heart)
##
## Residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -58.693 -10.107
                   1.754 11.778 47.909
```

##

```
## Coefficients:
##
                 Estimate Std. Error t value
                                                Pr(>|t|)
## (Intercept) 164.238103 32.557351
                                       5.045 0.000000804 ***
               -1.291530
                            1.176486 -1.098
                                                0.273212
## ï..age
## age2
                 0.004000
                           0.010892
                                       0.367
                                                0.713731
                           1.193009
## ср
                 2.154477
                                      1.806
                                                0.071971 .
## trestbps
                0.125473
                           0.064466
                                      1.946
                                                0.052583 .
## fbs
                 1.800014
                            3.048828
                                     0.590
                                                0.555387
## restecg
                -1.420493
                            2.054040 -0.692
                                                0.489769
## exang
               -9.406702
                            2.608682 -3.606
                                                0.000366 ***
## oldpeak
                -0.933972
                            1.183199 -0.789
                                                0.430548
## slope
                7.467407
                            2.127697
                                       3.510
                                                0.000520 ***
## thal
                 2.154899
                           1.861480
                                      1.158
                                                0.247972
## target
                 8.255172
                            2.883938
                                       2.862
                                                0.004511 **
                 0.035409
                            0.021088
                                       1.679
                                                0.094213 .
## chol
## sx_chl
                 0.004602
                            0.009708
                                       0.474
                                                0.635832
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 18.21 on 289 degrees of freedom
## Multiple R-squared: 0.3953, Adjusted R-squared: 0.3681
## F-statistic: 14.53 on 13 and 289 DF, p-value: < 2.2e-16
Remove age<sup>2</sup>
```

```
modl <- update(modl, .~. -age2)</pre>
summary(modl)
```

```
##
## lm(formula = thalach ~ i..age + cp + trestbps + fbs + restecg +
       exang + oldpeak + slope + thal + target + chol + sx_chl,
##
##
       data = heart)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
## -58.889 -10.012
                    1.787 11.560 47.931
##
## Coefficients:
                 Estimate Std. Error t value
                                                    Pr(>|t|)
## (Intercept) 153.131264 12.028995 12.730
                                                     < 2e-16 ***
## ï..age
                -0.862025
                            0.126517 -6.814 0.000000000055 ***
                            1.189092
                                       1.834
## cp
                 2.180698
                                                    0.067691 .
## trestbps
                 0.125405
                            0.064370
                                       1.948
                                                    0.052356 .
                            3.030189
## fbs
                 1.692430
                                       0.559
                                                    0.576918
                -1.407098
                            2.050650
                                      -0.686
                                                    0.493153
## restecg
                                                    0.000360 ***
## exang
                -9.404929
                            2.604783
                                      -3.611
## oldpeak
                -0.931317
                            1.181410
                                      -0.788
                                                    0.431159
## slope
                 7.501198
                            2.122533
                                       3.534
                                                    0.000476 ***
## thal
                 2.135826
                            1.857978
                                                    0.251279
                                       1.150
## target
                 8.278011
                            2.878963
                                       2.875
                                                    0.004335 **
                 0.035008
                            0.021028
                                                    0.097034 .
## chol
                                       1.665
## sx_chl
                 0.004474
                            0.009687
                                       0.462
                                                    0.644509
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.18 on 290 degrees of freedom
## Multiple R-squared: 0.395, Adjusted R-squared: 0.37
## F-statistic: 15.78 on 12 and 290 DF, p-value: < 2.2e-16</pre>
```

Remove: Sex vs Cholest.

