Assignment-1

Recurrence relation using Substitution Method.

1) $T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n-1)+1/n & \text{if } n>1 \end{cases}$

$T(n) = T(n-1) + 1/n$
$T(n-1) = T(n-2) + 1/n-1$
$T(n-2) = T(n-3) + 1/n-2$

$T(n) = T(n-1) + 1/n$
$= T(n-2) + 1/n-1 + 1/n$
$= T(n-3) + 1/n-2 + 1/n-1 + 1/n$

k times

$= T(n-k) + 1/n-(k-1) + 1/n-(k-2) + \dots + 1/n$

replacing $k=n-1$

$T(1)=1$ $= T(n-(n-1)) + 1/n-(n-2) + 1/n-(n-3) + \dots + 1/n$

$n-k=1$ $= T(1) + 1/\cancel{n}-\cancel{n}+2 + 1/3 + \dots + 1/n$

$n-1=k$ $= T(1) + 1/2 + 1/3 + \dots + 1/n$

$= O(\log n)$ by Harmonic Series

2) $T(n) = \begin{cases} 1 & n=0 \\ T(n-2)+n^2 & n>0 \end{cases}$

$T(n) = T(n-2)+n^2$
$T(n-2) = T(n-4) + (n-2)^2$
$T(n-4) = T(n-6) + (n-4)^2$

$T(n) = T(n-2)+n^2$
$= T(n-4) + (n-2)^2 + n^2$
$= T(n-6) + (n-4)^2 + (n-2)^2 + n^2$

$T(0)=1$ $= T(n-k) + (n-(k-2))^2 + (n-(k-4))^2 + \dots + n^2$

$n-k=0$ $= T(n-(n)) + (n-(n-2))^2 + (n-(n-4))^2 + \dots n^2$

$n=k$ $= T(0) + (2)^2 + (4)^2 + \dots + n^2$

$= O(n^3)$

3) $T(n) = \begin{cases} 10, & \text{if } n=0. \\ T(n-2)+\log(n) & \text{if } n>0 \end{cases}$

$T(n) = T(n-2) + \log(n)$

$T(n-2) = T(n-4) + \log(n-2)$

$T(n-4) = T(n-6) + \log(n-4)$

$T(n) = T(n-2) + \log(n)$

$= T(n-4) + \log(n-2) + \log(n)$

$= T(n-6) + \log(n-4) + \log(n-2) + \ldots \log(n)$

$= T(n-k) + \log(n-(k-2)) + \log(n-(k-4)) \ldots \log(n)$

$T(0) = 10$ then $= T(n-n) + \log(n-(n-2)) + \log(n-(n-4)) \ldots \log(n)$

$T(n-k) = 0$

$n = k$

$= T(0) + \log 2 + \log 4 + \ldots + \log n$

$= O(n \log n)$

Exercise: 2

Let $f(n) = n$ and $g(n) = n^{(1+\sin n)}$, where n is a positive integer.

Which of the following statements is/are correct?

$f(n) = n$, $g(n) = n^{(1+\sin n)}$

$f(n) = O\, g(n)$

$f(n) = \Omega\, g(n)$

for Big-O

$f(n) \leq c_1 \cdot g(n)$

$n \leq c_1 \cdot n^{(1+\sin n)}$

for omega

$f(n) \geq c_2 \cdot g(n)$

$n \geq c_2 \cdot n^{(1+\sin n)}$

None of the statement is correct.

Exercise: 3

Make them in the increasing order of asymptotic complexity of function f_1, f_2, f_3 and f_4?

$f_1(n) = 2^n$

$f_2(n) = n^{3/2}$

$f_3(n) = n \log_2 n$

$f_4(n) = n^{\log_2 n}$

$f_3 < f_2 < f_4 < f_1$

↳ complexity classes.