Data 605 HW 13

Vishal Arora

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Q1 :- Use integration by substitution to solve the integral below.

$$\int 4e{-7x}dx$$

u=-7x du=-7dx dx=dU/-7

$$4\int e^u \frac{du}{-7}$$

$$\frac{4}{-7}\int e^u du$$

$$\frac{4}{-7}e^u + \quad Constant$$

as u= -7x

$$\frac{4}{-7}e^{-7x} + \quad Constant$$

Q2:-Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of dN/dt = -3150/t^4 - 220 bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

N'(t)=(???3150/ t4)-220

$$N(t) \quad = \quad \int (\frac{-3150}{t^4} - 220)dt$$

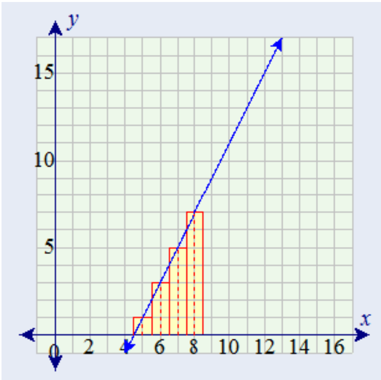
when N=1, then

N(1)=1050 - 220 + Constant=6530, hence Constant=5700

The function is

N(t) = 1050 / t3 -220t +5700

Q3:- Find the total area of the red rectangles in the figure below, where the equation of the line is f(x) = 2x - 9.



A : - Each square in the graph has an area of 1. Each rectangle has a width of 1. Counting each rectangle left to right the areas are

Area=1+3+5+7=16.

But an better way would be to use integral to find the area

```
findArea <- function(x)
  {
  2*x-9
}

integrate(findArea, lower = 4.5, upper = 8.5)
```

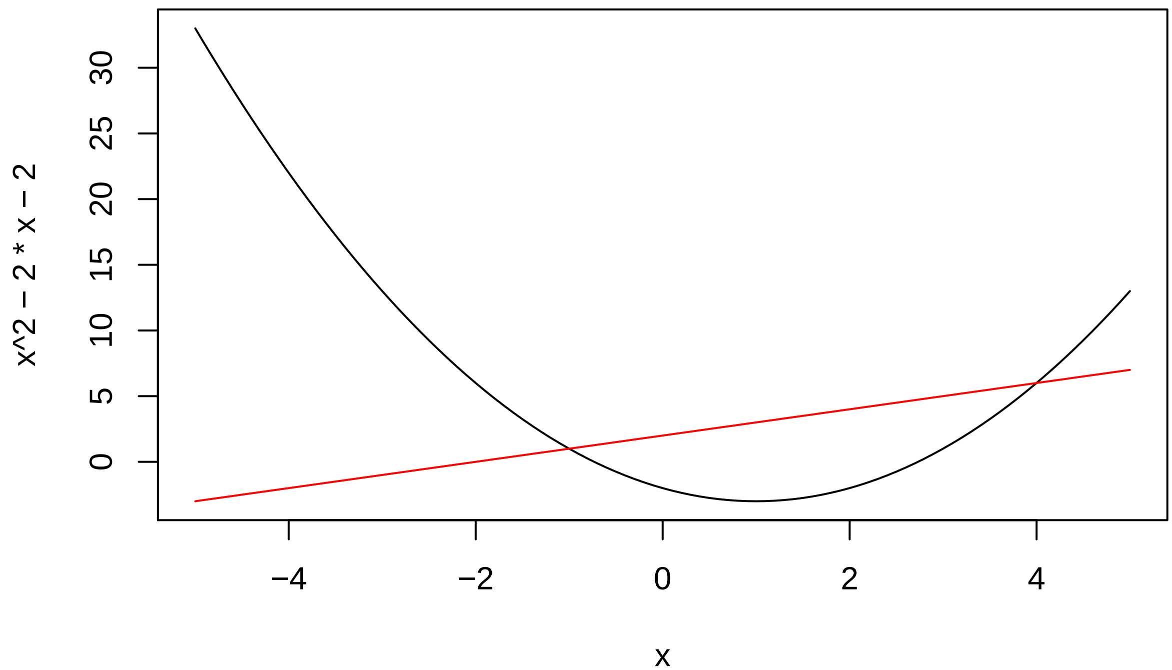
```
## 16 with absolute error < 1.8e-13
```

Q4 :- Find the area of the region bounded by the graphs of the given equations.

```
y=x^2???2x???2
y=x+2
```

A:-

```
curve(x^2 - 2*x - 2, -5, 5)
curve(x + 2, -5, 5, add=T, col="red")
```




```
((3/2)*4^2 +4*4 -(1/3)*4^3) - ((3/2)*(-1)^2 +4*(-1) -(1/3)*(-1)^3)
```

```
## [1] 20.83333
```

The formula for finding the area enclosed by two curves is as follows :-

$$\int_a^b (top - bottom)dx$$

Where a = -1 & b= 4 (We should get two values for x because of the quadratic term. x^2 -2x-2 = x+2 ~ (x-4)(x+10)

Function :-

$$\int_{-1}^4 (x+2) - (x^2 - 2x - 2)dx$$

$$\int_{-1}^4 -x^2 + 3x + 4)dx$$

Using R in the above function(x)

```
findArea <- function(x)
  {
  -x^{2}+3*x+4
  }
```

```
## integrate the function from 0 to infinity
integrate(findArea, lower = -1, upper = 4)
```

```
## 20.83333 with absolute error < 2.3e-13
```

Q5 :- A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store one flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

A:-

Assume Y = cost n = the no of orders per year x = no of irons in order. thus nx=110 so x=110n, assume half an order is in storage at on average. Such that,

$$Y \quad = 8.25n \quad + \quad \frac{3.75x}{2}$$

if nx = 100 , then x= 100/n

$$Y \quad = 8.25n \quad + \quad \frac{206.25}{n}$$

$$Y` \quad = 8.25 \quad + \quad \frac{206.25}{n^2}$$

if C`=0, then

$$n \quad = \quad \sqrt{\frac{206.25}{8.25}}$$

thus solving the above , we get n = 5 orders per year which will keep the inventory cost to minimum

Q6 :- Use integration by parts to solve the integral below.

```
$$ \int { ln(9x).{ x }^{ 6 }dx }  $$
```

A :-

$$\int udv \quad = \quad uv \quad - \quad \int vdu$$

u=ln(9x) dv=x^6dx

$$\frac{1}{7}ln(9x)x^7 \quad - \quad \frac{1}{7}\int x^6dx$$

$$\frac{1}{7}ln(9x)x^7 \quad - \quad \frac{1}{7}(\frac{x^7}{7}) \quad + \quad C$$

$$\frac{x^7}{7}\left[n(9x) \quad - \frac{1}{7}\right] \quad + \quad C$$

Q 7 :- Determine whether f (x) is a probability density function on the interval 1, e6 . If not, determine the value of the definite integral. f(x) = 1 / 6x.

A:-

$$\int_1^{e^6} \frac{1}{6x} dx$$

$$\frac{1}{6} \int_1^{e^6} \frac{1}{x} dx$$

$$\frac{1}{6}(ln(e^6) \quad - ln(1))$$

$$\frac{1}{6}(6-0) \quad = \quad 1$$

Hence the definite integral of the function on interval [1,e6] is 1