```
modl <- update(modl, .~. -sx_chl)</pre>
summary(modl)
##
## Call:
## lm(formula = thalach ~ i..age + cp + trestbps + fbs + restecg +
       exang + oldpeak + slope + thal + target + chol, data = heart)
##
## Residuals:
##
      Min
                1Q Median
                               3Q
                                      Max
## -59.491 -10.335
                   1.681 11.419 47.116
##
## Coefficients:
##
               Estimate Std. Error t value
                                                  Pr(>|t|)
## (Intercept) 154.31610 11.73640 13.149
                                                   < 2e-16 ***
                           0.12545 -6.927 0.0000000000277 ***
## ï..age
               -0.86896
                2.23120
                           1.18245 1.887
                                                  0.060165 .
## cp
## trestbps
                0.12324
                           0.06411
                                   1.922
                                                  0.055543 .
                1.75072
                           3.02346
                                   0.579
                                                  0.563007
## fbs
## restecg
               -1.46094
                           2.04457 -0.715
                                                  0.475462
               -9.34516
                           2.59805 -3.597
                                                  0.000378 ***
## exang
## oldpeak
               -0.93254
                           1.17981 -0.790
                                                  0.429928
                                    3.563
## slope
                7.54522
                           2.11752
                                                  0.000428 ***
## thal
                2.25434
                           1.83769
                                    1.227
                                                  0.220916
                7.93032
                                    2.858
                                                  0.004575 **
## target
                           2.77505
                0.03507
                           0.02100
                                   1.670
                                                  0.096035 .
## chol
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.16 on 291 degrees of freedom
## Multiple R-squared: 0.3946, Adjusted R-squared: 0.3717
## F-statistic: 17.24 on 11 and 291 DF, p-value: < 2.2e-16
```

#### Remove fbs:fasting blood sugar

```
modl <- update(modl, .~. -fbs)
summary(modl)</pre>
```

```
##
## Call:
## lm(formula = thalach ~ ï..age + cp + trestbps + restecg + exang +
```

```
##
       oldpeak + slope + thal + target + chol, data = heart)
##
## Residuals:
##
      Min
                               3Q
                1Q Median
                                      Max
## -58.072 -10.559
                    1.544 11.634
                                   46.962
##
## Coefficients:
##
                Estimate Std. Error t value
                                                  Pr(>|t|)
## (Intercept) 153.84255
                          11.69453 13.155
                                                   < 2e-16 ***
## ï..age
               -0.86332
                           0.12493 -6.910 0.0000000000304 ***
## cp
                2.30385
                           1.17444
                                    1.962
                                                  0.050753 .
## trestbps
                0.12846
                           0.06340
                                    2.026
                                                  0.043667 *
## restecg
               -1.52824
                           2.03893 -0.750
                                                  0.454140
## exang
               -9.27717
                           2.59244 - 3.579
                                                  0.000404 ***
## oldpeak
               -0.97787
                           1.17587 -0.832
                                                  0.406308
## slope
                7.48124
                           2.11223
                                    3.542
                                                  0.000462 ***
                                   1.207
## thal
                2.21444
                           1.83430
                                                  0.228316
## target
                7.88037
                           2.77055
                                    2.844
                                                  0.004765 **
                0.03482
                           0.02097
                                    1.660
                                                  0.097911 .
## chol
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 18.14 on 292 degrees of freedom
## Multiple R-squared: 0.3939, Adjusted R-squared: 0.3731
## F-statistic: 18.97 on 10 and 292 DF, p-value: < 2.2e-16
```

## Remove restecg: resting electrocardiographic results

```
modl <- update(modl, .~. -restecg)
summary(modl)</pre>
```

```
##
## Call:
## lm(formula = thalach ~ i..age + cp + trestbps + exang + oldpeak +
##
       slope + thal + target + chol, data = heart)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -58.930 -10.362
                    1.935 11.878 46.102
##
## Coefficients:
##
                Estimate Std. Error t value
                                                   Pr(>|t|)
## (Intercept) 152.29332
                         11.50182 13.241
                                                    < 2e-16 ***
## i..age
                -0.85946
                            0.12473 -6.890 0.0000000000341 ***
                                     1.974
## cp
                 2.31659
                            1.17344
                                                   0.049299 *
## trestbps
                0.13174
                            0.06321
                                      2.084
                                                   0.037993 *
                -9.25423
                            2.59032 -3.573
                                                   0.000413 ***
## exang
## oldpeak
                -1.01680
                            1.17384 -0.866
                                                   0.387080
## slope
                7.39037
                            2.10717
                                    3.507
                                                   0.000524 ***
## thal
                2.15370
                            1.83114
                                     1.176
                                                   0.240489
## target
                7.68187
                            2.75580 2.788
                                                   0.005658 **
                 0.03684
                            0.02078
                                                   0.077348 .
## chol
                                    1.773
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.12 on 293 degrees of freedom
## Multiple R-squared: 0.3927, Adjusted R-squared: 0.374
## F-statistic: 21.05 on 9 and 293 DF, p-value: < 2.2e-16
```

Remove oldpeak: ST depression induced by exercise relative to rest

```
modl <- update(modl, .~. -oldpeak)</pre>
summary(modl)
##
## Call:
## lm(formula = thalach ~ i..age + cp + trestbps + exang + slope +
       thal + target + chol, data = heart)
##
## Residuals:
       Min
                1Q Median
                                ЗQ
                                       Max
## -58.380 -10.718
                    1.829 11.223 45.912
##
## Coefficients:
                Estimate Std. Error t value
##
                                                  Pr(>|t|)
## (Intercept) 151.24588
                           11.43322 13.229
                                                   < 2e-16 ***
                -0.86624
                            0.12443 -6.961 0.000000000022 ***
## ï..age
## ср
                 2.27798
                            1.17209
                                      1.944
                                                  0.052908 .
                                      2.009
## trestbps
                 0.12630
                            0.06287
                                                  0.045450 *
## exang
                -9.41774
                            2.58233 -3.647
                                                  0.000314 ***
## slope
                            1.83572
                                     4.513 0.000009239010 ***
                 8.28532
                 2.01228
                                      1.104
                                                  0.270588
## thal
                            1.82307
                                      3.030
## target
                 8.17002
                            2.69642
                                                  0.002663 **
                                                  0.078012 .
## chol
                 0.03674
                            0.02077
                                      1.769
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 18.11 on 294 degrees of freedom
## Multiple R-squared: 0.3911, Adjusted R-squared: 0.3746
## F-statistic: 23.61 on 8 and 294 DF, p-value: < 2.2e-16
Remove thal: 3 = normal; 6 = fixed defect; 7 = reversable defect
modl <- update(modl, .~. -thal)</pre>
summary(modl)
##
## lm(formula = thalach ~ i..age + cp + trestbps + exang + slope +
       target + chol, data = heart)
##
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
```

1.905 11.671 44.759

## -59.288 -10.140

##

```
## Coefficients:
               Estimate Std. Error t value
##
                                                Pr(>|t|)
                                                 < 2e-16 ***
## (Intercept) 155.87051 10.64180 14.647
                          0.12444 -6.989 0.000000000185 ***
              -0.86973
## ï..age
## ср
                2.28549
                          1.17251
                                   1.949
                                                 0.05221 .
                          0.06288 2.021
                                                 0.04415 *
## trestbps
                0.12711
                                                 0.00040 ***
## exang
               -9.23031
                          2.57770 -3.581
                          1.83600 4.536 0.0000083551781 ***
## slope
                8.32783
                7.37733
## target
                          2.59998 2.837
                                                 0.00486 **
## chol
                0.03843
                          0.02072 1.854
                                                 0.06467 .
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 18.12 on 295 degrees of freedom
## Multiple R-squared: 0.3886, Adjusted R-squared: 0.3741
## F-statistic: 26.79 on 7 and 295 DF, p-value: < 2.2e-16
```

#### Remove chol: cholesterol

```
modl <- update(modl, .~. -chol)
summary(modl)</pre>
```

```
##
## Call:
## lm(formula = thalach ~ i..age + cp + trestbps + exang + slope +
##
      target, data = heart)
##
## Residuals:
      Min
               10 Median
                               3Q
                                      Max
## -61.079 -9.939
                   2.033 11.434 45.323
##
## Coefficients:
               Estimate Std. Error t value
##
                                                 Pr(>|t|)
## (Intercept) 161.75331 10.19977 15.859
                                                  < 2e-16 ***
                        0.12284 -6.736 0.0000000000846 ***
## ï..age
               -0.82749
## ср
                2.17615 1.17584 1.851
                                               0.065205 .
## trestbps
               0.13563
                          0.06297 2.154
                                                 0.032063 *
## exang
               -9.10341
                           2.58738 -3.518
                                                 0.000502 ***
                          1.84089 4.623 0.0000056542036 ***
## slope
                8.51057
## target
                7.32599
                           2.61053
                                   2.806
                                                 0.005343 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.2 on 296 degrees of freedom
## Multiple R-squared: 0.3815, Adjusted R-squared: 0.3689
## F-statistic: 30.43 on 6 and 296 DF, p-value: < 2.2e-16
```

#### Remove cp: chest pain type

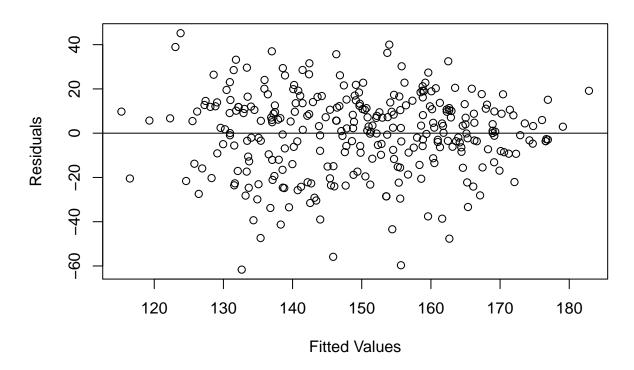
```
modl <- update(modl, .~. -cp)
summary(modl)</pre>
```

```
##
## Call:
## lm(formula = thalach ~ ï..age + trestbps + exang + slope + target,
      data = heart)
## Residuals:
             1Q Median
                            30
## -61.647 -10.178 2.408 11.865 45.190
##
## Coefficients:
              Estimate Std. Error t value
                                            Pr(>|t|)
## (Intercept) 161.87607 10.24111 15.806
                                             < 2e-16 ***
             ## ï..age
              0.14983 0.06276 2.387
## trestbps
                                             0.017598 *
## exang
             -10.35249 2.50799 -4.128 0.0000476278629 ***
              8.30084 1.84488 4.499 0.0000097920741 ***
## slope
## target
              8.92310
                        2.47380 3.607
                                             0.000363 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 18.27 on 297 degrees of freedom
## Multiple R-squared: 0.3743, Adjusted R-squared: 0.3638
## F-statistic: 35.54 on 5 and 297 DF, p-value: < 2.2e-16
```

#### Plot

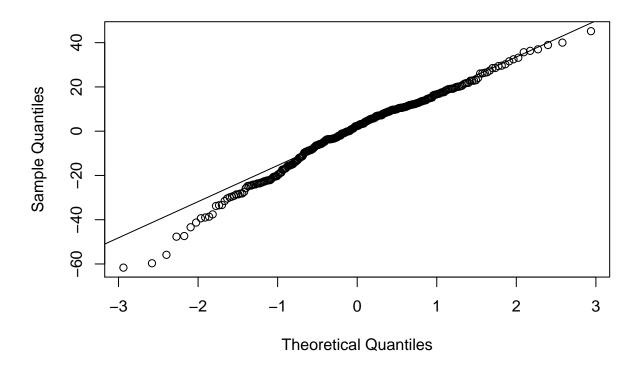
plot(modl\$fitted.values, modl\$residuals, xlab="Fitted Values", ylab="Residuals", main="Residuals vs. Fi
abline(h=0)

# Residuals vs. Fitted



qqnorm(modl\$residuals)
qqline(modl\$residuals)

## Normal Q-Q Plot



#### Conclusion

We included 5 variables in the final model that are used to predict the target variable, thalach: max heart rate achieved.

The results show: - The residuals median is 2.408 which is not very close to zero, indicating poor fit. - As min and max are not similar values, this indicates variability is not consistent. - The R^2 values explains only 37.43% of variability in the data - which again indicate poor fit. - The normal q-q plot shows us that the residuals do not follow a normal distribution.

So we may conclude this is as poorly fitted model.